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ORIGINAL ARTICLE

Factors in a professional learning program to support a teacher's growth in mathematical reasoning and its pedagogy

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Abstract
Improved pedagogical practice does not happen in a silo; it requires impetus. External influences and pressures and internal motivations are drivers for pedagogical change. Professional learning (PL) programs, in their multitude of forms, are key tools for affecting teacher change. In response to an increased focus on fostering students' reasoning in curriculum documents, our team developed the Mathematical Reasoning Professional Learning Research Program (MRPLRP) to support teacher change. This two-phased project aimed to build teachers' knowledge of the critical aspects of reasoning pedagogical approaches that foster students' development of reasoning. Phase One involved researchers' planning and demonstrating a lesson with a focus on reasoning. In Phase Two, a peer learning team (PLT) is formed to plan a lesson to elicit reasoning and to observe each other teach the lesson. This article reports on Phases One and Two of the MRPLRP from the perspective of one teacher who participated in both phases. The findings provide insights into the aspects of PL that were critical in shifting this teacher's understanding of reasoning and approaches to teaching reasoning. Whilst the results of a single case cannot be extrapolated to a larger population, we present and discuss the factors of this PL program in raising the awareness of critical aspects of reasoning, and thus this paper has the potential to impact future PL design.

Keywords mathematical reasoning · professional learning · Demonstration lessons · Peer Learning Teams · Primary

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Introduction

Mathematical reasoning is considered fundamental in the learning of mathematics in schools (Brodie 2010). The necessity of reasoning for the learning of mathematics is evident in the Australian Curriculum: Mathematics (AC:M) where reasoning is defined as: "… [a] sophisticated capacity for logical thought and actions, such as analysing, proving, evaluating, explaining, inferring, justifying and generalising" (ACARA 2019).

Many research studies report the significance of teaching reasoning (see Brodie 2010; Dreyfus et al. 2012; Kilpatrick et al. 2001; Jeannotte and Kieran 2017; Lannin et al. 2011; Lesseig 2016; Stylianides 2010). However, few studies address the nature of professional learning to enhance teachers' capacity to develop their students' reasoning. Clarke et al. (2012) found that many teachers held a limited understanding of the AC:M definition of reasoning above and recommended that PL about reasoning was required. Providing rich opportunities for students' reasoning requires strengthening of teachers' mathematical content knowledge (MCK), since effective teaching may be inhibited by a teacher's degree of content knowledge (Hill et al. 2008). The impetus to facilitate changes in pedagogy to enhance students' development of reasoning may be through a PL program (Wood 1999). Whilst there are many research studies that elaborate on effective PL programs (see Sowder 2007; Clarke 1994; Sullivan et al. 2012), we pondered what are the factors that constitute an effective PL program for the teaching of reasoning. Consequently, the research question driving this study was:

> *What are the factors that support teachers' professional learning for a mathematical reasoning–focused pedagogy?*

This paper reports on our initial steps in exploring this question through a case study of one teacher's growing *awareness of critical aspects* (Cope 2000) of reasoning through engagement in two phases of Deakin University's PL program *Mathematical Reasoning PL Research Program* (MRPLRP). The first phase of the MRPLRP involved observing an expert teach demonstration lessons followed by teachers' trialling of the lessons. A demonstration lesson is "a 'model' lesson with a prior discussion to clarify purpose, expectations and desired outcomes and a debriefing session to review the lesson and its outcomes" (Cole 2012, p. 12). In the second phase, a peer learning team (PLT) (including and supported by a researcher) planned and peer-observed their reasoning-focused lesson followed by critical reflection in a post-lesson discussion. A learning team develops its capacity through willingness and shared knowledge and ability as a team to produce results desired by its members (Lick, 2006). In this article, we refer to a learning team as a peer learning team.

First, key literature is discussed including, pedagogical content knowledge (PCK) of reasoning and relevant PL programs in mathematics. Second, Variation theory is presented as the theoretical framework for this study. Next, the results are set out according to a thematic analysis of transcripts of a teacher engaged with her peers in two phases of the MRPLRP, followed by a discussion of the results highlighting the influential factors of these phases. Finally, insights gained from this teacher's reflections on the impact of the two PL phases of the MRPLRP in her development of understanding of reasoning are outlined.

Background

Broadly, reasoning involves developing and communicating arguments with the intention to convince (Brodie 2010). Further, reasoning has been described as the "capacity for logical thought, reflection, explanation, and justification" (Kilpatrick et al. 2001, p. 5). Yet, much of the reasoning research has been focused on three main elements of reasoning: generalising (e.g. Ellis 2007; Kaput 1999; Radford 2008); justifying (e.g. Carpenter et al. 2003; Lannin et al. 2011) and proving (e.g. Lannin et al. 2011; Toulmin 1993). There has not been one single effective teaching approach for reasoning, but rather multiple approaches have been suggested by researchers (see Long et al. 2012; Stylianides 2010; Stein et al. 2008; Goos et al. 2007; Nardi and Knuth 2017). For example, Stylianides (2010) asserted an inquiry-based approach where learners generalise, justify and prove has potential to build new knowledge and validate new mathematical ideas. Similarly, Goos et al. (2007) advocated an investigative approach where students explore mathematical relationships giving explanations and justifications. Orchestrated discussions provide students with opportunities to articulate their reasoning (Stein et al. 2008). Preparing for an orchestrated discussion requires a teacher to anticipate student responses, monitor responses and select which students will present their thinking. During the orchestrated discussion, the selected responses are sequenced to provide opportunities for connecting the mathematical ideas presented.

Fostering reasoning in the classroom, regardless of the approach, requires planning. Planning for reasoning includes selection of tasks as well as determining purposeful teaching approaches which provide students with opportunities to reason. Teachers play an important role in the selection and creation of appropriate classroom tasks and cultures to foster students' expression of their reasoning (Nardi and Knuth 2017). Such cultures include those where students are expected to present their ideas, explain and justify their thinking and evaluate the ideas of others (Long et al. 2012). In building these cultures, teachers need to ensure that their planning includes sufficient time and rich experiences to facilitate the construction of valid arguments and the ability to critique others' arguments (Long et al. 2012). Teachers' actions are informed by their pedagogical content knowledge (Shulman 1986; Hill et al. 2008), their beliefs about mathematics (Beswick 2012) and the context in which they teach (Sullivan et al. 2012). Informed planning of lessons is essential for effective teaching (Roche et al. 2014; Superfine 2008), but planning for reasoning and incorporating it in lessons is challenging for many teachers (Stacey 2010).

Teachers' knowledge of reasoning and pedagogy

Despite reasoning's place in curriculum documents, many teachers have limited knowledge of the nature of reasoning and appropriate pedagogies to support students' development of it (Loong, Vale, Herbert, Bragg, L. A. & Widjaja, 2017; Clarke et al. 2012). Clarke et al. (2012) research indicated that the Australian Curriculum's reasoning statements were not well understood by teachers. This limitation is concerning as Australian teachers typically plan lessons by consulting curriculum documents (Sullivan et al. 2012).

