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ORIGINAL ARTICLE

Making sense out of the emerging complexity inherent in professional development

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Abstract This paper reports on a study of the process of professional development for mathematics teachers. The analysis connects two theoretical frameworks: the Meta-Didactical Transposition model developed by Arzarello et al. (2014), which describes the macro level, and, at the micro level, the idea of emergence, which has been around since at least the time of Aristotle and has been defined by Mill (1843), Lewes (1875), Blitz (1992), Huxley and Huxley (1947) and many others. The meta-didactical transposition model considers the evolution of teachers' practices as part of a community process, while the notion of emergence helps us to gain better insights into the details of the practices of individual teachers. This paper focuses on secondary school teachers' learning of new digital technologies to illuminate this theoretical framework.

Keywords Teachers' professional development · Technology · GeoGebra · Emergence · Meta-didactical transposition · Praxeology · Agent

Introduction

Mathematics teachers' educators and researchers share the widespread consensus that teaching practices have become increasingly complex, dynamic and socially demanding. This increasing complexity is partially due to the introduction of the digital technologies in the classroom environment. Ideally, such technologies are an integral feature of mathematics teaching and learning, but the significant research efforts dedicated to students' learning of mathematics with the support of technologies have not impacted classroom practices in a relevant way (Clark-Wilson et al. 2014). Our

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knowledge of “how teachers’ practices are impacted by the use of technologies, and subsequently how teachers embed them within their professional lives, for the purpose of improving pupils’ mathematical learning” is still quite underdeveloped (Clark-Wilson et al. 2014, p. 1). Past research on teachers’ integration of technology in mathematics classroom has focused on different factors as follows:

- The instrumental approach with its possible consequences on the teacher’s orchestration of classroom activities using instruments (e.g. Artigue 2002; Trouche 2005; Drijvers and Trouche 2008; Drijvers et al. 2010; Clark-Wilson 2014; Ruthven 2009)
- The identification of teachers’ pedagogical, content and technological knowledge (e.g. Hong and Thomas 2006; Koehler and Mishra 2009)
- Theoretical frameworks for describing the evolution of teachers’ practices in institutionally contextualised educational programmes fostering the integration of technology (Aldon et al. 2013; Arzarello et al. 2014).

Teachers using technology in their classroom have to cope with many factors (Artigue et al. 2009) (e.g. teaching practices in mathematics classrooms, integration of technology and professional development programmes) related to the above factors and “new, sometimes desterilizing situations, which might challenge their existing teaching practices and may invite the development of a new repertory of appropriate teaching practices for these technology-rich settings” (Drijvers et al. 2014, p. 190). Attempting to understand these factors, Trouche (2004) introduced the notion of instrumental orchestration for analysing how teachers organise available artefacts during their activity in a given mathematical situation. Later, Drijvers et al. (2010) and Drijvers (2012) highlighted the complexity of teaching processes, identifying three components within an instrumental orchestration—a didactic configuration, an exploitation mode and a didactical performance—by focusing on the design, the didactical context and the use of the technological instruments in the class. The aforementioned authors identified various instrumental orchestrations for whole-class teaching and for settings in which students work individually or in pairs with technology. In particular, their research studies distinguish between teacher-centred and student-centred orchestrations.

Teachers’ professional development has been mostly studied in the case of teachers working in communities of practice (Wenger 1998; Jaworski 2008). Teachers are viewed in the context of relationships and communities with other teachers (as mentors or tutors) and with researchers (Goos 2014). Opfer and Pedder (2011) consider teacher learning to be influenced by the different ways the elements of three subsystems (the teacher, the school and the learning activity) interact and combine. They recognised the multi-causal, multidimensional and multi-correlational quality of teacher learning and its impact on instructional practices. The existence of the myriad elements impacting teacher professional learning raises the challenge of conducting causal research studies on teacher professional learning that must be focused on explanatory causality and the reciprocal influences of the three subsystems. The complexity of teacher professional learning is consistent with the complexity of the teacher professional development that can be seen as both a product and a process. On the one hand, it is a finished product: a tangible set of professional activities, which cover a specific content within a limited period of time. On the other hand, it is the whole process—the whole sequence of professional activities during their implementation. In this study, we recognise that the process and the product are inextricably intertwined.

In this paper, drawing on the assumption that teaching practices embody a wide variety of different factors (cognitive, psychological, social, institutional, meta-cognitive, etc.; Wenger 1998; Robutti et al. 2016), we explore the complexity inherent in teaching practices using technology. We focus on secondary teachers' professional development programmes aimed at the use of digital technologies as part of their practices. In particular, this paper presents results from a study observing and analysing two professional development programmes for secondary mathematics teachers that intended to integrate GeoGebra into mathematics pedagogical practices, one in Australia and one in Italy. Other research studies have also reported the effect of GeoGebra professional development on in-service secondary mathematics teachers' practices (Hohenwarter et al. 2017; Kasti and Jurdak 2017; Prodromou et al. 2015; Prodromou and Lavicza 2017). It is noteworthy to point out that when we refer to the integration of GeoGebra in such programmes, we mean that our aim is not to simply introduce teachers to the technicalities of using GeoGebra software but to foster teachers' confidence and competencies when they teach mathematics using problem solving, modelling activities and exploration with the support of GeoGebra.

This paper helps open a window from which we study the actions and interactions among teachers, researchers and technological tools. The results of this study help researcher-educators detect factors that need to be organised and implemented in a programme to foster integration of technology (GeoGebra) as part of teachers' professional development.

Theoretical framework

This paper connects two theoretical approaches: the notion of *emergence* and the *meta-didactical transposition* model. Emergence is used to describe the micro-level interaction of independent *agents* that shapes the macro-level components—*praxeologies*—of the meta-didactical transposition (MDT), as defined in the following paragraphs.

We argue that teachers' professional development—the evolution of their praxeologies—emerges from the interactions amongst the variety of agents that affect teachers. Because macro events can be caused by micro ones (as in thermodynamics macro quantities such as internal energy or temperature are due to micro quantities as particles' velocity), in order to grasp possible evolutions in the teachers' praxeologies at a macro level, we try to have insights into the different interactions among agents at the micro level.

We will apply this combined theoretical framework to analysing one teachers' educational programme in Italy and one in Australia, both centred on the integration of GeoGebra in teaching mathematics.

The meta-didactical transposition

One theoretical element in our study is the meta-didactical transposition (Arzarello et al. 2014), which describes the process of professional development occurring in institutions when a community of teachers works with a community of researchers. The

MDT model is used to describe and analyse the evolution of teachers' and researchers' praxeologies over time. Starting with the assumption that institutional attributes (i.e. national curricula, national assessment resources, etc.) have a significant role in the school context, this model draws on the *Anthropological Theory of the Didactic* developed by Chevallard (1985, 1992, 1999). At the core of this theory are the notions of didactical transposition and praxeology. According to Chevallard, the didactical transposition consists of the transformation of knowledge through different stages: the knowledge as produced and used at university level, the knowledge that is expected to be taught based on national curricula and syllabuses and the knowledge taught by the teachers. The didactical transposition examines the classroom teaching praxeologies and the meta-didactical transposition examines the praxeologies developed by teachers in the process of training with researchers (Clark-Wilson et al. 2014). According to Chevallard, a praxeology consists of four interrelated components: task, technique, justification[1] and theory. The given task and the technique used to solve the task are the practical components of the praxeology, while the justification and the theory are the theoretical components that validate the use of that technique. In a mathematics classroom, we can identify the mathematical type of task (for example, *T*: determining the equation of the tangent to a generic function *f*) that the students have to solve, the employed technique and the more or less explicit justification for using it, all within a specific mathematical theory. These components constitute the *mathematical praxeology*. At the same time, we can identify the teacher's questions and actions to build such a mathematical praxeology with her students, which gives birth to a *didactical praxeology*. For instance, the teacher wonders about:

- introducing her students to the type of task *T* (didactical type of task)
- *how to* organise such an approach (didactical technique)
- *why she has to* organise it like that (didactical justification)
- *why she knows that she has to* organise it like that (didactical theory)

Praxeological components can be internal or external to a community. They are internal for a community when commonly shared and used by the members of the community, and external for a community when the members of the communities do not typically use them. The goal of a teachers' professional development programme is to transform praxeological components that are initially external to the teachers' community into internal praxeological components (e.g. activities using new technologies, such as new GeoGebra tools, or new pedagogical techniques, such as student-centred activities). Furthermore, the researchers participating in teachers' professional development may benefit from transforming external praxeological components for their community into internal ones. The MDT model allows description and analysis of these transformations. It describes the professional development in different contexts (including, for example, small professional learning groups, or big groups working for a long time), but some integration is needed in the case of broader contexts, such as Massive Open Online Courses (MOOCs) (see, e.g. Taranto et al. 2017).

