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ORIGINAL ARTICLE

Shifting more than the goal posts: developing classroom norms of inquiry-based learning in mathematics

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Abstract The 3-year study described in this paper aims to create new knowledge about inquiry norms in primary mathematics classrooms. Mathematical inquiry addresses complex problems that contain ambiguities, yet classroom environments often do not adopt norms that promote curiosity, risk-taking and negotiation needed to productively engage with complex problems. Little is known about how teachers and students initiate, develop and maintain norms of mathematical inquiry in primary classrooms. The research question guiding this study is, "How do classroom norms develop that facilitate student learning in primary classrooms which practice mathematical inquiry?" The project will (1) analyse a video archive of inquiry lessons to identify signature practices that enhance productive classroom norms of mathematical inquiry and facilitate learning, (2) engage expert inquiry teachers to collaborate to identify and design strategies for assisting teachers to develop and sustain norms over time that are conducive to mathematical inquiry and (3) support and study teachers new to mathematical inquiry adopting these practices in their classrooms. Anticipated outcomes include identification and illustration of classroom norms of mathematical inquiry, signature practices linked to these norms and case studies of primary teachers' progressive development of classroom norms of mathematical inquiry and how they facilitate learning.

Keywords Inquiry-based learning · Mathematical norms · Sociomathematical norms

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Introduction and rationale

It is to the nation's economic and social benefit to increase the mathematical capacity and disposition of the nation's workforce. Unfortunately, students are losing interest in mathematics largely due to a prevalence of tasks, pedagogies and learning environments that disengage students (Mills and Goos 2011). These practices minimise students' experiences with the relevance of mathematics, limit critical and creative thinking, inhibit discussions that deepen mathematical understanding and suppress intellectual risks for fear of getting an answer wrong—all issues that are addressed within inquiry (Barron and Darling-Hammond 2010; McPhan et al. 2008).

Inquiry-based learning has been shown to reverse the problems of disengagement and rebuild students' desire and capacity to learn challenging mathematics (ACOLA 2013; Bruder and Prescott 2013; Fielding-Wells and Makar 2008). Inquiry practices have been documented to promote engagement, resilience, flexible thinking, transfer of learning and capacity for addressing complex problems. Barron and Darling-Hammond (2010) in a review of inquiry-based learning reported that engaging in solving these complex, authentic problems predicted student performance better than any other factor, including students' family background and prior achievement. Despite evidence for the benefits of inquiry, teachers are not yet confident, or able, to invest the time needed to make lasting shifts in practice. These shifts in practice are complex; they involve changes in task design, changes in pedagogies and changes in classroom environments to reap the benefits of inquiry. In inquiry classrooms, norms—which dictate the learning environment in school mathematics—are particularly critical to productive class discussion and student engagement (Barron and Darling-Hammond 2010; Franke et al. 2015; Kazemi and Stipek 2001).

Declines in STEM education and competitiveness are considered urgent internationally; however, Australia has not reacted with the exigency of the USA, Europe or Asia (ACOLA 2013, p. 12). In 2015, Australia made a strong national commitment to improve the "M" in STEM education with its new *Mathematics by Inquiry* initiative (Australian Government 2015; Birmingham 2015). This Commonwealth project, implemented by the Australian Academy of Science (AAS) and the Australian Association of Mathematics Teachers (AAMT), aims to "promote innovative approaches to mathematics teaching in schools" by providing teachers with curriculum-based classroom materials and professional resources for teaching mathematics by inquiry (AAS 2016). Shifting to an inquiry focus would revolutionise the way mathematics is taught and require significant changes to classroom social and sociomathematical norms (Yackel and Cobb 1996). Providing teachers with resources is extremely important (AAS 2015), but shifts in classroom norms are equally important to bringing about the intended change needed to reap the benefits of an inquiry-based approach. Figure 1 provides a brief (hypothetical) illustration of how subtle shifts in teachers' questioning, for example, would change the norms of the classroom.

Norms of mathematical inquiry engage students in productive social interactions and improve their mathematical knowledge, as well as their interest, valuing and capacity to solve complex problems. The primary focus of the study is to characterise productive classroom norms of inquiry-based learning in mathematics and to develop guidelines for facilitating the establishment of these norms for teachers new to mathematical inquiry. This project therefore aims to create new knowledge about mathematical, Importance of Norms:

Illustration of a more traditional classroom:
Teacher: How would you solve this word problem?
Student: I would add the two numbers together.
Teacher: Good, adding the numbers would give you the correct answer.

