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ORIGINAL ARTICLE

Enhancing student engagement through the affordances of mobile technology: a 21st century learning perspective on Realistic Mathematics Education

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Abstract Several recent curriculum reforms aim to address the shortfalls traditionally associated with mathematics education through increased emphasis on higher-order-thinking and collaborative skills. Some stakeholders, such as the US National Council of Teachers of Mathematics and the UK Joint Mathematical Council, advocate harnessing the affordances of digital technology in conjunction with social constructivist pedagogies, contextual scenarios, and/or approaches aligned with Realistic Mathematics Education (RME). However, it can be difficult to create technology-mediated, collaborative and contextual activities within a conventional classroom setting. This paper explores how a combination of a transformative, mobile technology-mediated approach, RME, and a particular model of 21st century learning facilitates the development of mathematics learning activities with the potential to increase student engagement and confidence. An explanatory case study with multiple embedded units and a pre-experimental design was conducted with a total of 54 students in 3 schools over 25 hours of class time. Results from student interviews, along with pre-test/post-test analysis of questionnaires, suggest that the approach has the potential to increase student engagement with, and confidence in, mathematics. This paper expands on these results, proposing connections between aspects of the activity design and their impact on student attitudes and behaviours.

Keywords Mobile technology · RME · Contextualised learning · 21st century learning · Student engagement · Post-primary education

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Introduction

Although recent mathematics curriculum reforms have typically focused on developing students' conceptual understanding, problem-solving ability and productive disposition (Cai and Howson 2013; Conway and Sloane 2005), students' views of the subject at the end of compulsory education frequently remain fragmented and de-contextualised (Ainley et al. 2011). It has been suggested that within an appropriate pedagogical framework, the use of mobile technologies can make mathematics more meaningful, practical and engaging (Ainley et al. 2011; Drijvers et al. 2010; Hoyles and Lagrange 2010; Olive et al. 2010). Similarly, the use of context and the process of mathematisation in Realistic Mathematics Education (RME) have the potential to address some of the limitations associated with the more formal and abstract traditional mathematics education (Gravemeijer 1994; van den Heuvel-Panhuizen 2002). Social constructivist educational theories are advocated as aligning particularly well with the affordances of mobile technology (Bray and Tangney 2013; Patten et al. 2006), and are also highly compatible with RME. However, activities combining mobile technology, social constructivism and RME tend not to fit well with the didactic teaching and short class periods common in many conventional classrooms (Wijers et al. 2008). Therefore, pedagogical models more in line with 21st century learning (21CL), in which the skills of mathematical creativity, critical thinking, problem-solving, communication and collaboration, and technological fluency are emphasised, may be more appropriate (Tangney et al. 2015; Voogt and Roblin 2012).

This paper explores how a combination of a transformative, mobile technology-mediated approach, RME, and a particular model of 21CL (*Bridge21*) (Lawlor et al. 2015) facilitates the development of mathematics learning activities that have the potential to increase student engagement and confidence. Each of the three elements is discussed, and a set of heuristics for the design of mathematics learning activities based on their synergy is presented.

This explanatory case study places particular emphasis on the impact on students' engagement with, and confidence in, mathematics through participation in activities designed in accordance with the Design Heuristics. In order to explore this, the following questions are posed:

1. What effects on student engagement and confidence does participation in such activities have?
2. What are the primary factors that cause the changes in engagement and confidence?

A total of 54 students took part in three separate interventions, of varying duration, and in different schools. While a pre-experimental design, using pre- and post-questionnaires, is used to quantify the impact of the intervention on student engagement, qualitative analysis of focus-group interviews explores the transformation in greater depth, providing a rich, detailed description of how and why any changes have emerged.

Background

In order to ground this research within the wider context, this section provides a review of general issues associated with mathematics education and highlights a potential mechanism for addressing these problems through the appropriate integration of mobile technology.

Concern about the state of mathematics education can typically be broken down into two broad sets of factors: concerns about poor levels of understanding and achievement gaps on one hand, and a need for the development of key 21CL skills such as collaboration, communication, creativity and problem-solving on the other. These issues are reflective of the formal, abstract and assessment-driven approach to mathematics that is still prevalent in the *implemented* (as distinct from *intended*) curriculum in many countries (Ainley et al. 2011; Anderson et al. 2012; Ozdamli et al. 2013), in which content and procedure are often prized over conceptual understanding and numeracy. Anderson et al. (2012) query what the relationship should be between formal, abstract mathematics and the concept of numeracy laid out in the Australian Association of Mathematics Teachers' policy documents, which emphasises mathematical thinking, understanding and context (Australian Association of Mathematics Teachers 1997). They argue that, while elements of formal mathematics should be introduced through the curriculum, the main focus at school level should be on practical, usable mathematics. However, efforts to change teachers' pedagogical practice have not always matched the general shift in intended curricula towards a more skill-based one (Anderson et al. 2012).