[1] We use "justification" where Chevallard (1999) uses the term "technology" in the etymological sense of the term: *techne* + *logos* that is the discourse on the technique. To avoid confusion with common usage of "technology" to refer to digital tools, we use *justification* to refer to the technological part of a praxeology.

As shown in Fig. 1, within teachers' professional development, the researchers' praxeologies interact with the teachers' praxeologies and it is in such interactions that the transformation of praxeological components occurs. It is noteworthy that praxeological components for different teachers may evolve differently due to contextual factors (which are described by the theory of emergence, to be discussed next). If there is the same transformation from external to internal for all the members of the teachers' community, we have what we term a shared praxeology. According to the MDT, "shared" refers to both communities of teachers and researchers. This shared praxeology contains the praxeological components that the researchers planned to activate within the professional development.

Although the MDT model describes and analyses the process of professional development, this description does not shed light on the complex details of this process. To gain a better and deeper understanding of this complexity, we use the idea of emergence.

Emergence

Goldstein (1999) provided a current definition of emergence as "the arising of novel and coherent structures, patterns and properties during the process of self-organization in complex systems" (p. 49).

In this paper, we consider the notion of emergence as the process whereby behaviours of higher-level sophistication (higher-level patterns and regularities) arise through interactions among numerous smaller and simpler agents. Emergent behaviours create something greater and more complex than the simple sums of their parts (Johnson 2001). In mathematics, de De Freitas and Sinclair (2014) have used the term "agents" for referring both to the human and non-human entities involved in the mathematical activity.

Spurred by the necessity of capturing the dynamic interplay of the different factors involved in teachers' professional development in the MDT model, we adopt the perspective of emergence to explain the development of professional praxeologies. At the macro level, different communities (of researchers, teachers, educators, policy makers, etc.) come in contact during the *process* of professional development. The complex, unplanned interactions between the individual agents within these communities at the micro level produce behaviours of a higher-level sophistication that is the professional development as a *product*.

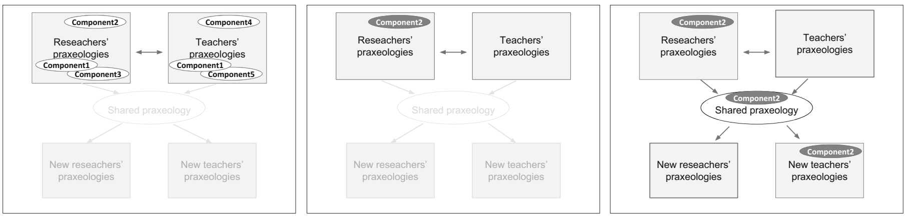

Fig. 1 Among the components of researchers' and teachers' praxeologies, let us consider Component2 which is initially internal to the researchers' praxeologies but external to teachers' praxeologies. The MDT model describes how it can be shared and become internal in the teachers' praxeologies, throughout the PD programme

The lens of this theoretical approach of emergence reveals three challenges faced by the teachers' and researchers' communities. The first challenge is for researchers to discriminate and move smoothly between professional development as a process of a series of outcomes at the micro level and the emerging professional development as a product at the macro level. The second challenge for teachers and researchers in mathematics education is to capture and highlight the evolution of participatory actions during the process of professional development that enhance the evolution of teachers' and researchers' praxeologies over time. The third challenge is to plan the interactions of the components of professional praxeologies at the micro level, aiming at certain professional development outcomes.

In this paper, we study professional development as both a process and a product, adopting a different attitude when examining professional development as a process and when examining it as a product. Our approach is coherent with Davis and Simmt (2003) who claim that "a different attitude is required,…one that makes it possible to attend to their over-shifting characters and that enables researchers to regard such systems, all at once, as coherent unities, as collections of coherent unities, and (likely) as agents within grander unities" (p. 140). We study micro-agents and the actions of the agents at the micro-level when examining professional development as a process to account for the complexity inherent in the professional development as a product.

Micro level of professional development

At the micro level of professional development, we study the teachers' praxeologies in terms of agents. (Agents can also be identified in the researchers' community, but we are not going to study the researchers' praxeologies in this paper.) For each teacher, at any given moment in her teaching, only some agents are active. We consider these active agents as the small elements whose interaction contributes to shaping the teachers' praxeologies. A list of possible agents coherent with the context of teaching is provided here. This list should not be considered as a rigid or exhaustive classification

- Methodological agents:

The commonly used teaching methods primarily fall into two extreme categories or "approaches"—teacher-centred (explaining, or lecturing, demonstration and direct instruction), and student-centred (class participation, inquiry-based learning, cooperative learning, discussions, etc.)—between which are more nuanced approaches that make use of the two extremes in different proportions. One particular student-centred methodology that we consider is the so-called "mathematics laboratory" (Robutti 2006), which is not necessarily a physical place, but mostly a set of activities through which the students work as in a *Renaissance artist's workshop*, in which students learn through collaboration among themselves, with expert students, and with the master. In our paper, the mathematics laboratory refers to this process of learning and teaching involving that collaboration and discussion among learners and experts, while using materials and instruments for constructing mathematical knowledge. Depending on context, the learners could be teachers in a course of professional development or students in a class. Many other elements can be considered *methodological agents*, such as the way teaching and learning materials are used to solve mathematical activities including geometric or algebraic constructions. These materials are related to practical inquiry in particular contexts of knowledge and theoretical inquiry aiming at producing generalised knowledge.

- Institutional agents[2]:

Institutional agents include national curriculum, national assessment and syllabuses proposed by mathematics associations (e.g. Mathematical Association of New South Wales Inc.—MANSW, Italian Unione Matematica Italiana—UMI). They also include congresses, seminars, national or local teachers' programmes of professional development—events organised by the institutions that bring people of different communities of practice together, providing them with opportunities to immerse in another practice (Wenger 1998). They may include professional development workshops, or peripheral experiences that "allow people who are not intending to become full community members to engage in the less demanding practices of this community, sometimes only via observation" (Goos 2014, p. 192).

- Material and technological agents:

These agents include simple materials (e.g. paper, pencil, compass and ruler), software (e.g. GeoGebra, spreadsheets) and hardware (e.g. display technologies, and interactive whiteboards), as well as the use of the web in passive (when one explores websites to find texts, videos and so on), or interactive modes (as the participation in a variety of platforms like virtual communities, etc.). They include also the instructions for using a particular tool (e.g. technical instructions for software use, or instructions for folding a piece of paper in a certain way), well-guided worksheets for using technology to solve a particular problem or investigate a particular content area, pre-built computer files, imagery instructions (Mason 2014), etc. We consider material and digital artefacts together (e.g. paper and pencil and GeoGebra) since it is shown in literature that they can mutually contribute to enhancing mathematical understanding, by constituting a "duo of artefacts", in which each artefact adds value to the use of the other artefact for mathematical learning (Maschietto and Soury-Lavergne 2013).

- Motivational agents:

Motivational agents influence action in the different communities. For example, they represent the teachers' community's motivation to join a programme of professional development or participate in a research project. For instance, the teachers' community is motivated to learn how to structure learning activities that enhance their students' mathematical thinking. While such motivations may be shared, they may manifest in

[2] These agents are particularly important for linking emergence and the MDT. Chevallard's theory "situates the mathematical activity, and so the activity of study in mathematics, within the set of human activities and of social institutions" (Chevallard 1999, p. 223).

different praxeologies by different teachers. The motivational agents are influenced by teachers' beliefs about the nature of mathematics, the nature of mathematics teaching and learning, themselves as teachers, their students as learners, other teachers as peers or the headmaster. Motivation and beliefs sometimes may be influenced by other communities: for example, the Ministry can influence teachers to attend a professional development promising them a promotion in their career.