Teachers' PCK of reasoning, that is knowledge of reasoning, curriculum and effective approaches to teaching it, impact on their planning for embedding reasoning in lessons, in particular choices of tasks. Task choice is influenced by teachers' understanding of the mathematics involved in the task, teachers' knowledge of their students, knowledge of suitable pedagogical approaches and students' possible responses to the task (Sullivan et al. 2012). Consequently, teachers' limited understanding of reasoning impacts on teacher's decision-making in planning for reasoning.

Teachers' ability to notice reasoning in students' actions is required to foster reasoning development and is dependent on their understanding of the multifaceted nature of reasoning and its pedagogy, including appropriate prompts to encourage students' complex reasoning (Bragg, Herbert, Loong, Vale, & Widjaja, 2016). Those teachers possessing an incomplete knowledge of the complexity of reasoning results in them not noticing the complexities of their students' reasoning (Carpenter et al. 2003; Dreyfus et al. 2012; Lobato et al. 2013; Bragg, Herbert, Loong, Vale, & Widjaja, 2016). Furthermore, teachers' focus specifically on content and/or procedural fluency diminishes their noticing and support of students' reasoning (Bragg, Herbert, Loong, Vale, & Widjaja, 2016). When teachers' understanding of reasoning is limited, it is likely that they will miss classroom opportunities to notice and to extend their students' reasoning.

Further support for teachers' understanding of reasoning may occur through PL programs (Goos et al. 2011). Wood (1999) found that PL building teachers' understanding of how to support reasoning in their classrooms is complex. Reasoning PL programs have been trialled (see Clarke et al. 2012; Rasmussen and Marrongelle 2006), and rich online mathematical resources to support the fostering and assessing of reasoning have been developed [see Top Drawer (Australian Association of Mathematics Teachers [AAMT], 2013); and reSolve: Maths by Inquiry websites, (Australian Academy of Science and AAMT, 2017)]. The MRPLRP sought to build teachers' knowledge of the complexity of reasoning and how to notice and foster reasoning. This article considers features of two PL phases within one PL program, namely the MRPLRP, from the perspective of a participant who engaged in both phases.

Professional learning

The aim of PL is to improve student learning through teachers' professional development. PL assists teachers bridge the gap between theory and practice so that they can combine their knowledge of students' needs with robust understanding of content, along with teaching strategies and activities that facilitate student learning (Loucks-Horsley et al. 2009). Chapman (1996) advised that impetus for change in teachers' practice requires internal pressure, such as school improvement plans, or external pressure, for example the explicit statements within curriculum documents about reasoning to be enacted in schools (ACARA 2019). Wood (1999) claimed for change to occur: teachers need to believe that a shift is required; PL experiences must model approaches new to the teachers whilst responding to prior habits, and building of new skills is supported. Wood concluded that this process requires teachers' commitment and energy. Quality PL offers teachers the opportunity to examine their approaches to tasks, assessment, observation and reflection whilst providing a nexus between their learning and classroom practice (Borko et al. 2010). Further, a successful PL program may include features such as, modelling of different classroom strategies, support for implementing strategies, teachers' active participation and ongoing support over an extended period of time (Clarke 1994; Sowder 2007). Next we describe the approaches taken in the two phases of the MRPLRP that is demonstration lessons and peer learning teams.

Demonstration lessons

A PL program incorporating a demonstration lesson usually begins with planning and a pre-lesson discussion of the focus for the lesson, including the role to be taken by the observers (Loucks-Horsley et al. 2009). During the lesson, the observers take no part in teaching the lesson, rather note features of the lesson including interactions and responses (Clarke et al. 2013). The lesson is followed by a post-lesson discussion to reflect on features of the lesson and observations of interest where the focus is on understanding the nature of student learning and how this was influenced by the demonstration of teacher's actions and questions (Loucks-Horsley et al. 2009). During demonstration lessons, interactive relationships between the demonstration teacher and students, the teacher and content, and students and content are identified by observers through noticing ways the learning was facilitated by the demonstration teacher (Loucks-Horsley et al. 2009). Teachers who bring their expertise, prior experiences and their understanding of their students to the PL and the cycle of pre-lesson discussion, observation and reflection are better placed to increase their understanding of the content and appropriate teaching strategies (Loucks-Horsley et al. 2009). Both Clarke et al. (2013) and Loucks-Horsley et al. (2009) earlier advice on demonstration lessons for teachers of mathematics was followed in the preparation of Phase One of the MRPLRP, as detailed in the Methodology below.

Whilst benefits from demonstration lessons have been noted (Loong, Vale, Herbert, Bragg, L. A. & Widjaja, 2017), teachers in Phase One of the MRPLRP reported a lack of autonomy as a consequence of the lessons being designed and taught by an expert in the field rather than planned and delivered by the teachers themselves. The demonstration lessons and design of delivery were imposed on the teachers rather than co-planned with the teachers. The starting point of a PL program should be from the processes teachers use to help improve their own practice and support their decision-making (Sullivan et al. 2012). As a result of our findings (Loong, Vale, Herbert, Bragg, L. A., & Widjaja, 2017), an alternative to researcher-led demonstration lessons was sought to allow teachers more autonomy and decision-making. Clarke (1994) advised broadening teachers' growth through observation of student learning, planning, reflection and feedback, teachers' ownership and goal setting. Peer learning teams' (PLT) planning and teaching a reasoning-focused lesson were pursued in Phase Two of the MRPLRP as an approach that would incorporate Clarke's recommendations for teacher growth.

Peer learning teams

There is strong advocacy for the formation of professional communities to support teachers seeking to implement change in their practice (Clarke 1994; Nickerson and Moriarty 2005; Sowder 2007; Sullivan et al. 2012). PLT, where teachers collaboratively plan, peer-observe and reflect upon their teaching, is a dynamic tool for PL which can offer positive educational change (Davidson 2017). PLTs offer teacher growth within a community of practice approach (Stein et al. 2013). Simplistically, a community of practice is a group of people with a shared interest who collectively engage in the learning process (Lave and Wenger 1991). Boud et al. (2001) described peer learning as a reciprocal two-way learning activity. Peer observation is a vehicle for reciprocal learning in a PLT. Observation of a peer's teaching affords space for constructive feedback and critical reflection by both the observer and the observed (Eri 2014). Peer observation has been found to be effective in improving student outcomes (Dufour and Eaker 1998) providing opportunities for teachers' self-reflection and exposure to a range of teaching approaches (Johnston and Cornish 2016). In Victoria, the Department of Education and Early Childhood Development (2004) supported peer observation and collaboration as a vehicle for improving teacher learning and student outcomes stating:

> One of the most effective ways to learn is by observing others, or being observed and receiving specific feedback from that observation. Analysing and reflecting on this information can be a valuable means of professional growth … [P]eer observation promotes an open environment where public discussion of teaching is encouraged and supported. (p.11)

In the large scale, *Researching Numeracy* project (Researching Numeracy Project Team 2004), researchers worked with teachers from sixteen Victorian primary schools to identify and describe a range of numeracy teaching approaches, using structured observation of a group of teachers by their peers. The researchers noted a substantial shift in teachers' knowledge as they acquired professional language in conjunction with opportunities for sustained, collective reflection on practice. A taken-as-shared understanding of the nature of good practice through discussion and reflection can facilitate shifts towards improvements in teaching (Byrne et al. 2010).

