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ORIGINAL ARTICLE

The affordances of using a flipped classroom approach in the teaching of mathematics: a case study of a grade 10 mathematics class

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Abstract Teaching secondary mathematics has a number of challenges, including the expectations that teachers cover the prescribed curriculum, help students learn difficult concepts, prepare students for future studies, and, increasingly, that they do so incorporating digital technologies. This study investigates a teacher's, and his students', perceptions of the benefits or otherwise of a flipped classroom approach in meeting these challenges, within a prescribed curriculum context. Data collection instruments included a survey designed to investigate the nature of students' engagement with the flipped approach and semi-structured student and teacher interviews. Analysis of these data indicated that the teacher and students were positive about their experiences with a flipped classroom approach and that students were motivated to engage with the teacher-created online mathematics resources. The study adds to the limited research literature related to student and teacher perceptions of the affordances of the flipped classroom approach and has implications for secondary mathematics teachers who face the challenge of the twin demands of covering the prescribed curriculum and catering for a range of students' learning needs.

Keywords Digital technologies · Flipped classroom · Technology · Adolescents · Engagement · Mathematics

Introduction

The flipped classroom provides an alternative to traditional classroom instruction where typically, in the former, students prepare for class by engaging with resources that have

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been pre-prepared by their teachers, and class time is used to do more targeted and individual instruction. The accessibility and affordances of digital technologies makes this possible by, for example, allowing teachers to record screenshots of worked mathematical problems from their computer screens and to overlay a narrative, create videos of themselves teaching and/or curate video lessons from internet sites such as TED-Ed and Khan Academy (Hamdan, McKnight, McKnight, & Arfstrom 2013). The approach is gaining popularity, particularly in mathematics classrooms, where it has been reported that it provides for greater student motivation and interest, as well as increased student-teacher interaction (Bergman & Sams 2012). It is also claimed that this approach capitalises on students' propensity to be online (Muir & Chick 2014) and reduces the time spent in class on teacher presentation and explanation (Hamdan et al. 2013). Other purported benefits include providing a medium that allows for differentiated teaching for a range of students' abilities (Herreid and Schiller 2013), allowing for the review of ideas and concepts, the option of "pausing and rewinding" of recorded presentations, greater transparency for students in relation to learning intent, and greater opportunity for teachers to be aware of students' progress (Bergman & Sams 2012).

The success or otherwise of the flipped classroom approaches to teaching and learning, however, has been the subject of limited empirical research and consequently remains under-theorised (Abeyseker & Dawson 2015). Further, previous research has typically been conducted in higher education settings (e.g., Abeyseker & Dawson 2015) and often reported in the form of reviews (e.g., Yarbro, Arfstrom, McKnight, & McKnight 2014). Findings from completed studies, however, show promise and indicate that flipped classroom approaches promote achievement, engagement (e.g., Hamdan et al. 2013), and positive affective outcomes (Muir & Chick 2014) for students.

The ways in which teachers integrate new digital technologies into mathematics teaching vary, and is dependent on a range of factors including teachers' beliefs about teaching and learning, their personal level of competence with technology use, and local influences such as school culture and the availability of resources (e.g. Geiger, Forgasz, Calder, Tan, & Hill 2012). Some teachers incorporate new technologies in ways that lead to innovative pedagogies, while others make use of cutting-edge digital tools within established approaches to teaching and learning in which they feel confident (Geiger 2011; Goos, Galbraith, Renshaw, & Geiger 2000). This study investigated the case of a single teacher's attempts to incorporate aspect of a flipped classroom approach into his teaching practice within the context of a grade 10 mathematics class. As part of this investigation, we explore the perspectives of the teacher and his students on the benefits, or otherwise, of a flipped classroom approach. In conducting this research, we were guided by the following research questions:

- What is the nature of a teacher's attempt to incorporate a flipped classroom approach into a year 10 classroom, working within a highly structured mathematics curriculum/subject?
- What are the teacher's and students' perspectives on the benefits or otherwise of the facilities offered by a flipped classroom approach within the context of a grade 10 mathematics class?

We will address these questions by first providing a brief overview of the relevant literature, including the concept of the flipped classroom, and the use of digital technologies in mathematics. Second, two theoretical frameworks are described, one used for analysing the teacher's role in facilitating mathematics instruction within the context of a flipped classroom (Flipped Learning Network (FLN) 2014) and the other for interpreting students' motivation to engage with the flipped classroom approach (Abeyseker & Dawson 2015). Third, a description of the methodology employed in the study is presented. Fourth, we provide an analysis of the teacher's and students' reported experiences of the flipped classroom. Finally, we draw conclusions and discuss implications for classroom practice and recommendations for further study.

Review of literature

In this section, we examine research literature related to digital technologies, flipped classrooms and flipped learning, and connections between the flipped classroom and student engagement and motivation. Additionally, we attend to the dynamics of a flipped classroom and what is entailed in flipped learning. Finally, the link between student engagement and motivation and the flipped classroom is discussed leading to the theoretical framework that guides this study.

The use of digital technologies in mathematics

The use of digital technologies is becoming increasingly common in today's mathematics classrooms, with some schools using them to replace or supplement teaching resources (e.g., Geiger, Goos, & Dole 2015). Digital technologies are an important aspect of the flipped classroom as they allow the teacher to produce and present learning materials in a variety of formats that can suit different learning styles and allow for access when and where it is convenient for students.