The use of mobile technology has the potential to address some of the problems highlighted above, opening up new, personal routes for learners to construct meaning and engage with mathematical concepts (Drijvers et al. 2010; Olive et al. 2010). This potential for meaningful engagement with mathematics increases when we consider mobile learning, defined by Crompton (2013) as "learning across multiple contexts, through social and content interactions, using personal electronic devices" (p. 4). The use of mobile devices can permit the traditional concept of the classroom to be expanded to include the environment and wider community; data can be realistic and activities genuinely problem-solving, and the potential for sharing data and the social construction of meaning across multiple contexts opens exciting possibilities for collaborative learning. Thus, the use of mobile technology has the potential to have a *transformative* impact on task design.

Theoretical framework

In this section, key features of the three elements of the framework that underpin the Design Heuristics—RME, the Bridge21 model and (mobile) technology usage in teaching—are outlined.

Realistic Mathematics Education

RME is an approach to mathematics education that involves students developing their understanding by engaging with problems set in contexts that engage their interest, with teachers scaffolding their re-invention of the mathematics that they encounter (Freudenthal 1991). Since its origin in the 1960s, RME has become internationally influential in mathematics curricula and pedagogy (Clements et al. 2013). Five characteristics are particularly associated with RME:

1. Use of meaningful contexts
2. Development of models to help move from the original context to the formal mathematical one
3. Students' guided re-invention of mathematical concepts
4. Interactivity between pupils and with the teacher
5. A view of mathematics as a connected subject

These characteristics guide a process called "progressive mathematisation", which involves the following:

- Starting from a problem set in context
- Identifying the relevant mathematical concepts
- Refining the problem so that it becomes a mathematical one
- Solving that problem
- Interpreting the solution in terms of the original situation

The Bridge21 model of 21st century learning

Bridge21 (www.bridge21.ie) is a particular model of 21CL developed in the authors' institution (Lawlor et al. 2010) and was originally designed for use in an out-of-school context, as part of an outreach project. The research team are in the process of adapting the model for the delivery of the mainstream curriculum through a series of professional development workshops, and in-school activities, with teachers from a number of post-primary schools (Johnston et al. 2014). The work is taking place against the backdrop of a systemic reform process in the Irish education system, which places increased focus on the development of certain key skills rather than solely on a terminal assessment (Department of Education and Skills 2012). The team-based pedagogical model is influenced by the patrol system of the World Organisation of the Scout Movement. Adults act as guides and mentors, scaffolding and orchestrating the learning experience. The physical learning space is configured to support a mobile, collaborative, project-based and technology-mediated approach, with an emphasis on individual and group reflection. Three examples of mathematics activities that adhere to the Bridge21 model are described in this article.

Technology usage (enhancement and transformation)

The use of mobile technologies in mathematics education has the potential to encourage meaningful student engagement with mathematics, by embedding the subject in authentic contexts. However, while technology can facilitate an emphasis on practical applications of mathematics, Olive et al. (2010) note that "it is not the technology itself that facilitates new knowledge and practice, but technology's affordances for development of tasks and processes that forge new pathways" (p. 154). The SAMR hierarchy (Fig. 1) (Puentedura 2006) offers a useful model for describing different levels of technology integration in activities. The Bridge21 approach focuses on the creation of activities that fall within the *Transformation* space—transformative uses of technology are those which allow significant task redesign (*modification*) or permit the creation of tasks that would not be possible without the digital tools (*redefinition*). The tasks described in this article make use of mobile devices and video tracking to create mathematical modelling activities that are transformed through the use of technology. The technology thus plays a transformative role in the conceptualisation and creation of the mathematical model, rather than simply being used to solve the problem after it has been abstracted (Geiger et al. 2010). In particular, mobile technology permits the collection and analysis of realistic data in authentic environments. Although the flexibility afforded by mobile technology is particularly well-suited to the modification and redefinition of tasks, within the field of mathematics education, the computational affordances of technology to *augment* traditional approaches—outsourcing the calculation, increasing speed and accuracy—are seen as equally important. The following section describes an approach to activity design that incorporates these elements.

Activity design

A combination of key elements of the framework described above, in conjunction with concepts emerging from a general literature review, provides the theoretical foundations for the development of a set of Design Heuristics for the creation of innovative, mobile technology-mediated, collaborative mathematics learning activities (Bray et al. 2013). These guidelines resonate with a view that mathematics should be presented as a problem-solving activity, involving the students' collaborative formulation and solution

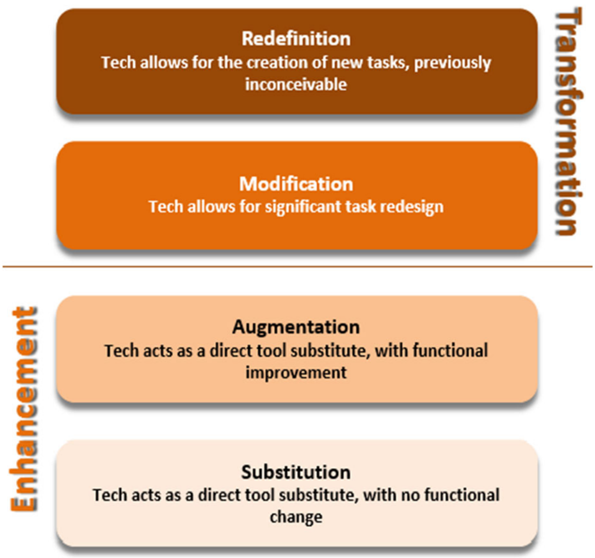

Fig. 1 SAMR hierarchy

of problems, while providing relevant, interesting contexts and compelling goals (Ainley et al. 2011; Confrey et al. 2010; Laborde 2002). The main concepts underpinning the Design Heuristics are summarised as follows:

1. Activities should follow a 21CL model such as Bridge21: they should be collaborative and team-based in accordance with a socially constructivist approach to learning.
2. They should make use of a variety of technologies (digital and traditional) suited to the task, in particular, non-specialist mobile technology such as smartphones and digital cameras that students have to hand. Emphasis should be placed on the transformative, as well as the computational, capabilities of the technology.
3. Task design should prioritise guided discovery, involving problem-solving, investigation and sense-making, and a move from concrete to abstract concepts. Tasks should be open-ended, allowing for different trajectories and solutions; they should have a "low-floor" and "high-ceiling", such that all students will be able to engage meaningfully with the problem, with the potential for more interested/able students to push its boundaries.
4. The context of the problem, and the learning experience, should be interesting and immersive/real, adapting the environment and class routine as appropriate.
5. Presentation, competition and reflection can be used for assessment purposes.

To date, five mathematics activities have been designed and piloted in accordance with the Design Heuristics, three of which have been implemented in diverse, but conventional school environments. The use of mobile technology permitted the classrooms to be rearranged into learning spaces that were suited to the collaborative, project-based activities and provided opportunities to learn across multiple contexts, within and outside the confines of the school. The activities are described in the following sections.

Activity 1: Barbie bungee

Although the Barbie bungee activity is not a novel concept,[1] it has been significantly redesigned through the application of the Design Heuristics. Student groups are provided with a doll and some rubber bands and are confronted with the problem of determining how many bands they will require to give Barbie a safe, but "exhilarating" jump from an as yet unknown height. Each team has access to smartphones and laptops with free spreadsheet[2] and video analysis software.[3] While estimation is encouraged, guesswork and trial-and-error approaches are disallowed. As the students do not initially know the distance their doll will need to fall, their problem-solving skills are put to the test while they attempt to develop a mathematical model of the relationship between falling distance and number of rubber bands.

The use of smartphones and video analysis facilitates accurate estimates of the distances that the Barbie falls with differing numbers of bands. The students use

[1] illuminations.nctm.org/Lesson.aspx?id=2157
[2] www.openoffice.org/product/calc.html
[3] www.kinovea.org

spreadsheets to create tables, scatter plots, line of best fit and linear functions representing the relationship between the distances and numbers of bands. This mathematical model represents the relationship between their Barbie and the number of bands required to drop her safely from any height. Throughout the activity, they are introduced to the concepts of correlation, causality, line of best fit and extrapolation, along with data collection and analysis. The activity concludes with a competition between the teams, as the Barbies "jump" from a designated height (Fig. 2).

Activity 2: the human catapult

In the catapult activity (Fig. 3), students work in groups to investigate the properties of projectile motion in order to solve the problem of getting a parcel of food across a river. Particular emphasis is placed on the following: functions relating height, horizontal distance and time; angles; rates of change; and velocity. Students use an oversized slingshot along with readily available, free software to conduct their investigations, moving from a concrete exploration of trajectory, to mathematical modelling of the activity, and then to verification of the results using a projectile motion simulation.

Students use smartphones to record videos of their team using the catapult to fire a foam ball. The trajectory of the ball is analysed using the free software Tracker[4] to trace the flight path and also to generate functions relating height, horizontal distance and time. GeoGebra[5] is used for analysis of the functions, enabling the students to estimate the angle of projection, distance travelled and initial velocity of the projectile. The investigations are guided and scaffolded by the mentors/facilitators, and a basic software instruction sheet is provided. Once the students have completed the calculations based on the data collected, a projectile motion simulation[6] is used to stage a competition and validate results. Group presentations and whole class discussion conclude the activity, providing scope for formative assessment as well as an opportunity for the students to demonstrate and consolidate their learning.

Activity 3: Plinko and probability

Plinko is a game of chance based on a Galton board: a board with evenly spaced pegs arranged in staggered order, to form a triangle (Fig. 4). Balls are funnelled onto the board from directly above the top peg. If the pegs are symmetrically placed, the marbles have an equal probability of bouncing left or right as they hit each peg. A number of evenly placed slots form the base of the board, into which the marbles fall.

Students are required to develop a game for a casino by devising rules and a scoring system in such a way that the game will be appealing to players and that the casino owners will win overall. They are provided with a Galton/Plinko board template, a cork board, some pins and marbles, smartphones, laptops with open-source spreadsheet software and Kinovea[7] (a free video analysis package), and a sheet of exploratory questions.