Cultivating collegiality through giving and receiving constructive feedback from colleagues has potential to improve teaching (Wilson 2013) and to reduce teacher isolation whilst encouraging teacher professional conversations (Johnston and Cornish 2016). Dos Santos (2017) employed participant interviews to research the effects of peer observation. Participants generally acknowledged that there are certain elements of a teacher's performance that only colleagues in the same or closely-related disciplines can accurately assess. Goos et al. (2007) concluded that teachers working together can have a powerful influence on the pedagogical knowledge of the teachers involved, as well as heads of department and principals, other schools through district officers and parents and community members. Lesseig (2016) invited four teachers to plan a lesson cooperatively then observe each other teach, eliciting student conjectures, generalisations and justifications. Observing a colleague teach the lesson provided the opportunity for other teachers to focus on the student–teacher interactions without the burden of responding in the moment or managing the classroom. Thus, teachers were able to witness the flow of the lesson and types of questions posed and attend to student thinking more deeply (Lesseig 2016).

Barriers to the success of peer observations of teaching are noted by some researchers. For example, Karagiorgi (2012) noted that critical reflection by the teachers was minimal, with most teachers focusing on what they did or saw rather than analysing critically. A further limitation of peer observations can be a perceived imbalance of power between participants (Gosling 2002). This is particularly troublesome when peer observations are utilised as a tool to appraise teaching performance or evaluate individuals (Byrne et al. 2010). Being observed can be confronting for teachers who typically teach in isolation, unobserved by peers. Consequently, the key to the success of the PLT involves building an atmosphere of respect (Wilson 2013) and trust between non-judgemental, cooperative colleagues with equal status (Schuck et al. 2008).

In summary, reasoning is considered "the glue that holds everything together, the lodestar that guides learning" (Kilpatrick et al. 2001, p. 129) and is essential for students' deep conceptual understanding. Increasingly, reasoning is the focus of curriculum documentation and PL programs. However, despite the importance of reasoning and its focus from key stakeholders, it is understood superficially by many teachers and poorly enacted in the classroom (Stacey 2010). This limitation in understanding and enactment negatively impacts on planning for reasoning in lessons and providing opportunities for fostering students' reasoning capabilities.

This article focuses on the provision of school-based PL in teacher's knowledge of reasoning and pedagogies which support it in the classroom. School-based PL may involve experts brought into the school for presentations or demonstration lessons, or internal arrangements via PLTs, where teachers in a school share their knowledge through critical and constructive feedback. This article reports on the MRPLRP which employed demonstration lessons, external experts and PLTs. Many features of the advice provided by previous research were implemented in the MRPLRP as experienced by the case teacher. Phase One involved demonstration lessons followed by post-lesson discussions and subsequent trialling by teachers of the same lesson in their own classrooms. In Phase Two, PLTs were formed to plan, peer observe and discuss pedagogical practices to promote reasoning, thereby, facilitating opportunities for constructive feedback and critical reflection (Eri 2014). Whilst there has been prior employment and support for PLT (Nickerson and Moriarty 2005), the process of this PL program to utilise a PLT to embed reasoning into mathematical lessons is relatively unexplored.

Theoretical framework: variation theory

Variation theory is a learning theory where learning is viewed as a change in an individual's conceptions of the phenomenon, that is, the particular concept, being explored (Cope 2000; Dahlgren and Marton 1978). The *object of learning* is the concept that the designer wishes the students to learn (Lo 2012). Advocates of variation theory (Runesson 2005; Dahlin 2007) assert that a concept should be explored in a wide variety of contexts, in this case the concept is mathematical reasoning. Learning may be seen as displaying a change to a deeper, more inclusive level of understanding of the concept. Initial descriptions of the phenomenon, in this case teachers' understanding of mathematical reasoning, may provide a starting point for PL, which results in a change in teachers' understanding of reasoning. Variation theory emphasises the importance of a learner's *awareness* of *critical aspects* of a concept (Cope 2000). Variation theory proposes that full understanding of a concept can only take place if the critical aspects of the concept are discerned and that discernment requires explicit exposure to the variation in the critical aspect, for example the nuances of reasoning are discerned only when reasoning is seen in a variety of experiences.

For this article we employ the reasoning actions listed in the AC:M (ACARA 2019) as the critical aspects of reasoning. Difficulties with the concept of reasoning may occur when teachers view reasoning only as explaining (Clarke et al. 2012). Therefore, for a teacher to discern critical aspects of reasoning it is necessary to vary that aspect alone whilst holding other aspects *invariant*. Runesson (2006) in her paper explaining the use of variation theory in a student's conceptions of graphs, claimed "to learn is to be aware of critical aspect*s* of what is learned" (p. 397). However, people discern various critical aspects depending on their prior experiences, so too, different teachers become aware of different aspects (Lo et al. 2005). Variation theory explains the differences in conceptions as differences in awareness. The simultaneous discernment of a greater number of aspects implies a more sophisticated level of understanding, with a person becoming increasingly aware of more aspects of a concept. In any given situation, people notice some aspects and not others (Marton et al. 2004), such as the multiple aspects of reasoning. Similarly, Kaput (1992) reminded us that the abstraction of invariance, that is seeing what stays the same in mathematical thinking, requires *awareness of variance*. However, Kaput warned that a learner's attention to variation cannot be guaranteed simply because the variation exists. He claimed that particular experiences are necessary to cause learners to attend to the educationally critical aspects. Therefore, the MRPLRP can be considered an application of variation theory, since various aspects of reasoning are experienced by the teachers and aspects of the approaches are varied.

To discern a particular aspect of reasoning, a teacher needs to become aware that there is possible variation in that aspect. Learning occurs when a teacher is able to discern more aspects, more variations of an aspect or connections between aspects. In this way, variation theory can be seen as a guide for designing learning activities which exposes teachers to the greatest possible variation in the widest range of aspects related to the object of learning. In PL program based on variation theory, the designers first determine the critical aspects of the object of learning and formulate activities which help to focus teachers' attention on them, so that the teachers experience varies for any particular object of learning. In the case of this study, the object of learning is the proficiency of reasoning. This study aims to determine the critical aspects of the MRPLRP in raising this teacher's awareness of the critical aspects of reasoning. Marton et al. (2004) suggested that the critical aspects should be found through interviews with learners, thus the participants of the MRPLRP, including the teacher participant in this paper, were interviewed to determine their perceptions of reasoning (Herbert, Vale, Bragg, Loong, & Widjaja, 2015).