How to best take advantage of the affordance of digital technologies is a challenge that has attracted the attention of researchers for over three decades (Hoyles & Noss 2003). Previous research in this area has focused on the potential of digital technologies to enhance teaching and learning within specific content domains such as number (e.g., Kieran & Guzmán 2005), algebra and calculus (e.g., Ferrara, Pratt, & Robutta 2006), and geometry (e.g., Laborde, Kynigos, Hollebrands, & Straesser 2006), or more general mathematical activity such as problem solving (e.g., Lesh & English 2005) and mathematical modelling (e.g., Geiger, Faragher, & Goos 2010). Work related to the affordances and constraints of digital technologies for instruction and learning continues to proliferate and is expanding beyond the above-mentioned domains for a range of reasons, one of which is the diversification of the types and forms of digital technologies (Hoyles & Lagrange 2010), for example, handheld technologies (e.g., Drijvers & Weigand 2010). More recently, there has been burgeoning research interest in the potential of more broadly conceived technology-rich environments such as those associated with Web 2.0 technologies (e.g., Gadanidis & Geiger 2010; Goos & Geiger 2012) and the use of mobile technologies, such as iPads (e.g. Attard & Curry 2012; Larkin & Jorgensen 2015), into technology-active classrooms. Flipped classrooms leverage these, and other types of digital technologies, in order to implement the restructuring and re-organisation of learning activities within and away from the classroom in both synchronous and asynchronous modes. Because the facilities of a flipped classroom approach can be accessed from anywhere, an appropriate digital tool is available (both static and mobile), in both real time and otherwise, the flipped classroom approach can also be considered a form of mobile technology in itself.

The flipped classroom

The Flipped Learning Network (FLN) was established by Bergmann and Sams in 2012 Bergmann and Sams differentiate between the terms 'flipped classroom' and 'flipped learning', stating that they are not synonymous and that flipping the classroom does not necessarily lead to flipped learning (FLN 2014). We explore the difference in these two terms below. According to Bergman, Overmyer, and Wilie (2013), initial versions of flipped classroom were learning environments where videos took the place of direct instruction, and teachers used class time to work with students on key activities. This pedagogical shift is enabled by digital technologies (Hamdan et al. 2013), providing students with access to preparatory resources outside of school hours, so that they can attend class with a basic understanding of concepts that are then explored in greater depth in timetabled contact time with their teacher. [Bergmann et al. (2013), para. 5] characterise flipped learning as a:

- Means to increase interaction and personalized contact time between students and teachers
- Space where students take responsibility for their own learning
- Classroom where the teacher is not the 'sage on the stage', but the 'guide on the side'
- Blending of direct instruction with constructivist learning
- Classroom where students who are absent due to illness or extra-curricular activities such as athletics or fieldtrips, don't get left behind
- Class where content is permanently archived for review or remediation
- Class where all students are engaged in their learning
- Place where all students can receive a personalized education

A definition of the flipped classroom, in contrast, is provided by Abeyseker and Dawson (2015, p. 3) who identify the following typical characteristics:

- A change in use of classroom time
- A change in use of out-of-class time
- Doing activities traditionally considered 'homework' in class
- Doing activities traditionally considered as in-class work out of class
- In-class activities that emphasise active learning, peer learning, and problem-solving
- Pre-class activities
- Post-class activities
- Use of technology, especially video

While Abeyseker and Dawson (2015) characteristics of a flipped classroom were identified from within higher-education settings, they are compatible with practices that occur in other teaching and learning settings such as our research context. Consistent with Bergmann et al's (2013) construct, the real potential of flipped learning lies in the provision for students to achieve mastery of topics as they are more able to self-pace their learning. In their vision of flipped classroom learning, students typically access video resources when ready, work through resources at their own pace, and demonstrate mastery through the completion of assessment tasks. This requires a major reconceptualisation practice for many teachers. For this reason, Bergman and Sams (2012) recommended that teachers begin by gradually adopting elements of the flipped classroom approach, rather than radically changing their practice from the outset. This is reflected in the approach taken by the teacher in this research.

A flipped classroom, therefore, does not necessarily mean flipped learning. There are different interpretations of this approach and associated variations in implementation strategies. For the purpose of this article, we will use the term 'flipped classroom' as defined by the FLN and endorsed by Bergman and Sams (2012), to refer to the mode of teaching and learning 'in which direct instruction moves from the group learning space to the individual learning space, and the resulting group space is transformed into a dynamic, interactive environment where the educator guides students as they apply concepts and engage creatively in the subject matter' (FLN 2014 para. 1).

The FLN propose a framework, the *Four Pillars of FLIP,* that identifies and describes key features necessary for learning to occur in a flipped classroom: *flexible environment*; a *shift in the learning culture*; *intentional content*; and *professional educators* (FLN 2014). This framework was conceptualised to account for a variety of learning modes, and so its implementation requires the creation of *flexible learning environments* that may involve, for example, the physical rearrangement of learning spaces and the increased use of digital technologies.

In a traditional teacher-centred model, the teacher is the primary source of information; within a flipped classroom, however, a *shift in learning culture* occurs, as there is a deliberate shift from a teacher-centred to a student-centred approach, with in-class time being used for exploring topics in greater depth and with increased interaction. According to FLN, this results in students being more actively involved in knowledge construction as they participate in learning that is personally meaningful.

The flipped classroom approach also requires teachers be *intentional* about their selection of content and so necessitates the evaluation of which content should be taught directly and what materials students should be allowed to initially explore on their own outside of the group learning space. This is evident when a teacher prioritizes concepts used in direct instruction for learners to access on their own, creates or curates relevant content (typically videos) for students to access, and differentiates content to make it accessible and relevant for individual students.

Finally, the role of the *professional educator* means teachers are available to students for individual, small group, and whole class feedback in real time as needed, conduct formative assessments during class time through observation, and record data to inform future instruction. With regard to this pillar, Hamdan et al. (2013) emphasise that instructional videos are not intended to replace teachers—instead, the teacher's role in a flipped classroom carries additional demands as they must determine when and how to shift direct instruction from the group to the individual learning space and how to maximise face-to-face class time.

The flipped classroom in practice

To date, empirical research related to the practices of flipped classroom approaches has generally taken the form of small-scale studies or large-scale surveys. These studies reveal both affordances and constraints for this approach.