[4] www.cabrillo.edu/~dbrown/tracker
[5] www.geogebra.org
[6] phet.colorado.edu/sims/projectile-motion/projectile-motion_en.html
[7] www.kinovea.org

Fig. 2 Barbie bungee

The aim of the activity is to encourage the students to make sense of what appears to be the random behaviour of the marbles—that is, although the path of each marble is arbitrary, if enough marbles are dropped, the emerging pattern formed by their destinations tends towards a normal distribution. In particular, they are encouraged to identify that, starting from the top, the number of routes to the pegs in the grid forms Pascal's triangle and also to understand the probability of a marble landing in a particular bin if the board were perfectly symmetrical. In addition, they analyse their own boards, using the spreadsheet to tabulate and visualise 100 rolls. They are thus able to see how well their game conformed to a digitally generated one,[8] introducing the notions of bias and fairness. They use video tracking to see if any of the marbles they roll followed the same path to any one bin, developing a practical understanding of the concept of probability.

Table 1 provides a rationale for the depiction of each of the activities as transformative.

Methodology

The interventions discussed in this paper constitute three embedded units within an overarching explanatory case study. A mixed methods approach to data collection and analysis has been employed, with emphasis placed on qualitative data (Creswell 2003; Yin 2014). Figure 5 provides an illustration of the research design.

An initial literature review informed the development of a research question and a theoretical framework to direct the research. This led to the development of the set of Design Heuristics and related 21CL activities described above. These activities were then trialled in three schools in which quantitative and qualitative data were collected. The analysis of the data and subsequent results are discussed in the following sections.

[8] http://phet.colorado.edu/en/simulation/plinko-probability

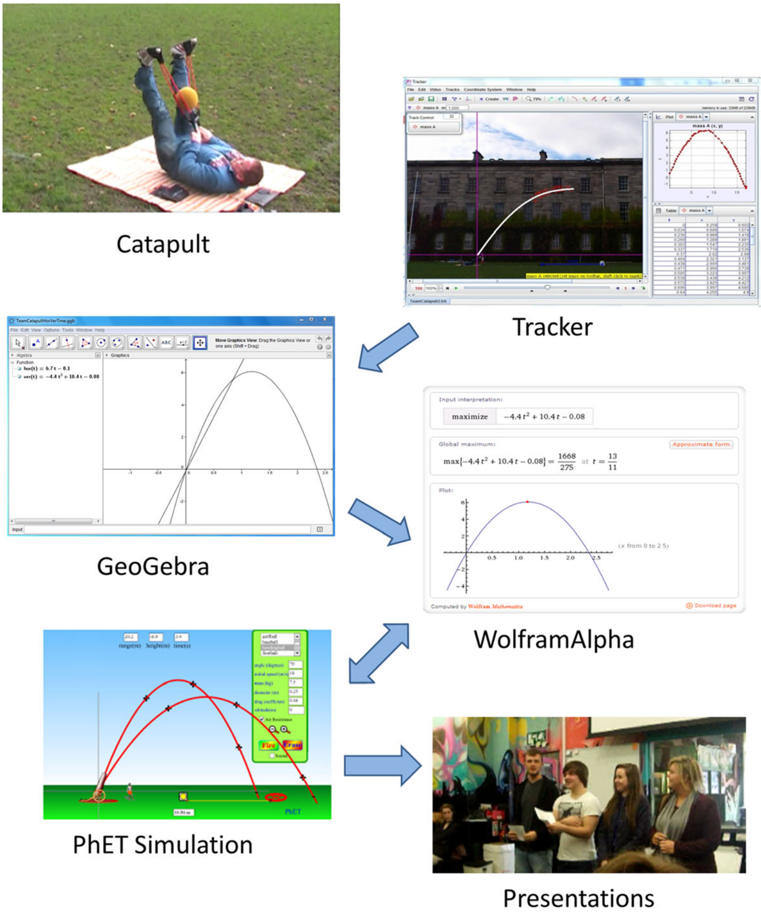

Fig. 3 Human catapult

Participants and interventions

Activities were conducted with 54 students in three secondary schools during the 2013/2014 academic year. Participating schools were drawn from a network of institutions that are working with our research centre to roll out the Bridge21 pedagogic model into mainstream classrooms (Johnston et al. 2014). All participating students had previously engaged in workshops in which they were introduced to the Bridge21 model of learning. The researchers provided laptops, smartphones and any other tools required for the activities. Ethical clearance was obtained, including permission to use the student images.

The students involved were from year 10 (age 15/16), known as "Transition Year" in the Irish system. This is a 1-year school programme that focuses on personal, social, vocational and educational development, providing opportunities for students to experience diverse educational inputs in a year that is free from formal examinations (Department of Education and Science 2004). Timetabling is more flexible than in other school years, facilitating teaching experiments that are not constrained by short class periods.