Illustration of a more traditional classroom on the introduction of Inquiry based on a semi-structured question:
Teacher: Can anyone see how we could approach this problem?
Student: It seems that we would need to gather data, perhaps through surveying.
Teacher: No, that won't work because we don't have access to the population we would need to survey. Anyone else?

The latter may involve a richer question but the teacher is maintaining the existing culture by curtailing the student's risk-taking through the use of an evaluative response. Contrast this with a response such as:
Teacher: Great, we have a possible approach. Let's note that down and then see if we can get a few more approaches before we break into groups to consider the advantages and disadvantages of each.

This response validates the student for attempting, makes no evaluative judgement and makes it clear that more responses are desirable and that the students will ultimately be evaluating these suggestions (NRC, 2002).

Fig. 1 Contrasting norms in three hypothetical classrooms

social and sociomathematical inquiry norms in primary mathematics classrooms, including expert teachers' signature practices, the development of norms and supports needed to aid teachers new to inquiry to adopt these practices. In addressing these aims, the project will theorise the role of inquiry norms in deepening students' mathematical learning.

Literature

Classroom norms

In any classroom, there exists a set of *norms*: explicit and implicit understandings that govern the behaviour of both students and teachers according to their knowledge of what others do and value. Norms develop through emergence and ongoing renegotiation by the classroom participants (Franke et al. 2007; Goos 2004). Cobb and his colleagues (Cobb et al. 1989; Yackel and Cobb 1996) have been dominant in the research area of classroom norms for student-centred mathematics, or inquiry as they often term it. They distinguish between *social* norms—those norms that exist across the classroom for all discipline areas—and *sociomathematical* norms, being those specific to mathematical practices. To illustrate, a classroom social norm may be to expect students to support a claim: in English, persuasive writing may involve the use of emotive devices to achieve this while in mathematics, evidence and reasoning are acceptable and the emotive device is not. Thus, in this example, the sociomathematical norms guide what is acceptable to offer in support of an argument *in mathematics*. Through such norms, children develop a view about the discipline of mathematics and what is valued and important in that discipline.

Once children have been in school for a few years, they tend to have an understanding of what it means to *do* or be *good* at mathematics: to answer questions quickly and correctly using the teacher's method (Stephan 2015). Often, students learn that the teacher will only question an incorrect answer and if no one answers, the teacher will provide the answer. Norms such as these have been found to be fairly consistent and stable at even year 1 (Franke and Carey 1996). As children progress through multiple years at school, such norms become ingrained. To introduce an approach like mathematical inquiry with significantly different goals and philosophies is unlikely to succeed without changes to classroom social and sociomathematical norms also. Kazemi and Stipek (2001) identified four sociomathematical norms which distinguish student-centred classrooms: (a) explanations are supported by mathematical reasons, (b) mistakes are seen as opportunities to engage further with mathematical ideas, (c) students draw mathematical connections between strategies and (d) consensus is reached through mathematical argumentation—that is, the recognition of authority for whether something is correct and sensible lies in the logic and structure of the subject, with resolution of disagreements being through mathematical argumentation (National Research Council 2002, pp. 344–345).

Research conducted on classroom norms of mathematics has largely centred on identifying norms of mathematics classrooms that are engaged in "instruction following an inquiry tradition" (Yackel 1995, p. 131). The illustrative excerpts used by Yackel and others typically rely on problems with relatively simple solutions with the discourse being about the choice of method. However, this research does not address students working with more complex problems that have neither a single correct answer nor a predetermined method, or those that require sustained collaboration. In the case of these complex problems, argument is truly the means of determining a justifiable solution based on the information known at a point in time (Fielding-Wells 2015; Makar et al. 2015).

There is a developing body of research in primary classrooms on the practice of inquiry-based pedagogies in mathematics. For example, Makar provides illustrated exemplars of teachers implementing mathematical inquiry as a pedagogy (e.g. Makar 2012, 2016; Makar et al. 2015) while Fielding-Wells has demonstrated the capability of young students of engaging with mathematical inquiry and of providing evidenced-based arguments across multiple contexts and strands of mathematics (e.g. Fielding-Wells 2015). However, the success of classrooms adopting inquiry practices depends heavily on their ability to create conducive classroom norms, and less is known about how teachers and students initiate, develop and maintain norms of mathematical inquiry in primary classrooms.