In a case study that compared the learning environments of a flipped statistics class with a traditional introductory statistics class at the same university, Strayer (2012) found that undergraduate students in a flipped classroom were less satisfied with how the classroom structure oriented them to the learning tasks in the course but were more open to cooperative learning and innovative teaching methods. In a study drawn from the secondary sector in the USA, Fulton (2012) reported that students enjoyed moving at their own pace, appreciated being able to review material by replaying videos, and could tackle more challenging problems in class, rather than at home. Within this study, survey-based findings indicated 87 % of parents and 95 % of students preferred flipped learning to the traditional teaching approach they had previously experienced (Pearson 2013). In addition, improvements were reported in teacher-student relationships, and academic results showed an increase in proficiency in algebra, pre-calculus, and calculus (Pearson 2013).

In a large-scale survey ($n = 521\ 865$) conducted as part of *Speak Up online* (FLN 2015), students, teachers, administrators, parents, and community members across the USA, responded to questions on flipped learning and the use of videos in classrooms. This sample included 431,241 students from kindergarten through to 12th grade, with over half indicating that they regularly used videos as part of their homework. From within this cohort, the majority of students in grades 3–12 agreed that a flipped classroom approach helped them to learn at their own pace (59 %) and exercise more control over their learning (50 %). Additionally, 49 % of students found the approach was compatible with their learning style (FLN 2015). Interestingly, these aspects are closely related to students' motivational factors for engaging with the flipped classroom (Abeyseker & Dawson 2015).

Surveys have also been used to document the degree to which teachers have adopted a flipped classroom approach. The Speak Up online survey (FLN 2015), for example, indicated there was a significant increase in the number of teachers adopting flipped classroom approaches. It was found that 32 % of teachers used videos found online while 29 % of teachers produced their own videos. In a survey of over 15,000 Science, Technology, Engineering and Mathematics (STEM) teachers, Herreid and Schiller (2013) reported that teachers were shifting towards a flipped classroom approach because it allowed students to become more actively involved in the learning process and also afforded teachers more time to spend with students on authentic research. Their research, however, also noted that students new to the approach were sometimes initially resistant and arrived unprepared for classes. This resulted in an additional time commitment by teachers as it became necessary to monitor and more carefully tailor 'homework' (videos, tutorials) to the needs of students.

In summary, the flipped classroom is an emerging field of instruction that brings with it both affordances and constraints. While the number of empirical studies conducted to date is limited, there is evidence that the adoption of a flipped classroom approach may lead to increased levels of student interest, engagement, and achievement (e.g., FLN 2015; Fulton 2012; Herreid & Schiller 2013). It has also been reported that the flipped classroom offers students' greater autonomy in relation to their learning, opportunities to self-pace their work, and the potential to engage more readily with homework tasks. Other studies have reported that the approach permits teachers to customise and update the curriculum more easily so that classroom time can be used more effectively and creatively (Herreid & Schiller 2013). There are also challenges associated with adopting a flipped classroom approach. Developing resources for a flipped classroom can be initially labour intensive as it is necessary to either create or source appropriate resources (such as videos). The approach is also dependent on students' preparedness to work on preparatory materials outside of class time, with the risk being that some students may come to class unprepared (Herreid & Schiller 2013). The approach might also meet with resistance from the home as some parents could be skeptical of the benefits offered by this mode of instruction because it is likely to be at odds with the way in which they were schooled themselves (Fulton 2012; Muir 2015). Other documented concerns include the accessibility for students to view the videos away from school and the availability of professional learning for educators to support the effective implementation of flipped learning (FLN 2015).

Engagement, motivation, and self-regulated learning and the flipped classroom approach

Previous research indicates that the use of technology, including iPads, has positive effects on student engagement (e.g., Attard & Curry 2012). Engagement, as used in this context, refers to a multi-faceted construct that involves behavioural, emotional, and cognitive engagement (Fredericks, Blumfield & Paris 2004). Behavioural engagement involves participation, emotional engagement encompasses positive and negative reactions and influences one's willingness to do the work, and cognitive engagement refers to 'investment' or a willingness to exert the effort necessary to achieve mastery (Fredericks et al. 2004).

Active learning, which can be generally defined as any instructional method that engages students in the learning process (Prince 2004), has been associated with increased student learning, improved student academic performance, critical thinking, and better attitudes towards learning (Hamdan et al. 2013). Similarly, self-regulation of behaviour and cognition is an important aspect of student learning and academic performance (Corno & Mandinach 1983). While definitions of self-regulated learning vary, Pintrich and DeGroot (1990) proposed a theoretical framework that includes reference to students' management and control of their efforts on classroom academic tasks and link it with motivational components, including students' beliefs about both their ability to perform a task and also with taking responsibility for their own performance. Taking responsibility for one's own performance is vital for the effective implementation of a flipped classroom approach because students are expected to be autonomous learners, self-directing, and self-pacing their learning.

Xu and Wu (2013) have linked self-regulated learning with completion of homework, suggesting that self-regulatory strategies may be influenced by goal orientation (purpose for engaging a task), task value (the importance and utility of a task), and task interest (the appeal of a task or activity). Consequently, students' levels of engagement with any homework task, including those set in the context of a flipped classroom, are likely to be influenced by these factors.

Homework, in general, can be a source of frustration for students (e.g., Civil 2006), and the role of motivation in relation to homework cannot be understated when evaluating the success of the flipped classroom approach as, 'it is the level of motivation that influences their focus and level of effort expended on a given learning activity' (Cole, Field & Harris 2004, p. 67). As the flipped classroom involves students completing out-of-class work, it seems logical to expect that students will only complete the tasks if they are motivated, either intrinsically or extrinsically (Abeysekera & Dawson 2015). Another aspect of motivation, highlighted by Ryan and Deci (2000), was that students tended to engage in learning behaviours that were valued by others with whom they feel an affinity.

In considering the research on flipped classrooms, we have adopted a theoretical model proposed by Abeyseker & Dawson (2015) (see Fig. 1). The model recognises the potential of the flipped classroom to instill a sense of competence, relatedness, and autonomy in students, leading to increased extrinsic and intrinsic motivation. Similarly, a flipped classroom approach that is tailored to students' different expertise and allows for self-pacing, leads to better management of cognitive load.