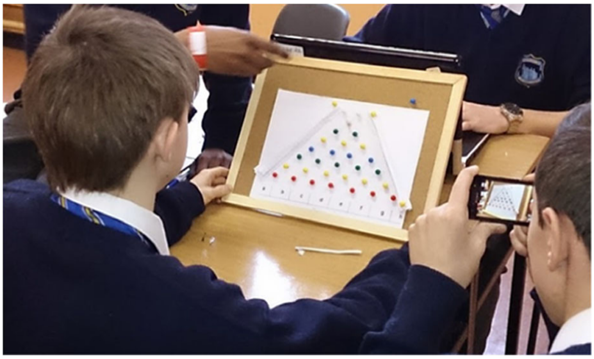

Fig. 4 Plinko

The first intervention took place in a co-educational, private school, for 2 hours per day, over the course of a week. The class consisted of 24 mixed-ability students, assigned to six groups of four students each. The working area comprised two interconnecting rooms, with easily moveable tables and chairs. Students participated in the Barbie bungee and human catapult activities. The second intervention was conducted in an all-boys school in a low socio-economic area and took place over the course of 2 days, running from 10 a.m. to 4 p.m. each day. Twenty mixed-ability students were organised into five teams. The work environment was a large, standard classroom, with moveable desks and chairs. During the course of this intervention, students engaged with the Barbie bungee and the Plinko activities. The final intervention was significantly shorter than the others. It was conducted in an all-girls school in a low socio-economic area and took place over the course of 2 hours in a single afternoon. Ten students participated in the Barbie bungee activity.

In each case, teams were selected to balance abilities amongst groups. In the first two interventions, the first author was the primary facilitator/teacher for the activities, accompanied by one classroom assistant. The last intervention was run by the class teacher, with the first author acting as classroom assistant in a participant-observer role.

Data collection and analysis

The Mathematics and Technology Attitudes Scale (MTAS) (Pierce et al. 2007) was utilised to collect quantitative data and focus-group interviews provided qualitative data. MTAS is a 20-item questionnaire with five subscales: affective engagement (AE—how students feel about mathematics), behavioural engagement (BE—how students behave when learning mathematics), mathematical confidence (MC), confidence with technology (TC) and attitude to using technology for learning mathematics (MT). Pierce et al. (2007) highlight that "students' vocabulary and behaviour indicating confidence and engagement will be dependent on local culture and context" (p. 289). As MTAS was initially designed and trialled in Australia, the reliability tests described in the original paper were applied to 148 responses from Irish students. Factor analysis confirmed the five-factor structure of the Australian scale, and while the Cronbach's alpha test highlighted some cultural differences, the scores remained satisfactory for each of the subscales (MC, 0.89; MT, 0.81, TC, 0.91; BE, 0.68; and AE, 0.74). The instrument was administered to students before and after the interventions, and paired t tests were used to analyse the data.

Table1 Activities and technologies

Activity	Digital technology	Usage/software	Transformative rationale
Barbie bungee	Smartphones Laptop	Video capture Kinovea Speadsheet Calculator/Internet resources, e.g. wolframalpha.com	A pencil and paper approach to the Barbie bungee activity forms the basis of the original task. However, the use of technology in the activity described herein has permitted significant task redesign, facilitating the generation of realistic data and mathematical modelling. It is thus classified at the level of *modification* on the SAMR hierarchy.
Human catapult	Smartphones Laptop	Video capture Tracker GeoGebra Simulation Calculator/Internet resource	The task involved in this exercise would not have been possible without the use of technology, leading to its classification as *redefinition* in terms of the SAMR hierarchy. The functions are generated through frame-by-frame analysis of video footage, which would not be feasible without the use of the software. The students are required to make extensive use of the computational facility afforded by the technology in a task that is designed to be engaging and immersive.
Plinko and probability	Smartphones Laptop	Video capture Kinovea Simulation Spreadsheet Calculator/Internet resource	The probability and Plinko activity could conceivably be conducted without the use of digital tools; however, the technological mediation of the activity permits a richer, more inquiry-based approach—allowing deep exploration of phenomena that would not be easily done without the use of technology—and placing the activity at the level of *modification* on the SAMR hierarchy.

Focus-group interviews were conducted between 2 and 4 weeks after each intervention. The duration of these interviews was between 20 and 40 min, and the groups were made up of between four and six participants. Namey et al. (2007) suggest that there is no single, right way to approach qualitative analysis of data, and an assortment of different approaches that build upon each other may be preferable. In this study, directed content analysis (DCA) was initially used; this is a theory-driven approach that provides a framework to focus on areas of particular interest (Hsieh and Shannon 2005; Krippendorff 2004; Namey et al. 2007; Yin 2014). This was followed by a re-examination of the data using constant comparative techniques (Glaser 1965; Glaser and Strauss 1967; Strauss and Corbin 2008).

Approaches that work the data from "the ground up", such as constant comparison (CC), contrast directly with DCA, in that they are not based on a priori theoretical propositions (Yin 2014). While DCA remains open to emergent coding (Namey et al.