Mathematical inquiry

A range of reform practices including inquiry-based learning, project-based or problem-based learning, design-based approaches and ambitious pedagogies have a similar aim of engaging students in actively constructing knowledge within a community of learners (Barron and Darling-Hammond 2010; Franke et al. 2007; Goos 2004; Hunter 2012). Most critiques of inquiry describe overly structured or open inquiry ("discovery learning"), which was long known to be ineffective (e.g. Confrey 1991; Hattie 2008). In this project, we use the term "mathematical inquiry" as a process that uses mathematical evidence to address complex problems that contain ambiguities (Makar 2012). For example, in *What is the best map?* (Fry 2013), a year-3 class used maps from zoos, shopping malls, parks and their local area to collaboratively generate a list of useful qualities of a map, then created their own map, justifying to peers why it fit the valued criteria of a *best* map. A year-6 class conducted an inquiry to collaboratively investigate the best brand of bubble gum by designing measures of elements they valued such as desirable taste, longevity of flavour and size of *bubble* (Makar 2012). The ambiguity of the word "best" in these mathematical inquiries created a need for mathematical discussion, connecting meaning to everyday understanding and designing an approach to respond to the question. In mathematical inquiry, students wrestle with and rework ideas alongside peers and with teacher support; they must cope with uncertainties, take intellectual risks and use mathematical evidence to persuade others of the quality of their solution. Lessons may not go to plan and students often spontaneously raise questions or ideas that teachers are not expecting or confident to respond to. Discussion and collaboration are central in inquiry, and teachers need to be confident in guiding student discussion and collaboration that maintains high intellectual focus.

Mathematical inquiry is associated with positive learning benefits in mathematical content and dispositions (de Corte et al. 2008). The approach is linked to improved communication, collaboration, creativity, sense-making, intellectual risk-taking and deep thinking in mathematics (e.g. Barron and Darling-Hammond 2010; Bruder and Prescott 2013). These qualities are highly valued in mathematics, but cannot be achieved using only direct instruction where teachers *tell* students what and how to think. Using mathematically rich problems in and of themselves is not sufficient to develop quality learning (Franke et al. 2007). Likewise, listening to students and encouraging them to describe their thinking and justify their responses can improve their mathematical understandings and reliance on mathematically based evidence and reasoning (Wells 2014), yet it alone is not sufficient to ensure student involvement and participation in mathematics (Franke et al. 2007). Franke et al. (2007) argued that there is far more complexity involved in understanding mathematical classroom practice such as *how* students engage in these practices. This is important because students' classroom interactions "impact not only their mathematical thinking but also their own sense of their ability to do and persist with mathematics, the way they are viewed as competent in mathematics, and their ability to perform successfully in school" (p. 226).

Teachers need time and support to adopt mathematical inquiry confidently and in some instances, even with support, this may not occur (Makar 2011; Makar and Fielding-Wells 2011). This is unsurprising, as research has suggested that changing curricula is not sufficient to change classroom practice. What teachers do in the classroom *and* how this is experienced by students must be studied together to develop meaningful insight.

Methodology

This design-based study has been designed to draw on resources developed over the past decade to fuse existing bodies of research on inquiry-based learning, including teachers' evolving pedagogies and experiences in adopting mathematical inquiry practices, students' affective and cognitive engagement with inquiry and students' facility with inquiry-based argumentation in mathematics. The project seeks to develop new knowledge about the development of classroom norms of mathematical inquiry and does so by drawing on previous research, including the analysis of archival data. In the first instance, selected videos from a video archive of inquiry lessons (~1000) will be drawn upon to generate potential classroom norms productive for developing mathematical inquiry, including interviews with the teachers about these lessons. Second, expert inquiry teachers will be engaged to collaboratively design teacher professional development that promotes classroom norms of inquiry. Third, new mathematical inquiry teachers will be supported to develop norms of mathematical inquiry in their classrooms. The detail of this process is described below.

The overarching research question for this project is, "How do classroom norms develop that facilitate student learning in primary classrooms which practice mathematical inquiry?" In particular, this would be addressed through the following sub-questions:

1. What are key norms of mathematical inquiry across diverse primary classrooms?
2. What signature pedagogical practices develop and enhance an establishment of classroom norms of mathematical inquiry?
3. How do teachers new to mathematical inquiry initiate, develop and maintain norms of mathematical inquiry?