A focus of our study was on students' engagement with learning and with mathematics learning in particular, the model was selected as a lens to analyse data relevant to this aspect of the investigation. As students' experiences included accessing learning resources away from school, their motivation to do so was of particular interest. According to the framework, motivation to complete homework tasks is more likely to occur if a flipped classroom approach helps foster a sense of competence, relatedness. and autonomy in students. In addition, motivation may be maintained through better management of cognitive load by tailoring activities to students' expertise and through the provision of opportunities for self-pacing. These components are particularly applicable to students' engagement with homework tasks, including those assigned as part of the flipped classroom approach.

In summary, we use two theoretical frameworks to interpret the results of the study. The Four Pillars of FLIP framework is appropriate for understanding how the flipped classroom approach is implemented in practice and is particularly relevant when analysing the data in relation to the teacher's role. As we were particularly interested in students' engagement with the flipped classroom approach, the application of Abeyseker and Dawson's (2015) framework was considered necessary for interpreting data pertaining to students' engagement.

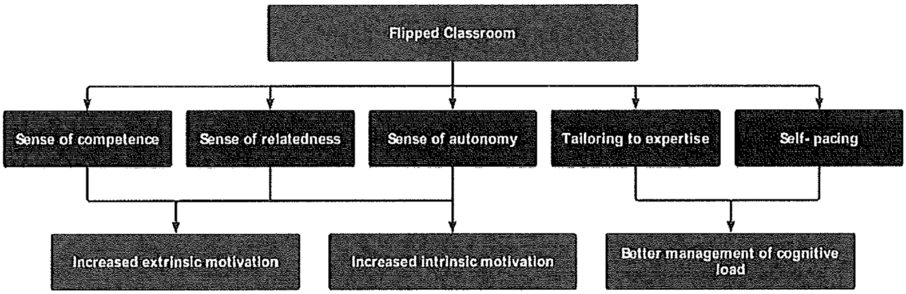

Fig. 1 Theoretical model for the flipped classroom

Methodology

An exploratory case study methodology was selected as it involves the detailed study of a group in order to bring new understandings to the fore (O'Leary 2010). The group, or 'bounded system' (Burns 2000) was the grade 10 class that is described further in this section. This was an appropriate methodology to use in this context as the researchers were interested in describing and analysing one particular group's experiences. Within the case study, both qualitative and qualitative data collection and analysis methods were employed (Creswell 2003) as these provided richness and depth to the study of a single classroom—the teacher, his entire class, and teaching and learning resources that were utilised by the students. Consistent with this approach, the researchers used sequential procedures where data collected from the surveys were used to inform the interview schedule, allowing more detailed exploration with a few cases or individuals.

The selection of the teacher and his class was purposive in that the teacher had indicated a strong desire to improve aspects of his teaching related to student engagement, motivation, and self-regulation in relation to mathematics learning, and he had already begun trialling a flipped classroom approach. Furthermore, availability of relevant digital technologies and technological infrastructure was in place, which allowed all students access to teacher generated resources.

School and classroom context

The study was conducted with a teacher and his grade 10 'mathematics extended' class from a large metropolitan secondary college in Tasmania. The school is co-educational, caters for students from grades 7–10, and has an enrolment of just over 500 students. It had been an 'iPad school' since 2012 and all students had their own iPads. In addition, students had access to other resources including the following: an electronic version of their mathematics text-book preloaded onto their iPads; access to iTunesU for downloading teacher generated videos, a memory stick onto which they could download relevant videos, and separate workbooks where they could make notes. While such technologies had been freely available, the majority of students had little to no experience of a flipped classroom approach until they became members of the classroom in which this study took place.

The 'mathematics extended' class was offered as an elective for grade 10 students. There were no pre-requisites meaning that every student had the opportunity to enrol in the class, irrespective of prior attainment in mathematics. In practice, students were motivated to enrol in this class because they were aiming to study senior secondary mathematics in the future. As such, topics studied included algebra, functions and their graphs, calculus, and probability.

The teacher of the 'mathematics extended' group was responsible for covering the required grade 10 curriculum, with the aim of preparing his students for the pre-tertiary subject of Mathematics Methods. Within his school, this meant a prescribed textbook (Greenwood, Woolley, Vaughan, & Goodman 2014) was used to guide decisions about the sequence and approach to teaching mathematics topics. The text had been chosen by the school because it aligned to the Australian Curriculum: Mathematics and all teachers based their mathematics program on the textbook relevant to their grade.

Despite these constraints, the teacher had decided to adopt a flipped classroom approach that allowed him to capitalise on the affordances offered by digital technologies while, at the same time, conforming to the school expectations regarding coverage of the Australian Curriculum. Thus, in preparing video tutorials, demonstrations, and electronic resources, he was mindful that they were all based on topics and excerpts from the school-selected text. Students accessed these materials from home using the iTunesU platform.

All students had access to the internet at home, but the online materials were also available for downloading directly onto students' devices at school. In the lessons observed as part of this study, students were typically working through individual examples and problems on their iPads with the teacher assisting individuals when required. There were very few whole class demonstrations or explanations. The teacher indicated that when whole class demonstrations were used, they were targeted at a specific mis-conception and certainly less frequent than before he implemented the flipped classroom approach. As the year progressed, the range of topics that different students were engaged with increased considerably, making whole class instruction less relevant and effective. Students were encouraged to demonstrate mastery, through accessing the video tutorials, completing the exercises, and assessment tasks for each unit, and then moving onto the next topic.

Participants

Participants included one teacher (Mr. Hill, pseudonym) and his entire class of 27, grade 10 students (approximately 15 years of age; 17 male and 10 female). All students completed the online survey, and six students participated in semi-structured, paired interviews (two girls and four boys). The students indicated at the end of the survey that they would be willing to participate in an interview, and all had received written parental consent to do so. Ethical approval was granted for the research.