Macro level of professional development

At the macro level of professional development, the theoretical concern is to understand how the multiple interactions of agents at the micro level—the collaborations of teachers and researchers in professional development programmes—result in the emergent outcomes of new praxeologies. Such emergent phenomena might constitute: (1) praxeologies of teachers in the classroom; (2) new educational programmes where the communities of researchers, teachers and institutions are involved; (3) new praxeologies of researchers; and (4) new research projects during which researchers and teachers collaborate in researching and designing lesson plans that integrate technology.

We discuss emergence looking through the lens of the analytical approach introduced by Goos (2014) for understanding researcher-teacher relationships in the context of different types of research and development. This lens provides further insight into the emergent change in the partnerships between the communities of researchers, teachers and institutions. Combining the MDT model with the idea of emergence provides opportunities for understanding the interactivity of the complex factors involved in the processes of teachers' professional development.

Considering the beginning of the partnerships between teachers and researchers, Goos (2014) identifies three categories: (a) researcher seeks teachers as participants, (b) teachers seek researchers and (c) funding institution (school system) invites schools or teachers to participate to a programme. In our study, we used mostly category a, and also some c (but no b). Initially, at the micro level, motivational agents of the members of each community may interact with institutional agents (e.g. professional development workshops and training modules for in-service teachers). As an example of (a), a university-based researcher attends an annual conference organised by an association of Mathematics teachers and delivers a workshop for the participating teachers. The researcher's motivation is to conduct a research study related to the topic of the workshop. At the end of the workshop, he invites teachers to participate in his research study that has already been planned. As an example of (c), the Ministry of Education supports national/local plans of professional development involving researchers as educators, and in-service teachers as learners. The interaction of the motivational agents of the researcher, the methodological agents of researchers and teachers and the institutional agent (respectively, the workshop or the national/local professional development plan) yields the initiation of a research and a training partnership between the two communities of researchers and teachers.

Once the research study has been planned, researchers might work with teachers who participate in the research as a researcher's aide to develop theoretical and practical knowledge for the research and the teacher professional development programme. The set of research questions and the materials for the training are defined in the researchers' community, and emerge at the macro level as an outcome of the interaction among the institutional, motivational and methodological agents that each community possesses. After that, when the professional development programme is carried out, researchers and researcher's aides enter into contact with the participating teachers. In this way, methodological and motivational agents interact at the micro level to give rise to a shared repertoire of emergent praxeologies. Such repertoires at the macro level may also include formal documents with negotiated roles, work, meanings, and rules, standards of behaviour, research ethics, and ethical implications of such roles. These products result at the macro level out of the interaction between many different agents at the micro level. They may deeply influence the researchers' and researcher's aides' communities and the individuals involved in those communities.

Meta-didactical transposition and emergence

The theoretical framework of the MDT focuses on the development of the praxeologies of teachers and researchers involved in a professional development programme. At the beginning of the programme, the initial praxeologies of the teachers come from the teachers' experiences and knowledge (qualification, professional development programmes, pedagogical competencies, personal studies, etc.). They can be characterised as the combination of internal components, employed in different ways. In the dialectics between teachers and researchers, the initial praxeologies (or one/more of their components) of a teacher could evolve in new praxeologies. The new praxeologies of the teacher are constituted by new components that can replace the previous components.

In combination, MDT and emergence allow us to understand how the praxeological components have been constituted, or changed. In particular, we use the notion of agents at the micro level: for example, a component could be a certain way of integrating GeoGebra in math class (according to some strong ideas of mathematics education research) that can be considered, in turn, as the product of the interaction of three different types of agents: the methodological agents (e.g. the teacher showing a construction with GeoGebra or giving to students a construction to explore), the technological agents (e.g. GeoGebra software on computers or tablets and activities with GeoGebra) and the institutional agents (e.g. indications provided by the national curriculum and standards regarding both the mathematical content and the use of technology in class). Emergence is the crucial aspect in the development of praxeologies and in the transformation of praxeological components from external to internal (Fig. 2).

Approach

This study reports on a common project implemented in two countries taking into consideration contextual factors (e.g. institutional constraints: national curriculum, assessment, school system, teacher education) that influence the implementation of the project in each country: Italy and Australia. Because of differences in individual agents affecting each participant, one concern for the research is to understand how to activate the desired components for participants, despite contextual and personal differences that might interfere.

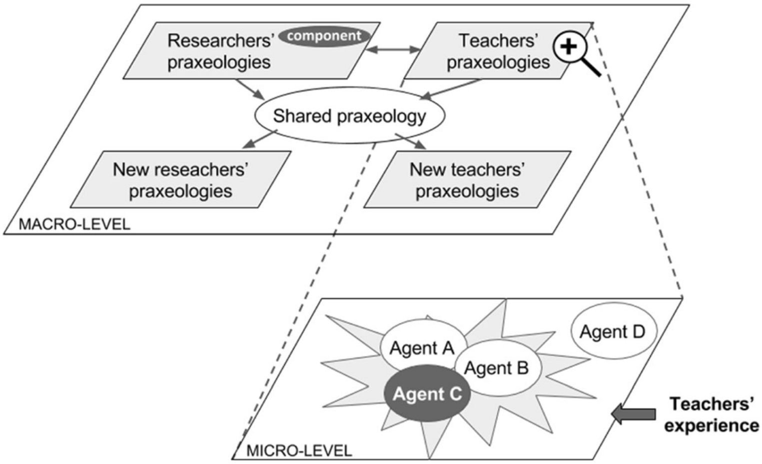

Fig. 2 The Meta-Didactical Transposition model (at the macro-level) and emergence (at the micro-level): with the aim of transposing a researchers' praxeological *component* to teachers' praxeologies, a related agent (*Agent C*) has to be activated and to interact with other agents at the micro-level

In Piedmont (Italy), the project was developed as a teachers' professional development course with GeoGebra as part of a national programme (Piano Lauree Scientifiche—Scientific Degree Programme), promoted by the Ministry of Education and implemented by the Department of Mathematics of the University of Turin. In Australia, the project was developed as a GeoGebra training course for secondary mathematics teachers.

Aims of the professional development programme and of the research

The common aims of the professional development programme implemented both in Italy and Australia were as follows:

- To develop teachers' competencies in the integration of GeoGebra in mathematics problem-solving activities at the secondary level
- To support teachers in the use of various teaching methodologies (mathematics laboratory, discussion, problem solving in working groups)
- To guide teachers to implement the mathematics laboratory activities using GeoGebra in the professional development programme within the context of the appropriate National Curricula

The professional development programme in both countries consisted of three face-to-face meetings of 3 h duration among teachers and educators (who include both the researchers who authored this paper as well as expert teachers who are experienced working as researchers' aides), collaborative work through the Moodle platform, teachers' implementation of the activities in their classrooms. The last meeting was devoted to the presentation of the different experiences of teachers in their classroom and to a discussion of their experiences. During the 4-month programme, teachers participated in regular discussions on the Moodle platform.

Within this context, the common aims of the research were as follows:

- To design the activities as tasks for the students and the teachers when integrating GeoGebra in classrooms
- To observe the processes of the professional development and analyse teachers' praxeologies and the evolution of those praxeologies

This article comes from a larger design-based research study in Italy and Australia that has developed to implement GeoGebra in classrooms. We report on four specific participants involved in programs related to this larger design-based research project who are considered as significative of how professional development occurs and how praxeologies evolve through the interactions of particular agents.

Instruments for data collection

A questionnaire was given before the beginning of the teacher professional development programme, and another at its end. Questions were grouped in the following categories: teaching practices, different uses and frequencies in the use of technological tools for teaching mathematics in secondary classrooms and uses of GeoGebra in mathematics problem solving. Some teachers both in Italy and in Australia were also interviewed before and after they attended the programme and implemented what they were taught in their teaching practices in classroom. A sample of the group of volunteers were selected by researchers according to the following criteria: years of experience of teaching in the school, competencies in the integration of technologies in teaching mathematics and use of different kinds of teaching practices. We used the information from the questionnaire (which will be presented in another paper) to make the selection of the sample. This paper focuses on the data coming from the interviews as being more relevant for examination of the process of the development of the personal praxeologies of the teachers.

For this sample, we conducted audio-recorded clinical interviews lasting 30 min. During the teacher professional development programme, the participants were first given an opportunity to explore the use of GeoGebra. Then they were presented with two classroom activities designed for secondary school classes to solve and discuss among themselves. The activities were designed by the researchers and so the activities represented an opportunity for members of the teachers' community to be exposed to praxeologies from the researchers' community.