Variation theory characterises learning as changes in a person's perception of the phenomenon under investigation (Cope 2000). Learners, such as the primary teacher in this study, become aware of additional features or aspects of this phenomenon, which had not previously been discerned, (Bowden and Marton 2004), in this case mathematical reasoning. Learning occurs when they "see the phenomenon differently" (Bowden and Green 2005, p. 30). Consequently, PL is the vehicle for teachers' increased awareness of the complexity of reasoning and its pedagogy by drawing attention to critical aspects of reasoning. Full understanding of a phenomenon, such as the concept of reasoning, is reliant on the discernment of all critical aspects through experience with the variation in critical aspects, such as different forms of PL where students' reasoning is foregrounded. The two phases of the MRPLRP exposed the teacher in this study to the critical aspects of reasoning, because variation theory was applied in the creation of the program. The design of the activities considered the widest range of critical aspects, such as the key reasoning actions of analysing, generalising and justifying, thus providing teachers with variation in object of learning (Cope 2000) that is reasoning. The program was arranged so that the teacher came "to experience something in a qualitatively new and more powerful way, so that it can be accomplished in different circumstances, in different ways, and facilitate doing altogether new things" (Booth 2004, p. 10).

Methodology

This article employs a case study approach because it presents an in-depth inquiry into a single case (Merriam 1988), that is a primary classroom teacher's perspective of her experiences in the two phases of the MRPLRP. Olive's participation and the development of her knowledge through engagement in the program is reported. MRPLRP aimed to improve teacher knowledge of reasoning and appropriate pedagogies to develop students' reasoning capacity. The data were collected over an extended period of time and included three audio-recorded interviews from Phase One, and the peer-planning, observation and reflection from Phase Two. Olive and her PLT, Don and Kerry (Pseudonyms used throughout), were chosen for this case study because Olive demonstrated substantial progress in the development of her understanding of reasoning to take the lead role in Phase Two. Changes in Olive's use of reasoning terms and other descriptions of her understanding of reasoning are included in the data to illustrate the impact of the program had on her increased *awareness of the critical aspects* of reasoning.

The 'case' teacher, Olive, was in a leadership position at a small rural primary school in Victoria, Australia. The PLT, which is discussed in this article, comprised the 'case' teacher and two other year 5 classroom teachers from her school. Don was the school's numeracy leader and Kerry was a teacher with many years of experience. During Phase Two, Olive was teaching a composite class of years 4 and 5 children. This PLT was selected as Olive had participated in Phase One whilst team teaching a composite Years 3 and 4 class and so had an emerging understanding of reasoning to support and guide the other members of the PLT. She reported as being proactive in seeking external PL in mathematics and *always had a passion for maths*, engaging in independent study in teaching mathematics, thus demonstrating her willingness to learn more about teaching mathematics (Beswick and Jones 2011).

Variation Theory guided the development of both phases of the MRPLRP, to build teachers' awareness of the critical aspects of reasoning (Booth 2004). Phase One was designed to expose teachers to the critical aspects of reasoning – analysing, proving, evaluating, explaining, inferring, justifying and generalising. In Phase One, Olive and other teacher participants in the school observed two researcher-taught demonstration lessons following lesson plans designed by the research team and provided to the participants before each demonstration, thus encouraging them to be alert for instances of reasoning by the year 3 and 4 students (aged 8–10). Variation in the critical aspects of reasoning was purposefully included in the design of the lesson plans to draw the teachers' awareness to differing aspects of reasoning. Each lesson was immediately followed by a researcher-led post-lesson group discussion with the teachers and the researcher who taught the demonstration lesson, focusing on the teachers' observations of students' reasoning and the way the lesson structure and the researcher fostered the students' reasoning. After both demonstration lessons, the teachers trialled the lesson in their classroom. To track any changes in the teachers' perceptions of reasoning (Loong, Vale, Herbert, Bragg, L. A., & Widjaja, 2017) against the *Mathematical Reasoning Framework* (Herbert, Vale, Bragg, Loong, & Widjaja, 2015), the teachers were interviewed individually three times: before observing the first demonstration lesson, after trialling the first demonstration lesson with their class, and after trialling the second demonstration lesson with their class. The findings from Phase One of the project indicated that further PL was required to enhance teachers' knowledge of reasoning to enact it effectively in the classroom (Loong, Vale, Herbert, Bragg, L. A., & Widjaja, 2017). Consequently, Phase Two of the MRPLRP was developed to supplement and enhance teachers' existing knowledge of reasoning through a potentially more autonomous, cost-effective and feasible design of PL, guided by variation theory. The aim of this paper was to reveal factors in this PL program that supported primary teachers' growth in understanding mathematical reasoning and its pedagogy. Applying the principles of variation theory in Phase Two, some aspects of the PL were kept invariant, whilst others varied. Invariant aspects included continued focus on reasoning, lesson planning for reasoning and observation of lessons. Variations to the PL included the formation of a PLT, shifting the responsibility of the planning of a lesson from the researchers to the teachers, and the teaching of the lesson moved from researcher-led to teacher-led. Further, the number of observers in Phase Two was reduced from several teachers and researchers to the two teachers from the PLT. Table 1 displays the variation in designs between Phase One and Phase Two.

Phase One lessons

In Phase One, the first demonstration lesson, observed by Olive and the other participants, was based on an adaption of Small's (2011) task *What else belongs?* where years 3 and 4 students were challenged to find other numbers which belonged with a set of numbers {30, 12, 18}, hence *analysing* the problem by finding additional examples. In a whole class orchestrated discussion (Stein et al. 2008) midway through the lesson, selected students *explained* the numbers they had found and *justified* their choices of numbers. Next students were required to work with their partner to create a set of three numbers which were formed through one unifying feature, e.g. even numbers, multiples of 5, etc. The lesson closed with a final class discussion where selected pairs *explained* their three numbers and *justified* why they belonged together, whilst other students *evaluated* their justifications. This demonstration lesson modelled the conduct of orchestrated discussions, use of appropriate prompts and sequencing of a lesson to facilitate reasoning, and exposed Olive to the critical aspects of reasoning, analysing, evaluating, explaining and justifying. *What else belongs?* was then trialled in the participating teachers' classrooms. This process was repeated for the second demonstration lesson, based on the Magic V task (University of Cambridge 2019). In this task, pairs of students were given five cards numbered 1–5 and asked to arrange them in a V shape so that the two arms of the V have the same total when summed, and thus termed

Table 1 Variation of Critical Aspects of PL Design in Phase One and Two

Phase One	Phase Two
• Researcher-designed lesson plans • Researcher teaches lessons • Lesson observed by researchers and teachers • Post-lesson discussion – led by researcher • Teachers trial researcher-designed lessons • Teacher trialled lessons not observed	• PLT collaboratively designed lesson plan • PLT members teach lesson • Lesson observed by PLT members only • Post-lesson discussion – no lead

a Magic V. Students analysed this task by generating examples of Magic Vs. Orchestrated class discussions provided students the opportunity to present and justify their solutions. This task concluded with a generalising task of responding to a conjecture:

> Sam said, "It is impossible to make a Magic V with an even number at the bottom". Is Sam right? Explain why or why not?