The teacher, Mr. Hill, had been teaching at the school for 3 years and had a total of 7 years' teaching experience and was qualified to teach both mathematics and science. At the time of the study, he was teaching mathematics from grades 8–10, including the grade 10 'mathematics extended' class. This was his third year of using iPads to flip his mathematics extended class. He also was using aspects of the flipped classroom with his other mathematics classes, but not to the extent to which it was happening with the grade 10 class.

Instruments, procedure and data analysis

Data collection instruments consisted of an online survey using Qualtrics[1], semi-structured student and teacher interviews, and classroom observations. Data collection took place over a number of months in the second half of the school year. The purpose of the survey was to gather information concerning students' perceptions of using online tutorials as a resource for learning mathematics. These perceptions were naturally informed by their previous experiences with learning mathematics. Semi-structured interviews were designed to allow the researchers to probe, more deeply,

[1] An online survey construction tool: http://www.qualtrics.com/

the students' experiences of the flipped classroom. Classroom observations and a teacher interview were used to complement and triangulate the data collected from students.

The survey contained 24 questions. Question 6 required responses to 20 statements about the use of online resources generally, and Question 22 required responses to 10 statements about the use of their teacher's pre-prepared resources. Responses were recorded against a five-point Likert scale (see Table 1 for example items). The items were adapted from an existing instrument designed to investigate students' self-initiated use of online resources (see Muir 2014) and was structured around themes related to students' use of the video tutorials including logistical and attitudinal aspects. There was also the provision for open-ended responses to seven questions. Most relevant to this article, were items 5, 15, and 23, as these required students to list the advantages of using online resources instead of the textbook (5) or asking their teacher (15) and if they would recommend the videos to others and why (23). The survey was administered by the teacher during a scheduled lesson and took approximately 15 min to complete.

A semi-structured interview of approximately 40 min' duration was conducted with the teacher prior to the lesson observation and administration of the student surveys. Semi-structured interviews were also conducted with three pairs of students following

Table 1 Students' use of online resources ($n = 27$)

Item	SA/ Agree	Undecided	Disagee /SD
Q1. I use online resources to help me with my learning	93 %	7 %	0 %
Q2. I have used online resources (not prepared by my teacher) to help me with my mathematics this year	26 %	11 %	63 %
Q3. I have used online tutorials prepared by my teacher to help me with my mathematics this year	93 %	0 %	7 %
Q4. I have used online tutorials prepared by my teacher to help me with my mathematics this month	93 %	0 %	7 %
Q5. I have used the internet to help me with my mathematics	60 %	7 %	33 %
Q6. The tutorial helped me to understand a concept	96 %	4 %	0 %
Q7. The tutorial was about the right length	96 %	4 %	0 %
Q8. I watched all of the tutorial from beginning to end	87 %	13 %	0 %
Q9. I found the tutorial helpful	100 %	0 %	0 %
Q.10. I found the tutorial boring	17 %	34 %	49 %
Q11. I think I did better in the test because I watched the tutorial	87 %	13 %	0 %
Q12. I think I understood the work better in class because I watched the tutorial	100 %	0 %	0 %
Q13. I used the tutorial to explore mathematics of my own	43 %	43 %	14 %
Q14. I used the tutorial to explore ideas about mathematics begun in class	48 %	48 %	4 %
Q15. I used the tutorial as a last resort because I was stuck on problem	13 %	22 %	65 %

completion of the online survey and the observation of an in-class lesson that occurred approximately 2 weeks after the teacher interview. All interviews were audio-taped and transcribed.

Quantitative data from the survey were subject to descriptive statistical analysis through Qualtrics. Qualitative data from the surveys and interviews were transcribed and analysed using reflexive iteration (Srivastava 2009). In this process, each sentence in the transcript was coded, initially using themes that emerged from the data. The transcripts of the students' interviews were then re-read to identify instances of the components identified in Fig. 1. This process helped to eliminate researcher bias in that the researcher was open to other themes that might emerge from the data. Initially, 11 codes were ascribed to the data, which were then matched with the five categories identified in the Abeysekera and Dawson (2015) model (Fig. 1). For example, comments originally assigned a code of 'convenience' or 'easily accessible' were reassigned to 'sense of autonomy' because they indicated that students could access the resources when and where they were required. Similarly, references to 'targeted work' were reassigned to 'tailoring to expertise'. In addition, references related to the implementation of the approach and the role of the teacher were also analysed using the Four Pillars of FLIP framework. The following section presents the results and discussion from the data, primarily from the perspectives of the students, and supplemented with the teacher interview data.

Results and discussion

In this section, we present an analysis and interpretation of the data. Data have been grouped according to the various themes that emerged from the data.

The nature of Mr. Hill's flipped classroom

In this section, we draw on survey and interview data to explore the nature of Mr Hill's flipped classroom. Data are analysed by utilising the *Four Pillars of FLIP* as a framework to identify and describe key features of the teacher's approach against the criteria *flexible environment*; a *shift in the learning culture*; *intentional content*; and *professional educators.*

Teaching and learning in Mr. Hill's flipped classroom

Mr. Hill had arrived at the school in 2012 during the initial stages of the school's iPad trials. He had read about flipped classrooms and some of the techniques associated with this approach in teacher professional learning materials and had been excited by the prospect of working in this way. To support his approach to the flipped classroom, he typically produced video clips of 7–8 min duration. At the time of the study, he had produced around 140 videos and uploaded them into iTunesU for students to access. He used Camtasia Studio software to record and edit the videos and SmoothDraw 3 to write on the screen. The resulting videos were a combination of mathematics content and worked examples that students were expected to watch prior to attending class. Loading the videos to iTunesU meant that students could download and view the videos on a mobile or desktop device, at a time and place that was conducive to their own learning.