One activity concerns the circumcenter of a triangle and the other is about parabolas and their properties. The first activity refers to a triangular pedestrian area in which the students need to place a streetlamp to best illuminate the entire area. The teachers were asked, first, to use a torch, paper and pencil to identify the best position to place the lamp, and then to solve the problem using GeoGebra. The second activity explored the construction of a parabola according to the definition of parabola as those points equidistant from the focus and the directrix, first in an exercise using a folded piece of paper, then using GeoGebra. The teachers were asked to explain their choices as learners, and then to discuss how they could integrate the activities in their teaching practices using GeoGebra to teach this material. Finally, teachers had to experiment using the activity they preferred in one of their classrooms. During the entire experience, the teachers were invited to write a logbook containing their observations about class activities in terms of students' processes, use of technology, approaches to problem solving, self-observed teaching processes, etc. The content of such logbooks became the basis for discussions in the programme meetings with other teachers, researchers and educators.

Participants

The directors of the GeoGebra Institute in Turin and in Australia, who are also among the authors of this paper, invited secondary schools to participate in this project for secondary mathematics teacher training. Sixty teachers from each country volunteered to undertake the professional development programme. From these 60 volunteers from each country, we selected 10 (20 in total) to interview before and after the programme. In this paper, we are going to present two cases from Italy and two from Australia. Next, we will introduce the Italian and Australian participants selected for interviews in terms of their experience both as teachers and with the integration of GeoGebra in their classrooms.

The Italian teachers

Riccardo teaches mathematics in a professional institute in grades 9, 10 and 13. He did not attend a scientific high school[3] and, since the two GeoGebra activities proposed in the programme are about geometrical constructions, we learned with interest that he was at university when he studied Euclidean geometry for the first time. In his classrooms, he teaches basic geometrical concepts and properties but without the use of technology. We selected him because he represents the group of teachers who teach without using any technological tools in their teaching practices or using them only sporadically for showing something to the students.

Lara has been teaching mathematics in a scientific high school for 2 years; before that, she worked in a technical institute. She started to teach after having achieved a PhD in Physics. In her two grade-10 classrooms, she employs the interactive whiteboard (IWB) in order to show geometrical constructions and properties using GeoGebra. We chose her because she represents teachers who only use technology for demonstration.

The Australian teachers

Brett has taught mathematics in grades 7, 8 and 9 of a Catholic secondary school for the last 6 years, after receiving a master's degree in pure mathematics. In his classrooms, he uses technologies such as spreadsheets, Cabri II plus and Cabri 3D to demonstrate geometrical constructions. He is a proficient user of computers. We selected him because he represents teachers who are proficient users of computers but who use digital technologies in teacher-centred approaches.

[3] The expression "scientific high school" denotes the Italian *Liceo scientifico*. It is a type of secondary school whose curriculum emphasises the study of mathematics, physics, chemistry and biology, by highlighting the link between the humanistic tradition and scientific culture.

Gina is a secondary school mathematics teacher with 10 years of experience of teaching at public secondary schools in urban areas of the state of New South Wales. She teaches in grades 9, 10 and 11 and her use of technology in the classroom started when GeoGebra software became publicly available in 2008. Although she has not taken previously any training course, she taught herself to use GeoGebra. Gina mentioned that she likes using GeoGebra because it is helpful to create geometrical constructions, algebraic graphical representations included in the worksheets and assignments used with her students in the classroom. Gina was selected because in contrast to Brett she represents the group of teachers who, although they are proficient users of computers, do not use digital technologies in their teaching practices.

Data

The audio recordings of the participating teachers were fully transcribed shortly after the end of each interview session. The researchers kept reflective notes, setting out impressions and feelings about the teachers' responses to the interview questions. When issues or themes emerged from teachers' responses, the researcher wrote analytic memos. Those memos along with notes helped to supplement the interview transcriptions.

By drawing on the notes and memos, the transcripts of each teacher interview were examined with an interpretative analysis developed and discussed by the authors, who commented on how the interactions of agents at the micro level might have caused a change to the participants' praxeologies.

We focused on the detailed evolution of each teacher's praxeologies when they integrated GeoGebra in their teaching practices, and report in this paper on the different ways in which each teacher's praxeologies evolved within the process of professional development.

Results

In this section, we analyse some extracts of the interviews before and after the professional development programme. This analysis attempts to show how similar agents, acting in different ways for different teachers, produce different evolutions of praxeologies in the professional development process. Such differences depend on how these agents interact in the teachers' praxeologies. The teachers' beliefs, shaped by their personal experience, have an important role in this phase of the process, because we cannot separate the professional life of teachers from their beliefs, experiences and opinions. It is noteworthy to point out that we analyse the evolution of teachers' praxeologies not by observing teachers' activity in the classroom, but by observing the teachers' self-reflection on the process of their professional development. It is our aim—in a future paper—to observe teachers' activity in the classroom, looking at the didactical praxeologies they use with their students.

We observed which agents the researcher-educators put into play in the professional development programme and, through studying their interactions at the micro level, we try to follow the evolution of a particular praxeological component at the macro level.

Specifically, the praxeological component we follow is the use of GeoGebra in the teaching of secondary mathematics. For the researcher-educators, using GeoGebra for mathematics laboratory activities is an internal component, and the teachers' professional development programme tried to integrate this component into the participating teachers' praxeologies.

In the cases of Riccardo, Brett and Gina, this aim was accomplished, while in the case of Lara it was not. The outcome depended on how the agents triggered in the MDT process by the researchers interacted with other agents active in each teacher's praxeologies, especially personal beliefs as motivational agents.

The case of Riccardo

At the beginning of the programme, Riccardo explicitly declares, in the pre-training interview, that he has used GeoGebra in his classrooms only on some sporadic occasions and "in an even less than basic way":

> With grade 13 students, I went [to the digital laboratory] in order to show them how to draw graphs of functions with GeoGebra, whereas, with grade 10 students I made something of statistics, but with grade 9 students, I haven't gone yet [to the digital laboratory], but I intend to show them something of geometry with GeoGebra. The training course can help me in this sense.

In Riccardo's praxeologies, at this micro level, we can identify an interaction between the GeoGebra agent and the lecture as a teacher-centred methodological agent. After the implementation of the proposed parabola activity in his classroom, in the last meeting, Riccardo describes his experience:

> I used the video-projector and I started showing the functionalities because they [the students] didn't know how to do. I had some problems too, so we helped each other. By the way, I surfed the Internet to try to understand how to do some things. Someone suggested me: "Try with YouTube", and we found several videos […] In that way, basically they followed [the activity], they worked in groups and, […] I induced them to get to understand that they were actually constructing the parabola.

Here and before Riccardo is using some verbs referred to his action as protagonist: "I went in order to show", "I intend to show them", "I used the video-projector", "I started showing the functionalities". Only after having interacted with his students, welcoming their suggestion of using YouTube, Riccardo started to refer not only to himself any more. And he used other words such as "We found", "They followed", "they worked". Riccardo with his words is revealing a shift in his methodology from attention on the teacher to attention on the students or the students together with the teacher. In particular, he decides to involve the whole class in the activity with his engagement as both a guide—when he institutionalises the knowledge at stake—and a co-learner, relating to the use of GeoGebra.

At the micro level of analysis, this methodological agent interacts with two technological agents—GeoGebra and YouTube videos—as both teaching and learning materials and instructions for using GeoGebra. Specifically, the teacher and his students use GeoGebra on their computers. As Riccardo noted in his logbook:

> The activity could confirm that the definition of parabola as a geometric locus is well-established. When the teacher gave the definition of the parabola [...], the students were puzzled and did not understand how it could be true that the points in the plane that are equidistant from a given point and from a given line could build a parabola. The teacher tried hard to justify and comment every action made with GeoGebra referring to fundamental notions of Euclidean geometry. [...] [One positive aspect of this experience is] the verification of the validity of the parabola definition, which would not be possible in practice.

In another moment of the last meeting, Riccardo adds:

> I believe it was a positive experience because, let me say… from the next year on, the first time I can use the digital laboratory, I will do it. In fact, with grade 9 students, I found a video on YouTube […] we studied the first Euclid's theorem and we proved it practically in a particular case, but then, since we could move [the points in the construction], we proved it in a practical way.