The Magic V demonstration lesson exposed Olive to the critical aspects of reasoning; analysing, evaluating, explaining, justifying and generalising. Olive's trialling of the demonstration lessons exposed her to critical aspects of teaching reasoning by this practical experience of teaching a lesson where reasoning had been specifically embedded. These experiences afforded Olive insights into students' reasoning and strategies relevant to the teaching of reasoning.

Phase Two lesson

In Phase Two, Olive and her PLT participated in collaborative planning to design a reasoning-embedded lesson for their classes. In this way, the Phase Two varied from Phase One, where the researchers planned the lesson. However the focus of the lesson remained the same that is with a focus on reasoning. The researcher's role in the planning meeting was as a participant observer, responding to teachers' requests for advice or questions and refocusing the team on reasoning where necessary. This PLT chose to base their lesson on the picture book, *Spaghetti and Meatballs for All* by Marilyn Burns (1997). The story describes a problem about arranging tables and chairs to accommodate a large number of people for a dinner. The task was:

> *Today we would like to keep both Mrs. and Mr. Comfort happy. We need to fit 32 people but not all at the same table. How many arrangements can we make? Show two examples with square counters and record in workbooks, showing the number of tables used.*

Restrictions were placed on possible solutions: tables must be square or rectangular, and the number of people will always be 32. Later in the lesson, the task was modified to:

> *Show as many examples as you can find, showing the number of tables that could be used.*

The reasoning intentions for this task were as follows: explore and notice relationships between the number of people and the number of tables (analysing), use trials to develop conjectures (analysing) compare and contrast examples to form conjectures (generalising), explain conjectures using an example (explaining *why*), and test conjectures using examples to verify (justifying).

Following the planning meeting, each teacher in the team taught this reasoning-focused lesson with other teachers in the team observing. This varies from Phase One where teachers observed a lesson presented by a researcher; however, the intention of the observation was the same, that is with a focus on reasoning pedagogy. The intent of the peer observation process was for only fellow staff members from the PLTs to observe each other, a process that could be undertaken in any school without researcher intervention. During the lesson, the observing teachers recorded on an observation schedule any reasoning actions noticed; afterwards, all teachers completed reflective notes. Once all lessons had been conducted and observed, the PLT along with a researcher met for a post-lesson discussion to share the teachers' observations of students' reasoning, teachers' actions to support reasoning, and the lesson outcomes.

Data analysis

Data analysed in this paper includes: Phase One – the three interview transcripts described above, Phase Two – the transcripts from the video recordings of the planning and post-lesson discussions. The data presented from both phases of the MRPLRP provided evidence of Olive's opinions of influential features which she reported shaped her growing awareness of reasoning. The recordings from the Phase One interviews and Phase Two planning and post-lesson discussions were transcribed verbatim, checked for accuracy by the researchers and entered into NVivo (QSR International 2017) to facilitate data analysis. Variation theory was employed in the analysis of the data by first coding according to the critical aspects of reasoning: *analysing*, *proving*, *evaluating*, *explaining*, *inferring*, *justifying* and *generalising*. Then these data were coded according to features of the MRPLRP: *planning* and *observation*, and the PCK for reasoning, such as *orchestrated discussions* and *effective prompts*. The researchers independently coded the one transcript of a post-lesson discussion according to these codes, thus confirming interjudge reliability through verification of each other's coding. Further scrutiny of the transcripts was undertaken to refine coding and create subcodes. The findings arising from analysis of the data from peer-planning meetings and post-lesson discussions with the PLT are presented in the Results section.

Results

Analysis of the transcripts over Phases One and Two of the MRPLRP highlighted Olive's changing awareness of the critical aspects of reasoning and her growth in pedagogical knowledge of reasoning. Further, there were indicators that Olive, as a more experienced member in the PLT, provided guidance to her PLT members and motivated them to think differently about the teaching of reasoning. The following results present Olive's experience within both phases of the MRPLRP and explore the factors each phase offered Olive in building her knowledge and teaching of reasoning.

Phase One: Demonstration lessons

When Olive was interviewed before the commencement of MRPLRP, limited understanding of reasoning was evident in her responses. Reasoning was not considered when planning her mathematics lessons. In the first interview, when asked "*What do you see reasoning to be?*" Olive replied:

> The thinking behind why things work. I don't think when we're planning we purposely think about reasoning. We do a lot of work in number and if reasoning happens to happen [it is] within that.

During Phase One, Olive's developing understanding of reasoning is evidenced in her comments from the second interview indicating her growing awareness of more aspects of reasoning. When asked again, "*What do you see reasoning to be?*" Olive replied:

> The thinking about the thinking, analysing what they're actually thinking about. [Students] talk about thinking and infer with evidence. So within reasoning it's getting them to think about their own thinking and why does this work. It's how they think, and how well they think, and what they know. If they know more, they can think more, if you [students] can give your [their] reasons you [they] seem to have a higher level of reasoning.

Olive's quote indicated that she considered reasoning occurred when students engage in metacognitive thinking. Key features of Olive's awareness of critical aspects of reasoning are seen in her reference to inferring from evidence, talking about thinking (explaining) and justifying solutions. Illustrative quotes from the second interview indicated aspects of Phase One that influenced Olive's growing awareness and knowledge about reasoning and its pedagogy, that is factors in Phase One that supported her growth in understanding of mathematical reasoning and its pedagogy. When commenting about the demonstration lessons Olive stated:

> … just reading someone's notes [lesson plans] you can interpret it differently, but watching someone do it, you've seen the conversations, you've seen how they've done it, so you get so much more out of it. When I read it [lesson plan] maybe I didn't understand it very well but we had sat down with you before the lesson and you'd asked us what we thought some of the answers might be and we said hopefully they'll say 2 digit numbers and hopefully they'll say even numbers.

This quote emphasised the benefits of witnessing teaching-in-action to possess a deeper understanding of the complexity of the lesson plan. Olive had not previously experienced the orchestrated whole group discussions in the way it was utilised in the demonstration lessons:

> … when [the researcher teaching the demonstration lesson] put it [table of students' solutions] on the board you could see it set out within the levels. [I could see that] two digit numbers was the simplest [reason] or even numbers and then, through to higher level thinking [of multiples].

Olive had witnessed these teaching actions modelled in the demonstration lessons. Olive's comment illustrated her learning about this strategy, demonstrating Olive's awareness of teacher actions to facilitate the development of students' reasoning. This quote demonstrated her learning and appreciation of how to purposefully sequence student responses, thus assisting students to notice mathematical connections between different ideas and advancing their mathematical thinking. When commenting about the post-lesson discussion, Olive reflected:

> Yes, the discussion on reasoning and the lesson itself, you were hearing ideas from someone else that you didn't necessarily have, but it also confirmed what we were thinking … we got a lot out of the discussion, all the questions were answered and everyone had a chance to speak and everyone did speak.