Design

Complex classroom-based research challenges most research designs because they neglect the contingencies of classrooms during data collection and the outcomes typically do not adapt easily to messy classroom contexts. Design Research was purpose-built as a methodology to anticipate and embrace the unpredictability of classroom dynamics. "In contrast to most research methodologies, the theoretical products of design experiments have the potential for rapid pay-off because they are filtered in advance for their instrumental effect" (Cobb et al. 2003, p. 11). Although it shares its cyclic nature with action research, design research is both more structured and more intensive. Design research maintains rigour while allowing researchers to adapt to and engineer the classroom conditions by adhering to five core principles:

1. A primary focus on *developing theories* to support learning. The project will develop robust theory characterising development of mathematical inquiry norms with teachers experienced with and those new to mathematical inquiry by iteratively studying teachers' inquiry practices.
2. Innovative *intervention* for educational improvement. The research aims to simultaneously improve and study classroom norms of mathematical inquiry. The project will partner with teachers through professional development, classroom observations, collaborative planning and reflection to co-construct norms of mathematical inquiry.
3. Concurrent *prospective and reflective processes* with postulated theories continuously scrutinised, capitalising on unanticipated opportunities. Data collection and

analyses are continuous and ongoing, so reflection and feedback from data, teachers and professional colleagues are critical to feed into subsequent planning.
4. Reliance on *iteration* (feedback cycles) to refine conjectures and learning environment. Annual cycles are informed by multiple smaller ongoing cycles within classrooms. Data collection and analysis occur all through the year, so conjectures and methods of evidence can be refined and adapted.
5. Acknowledgement that theories developed are humble and intermediate but *put the theory to work* (Cobb et al. 2003). As iterations build, theories are tested in subsequent stages. Intensive data analysis carried out early in the research creates foundations for later interventions. This ensures that theoretical principles at all three stages authentically address issues of classroom practice.

In this project, these principles of design research are evident in all aspects of the study including planning, professional development, data collection and analysis. These aspects are constantly revised and improved through iterative and ongoing reflection. Design research therefore acts as an overarching methodology to theorise how mathematical inquiry norms facilitate student learning.

Participants and professional development

The participants are drawn from several Australian primary schools that vary in terms of location (rural/urban) and school advantage (low to moderately high ICSEA values). These participants consist of three groups: teacher-researchers (n = 6) who have previously developed expertise in mathematical inquiry and students' argumentation practices, (2) new teachers (n = 6) who are yet to develop inquiry as a teaching pedagogy and who will work closely with the teacher-researchers to develop an inquiry culture in their classrooms and (3) students (~600 over 3 years) of the 12 primary teachers. Data are collected annually rather than longitudinally, making attrition unproblematic methodologically.

Professional development and working meetings involving teacher-researchers and new teachers are designed to (1) advance teachers' understanding of the many ways in which norms of inquiry can develop, (2) collaborate with teachers to design resources specifically to initiate, develop and maintain classroom inquiry norms and (3) draw on teachers' experiences to contribute to, elaborate and challenge emerging outcomes of the research. In line with our previous research, professional development typically includes time for teachers to share their classroom experiences with peers, engage in activities that promote a deeper understanding of mathematical inquiry (such as working on an inquiry problem as learners or designing classroom assessment or resources) and collaboratively planning future inquiry lessons. The professional development and data collection also ensure that they align with school initiatives so that the project engages with social and organisational contexts of schools such as teachers' classrooms and support of school administration critical for productive pedagogical innovation (Resnick et al. 2010).

Data collection and analysis

Data will focus on three key sources: an existing archive of video data from previous research by the authors, videos of classrooms during the project of both expert inquiry teachers and those new to inquiry and interviews with teachers (both those in the archive of classroom videos and teachers actively teaching in the project).

Video database This project initially draws on a database of over 1000 videotaped lessons from 54 teachers. Previous research has reported on selected lessons in this database which illustrate rich examples of mathematical inquiry (e.g. Fielding-Wells 2015; Fielding-Wells and Makar 2012; Makar 2012; Makar et al. 2015). To date, 400 of these videos have been coded using the Productive Pedagogies framework as a guide to identifying lesson quality (QSRLS 2001). Measures most sensitive to inquiry norms are intellectual quality (high cognitive engagement), connectedness (emphasising relationships, applications and utility of mathematics) and social support (positive learning environment). Three groups of lessons are to be identified in the database: (1) high-scoring lessons, (2) low-scoring lessons and (3) lessons from teachers who improved significantly to understand aspects that changed as they improved. Lessons will be randomly selected from group 1 to be transcribed (~5 initially) and undergo open coding, focusing on observable norms and actions used by the teacher to develop norms. Videos of these lessons will be re-watched to add additional codes from visual clues in the videos. These codes will be discussed within the research team and organised into categories. This process will be repeated until categories are saturated. The outcome of coding will be draft sets of desired inquiry norms and links to *signature practices* from which to build and elaborate. This process will be repeated using lessons in groups 2 and 3 ("low-scoring" and "improvement") in order to deepen insight into contrasts between signature practices and actions which limit desired inquiry norms to develop.