Riccardo seems to be determined to use GeoGebra for additional mathematics laboratory activities different from those proposed by the researchers because he sees benefits for his students and has experience that suggests that students can learn GeoGebra without undue difficulty. Moving on to the macro-level of analysis, we can observe that Riccardo's praxeology is changing. He is moving from the use of GeoGebra for reproducing sketches he could have drawn on the blackboard to the use of GeoGebra as a tool for the mathematics laboratory. Thus, we can infer that the previously external praxeological component promoted by the teachers' development programme has become internal. This happens because a different methodological agent interacts with the GeoGebra agent: a more student-centred methodology. Before the course, Riccardo never thought about using GeoGebra in a mathematics laboratory because of "laziness but also a little bit of worry" as he declares in the first interview. At the end of the course, his feelings towards the use of GeoGebra have changed, and he believes that the use of GeoGebra can enhance students' imagination, as we can see in this extract:

> there is a childish attitude in me, that is … the child who has not discovered the geometry has to manipulate [GeoGebra] … You can use this with students […] In my opinion, working with GeoGebra, you can construct on their imagination. And some of them have got a lot of it [imagination], it is sufficient to train it someway.

The above extract suggests that a more student-centred methodology component has been activated at the micro level. We can observe that the emergence at the macro level of an evolution of Riccardo's praxeology is due to the interactions at the micro level of the technological agent GeoGebra and methodological agents that have been enriched (GeoGebra, YouTube, ...) or changed (towards more student-centred methodologies) due to the influence of motivational agents: Riccardo felt “lazy” and “worried” before but during the classroom experimentation, his “childish attitude” prevailed. The articulation of Riccardo also supports our conjecture that the use of the technological agent GeoGebra in a mathematics laboratory at the micro level has been changed from external to internal in his praxeology.

The case of Lara

In the first meeting, Lara shows an approach to the use of GeoGebra that is similar to the starting approach of Riccardo:

> I use it in a very elementary way, maybe this is the right time to dig up, it is exactly for this reason that I decided to join [the programme]. I am in an high school and every classroom has an IWB, so it [is] very easy to use [GeoGebra] in a demo mode, [but] we don’t use the digital laboratory […] Through the IWB, I propose one of the several problems taken from the textbook, I make the construction step by step and then [the students] see that effectively some properties are conserved, and why they are conserved […] with the IWB they take the points, they drag them, they do a lot of things by now, they are already autonomous, automatically they stand up, they come to the white board and they do.

Like Riccardo, Lara uses GeoGebra on the IWB for reproducing the sketches she would have drawn on the blackboard, in order to stress invariant properties of the construction. In particular, at the micro-level, we can observe the interaction of the GeoGebra agent and the lecture as teacher-centred methodological agent. Unlike Riccardo, however, Lara uses GeoGebra on the IWB (technological agent) available in the classroom, sometimes requiring the students’ intervention.

In the last meeting of the programme, Lara speaks of her preparation for implementing the proposed activity about parabolas in one of her classrooms:

> I looked at my students and I said: what do I really have to do? Number one: I must explain them what GeoGebra is. […] During the Easter holidays, they had to download the software at home. Number two: in the class mail I sent this [booklet], it is 7 pages long, there is the essence of GeoGebra in it. And I said: learn to get the hang of it.

From her words, we can say that the intervening technological agents are GeoGebra on the computer and the handbook with the instructions for using GeoGebra as a teaching and learning material, always in a teacher-centred methodology.

Then, she implements the parabola activity proposed by the researchers in two classrooms in different ways. The former follows all the phases designed by the researchers and every student works on a computer within a mathematics laboratory methodology. The latter does not require the active involvement of the students, since the teacher reproduces part of the activity on the IWB, following her usual teaching with GeoGebra. In one case, Lara wanted to test the mathematics laboratory as a methodological agent, but in the other case, she wanted to teach using her usual teacher-centred methodology because she felt more confident with that methodology.

In her logbook, Lara reports on the classroom experience and enthusiastically writes:

> Only at the end of the hour three students, using computers that were far apart, discovered, by using the tool “distance or length” from the geometrical tools bar, that the points in question were equidistant from the line d and from the point F. … and the discovery of few immediately became the discovery of almost the entire class group! [...] Thanks to this activity the students became “leading actors”. They had to involve all their knowledge and skills to solve the proposed activity. And we know… Being the “leading actor” is not easy! Let’s say that it needs time!

However, this question of time influences negatively Lara’s answer, in the final meeting, to the question “Would you repeat this experience?”:

> I don’t believe I would repeat it: number one for time reasons, number two because I have neither the availability of [a] digital laboratory nor of a technician who could help me. I believe I would employ it more in the second modality, which I found myself to adopt in the other grade 10 classroom I have: a moment of discovery, of construction, of internalisation through an origami, I would do that. GeoGebra, as after all I’m often using it in the classroom with the IWB, but, indeed, everything is constructed by me. Here the only difference for them is that I don’t show a ready-made construction, we build it together, starting from zero.

After the professional development programme, reflecting on her experience in the two classrooms, Lara prefers the case in which GeoGebra as an agent interacts with a more teacher-centred methodology. Indeed, as Riccardo, she uses expressions centred on herself: “I use it”, “I propose”, “I make the construction step by step”, but unlike Riccardo, she does not change this point of view by the end of professional development programme. She uses expressions more centred on students only when she describes her experience with the mathematics laboratory activity with GeoGebra. At the macro level of analysis, in fact, Lara has integrated GeoGebra in mathematics laboratory activities only in one classroom that was officially selected in the professional development module. So, differently from Riccardo, we cannot say that Lara’s praxeology has changed, because from her words, nothing let us think she has a new internal component based on mathematics laboratory activity with GeoGebra. She prefers to continue with her usual praxeology, grounded on a teacher-centred methodology. We can justify this lacking evolution with motivational agents which interact with the other agents all along the professional development programme, namely Lara’s belief that:

> Having taught in a technical institute, I carry a heavy burden, that is doing a few things but very stable. I have this belief. […] I have a little bit different way of working from “I propose and then collect”, firstly I have to manage the didactical situation by myself.

In conclusion, the Italian cases of Riccardo and Lara show us how the same initial agents interacting differently among them and with different motivational agents can cause different effects in the teachers’ praxeologies. The differences in outcome might depend on many factors, including practical concerns, like limitations of time and resources that can influence the teacher's beliefs about the agents triggered by the researchers' community during the professional development programme.

The case of Brett

Comparing his experience at the beginning and the end of the professional development programme, Brett states:

> Before taking the GeoGebra training, I was not confident enough to use GeoGebra during my teaching to draw any graphs of functions. After taking the GeoGebra training I used GeoGebra while teaching to grade 7 students to demonstrate how to draw a straight line and find the gradient of the line.

From this excerpt, Brett seems to be influenced by GeoGebra—his statement that he was not confident enough to use it suggests that at the very least he had given some thought to using it, and perhaps had an active desire to use it.

GeoGebra was not among the agents influencing Brett's praxeologies before the professional development programme, but after it, the GeoGebra technological agent begins to have an important role while it interacts with the related teaching and learning materials. This change of the status of GeoGebra in Brett's praxeologies can be seen as one implication of the meta-didactical transposition process because it creates an occasion for the activation of the technological agent, GeoGebra. After the implementation of the GeoGebra activities in his classroom, Brett reflects on his use of GeoGebra during his teacher-centred lesson:

> I demonstrated construction of the slope of the line and the intercept on the graph of a function of one variable; and I realised that it would be most appropriate to encourage my students to become involved in the construction of their own knowledge, by encouraging them to investigate mathematical problems in problem-based lessons. I encouraged students to investigate the relation of the slope of the line, and the intercept on the graph of a function of one variable. I asked the head maths teacher to contact a researcher and asked whether there were any applets that simulate the dynamic relationship amongst gradient of the straight line, intercept and the graph of a straight line.

After the demonstration of drawing graphs and calculating the gradient of a straight line with GeoGebra, he wanted to provide his students with opportunities to use GeoGebra in an investigative lesson to study the impact of slope and intercept on the graph of a one-variable linear equation. Therefore, he contacted the head mathematics teacher to ask the researchers to create two GeoGebra dynamic worksheets (Fig. 3) and upload them to the internet. These online worksheets allow the manipulation of slope and intercept by the users who were able to observe the impact of their on-screen actions and interactions with the GeoGebra tools. The online worksheets emerge at the macro level as an outcome of the micro-level interaction of technological and methodological agents and the related teaching and learning materials for teaching the mathematical content at stake (Figs. 4, 5, 6 and 7).