When asked about the advantages offered by the flipped classroom approach, Mr Hill commented that:

> *the great thing about it is that it frees up time rather than spending half the time standing at the board, writing something down or copying, or read this or watch something in class which wastes precious class time, you can maximise the time, which is what I do – it's the only way I can get through the course.*

Students confirmed that this was Mr Hill's mode of working in his mathematics classroom. For example, Jill (pseudonym) commented during an interview:

> *And if some are like all up to one section and a few people have asked him how to do this and if there's a few people stuck, he'll say listen up, I noticed there's a few people stuck on this and he'll do just one or two things on the board, but not generally much on the board unless everyone's kind of stuck*

Fundamental to freeing up time was the use of video based resources that students were expected to view out of class time. Students reported to making good use of videos as home and that these were helpful for their learning. As observed by two of Mr. Hill's students John and Frieda:

> John: *I go straight to the videos – I think the videos are really helpful – if you watch it carefully he actually explains everything so there'd be nothing you're missing out on, but yeah, that's where I'd go straight to.*

> Frieda*: With the videos, firstly I watch the whole thing through without stopping it, then I'll watch it and stop it and write down what I need to write down and then if I need to go back to it, I'll just go back to the part I need to watch so yeah, I do watch it the whole time, then I'll stop it and write down if I need to refer back.*

Like other users of the flipped classroom approach, Mr Hill relegated the more routine, procedural presentations and explanations for the students to complete in non-school time. His comment that prior watching of these aspects on the videos 'frees up time' so that he no longer spent half his class time standing at the board is indicative of a *shift in learning culture* whereby direct instruction moves from the group learning space to the individual learning space and from a teacher-centred approach to a student-centred approach (FLN 2014). Mr. Hill also had to select what content to provide, the way it was presented, and what resources would be appropriate to access before class—demonstrating his use of *intentional content*. His approach included the use of technology to transform the material and examples from the textbook into video format that allowed him to expand on examples in class and unpack the mathematics in more depth and also relate it more directly to the learning context of these students. Students commented that the technology employed, as well as the availability of Mr. Hill's videos, allowed for access to step-by-step instruction as their teacher had intended. As noted by John, when asked if there was anything about the way Mr. Hill presented the videos that made it particularly effective:

> *Definitely the technology he uses…like he has the screen so we can see him, and he also does all the steps step by step so that he explains exactly what he's doing and the quality of the sound and everything*

Harold also mentioned that videos were more engaging than working with the textbook alone:

> *I really prefer the videos – I think they're definitely more engaging and you actually pay more attention to it. They're just more appealing to you and using today's technology – they're just really good.*

As indicated, although the videos could be accessed on a desktop computer, accessibility was particularly facilitated through students' individual ownership of iPads. The convenience of having individual iPads was referred to in students' open-ended responses to the survey questions and in their interviews. For example, in one survey response, a student indicated iPads were preferable to textbooks, due to their portability and accessibility:

> *All the online resources are on my iPad so it when I take it home to do homework I can just take my iPad instead of taking a big textbook and making my bag heavier*

The preparation of the material used in direct instruction for students to access on their own exemplifies the enactment of the second and third pillars of the FLIP framework; there was a shift from a static to a dynamic representation of the content and a deliberate selection of which aspects of direct instruction could be accessed in students' individual spaces.

Students' use of online resources

A summary of student responses to those Likert items that specifically related to their use of online resources, including those prepared by their teacher, is presented in Table 1. In order to highlight agreement of disagreement with item statements, strongly agree/agree (SA/agree) and disagree/strongly disagree (disagree/SD) have been collapsed from four categories to two. Students' responses provide insight into how often, and in what ways, the resources of Mr. Hill's flipped classroom were accessed.

Students indicated that they made use of online resources to help with their learning (Q1, 93 %). The resources prepared by their teacher were favoured over other online materials (Q2, 26 %; Q3, 93 %; Q4, 93 %). Students also used additional internet resources to assist them with their mathematics learning (Q5, 60 %). Responses to the survey also indicate that students believed the tutorials provided by their teacher were engaging and helpful in terms of their mathematics learning (Q10, only 17 % boring; Q6, 96 %; Q9, 100 %), and of the right duration (Q7, 96 %; Q8, 87 %). Importantly, students also believed watching online tutorials promoted their learning and achievement on topic tests (Q11, 87 %; Q12, 100 %). Interestingly, responses indicated that students were less likely to make use of the tutorials to explore the mathematics begun in class on their own (Q14, 48 %; Q13, 43 %) and that they rarely used tutorials when they were stuck on a problem (Q15, 13 %)—a possible consequence of the content focused approach adopted by their teacher.

The open-ended section of the survey provided additional insights with 60 % of students indicating they had watched all the available videos. The remaining responses included 'lots', 'too many to count', and 'about 90 of the 150 videos'.

In summary, students' indicated that Mr. Hill's flipped classroom approach was more engaging than a textbook only mode of teaching, supported their learning, and that they were satisfied with the quality of the resources produced by the teacher. This approach meant that routine tasks and examples were worked on at home, prior to class, and class time was devoted to individual support for students' challenges and difficulties, rather than whole class presentations. Mr. Hill's approach was consistent with the first Pillar of FLIP, in that the *environment was flexible* in terms of both classroom arrangement and extending the classroom into the home. While acting within the constraints of using a prescribed textbook, Mr. Hill showed evidence of using the third Pillar of FLIP, *intentional content*, in that he was selective in the content he expected students to cover before attending class and how this material was presented in his videos.

Students' and teacher's perceptions of a flipped classroom approach

Data analysed in this section is drawn from student survey responses and teacher and students' interviews. The analysis has utilised Abeyseker and Dawson's (2015) motivation framework as a theoretical lens with respect to the following elements: *sense of competence*, *sense of relatedness*, *sense of autonomy*, *self-pacing*, and *tailoring to expertise*.