The post-lesson discussions provided an avenue to share colleagues' views which supported Olive's growing understanding of reasoning. When trialling the demonstration lessons Olive followed the lesson plans closely, orchestrated the discussion which she witnessed in the demonstration lessons and employed questioning techniques she had seen modelled in the demonstration lesson.

> I followed the script [lesson plan] … [and] orchestrated discussions in the same way. I walked around the tables taking copious notes and picked a few of their reasons, put them in order according to easiest reason, to more developed reasoning. I asked a mix of children to get a variety of answers. The table was already drawn on the board, same as XXXX did… while they [students] were speaking we filled it in.

Olive's most powerful strategy to support and encourage reasoning was the use of counterexamples in her probing questions. This strategy explicitly highlighted for students critical aspects of the concept of even numbers because students had other examples of when a number is not even, thereby deepening their understanding of this concept. The use of this strategy illustrates Olive's enhanced PCK as a result of observing the demonstration lessons where orchestrated discussions were modelled by the researcher teaching the lesson who used student answers to elicit reasoning. When reflecting on her trial of the demonstration lessons, Olive commented:

> We weren't necessarily teaching them something, so from a teacher's point of view I found that hard, I wasn't teaching them anything. I was trying to get information from them to understand their reason. We were [teaching] but the learning was, the teaching was coming from each other [the students] not me as a facilitator, not me as the person standing at the front. I didn't necessarily feel that we were teaching the children something, it was more collecting data and seeing how they think which was useful for us, but it wouldn't be how we teach all the time.

This quote, especially "*but it wouldn't be how we teach all the time"* and *"I didn't necessarily feel that we were teaching the children something"*, indicated that Olive is still viewing her role as teacher to be the teaching of mathematical content and does not include developing the students' mathematical proficiency of reasoning and would not usually be included in her practice of teaching mathematics. However, countering this comment Olive later stated:

> I think I'll read AUSVELS and VELS [curriculum] documents more closely and actually have a better understanding and talk to our team about teachers having a better understanding of that part of maths [reasoning]. It's not just about place value, it's not just about Number, this [reasoning] is another important part of it.

This explanation showed Olive's emerging realisation that, in addition to content, it is important to incorporate reasoning into mathematics lessons. It is her *aha* moment where she understood the necessity to develop the reasoning proficiency as called for in the Australian Curriculum. When asked to comment about the main ideas or strategies that she had learnt about teaching reasoning, Olive replied:

> Giving the kids more chance to talk to a partner. I would set more activities that were problem-solving type activities, where you are working with a partner and discussing how and why you're going to do it the way you are, and working together. I actually liked the journal entry at the end and I think that would be useful because we don't always have time for that share time. And then it becomes a record of their learning as well. So I think it [reflective journaling] would be a powerful thing, and they would get better at that the more times they saw that.

Olive's references to '*problem-solving type activities*, *'working with a partner* and reflective journaling '*it would be a powerful thing* along with the expectation that students would be required to justify their solutions *'why you"re going to do it the way you are'* indicated improved knowledge of new teaching strategies to facilitate reasoning. Trialling the same lesson as the demonstration lesson was instrumental in raising Olive's growth, including the use of probing questions to challenge and extend students' thinking. At this stage, Olive appeared to be considering other ways of teaching that facilitate students' thinking and reasoning.

Phase Two: Peer learning teams

The purpose of Phase Two of the MRPLRP was to gain further understanding of the interacting factors, that is critical aspects, supporting PL through engagement in a PLT to foster a reasoning-focused pedagogy. When considering anticipated students' responses to the Spaghetti for Meatballs task, Olive demonstrated her growing awareness of the critical aspects of reasoning when she commented "*For them [the students] to actually generalise what's going on there,[that] will be reasoning*".

Following Olive's lead, other teachers in the PLT offered practical solutions and ideas for eliciting reasoning.

> Kerry: So would you just give the kids eight counters and say "Find all the combinations that you can. What's the same? What's different?"

Kerry's comment is an illustration of comparing and contrasting mathematical ideas that offer students the opportunity to reason through analysing and generating examples. Initially, Kerry revealed her limited understanding of the complexity of reasoning, focusing instead only on explaining "*Yeah so the kids are sharing it and explaining it*". However, Olive understood that reasoning goes beyond students describing what they did. Her knowledge of a reasoning pedagogy indicated that she knew to shift students' reasoning to more complex levels, such as justifying, and shared her insights with the PLT.

> Olive: For them to be able to talk about, “You can fit a person here, you can fit a person here,” that’s an explanation which is explaining why and it is part of the ‘convince me’ and ‘justify’.

Olive moved her PLT beyond the idea of explaining as reasoning, to develop a more complex level of reasoning by seeking students to *convince* and *justify* their positions through asking the powerful question, *Why?*

> Olive: And that’s that, the justifying, “Well why does yours …?”

Olive cautioned the group against stifling opportunities for reasoning, “*You don’t want to give them the solution. The solution isn’t where we’re heading*”. Olive’s advice recommended providing a space for students’ thinking *opportunity to justify their findings*. Olive’s comment provided an indication of her awareness of the critical aspect of justifying.

Conversely, Don’s focus on correct answers and correct procedures is reflected in his comment, “*You want the most efficient way*”. The following exchanges demonstrated the interchange between two teachers with differing views of mathematics and its pedagogy.

> Don: Well you have to give them some boundaries, don’t you?

> Olive: It needs to be a little bit open doesn’t it? So that they can reason. That’s where the reasoning comes in.

When considering students’ responses to analysing the task, this exchange demonstrated Olive trying to shift Don’s focus to foster pedagogies more suitable for eliciting students’ reasoning.

> Don: I'm thinking they're going to fiddle with this [task - table arrangements for 32 people], right?

> Olive: You have to explore first

> Don: Okay, we’ve got reasoning. We’re having a lot of chats. Where do our formulas come into it?

> Olive: If we tell them then we’ve taken all of that opportunity away from them to discover for themselves.

Olive highlighted to Don the limitations she perceived with focusing on direct instruction which offers students a formula rather than exploration of these mathematical concepts. Perhaps her understanding of suitable pedagogies to foster students’ reasoning may have been enhanced by previous involvement in the Phase One of the project where she engaged in observing demonstration lessons with open-ended tasks and problem-solving which facilitated the observation of students’ reasoning.

Emerging from the data was the impact that Olive had on the PL of others within her PLT. She demonstrated awareness of more critical aspects of reasoning than the other teachers and advocated its implementation in the classroom. For example, Olive suggested prompts to foster students' reasoning by encouraging comparisons and justification:

> Olive: "Why does that work and why doesn't that one?" … "Why are those two the same number of people?"