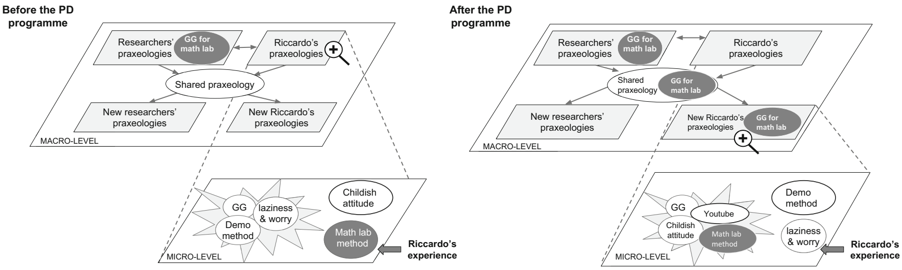

Fig. 3 The integration of GeoGebra for mathematics laboratory activities as a component in Riccardo's praxeologies before and after the professional development programme

For the researcher, the use of GeoGebra dynamic worksheets in teaching was an internal praxeological component, but for the teachers' community it was initially external. In the case of Brett, interaction between researcher and teacher led to the development of shared praxeologies (specific uses of GeoGebra worksheets directed to exploring, discovering, conjecturing) between teacher and researcher. Indeed, shared praxeologies resulted from the intervention of the head mathematics who facilitates the sharing of practices (specific uses of GeoGebra worksheets as written above) from the researchers' community to the teachers' community. Brett adds:

> I used for the first time with these students the computer room…I was hesitant to implement GeoGebra when teaching mathematics in my classrooms. The use of the worksheets by the students who worked in pairs and used the two GeoGebra worksheets, … provided them with opportunities either to adjust two sliders (slope and intercept) and observe the dynamic change of the graphical representation of the line, or move the line on the Cartesian plane by dragging it and observe the dynamic change of the two constants (slope and intercept)… after having students working in pairs in front of the computer, we had a whole class discussion about their observations…

We consider the two GeoGebra worksheets as teaching and learning materials emerging from the interaction (at the micro-level) of the following agents: technological agents (computer room, GeoGebra), methodological agents (investigative lesson, students working in pairs) and institutional agents (head maths teacher).

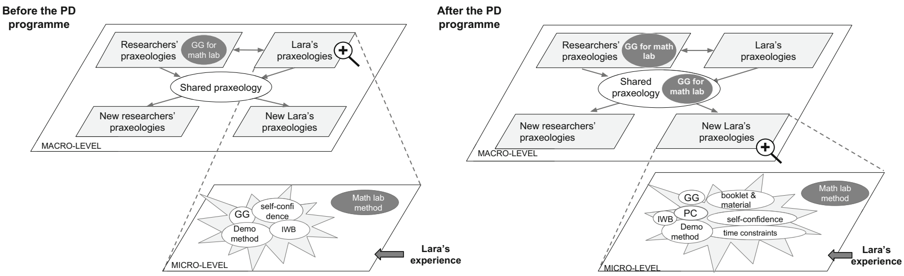

Fig. 4 The sharing but non-integration of GeoGebra for mathematics laboratory activities as a component in Lara's praxeologies before and after the professional development programme

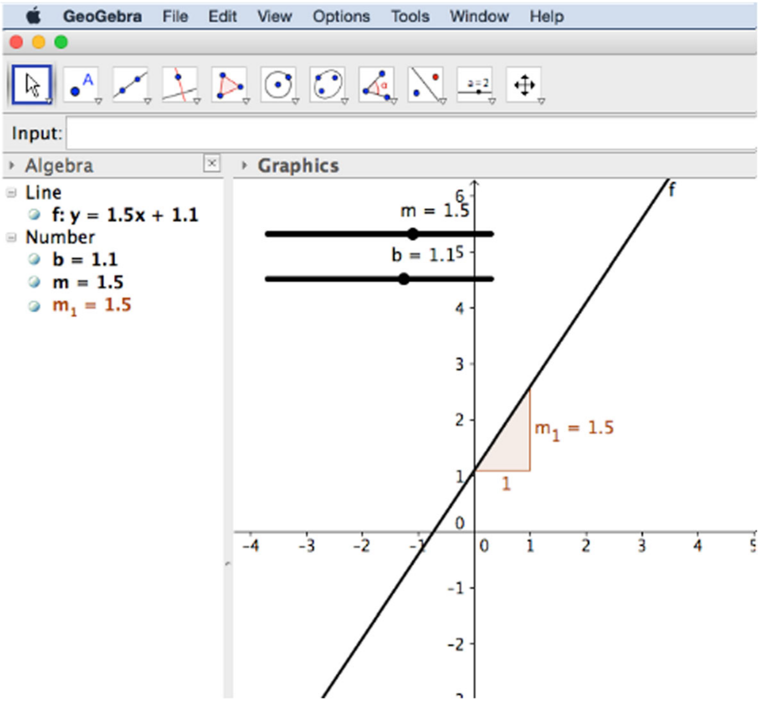

Fig. 5 The GeoGebra dynamic worksheets

The micro-level interaction of GeoGebra, a technological agent, with the use of these worksheets as teaching and learning materials, and the instructions for using GeoGebra, leads to a combination of different orchestrations for class teaching. Brett exploits methodological agents as a combination of student-centred approaches, when students work in pairs, and class discussions.

> GeoGebra worksheets are excellent for teaching, however I realise that students needed guidance to construct the meanings of slope, intersect and their relation to the equation of the line. I should have provided students with a series of explicit questions about how to use more effectively the two GeoGebra worksheets and I asked the head maths teacher to contact the researcher and ask whether there were

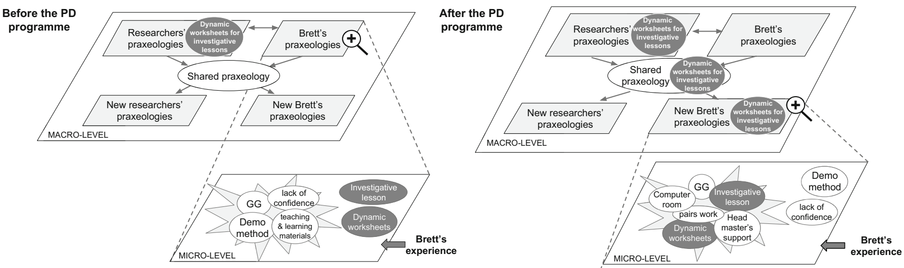

Fig. 6 The integration of GeoGebra dynamic worksheets for investigative lessons as a component in Brett's praxeologies before and after the professional development programme

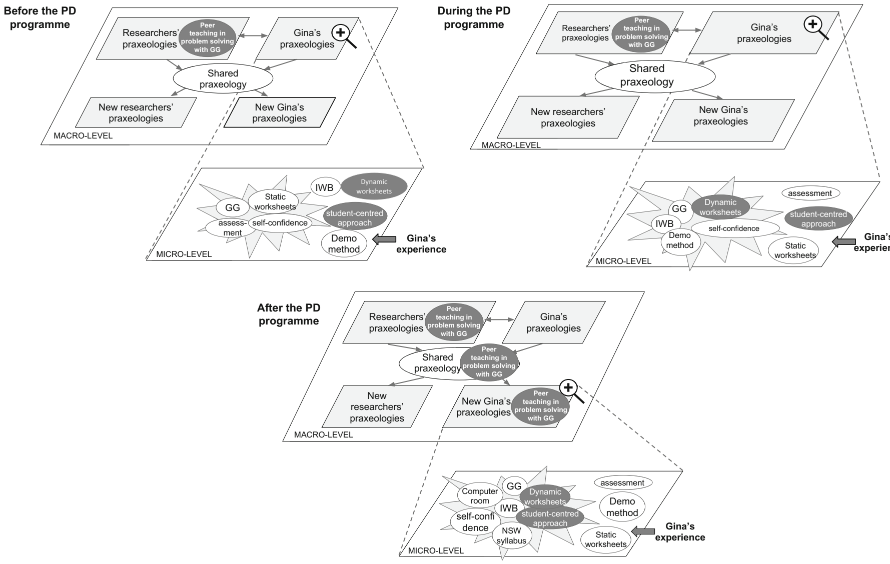

Fig. 7 The integration of peer teaching in problem solving with GeoGebra as a component in Gina's praxeologies before, during and after the professional development programme

> any instructions/questions to accompany the GeoGebra worksheets. The researcher visited my school and we discussed and developed questions that accompanied the GeoGebra worksheets. I will be using these GeoGebra worksheets and their corresponding questions every year when I teach the graphs of linear equations. I also used GeoGebra worksheets with Grade 10 when I taught them parabolas…From now on I will use GeoGebra worksheets very often with all the grades I teach, although in the past I did not even dare to think that I could use a software to teach students how to construct mathematical graphs.