Interviews Interviews of teachers from the database will be undertaken regarding the development of classroom norms. In year 1, teachers will be interviewed from groups 1 to 3 to discuss their strategies in initiating, developing and maintaining classroom inquiry norms. If agreed by the teacher, she/he will observe a lesson with the researchers to explain their strategies for creating classroom inquiry norms, with evidence from the videos. In years 2 and 3, interviews at three intervals during the school year—beginning, middle and end—will ask the teachers how they initiate, develop or maintain inquiry norms in their class (and examples). These interviews will be transcribed and linked to lessons in the database.

Classroom videos Mathematical inquiry lessons (~15 lessons/year) will be videotaped from each teacher in years 2–3. These videos will undergo a process of analysis adapted from Powell et al. (2003): careful observation, creating video logs and identifying rich segments, annotating actions to identify norms, re-watching videos in reverse order to trace threads of how practices emerge, selecting excerpts to illustrate these threads and writing narrative.

Anticipated outcomes and contributions

The anticipated outcomes of this project aim to raise capacity for frontier STEM momentum critical to the future of the economy. In particular, the project anticipates three key outcomes and their related contributions to the field.

Identification and illustration of classroom norms of mathematical inquiry

Nearly all research on mathematical norms focuses on closed questions. The open-endedness of inquiry requires additional or different norms in which students see grappling with challenging ideas as a normal part of learning, where obstacles encountered are proactively used to deepen students' understanding and in which peer collaboration is used to share and strengthen students' own ideas and those of other students. This project will create new knowledge of classroom norms of mathematical inquiry, which substantially extend beyond strategies reported in literature. The contribution of this outcome to research includes identification, classroom illustrations and evidence documenting norms found in inquiry-based classrooms.

Pedagogies linked to identified norms

Educational research rarely involves immersion in classrooms. Research in practice-intensive contexts over time requires new ways of thinking about paradigms of research focused on students within classrooms, challenges what counts as *change* and questions pure approaches which neglect complexities of working with schools (e.g. data collection respecting unpredictability of classrooms). Signature practices articulate productive strategies for teachers and teacher educators to improve pedagogies for developing classroom norms of inquiry. Because developing classroom culture is one of the most challenging aspects of teaching mathematics through inquiry, these signature practices can greatly facilitate its uptake. Key norms of classrooms which practise inquiry-based learning, detailed and illustrated through a diversity of practices, will contribute a deeper understanding in the field as to how pedagogies can promote or limit classroom norms of inquiry. Signature practices linked to norms further acknowledge teachers' diverse but high quality pedagogies for developing norms of mathematical inquiry. Access to an already existing video database of inquiry lessons of this size is unattainable in other research projects. This database allows the project to respect the diversity of teachers' practices in its search for understanding inquiry norms. Being able to interview most teachers in the database adds enormous strength of evidence to the outcomes by drawing on the teachers' thinking about their classroom norms rather than relying only on researchers' inferences.

Case studies of teachers' progressive development of classroom norms of mathematical inquiry

Teachers new to mathematical inquiry need guidance to get beyond ideals of inquiry to change their practice. The complexities surrounding *how* teachers develop classroom norms are often neglected. Teachers and policymakers may expect new practices to be quickly taken up by students and are disappointed if they do not appear as expected. Milestones and case studies document inquiry norms expected initially, mid-year and by the end of the year (e.g. Makar et al. 2015). Case studies with these milestones will boost teacher efficacy in facilitating norms of inquiry by identifying reasonable targets across diverse contexts. Partnerships with both experienced teacher-researchers and those new to inquiry will contribute fresh practice-oriented insight into classroom norms and related pedagogies. Illustrations of practice will also contribute evidence-based examples for researchers and teachers to identify practices that align with diverse teaching styles and localised contexts. This innovation bridges a theory-practice gap, with credible classroom-relevant findings.

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