Olive's high energy and ability to motivate others in the team was evident in her interactions within the PLT even though she was not the designated numeracy leader/ mathematics coordinator within the school. Nevertheless, Olive was able to offer guidance in teaching reasoning due to her knowledge of reasoning. Peer observation of her lesson led to the other teachers in the PLT noticing modelling of effective reasoning prompts:

> Kerry: I deliberately went last [to teach the lesson]. It was great to be able to watch Olive and Don. Probably I didn't think my language – I wasn't prompting as well as Don and Olive was just because I wasn't tuned into it [reasoning]. One of the interesting things was one of the conversations the kids were having when they were partnering together".

Kerry is suggesting that a PL program that exposes teachers to other's teaching practices which orchestrated class discussions to elicit reasoning provided her with an opportunity to grow professionally from this PLT experience. Peer observation and post-lesson discussion afforded Olive with a voice for positive change. Peer observations offered teachers the opportunity to examine and scrutinise students' conversations that may have previously gone unnoticed during the typical course of teaching.

Whilst the findings above validated the usefulness of a PLT as an opportunity to enhance teachers' PL, constraints, such as personal discomfort and financial cost, were raised by the participants during the planning meeting and post-lesson discussion. These participants felt nervous when observed by their peers despite the notable collegiality in this PLT.

> Kerry: We were still a little bit anxious about it but it was a much more natural feel to it.

Budgetary considerations in the manner of funding time release were viewed as another constraint to working synchronously as a PLT.

> Olive: It was really powerful those couple of years ago when Don and Bonnie did have release to help with planning but it meant that they could also release someone to go and watch someone else teach. If there is a budget for it.

Much of the focus during the planning for reasoning was in the consideration of prompts to elicit reason. Together the PLT brainstormed various prompts, such as:

> Kerry: Explore and then explain.

Don: Elaborate.

Olive: Why does that work and why doesn't that one?

Olive: And as a class, what have we found so far, has someone got something different, has someone, come up and draw yours, has someone got something different?

Kerry: So every time the child sees something like that you go Oh let's try it, does it work?

Don: Prove it, [that's] where your reasoning comes in.

The suggested prompts revealed Olive's knowledge of the importance of communicating reasoning and fostering dialogue between themselves and their students. Further, the prompts offer teachers tools to elevate the complexity of their students' reasoning.

The peer observation of Olive teaching the lesson the PLT had planned together was significant in building peers' knowledge and understanding of reasoning and appropriate pedagogy to facilitate students' development of reasoning. In addition, it appeared that Olive considered Phase Two to have been more useful for growth in her awareness of critical aspects of reasoning.

Olive: Planning with each other and observing each other was much better than just watching and trialling the demonstration lesson. It [being observed] definitely made you stay focussed and on task with the reasoning. It's much easier to work with other people.

The results aid in identifying the factors of the two PL phases influencing Olive's growth in knowledge of reasoning and its pedagogy. PL was instrumental in raising her awareness of the multifaceted nature of reasoning. The results highlight aspects of these two phases (demonstration lessons and a PLT) that were influential in the development of a robust understanding of reasoning, namely: focus on fostering reasoning, planning, observation and post-lesson discussions.

Discussion

This study sought to identify factors in these two PL designs that supported this primary teacher's growing *awareness* of mathematical reasoning and its pedagogy. The focus on reasoning and lesson observations were kept invariant whilst planning, and teaching a lesson varied with the teachers in the PLT taking these roles rather than the researchers. Phase One and Phase Two of the MRPLRP were developed according to the advice found in the literature regarding effective PL in mathematics. This article explores the factors in this two-phased PL program in their relative suitability for raising teachers' knowledge of reasoning and its pedagogy.

In Phase One, the use of demonstration lessons followed the advice of Clarke et al. (2013) and Loucks-Horsley et al. (2009) with a lesson plan prepared by the researchers with tasks specifically designed to encourage reasoning and made available to observing teachers before the lesson; observation of the lesson presented by an expert teacher who held a deep understanding of reasoning and used pedagogical strategies for supporting student reasoning, such as appropriate prompts, and orchestrated discussion; and a post-lesson discussion to reflect on the observations. Observing teachers were able to focus on noticing instances of the students' reasoning without the need to manage the class. Olive's comments above indicated that the observation of the researcher teaching lessons had a lasting and positive impact in her advice for her PLT in the second phase of the project. In addition to observing the lesson, teachers were expected to trial the lesson with their class. Whilst positive aspects of demonstration lessons were noted by Olive, this form of PL has some limitations, particularly teacher autonomy. Teachers did not have an opportunity for input into the lesson planning and did not have ownership of it. The content of these imposed lessons may not fit into the content of their planned sequence. Another issue raised is the instances of the pedagogical focus may go unnoticed during observation, and observers may focus on teaching and class management rather than the intended focus of the lesson, reasoning. Further, there were school logistical issues relating to time release for observation. Whilst neither raised by Olive nor by her PLT members, the logistics of the researchers' availability and the financial costs to their institutions are compounding issues that negatively impact researcher-led demonstration lessons as a viable design of PL.

In Phase Two, advice from Sullivan et al. (2012), Eri (2014) and Johnston and Cornish (2016) guided the PL design. In their PLT, the teachers planned a lesson designed to embed reasoning, hence resulting in ownership of the lesson and investment in its success. The PLT was able to tailor the lesson to their students' needs and fit into their curriculum plans. At this PLT's school, teachers usually planned lessons collaboratively so that they were experienced in supporting their colleagues. Each member of the PLT taught the lessons observed by their peers, and when all had taught the lesson, they came together for a post-lesson discussion on the lesson to share their observations and reflections. Similarly to the demonstration lessons, the observing teachers focused on noticing instances of students reasoning and strategies for supporting reasoning without the need to manage class. Some discomfort with being observed was mentioned in the post-lesson discussion and again there were some school logistical and financial issues relating to time release for observation.

In Phase Two, a significant opportunity was for teachers to work in a team to plan a lesson specifically designed to include tasks which foster students' reasoning, rather than a lesson planned by the researchers. The opportunity of participating in the MRPLRP enabled Olive to implement changes in her practice as similarly noted by Nickerson and Moriarty (2005). Further, Johnston and Cornish (2016) supported the employment of peer observation in offering exposure to varied teaching approaches. This diversity of teaching approaches was evident when the teachers were actively engaged in observing good pedagogical practice. Byrne et al. (2010) advocated peers working together to improve practice facilitated by discussion and reflection. Olive's greater knowledge of reasoning enabled the other two teachers in her PLT to observe her modelling of reasoning pedagogy including prompts to extend and challenge students reasoning hence creating a successful learning situation (Pietilla 2003). Throughout the planning session, the teachers demonstrated their understanding of the need for students to be provided with opportunities for comparing and contrasting mathematical ideas that offered students the opportunity to reason through generating examples, that is analysing (Lo 2012; Lobato et al. 2013). Analysing occurs when comparing and contrasting examples to notice what is same and what is different and to sort and classify them (Mason 2002). Whilst many teachers focus on explaining being a common reasoning action in the primary years (Clarke et al. 2012), as indicated by Kerry's comment, "*Yeah, so the kids are sharing it and explaining it*". However, later in the planning session, Kerry commented "*So would you just give the kids eight counters and say "Find all the combinations that you can. What's the same? What's different?*" suggesting her growing awareness of analysing through working with Olive in the PLT. Peer observation of the teaching has the potential to support other members of the team to notice students' reasoning and the prompts which are useful in eliciting it. These teachers found peer observation advantageous in offering a space to learn from their colleagues' pedagogical actions (as suggested by Clarke et al. 2013).