First, we observe in Brett's excerpt that his students have difficulties in constructing or attaching any mathematical meanings to the target didactic situation, and students need more explicit guidance. Brett reflects on how to use the GeoGebra worksheets and he realises that a collective discussion among him, the head maths teacher and the researcher was required to design appropriate questions for students' use of the GeoGebra worksheets. This is an example of the first steps of the process of creating a new shared praxeology. The head maths teacher in turn helps to foster the collaboration of the teachers and researchers. In particular, he acts as an institutional agent that brings together the two communities of practice providing them with an opportunity to engage in creating questions for students' use of the GeoGebra worksheets. The interaction of this agent with the knowledge of the community of researchers at the micro level gives rise (at the macro level) to the design of the research questions that accompany the GeoGebra worksheets. These research questions are part of a new shared praxeology.

Second, Brett's statement that he will be using these GeoGebra worksheets and their corresponding questions every year when teaching students the graphs of linear equations shows that the GeoGebra worksheets and the basic features of slope and intercept are chosen by the teacher as teaching tools that will be integrated into his future teaching.

Third, as Brett articulates, he will incorporate GeoGebra worksheets in his teaching of other mathematical topics. This intention shows that the use of GeoGebra worksheets has been transformed from an external to an internal component of Brett's praxeologies. It will become an integral tool of his classroom teaching practices, impacting, as well, his view on mathematics. The interaction between the researcher and the teacher, facilitated by the head maths teacher who acts as an institutional agent, has led to the development of a shared praxeological component between this teacher and the researchers. At the same time, the use of the GeoGebra worksheets strengthens the use of the knowledge at play (dynamic interplay among parameters of a linear equation and the graphical representation of a straight line) relating to the use of GeoGebra by other members of his institution.

The case of Gina

At the beginning of the programme, Gina affirmed that she uses GeoGebra to create geometrical constructions or algebraic graphical representations that are included in the paper-worksheets she creates for her student assignments. It is important to point out that Gina's worksheets are not dynamic like the worksheets that Brett had prepared for his students, which allowed students to interact with the GeoGebra constructions. Gina's worksheets are static exercises in Word documents. She adds that she does not use GeoGebra on the IWB to demonstrate mathematical representations in her classroom because she is not confident with it.

At the micro-level, we can observe the interaction of the GeoGebra agent and the material agents which are static paper worksheets and student assignments.

During the last meeting of the GeoGebra professional development programme, Gina described how she uses GeoGebra now in her classroom:

> After attending the GeoGebra training I felt more confident so I introduced GeoGebra in the teaching of geometry. I initially demonstrated on the IWB some dynamic applications that allow us to view different alterations, for example … I showed to my students that when we have parallel lines intersected by another line, the opposite angles and vertical angles vary when we move the line that intersects the parallel lines…I discuss these applications and explore other dynamic constructions that I believe help with developing students' dynamic imagery skills.

After Gina had attended the GeoGebra professional development programme, she was able to use GeoGebra in her teaching of geometry. At the macro level, we observe that a different technique emerges when she used GeoGebra for demonstration in her praxeologies, from the interaction of the material and technological agents (worksheets, assignments) at the micro level. Moreover, Gina started using dynamic worksheets with her students as teaching and learning materials that interacted with technological agents like the IWB and GeoGebra. The aforementioned agents also interacted with the motivational agents (self-confidence).

Gina states:

> My students asked me if there was any chance for them to try these applications during mathematical lessons. I arranged to take my students in the computer room, where I instructed them to use GeoGebra and they worked in groups to solve geometrical problems on the computer. They enjoyed working in groups (3 in each group) in front of a computer. At the end of the session we discussed all together and a representative of each group explained on IWB how they explored a mathematical problem.

Gina described how she used the computer room to organise group work for her students who engaged in the exploration of a mathematical problem. Her teaching methodology changed from teacher centred to student centred (group work) due to the interaction of methodological agents (group work), material and technological agents (dynamic worksheets) and motivational agents (students were motivated and enjoyed working in groups). Gina says:

> After my first experience of taking my students in the computer room and organising them to work in groups, I found the group work very beneficial for my students. The last weeks, I take all my classrooms in the computer room at least once per week where students solve problems and explore alternative solutions of the problems. When my students worked in the computer room, they worked in groups and peer teaching was taking place because one student instructed another student in the construction of the graphical representation or in mathematical content or in the use of GeoGebra.

Gina found group work and peer teaching beneficial for her students. At the micro level, we observe that a more student-centred approach as a methodological agent interacted with GeoGebra and the teaching and learning material agents (mathematical problems). Whereas at the macro level, the peer teaching in problem solving with GeoGebra emerges as a praxeological component from the interaction of the above agents. Gina also adds:

> I wish they have included activities with GeoGebra in the NSW curriculum materials so students could explore alterations of dynamic properties, and we had lessons plans about using GeoGebra to explore mathematical concepts or engage students in an inquiry learning experience during which students develop questions and investigate scenarios to find solutions of those questions.

Gina's articulations show that, at the micro level, the use of dynamic environments of GeoGebra (dynamic worksheets) that are technological agents interacted with the material agents (mathematical problems to explore, scenarios) and with the institutional agents (NSW syllabus materials). The interaction of those agents gives rise to what Gina calls "inquiry learning experiences" realising that the students were engaged in a new way into the exploration and construction of the mathematical knowledge.

From Gina's words, it seems that the integration of GeoGebra in mathematics teaching and learning—allowing students to use the software and engage with

GeoGebra constructions in order to explore mathematical problems—has been transformed from an external to an internal component of her praxeologies. She seems confident in changing her role slowly from the transmitter of knowledge to a kind of facilitator of the class and promoting peer teaching. From Gina's articulations, we can see that peer teaching in problem solving with GeoGebra emerged (at the macro level) from the interaction of different agents, as discussed above and became an internal component for both her teaching praxeologies and her students' learning. She also articulates that GeoGebra should officially become an integral element of the new Syllabus of the New South Wales Board of Studies.

Gina's use of GeoGebra's dynamic worksheets in her mathematics teaching has been transformed from an external to an internal component of her praxeologies and also led to her changing her role from a transmitter of knowledge to a facilitator of knowledge for her classrooms, promoting inquiry-based learning experiences for her students. We observe that peer teaching in problem solving with GeoGebra emerged (at the macro level) from the interaction of different agents (static worksheets, GeoGebra, assessment, dynamic worksheets, Interactive white board, student-centred approach, demonstration method, etc.) and became an internal component for both Gina's teaching praxeologies and her students' learning.

Discussion

The research interest in this professional development programme was to observe the integration of GeoGebra as a praxeological component that, at the beginning, was external for the teachers' community, and then became internal for the teachers' community (or at least for some of the individual teachers). To foster the transformation of this praxeological component from external to internal, the researchers implemented activities involving different kinds of agents: material (e.g. paper, pencil), technological (e.g. GeoGebra, IWB), methodological (e.g. mathematics laboratory), institutional (National curriculums, other teachers) and motivational (e.g. personal beliefs).

For the participating teachers, these agents can be already active or be activated by the professional development programme by interacting with other agents. This level of observing the interaction of agents corresponds to the micro level of analysis. This level is useful to understand the emergence of new phenomena at the macro level, which is the level of the praxeologies. In fact, when different independent agents were active in teachers' activities, the researchers observed some changes in teachers' praxeologies at the macro level. For that reason, we need both theoretical frameworks: the MDT model helps us to identify the praxeological component at stake, and the idea of emergence gives insight into how this component arises from the action/interaction of agents. What we observe in the emergence of a new phenomenon may be one component of a praxeology, or several components, or an entire praxeology. For example, teachers can introduce only the technique, or also its justification, or the justification for using a task that they already used. During the professional development, it is not assumed that an entire praxeology emerged in all the teachers involved. The new praxeologies can be either individual, or shared in a small group of teachers, or in the entire community of teachers with the researchers. The end result is the teachers' professional development that emerges from the evolution of praxeologies (Fig. 8).