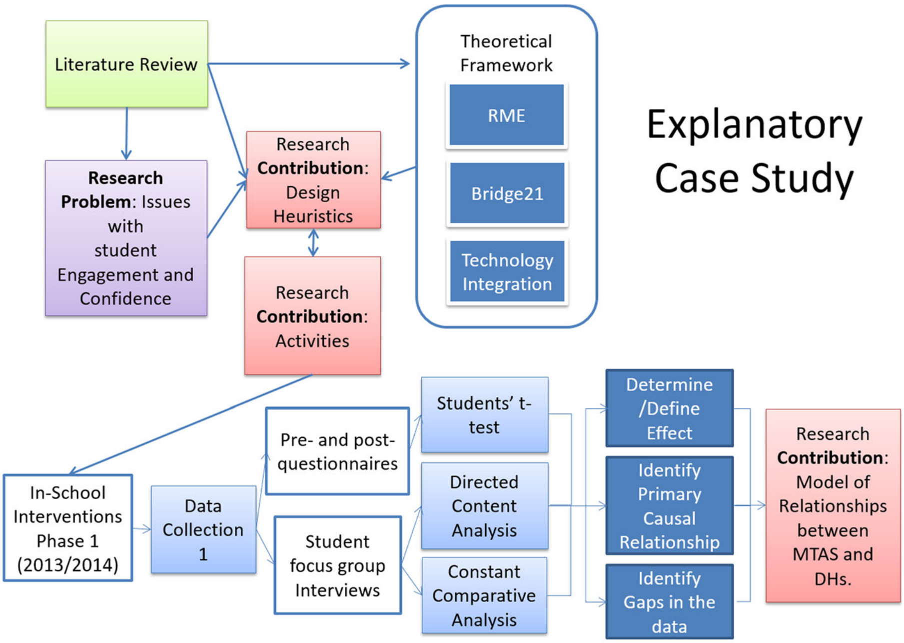

Fig. 5 Research design

2007), this is generally restricted to portions of the data that have not been already coded according to the theory. CC considers the entire data set free from pre-conceived ideas and thus allows for a broader spectrum of emergent themes and a fuller understanding of the properties of, and relationships between, such themes. As the embedded units within this case study are diverse in many ways (location, duration, organisation, number of students, socio-economic background and so on), CC has been used in addition to the directed approach, in order to be sensitive to emerging themes and to give a richer understanding of the relationships between the codes.

Statistical analysis

Paired t tests were used to analyse the pre-/post-test change on the MTAS scores for the 54 participating students. There were gains in all subtest scores, with significant

Table 2 t test results

	Mean-pre	SD-pre	Mean-post	SD-post	$t(53)$	p
AE	12.4	3.6	14.2	3.3	−4.267	0.000
BE	14.2	2.9	14.8	2.8	−1.867	0.068
MC	12.9	3.9	13.4	3.8	−1.947	0.057
TC	15.0	3.1	15.4	3.5	−1.410	0.165
MT	13.7	2.8	15.4	3.7	−3.140	0.003

differences identified in the AE and MT subscales (Table 2—pre- and post-tests are scored out of 20).

Directed content analysis

The MTAS subscales were used as a priori codes to direct content analysis of the interviews using the qualitative analysis software tool, NVivo10. Use was also made of codes drawn from the set of Design Heuristics described above. Figures 6 and 7 show the categorisation matrices used for the directed coding (Elo and Kyngäs 2008). The generic categories in Fig. 6 differentiate between the five components of MTAS (AE, BE, MC, TC, MT). The MTAS categorisation matrix subcategories differentiate between positive and negative references, and between the students' experience of their traditional class, and the mathematics learning activities (MLAs) of the intervention (e.g. BE_MLAsPos relates to positive references to behavioural engagement associated with one of the mathematics learning activities previously described). The categories defined in Fig. 7 are taken directly from the set of Design Heuristics. Coding matrices generated by NVivo10 facilitated comparisons of the subcategories of the Design Heuristics and the MTAS subscales permitting the generation of tentative conjectures as to the primary factors that cause the change in student engagement and confidence evident in the MTAS scores.