Teachers play a crucial role in developing classroom cultures where students are expected to communicate their reasoning whilst feeling safe to do so (Nardi and Knuth 2017). Whilst the PLT teachers voiced some discomfort in being observed, the atmosphere of mutual trust between the teachers in the PLT assisted in overcoming it (Schuck et al. 2008). Further, mutual respect and responsibility underpin sustained improvements to pedagogical practice (Byrne et al. 2010). For teachers new to embedding reasoning in their mathematics lessons, it can be as simple as using the prompt *Tell me why?* Any teacher who continually questions *Why?* from their students and encourages them to justify their thinking is promoting reasoning. Open-ended questions or problem-solving tasks lend themselves to fostering and valuing reasoning; however, closed questions can still offer reasoning opportunities. For example, a generalising prompt to promote reasoning that could be used with a page of addition problems from a traditional textbook might be *What patterns are you noticing in the addition problems?*

Variation theory was employed to investigate changes in Olive's awareness of the critical aspects of reasoning as listed in the AC:M (ACARA 2019): analysing, proving, evaluating, explaining, inferring, justifying and generalising. Olive's comments throughout the interviews in Phase One and the planning and reflection sessions from Phase Two indicated changes in her awareness of reasoning. The data collected for this study suggest that Olive's awareness of a wider range of critical aspects of reasoning was effective in supporting her PLT to design and implement lessons focused on reasoning. In Phase Two, she understood that the importance of providing a space within the task for students to express their reasoning is an important element for promoting a coherent argument to foster students' reasoning (Stiff and Curcio 1999). In addition, she had built her understanding of orchestrating discussion Stein et al. (2008) through participation in Phase One where this was modelled in the demonstration lessons. Consequently the conduct of her whole class discussion was consistent with Stein et al. (2008) five practices of orchestrating discussions, i.e. "anticipating, monitoring, selecting, sequencing and making connections between student responses" (p. 314). The PLT's planning, with a focus on reasoning, was influenced by their collective knowledge of reasoning and how to assist students in developing reasoning and the opportunity to plan together and peer observe. Olive's understanding of an appropriate reasoning-focused pedagogical approaches including orchestrated discussions and effective prompts was evident in the probing questions she used and influential in building the other teachers' understanding of reasoning and its pedagogy. The variation in the critical aspects of mathematical reasoning Olive experienced in MRPLRP was influential in her growth in MCK and PCK of reasoning, and perhaps may extend to teaching mathematics more generally. However, Olive's preference for Phase Two may have been influenced by her existing awareness of critical aspects of reasoning established during Phase One; however it is not clear if Phase One is the only design of PL necessary to prepare a teacher to lead a PLT focused on reasoning.

Since this paper reports on a case study of one teacher, it is necessary to acknowledge the limitations of this approach and the data it offers. For researchers attempting to conduct their own small-scale action research investigations on a limited budget, a single case study of one participant offers valuable data. We acknowledge that the findings of this study are idiosyncratic, but consider that the close examination of this one case does provide insights into factors of PL influencing teachers' awareness of mathematical reasoning. These factors appear to be lesson planning for reasoning, resources including suitable prompts, lesson observations and post-lesson discussions to reflect on observations.

Conclusion

Teachers' growing attention to fostering students' mathematical reasoning has resulted from its prominent place as one of the four key proficiencies in the current AC:M (ACARA 2019). Understanding this key proficiency more deeply has been an impetus for driving teacher change. This driving change has led to the recent influx of multiple approaches to PL focused on reasoning. In response to research indicating teachers' limited knowledge of reasoning, further support for teachers was required. Our team created and implemented the MRPLRP to assist teacher change. Two differing designs of PL were utilised in the MRPLRP; demonstration lessons and peer learning teams. In this paper, we identify critical factors of these phases in supporting teacher change through examining the experience of one teacher who participated in both phases. The evidence from this study is taken from an analysis of Olive's experiences of both phases points towards the critical factors towards change being: planning a lesson with a reasoning focus; observing a lesson with a reasoning focus; teaching a reasoning-focused lesson and reflecting on the lesson. These critical factors were common in both phases of the MRPLRP.

Additional influences on Olive's expanding awareness of reasoning and its teaching were her interest and intent to learn about mathematical pedagogy and ownership of the planned lesson. Olive's response to her experience highlighted the effectiveness of peer planning and peer observation in supporting the development of her peer's understanding of reasoning within the PLT. Olive voiced a personal preference for Phase Two as it offered greater autonomy, thus allowed for more focused planning for students' needs. However, one teacher's preference does not diminish the usefulness of demonstration lessons in supporting professional growth. Further studies, which take into account the role of the expert in the PL design, will need to be undertaken.

Whilst engagement in both phases of the MRPLRP was instrumental in raising Olive's awareness of the critical aspects of mathematical reasoning, it may be that

Phase Two would be sufficient in building a PLT's knowledge of reasoning and its pedagogy. The design of PL utilising a PLT appears to be equally successful and would offer schools and education departments an effective and financially viable avenue forward for future PL. However, for PL like Phase Two to be successful, the leader of the PLT would require knowledge of the critical aspects of reasoning. Our findings have led us to conclude that a key desirable characteristic of successful PLTs is the inclusion of a teacher with sufficient knowledge of reasoning (a reasoning champion) and how to elicit it from students; Olive was one such teacher. Her emerging expertise with respect to teaching reasoning elevated the PLT's success in understanding and identifying reasoning opportunities when planning their lessons. Further exploration is needed to examine the role of a reasoning champion within a PLT and the degree of impact a knowledgeable member has on the team. Further, what potential for harm on a PLT does the input of a dominant teacher who holds a limited or faulty understanding of reasoning have on a group? Whilst the use of PLTs in schools requires further investigation, teachers, schools and education departments can be encouraged that in this case the PLT actively supported the learning of a mathematical reasoning-focused pedagogy.

This study has gone some way towards enhancing our understanding of the critical factors of PL in supporting teachers' understanding of mathematical reasoning. The importance of these findings serves to address concerns raised in previous research that showed teachers hold incomplete conceptions of reasoning by providing guidance for PL programs focused on building teachers understanding of reasoning. The present findings have implications for future considerations when designing and implementing PL for supporting reasoning in primary mathematics classrooms.

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