Sense of competence

According to Abeyseker and Dawson (2015), learning environments created by flipping the classroom are likely to satisfy students' need for competence. Responses in this category included references to being helpful generally, as well as providing assistance to students in understanding and making use of mathematical techniques. For example, in response to question 23 on the survey, which asked if students would recommend the video tutorials to others and why, approximately 70 % of responses mentioned that the videos supported their learning. Indicative student survey responses include the following:

> *It is easier to understand as the teacher is working through the question with you on the video and you can follow along.*

> *Because it shows you step by step how to do a problem.*

> *It helped me to understand the concept better because it was presented in a different way.*

Students' responses to the survey indicated that 93 % agreed that online resources were helpful to their learning, and 100 % agreed that they found the tutorial helpful (see Table 1, Q9). The majority of students agreed that they performed better in tests as a result of watching the tutorial, and there was 100 % agreement that they understood the work better indicating students believed the videos were supportive of their development of mathematical competence. There was a greater variation in responses to the items 'I found the tutorial boring' (SA/A 17 %, U 34 %, DA/SD 49 %), 'I used the tutorial to explore mathematics of my own' (SA/A 43 %, U 43 %, DA/SD14%), and 'I used the tutorial to explore ideas about mathematics begun in class' (SA/A 48 %, U 48 %, DA/ SD 4 %). This may indicate that students may not have found the tutorials inherently interesting or engaging or inspired students to explore mathematics further on their own—to move beyond competence. This is an area of potential future development for Mr. Hill as students may also require videos more targeted at remediation of common misunderstandings or extension activities. These less structured (in terms of content presentation) videos may begin to address the needs of the students, as seen in responses to Q13 and Q14, who were undecided in their views regarding the usefulness of videos for exploration of mathematical ideas.

Understanding was also mentioned in the survey responses related to advantages, or otherwise, in using online resources as compared to the textbook alone. Approximately, 65 % of students specifically mentioned understanding, as the following responses show:

> *Yes, it is easier to understand as the teacher is working through the question with you on the video and you can follow on.*

> *A book doesn't really walk through the steps on how to do something as well.*

The data provides evidence that, for the majority of these students, the flipped classroom approach satisfied their desire for competence, likely leading to increased extrinsic motivation.

Sense of relatedness

Although discussed in relation to tertiary students, Ryan and Deci (2000) found that students tended to engage in learning behaviours that are valued by significant others with whom they feel an affinity. It was evident through these students' survey and interview responses that they valued Mr. Hill's personal approach and the time and effort that he put into preparing the resources for them. John, for example, was aware that

> *Mr Hill stays up late and does all the videos and gets them all up for the next day.*

There was also regular reference to Mr. Hill's attempts to make the videos engaging, as the following comment from Harold illustrates:

> *Mr Hill did a bit of a song at the start of the videos which had like a theme … and he cracks jokes – not that funny, but … he tries to make it fun.*

When asked, for example, if Mr. Hill's videos would appeal to other students, Harold responded that:

It's probably more appealing if it's your own teacher ... like other teachers around here wouldn't have as much success with them, with the videos, so I think it depends upon the teacher.

While some flipped classroom approaches involve regular use of external online resources, Mr. Hill limited the links to other sites and preferred to produce his own videos, as he observes:

[What they access at home] is a substitute for me being in the room – you've got to have that personal link ... so I think it's important that the content does come from the teacher.

Students also recognised that the videos were effective, at least in part, because they were prepared by Mr. Hill. As Harold, stated:

[Mr Hill] adds his own personal touch to it and it's far better to watch [than Khan Academy] I think.

It would appear that Mr. Hill's students appreciated the approach he adopted in building his flipped classroom around self-authored video resources. Their comments suggest that his personal touch helped build a sense of *relatedness*, which contributed to their motivation towards, and engagement with, the 'extended mathematics' course.

Sense of autonomy

Students were also asked whether or not the online resources had advantages over asking the teacher in a face-to-face context, (open-ended survey question 15). Nine students indicated that there were advantages to online resources, five responses were in the negative, and six recorded answers that online resources held advantages some of the time. Eight of the positive responses made reference to accessibility or convenience, and these were grouped under *sense of autonomy*. The following are illustrative of the comments received in the survey:

Sometimes the teacher is busy with other students so you can watch it while you are waiting and it might help you understand better.

You can try and work it out on your own and if you are still stuck, then you can ask a teacher later.

These comments indicate that the online resources provided by Mr Hill, allowed students to exercise a degree of *autonomy* over their learning.

Trisha, during student interviews, also recognised Mr. Hill's approach to promoting students' *autonomy* in relation to their learning, stating the following:

> *Well you've got to do it by yourself – like he tells us when the test is, and ...he'll give us guidelines on where we need to be up to ... but he doesn't say if you're not up to here, you've got a detention, or you have to do it, kind of thing, it's more up to you ... he's not going to start yelling at you if you're behind or whatever.*

This observation is consistent with Mr. Hill's espoused view on the need for students to be self-motivated and his trust in them to watch the videos:

> *You could chase them up, but I think you've just got to trust the kids ... I know that most of the students in my class do the work at home and ... I also like them to learn the self-motivation skills.*

These comments suggest that Mr. Hill made a conscious attempt to promote students' *autonomy* and that this worked with most students.

Self-pacing and tailoring to expertise

In the framework shown in Fig. 1, *self-pacing* and *tailoring to expertise* led to better management of cognitive load. As indicated previously in relation to developing a sense of competence, relatedness, and autonomy, the flipped classroom created an environment where self-pacing and tailoring to expertise could be managed. Two facilities the flipped classroom offers over traditional instruction (including homework tasks), are the opportunity for students to select videos relevant to their learning needs and then to pause and rewind the material in order to work at their own pace. Such affordances have the potential to support the development of students' competence and autonomy. This potential is evident in a number of student comments recorded in the open response sections of the questionnaire (5 and 15):

> *Yes they help you and you can rewind or watch the whole video again until you understand the concept.*

> *...[you can] interpret at your own rate which would be different to other people*

These views were also reflected in comments made by students during interviews, for example:

> Harold*: Well if you're not working at the pace of the class, you can actually work at your own pace – it's not like they're going to go on without you and you're trying to keep up, and you're struggling or you're way ahead, you can go at your own pace*

> John: *I think it's good that you can work at your own rate and you can just slowly progress through all the work, like basically no one in the class is up to the same question or anything like that ... I don't think any two people are alike with their*

work, so like people can figure it out in their own time, and just go through it at the time they need it and at the rate they want

It would appear that students genuinely appreciated the opportunity to make selections of material that suited their learning needs and to progress through material at their own pace. By being provided with the opportunity to make choices, students were also allowed to exercise a degree of autonomy in developing their own competence with mathematics.