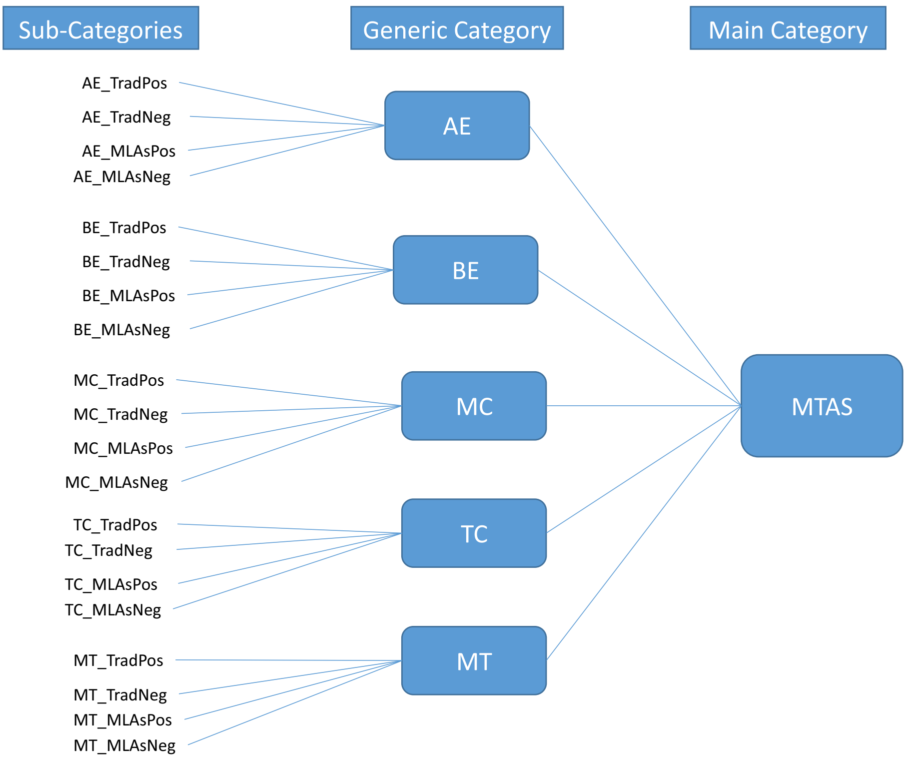

Fig. 6 MTAS categorisation matrix

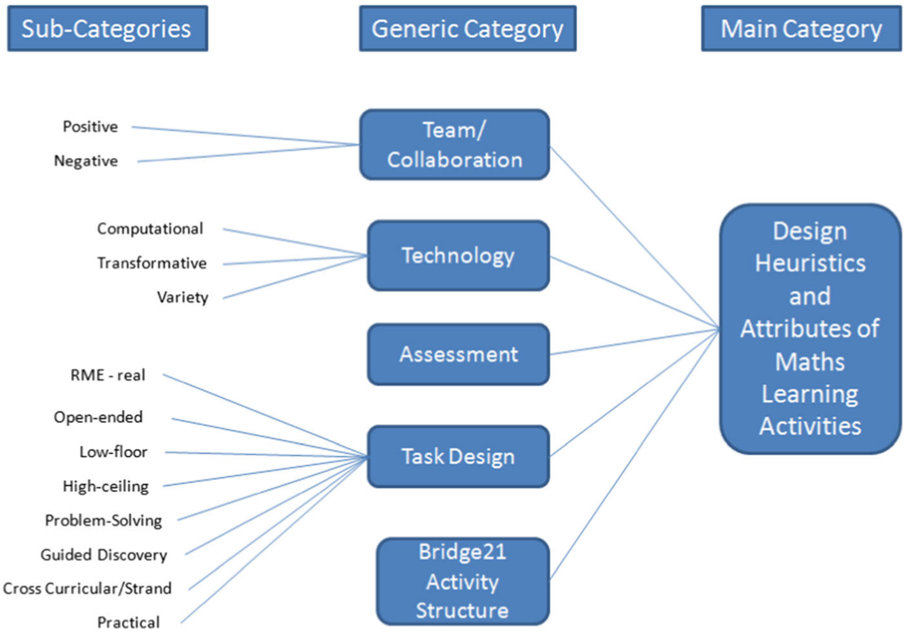

Fig. 7 Design Heuristics categorisation matrix

Coding comparison

In order to ensure reliability of the results of the directed content analysis, a second coding of the data was conducted by a researcher from outside the project. Coding matrices and schema were supplied to the researcher in order to direct the analysis of the interviews. An NVivo coding comparison query was used to compare the two analyses. The average Cohen's kappa coefficient across all of the data was 0.8, which Landis and Koch (1977) suggest demonstrates a "substantial" to "almost perfect" agreement between the coders.

Constant comparative analysis

After completion of the process of directed coding, and owing to the richness of the data that stemmed from the diversity of the embedded units of analysis in the case study, a second analysis of the data using constant comparative techniques was conducted. The purpose of this was to attempt to fully grasp emerging themes, providing a more complete description of the case.

Constant comparative analysis involves generating initial codes and categories; comparing these codes; reducing the codes into appropriate, comprehensive categories; and analysing emerging relationships (Glaser 1965; Merriam 1998; Strauss and Corbin 2008). The end product of the second analysis was a set of emergent categories, subcategories and associated codes, which were useful for the development of a rich description of the case, and provided an opportunity to develop further hypotheses relating to the relationships between the Design Heuristics and the perceived increases in engagement and confidence.

The primary categories that emerged throughout the constant comparative process were motivation, task design, learning, negative attitude and traditional approach, each with a number of related subcategories. Many possible relationships between codes and categories were highlighted through the analysis, but those that emerged most frequently, and which were most relevant to the research questions, were between the elements of the task design and their impact on motivation and on learning outcomes.

Congruence between constant comparison and directed content analysis

There were two motivators for analysing the qualitative data using these techniques. One was to ensure that nothing relevant had been missed during the initial directed content analysis of the interviews, and the other was to further tap into the rich interview data in order to deepen our understanding of the relationship between the MTAS results and the Design Heuristics. At first glance, the only theme common to both methods of analysis was task design. However, the categories of motivation, learning and Beliefs were easily mapped to different aspects of affective or behavioural engagement, mathematical or technical confidence, and attitude to using technology for learning mathematics. The process of mapping (described in the "Findings" section) served to deepen the understanding of the different motivations for, and learning outcomes of, the MTAS subcategories.

In order to probe these relationships, the codes and categories developed through constant comparison were compared and mapped to the codes that directed the initial content analysis. NVivo10 matrix coding was used to identify crossover of the different areas of the analysis. Table 3 provides a clear indication of the crossover of references coded using the Design Heuristics categorisation matrix and the relevant codes from the constant comparative approach. Similar tables were generated to identify relationships between the analysis directed by MTAS categorisation matrix and the relevant aspects of the comparative approach. The relationships between the elements of the Design Heuristics and students' engagement and confidence are discussed in the following section.