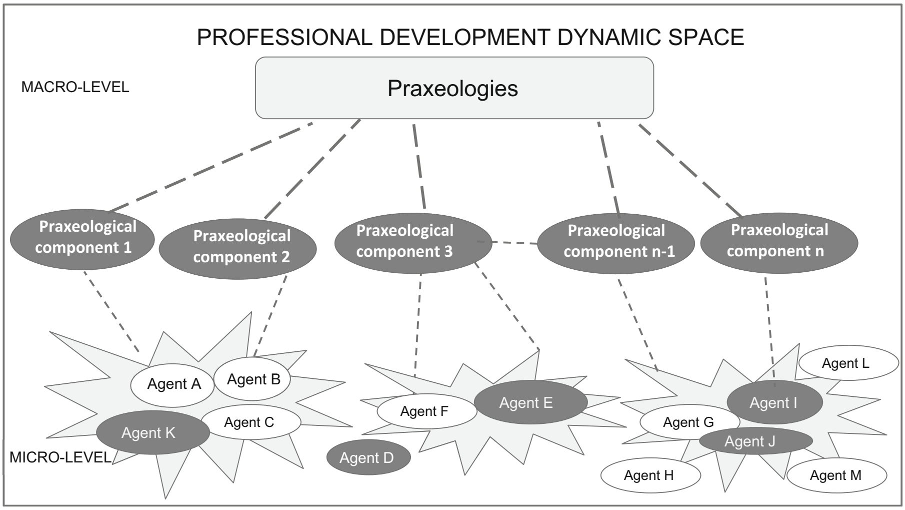

Fig. 8 Different levels of emergence from the micro-level (agents) to macro-level (professional development)

One example of a praxeological component is the integrated use of GeoGebra for mathematics laboratory activities, which was internal to the Italian researchers' community. The participating Italian teachers who are analysed in this paper were not new to the use of GeoGebra. It was already active as an agent in their teaching practices. However, they used it for demonstration, reflecting the interaction of GeoGebra agent with teacher-centred methodological agents. The aim of the researchers was to make the use of GeoGebra for mathematics laboratory activities become an internal praxeological component for each of the participating teachers (and so, hopefully, for the entire community of teachers in a shared praxeology). To do so, since GeoGebra was already active as a technological agent in teachers' praxeologies, the researchers triggered the mathematics laboratory methodology as a methodological agent, by giving a great importance to discussing the proposed activities with the teachers. The researchers, guiding the teachers in doing activities as if they were students, let them experience by themselves the laboratory methodology as a means to approach problem-solving activities. Indications about how to use GeoGebra were given when required during the teachers' workshops. Meetings and debates among researchers and teachers provided the opportunity for discussing the contribution of such a methodological agent in teaching practices, and for gaining insights into the teachers' possible activation of it in their practices. After the implementation of the activities in the classrooms, for some teachers (as in Riccardo's case), the interaction of GeoGebra agents with the new explored laboratory methodology caused the emergence of the targeted praxeological component. For other teachers (as in Lara's case), this did not occur. We interpreted this difference as the intervention of motivational agents coming from the teacher's beliefs and personal experience as well as of institutional agents (e.g. time, school context and curriculum constraints). They interact with the other agents, sometimes causing the emergence of the use of GeoGebra in a laboratory way as an internal praxeological component for teachers.

Another praxeological component particularly developed in the Australian context was the use of dynamic GeoGebra worksheets, which were internal for the community of the researchers. The design and the use of the two dynamic GeoGebra worksheets constituted external components in the praxeologies of the teachers' community when initially introduced by researchers to support teaching practices for graphing a straight line, and exploring its basic features of slope and intercept. For example, Brett selected dynamic GeoGebra worksheets as fruitful tools for teaching mathematics and he integrated them in his teaching practices for that specific mathematical topic (straight line graphing, slope and intercept). Other dynamic GeoGebra worksheets to teach other mathematical topics were also integrated in his teaching practices. Hence, the use of those dynamic GeoGebra worksheets by the participating teacher became internal praxeological components that emerged from the interaction of methodological agents, the related teaching and learning materials and the knowledge used by the two different communities of practice (teachers and researchers). The interaction of agents in the two communities working together supported the emergence of a shared praxeology (between Brett, who was the participating teacher, and the researcher), as a dialectical process where the teachers' and researchers' professional competences could meaningfully evolve. These dynamic GeoGebra worksheets have a specific meaning for the community of the researchers, resulting from researchers' praxeologies that in this example incorporate mathematical knowledge, expertise in designing GeoGebra worksheets, etc. There is an interesting dialectic between:

– the interpretations that the teacher gave to his students' understanding regarding the use of the GeoGebra worksheets (at a didactical level) to link the actions while manipulating the dynamic features of the worksheets to construct mathematical meanings
– and the interpretation attributed by the community of researchers (at a meta-didactical level) to this functionality of the GeoGebra worksheets.

At a meta-didactical level after the collaboration of the teachers and researchers, a shared meaning was attributed to GeoGebra worksheets. For that reason, we can say that the shared praxeology emerged during the professional development process after meta-reflections on teacher's praxeologies.

In the case of Gina, we observed that, although she did not exhibit the hoped-for internalisation of mathematics laboratory use of GeoGebra, she did internalise some components (more student-centred learning), and this may be true in many cases: teachers adopt certain praxeologies and not others. Such unexpected outcomes offer an opportunity for researchers to better understand their own praxeologies, possibly influencing future praxeologies in the researchers' community.

Analysing the process of professional development through the lenses of the MDT and emergence, we observe that, in order to achieve the objective of a professional development programme, researcher-educators need to identify agents to be activated in the practices of the participating teachers, reflect carefully on their possible interactions and propose activities that may cause the interactions of such agents. In doing so, researchers might succeed in causing the desired transformation of the new (external) component becoming internal in teachers' praxeologies, not only in the techniques used but also in the justifications and meta-reflections about those techniques. Moreover, we highlight the role of motivational agents that influence whether or not the targeted praxeological component will transform from external to internal for specific teachers.

Conclusions

To study the processes of teachers' professional development, we used the meta-didactical transposition model to analyse the evolution of teachers' praxeologies. To investigate which agents intervene in this evolution, we integrated the MDT model with the idea of emergence. We highlight the strength of this analytical instrument, which provides a theoretical description of the link between the interaction of agents at the micro level and the transformation of the praxeological components at the macro level. In fact, if the meta-didactical transposition is useful in describing the evolution of praxeologies—for example in the transformation of a praxeological component from external to internal for a teacher or for a community of teachers—nonetheless it is not sufficient for explaining which elements may cause this evolution or lack of evolution. The concept of emergence clarifies this evolution as the interaction of various elements—the agents of various types. The researchers, observing teachers in their professional development, can analyse the macro level of the praxeologies in light of what happens at the micro level. At the micro level, the researchers observe a variety of agents interacting, giving rise to different praxeological components, which in turn give rise to the praxeologies at the macro level (Fig. 8). The researchers could carry out the aim of supporting the transformation of praxeological components from external to internal at various levels of scale from the individual classroom to a national level (Schoenfeld 2015).

The awareness of how the macro and the micro levels of professional development relate supports researchers in designing teachers' professional development programmes and also in training teachers. The analytical instrument of the MDT model combined with the notion of emergence enables researcher-educators to go deeper into the praxeologies. More specifically, they can potentially activate those agents that can fruitfully interact with teachers' agents for supporting the evolution of praxeologies. For example, educators can claim for a weak methodological agent (e.g. mathematics laboratory methodology) or for a not yet integrated technological agent (e.g. GeoGebra).

Because the outcomes of interactions among agents are unpredictable, and the agents affecting each teacher differ (especially motivational agents), it is not to be expected that all the teachers involved in the educational programme evolve in the same way with the same transformation of components. Different teachers may evolve in different ways in terms of changing certain praxeologies with respect to their expertise, their experience, their background, their motivation and other factors. Therefore, further research is necessary to investigate the factors that influence these different trajectories in the praxeologies. Additionally, further research could explore the researchers' community and their praxeologies according to their institutional context in which they design the educational programme. In Italy and Australia, for example, the impact of this research is already evident in researchers' design of new educational programmes in recent years.

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