Conclusions and implications

Mr. Hill's desire to implement a flipped classroom approach to his teaching was initially made possible due to the robust technological infrastructure in the school; however, the real impact on his students' engagement with, and motivation toward, the study of 'extended mathematics' was a consequence of the resources he developed to support his students' learning. While preparation of video resources was time consuming for Mr. Hill, his students appreciated the opportunities he provided for them to make selections of resources most aligned to their learning needs and to proceed at their own pace in a learning space of their own choosing. In particular, his students indicated their positive view of Mr. Hill's instructional techniques, such as his step-by-step approach to explaining the problems.

Findings from this study were consistent with those of other research that found relevant content from mathematics courses of study could be covered and made accessible to students via pre-prepared video resources (e.g., Muir & Chick 2014). This was particularly pertinent for school-based assessment processes, including tests. In addition, other than the time and resources required to produce video resources, there were no concerns expressed or disadvantages identified by the teacher or his students related to the implemented approach to the flipped classroom. This stands in contrast to the findings of other studies, which noted problems related to student access to online materials and parental concerns related to the novelty of the approach (FLN 2015; Fulton 2012). Likewise, student resistance to a flipped classroom approach, especially a lack of pre-class preparation (Herreid & Schiller 2013), were not issues identified in this study.

This article adds to knowledge in four ways: use of the Four Pillars Framework (FLN 2015) as a means of evaluating the effectiveness of attempts to implement a flipped classroom approach, use of Abeyseker and Dawson's (2015) model of the flipped classroom to identify the factors that contributed to students' engagement with and motivation towards mathematics learning, importance for students of relatedness to their teacher in relation to prepared resources, and the way in which Mr. Hill was able to implement aspects of a flipped classroom within a highly structured teaching and learning context.

The Four Pillars Framework provided insight into the specifics of Mr. Hill's adoption of the principles of flipped learning. Examination of his practice and analysis of the data showed that he provided students with a *flexible environment*, enacted a *shift in the learning culture* by removing direct instruction from the classroom to the home environment, selected *intentional content,* and made himself regularly accessible to students in his role as a *professional educator*.

Abeyseker and Dawson's (2015) model was used to identify the factors that contributed to students' engagement with, and motivation towards, Mr. Hill's flipped classroom approach. While Abeyseker and Dawson's (2015) framework was developed within the context of higher education, this study demonstrated it is applicable to the context of secondary mathematics classrooms. Student survey and interview data revealed regular references to aspects of the approach that involved *competency, autonomy*, and a *sense of relatedness*. Students were able to use the tutorials to access learning material ahead of class time and could revisit the tutorials to consolidate learning and to prepare for assessment tasks. The nature of the tutorials, in terms of Mr. Hill's instructional techniques, helped to facilitate this aspect and satisfy students' need for competence.

Students reported that open access to pre-prepared video resources provided them with the opportunity to take more responsibility for their learning and thus promoted their autonomy. Students also indicated that the ability to select relevant resources, and to work at their own pace, was a benefit of Mr. Hill's approach, a finding consistent with previous research (e.g., Muir & Chick 2014).

There was evidence that students were motivated to access, and engage with, video resources, in part, because they were prepared by their teacher—highlighting the importance of relatedness between students and teacher provided materials in fostering Mr. Hill's version of a flipped classroom. It would appear that this was because students believed the videos Mr. Hill prepared directly targeted the relevant learning outcomes of the course they were studying—a finding consistent with research conducted by Ryan and Deci (2000) and Muir and Chick (2014) who both found that a teacher's pedagogical approach and classroom manner are critical to the success of flipped classrooms.

Mr. Hill also demonstrated how a flipped classroom approach could be implemented without radically reforming a teaching practice. This may be an encouraging finding for teachers who wish to consider incorporating advantageous aspects of this pedagogy into their teaching and learning practice without abandoning approaches that experience has shown are effective.

This study has implications for teaching and learning practice. Given the range of diversity that exists in any classroom, and the pressure to cover core curriculum content, the flipped classroom may be a pedagogical approach that offers the flexibility to cater for a wider range of student needs and maximises the use of available time for learning. Mr. Hill rarely engaged in whole class teaching, adopting instead a pedagogical approach that allowed him to differentiate individual instruction. The study also has implications for the support of students in completing homework tasks. As students move through secondary school, mathematical homework tasks typically become more challenging, and parents are less able to assist. Research studies have provided evidence that homework tasks are often a source of tension between students and parents (e.g., Civil 2006), with difficult subject matter leading students to seek assistance from online sites (Muir 2014), which may or may not be credible sources of information. Watching a relevant online tutorial, prepared by one's own teacher, seems to be a preferable option compared to seeking out information or assistance from the less specific plethora of online material available via the internet.

The flipped classroom has been under-researched and under-theorised. This study has provided empirical evidence of the potential of the flipped classroom to support student learning in new ways. While these findings are promising, it is important to acknowledge that the study took place in one 'extended mathematics' classroom and involved motivated students and their enthusiastic and conscientious teacher. It remains to be seen whether or not such an approach would be as effective if implemented with secondary mathematics classrooms in different contexts. Further empirical research also needs to be conducted on the flipped classroom in terms of student outcomes, as this was not explored within this study. Future studies could examine the suitability of the flipped classroom approach in other contexts, such as project-based inquiries, whether or not it would be suitable for younger students, and whether or not it could be used to teach a range of disciplines.

Compliance with ethical standards The students indicated at the end of the survey that they would be willing to participate in an interview, and all had received written parental consent to do so. Ethical approval was granted for the research.

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