Findings

Statistically significant differences can be difficult to register in small-scale interventions. Bearing this in mind, the fact that the pre-/post-improvement was statistically significant on two of the MTAS subscales is a noteworthy result. However, it is important to also investigate any confounding factors that emerge through the qualitative analysis.

The pre-/post-test changes recorded on the subscales of the MTAS instrument were initially explored through directed content analysis of focus-group student interviews. Coding matrices facilitated comparison of the subscales with the Design Heuristics, permitting the generation of conjectures as to the primary factors that may have encouraged student gains in engagement and confidence evident in the quantitative data.

Table 3 Crossover between directed content and constant comparative coding relating to Design Heuristics

Directed Coding v Constant Comparative	Assessment	Bridge21 Activity Structure	Task_ cross-strand	Task_ guided discovery	Task_ high-ceiling	Task_ low-floor	Task _open-ended	Task_ practical	Task_ problem-solving	Task _RME-real	Team	Tech_ Computational	Tech_ Transformative	Variety of Tech
Adaptable	0	0	0	0	0	0	0	0	0	0	0	0	0	2
Enabled by technology	0	3	0	1	0	0	0	0	1	2	0	7	5	1
Task_Active-Hands-on	0	4	0	5	0	0	0	10	2	5	0	0	2	1
Task_ Assessment	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Task_ Bridge21	0	8	0	3	1	0	1	1	3	1	2	0	0	0
Task_ Context	0	0	1	2	0	0	0	7	0	7	0	0	3	0
Task_ Cross-strand	0	0	4	0	1	0	1	0	0	0	0	0	2	1
Task_ High Ceiling	0	2	0	1	4	0	4	1	3	1	2	0	0	0
Task_ Meaningful	0	4	1	6	4	0	4	9	6	9	0	0	4	0
Task_ Open-ended	0	7	0	3	5	0	5	1	7	2	2	0	1	0
Task_ Preparation	0	3	0	1	0	0	0	0	0	0	0	0	0	0
Task_Presentation	2	4	0	0	0	0	0	0	0	0	0	0	0	0
Task_Problem-solving	0	5	0	4	2	0	2	2	6	2	2	0	0	0
Task_Real life	0	0	0	1	0	0	0	6	0	3	0	0	1	0
Task_Team	0	5	0	1	0	0	0	1	2	1	11	0	0	0
Task_Technology-mediated _Outsourcing	0	0	0	0	0	0	0	0	0	0	0	4	4	0
Task_Technology-mediated _Transformative	0	3	0	0	0	0	0	0	1	1	0	5	7	3
Task_Timing	1	11	1	0	0	0	0	0	0	0	0	1	1	0
Task_Useful - Practical	0	0	0	1	0	0	0	5	1	1	0	0	2	0

Legend
Highest score in both rows and columns
Highest score in rows (CC codes)
Highest score in columns (DC codes)

Initial results suggest that the aspects of the Design Heuristics most associated with affective engagement are the realistic (in the RME sense), cross-curricular and guided discovery aspects of the task design, the Bridge21 activity structure and the transformative use of mobile technology, which all facilitated the realistic nature of the tasks. Behavioural engagement is also positively associated with the realistic, practical and guided discovery aspects of the task design, the activity structure and the transformative use of mobile technology; additionally, the impact of working in a team also appears to have had a positive effect. Mathematics confidence is positively associated with real, guided and practical tasks, with use of technology also appearing influential. Not surprisingly, the use of technology, both transformative and computational, is most significantly related to confidence using technology, with the variety of technologies noted as adding to flexibility and adaptability. The transformative and computational use of technology, in conjunction with the task design, appears to have the most influence on students' attitude to using technology for learning mathematics.

The majority of negative responses emerged from the students' experiences of their conventional mathematics lessons. However, some negative associations were also drawn between behavioural engagement and teamwork—in particular if the allocation of the workload was not considered fair—and between technical confidence and the use of technology, particularly when devices did not work as planned. A more in-depth exploration of the MTAS subscales, the Design Heuristics, and how they relate, was permitted through the comparison of the results of the two methods of qualitative analysis (e.g. Table 3). Modelling the relationships that emerged from this comparison leads to a number of insights.

Task design

One significant point to emerge from comparison of the methods was that the task design aspect of the Design Heuristics could be more subtly differentiated, leading to a more comprehensive guide for teachers, and clearer descriptions of the attributes of the tasks. Figure 8 illustrates the relationship between the coding processes, with the bold line indicative of the strongest overlap.

Using this more comprehensive model of the Design Heuristics, it is possible to give more detail to the aspects of the task design that seem to be having the greatest impact on the MTAS subscales.

MTAS and Design Heuristics

Affective engagement

AE presented a statistically significant change between pre- and post-tests (Table 1). Comparison of the analyses indicates that positive AE was generated by interest, a desire for understanding, curiosity and a sense of ownership within the student cohort. The connectedness, practicality and realistic nature of the learning outcomes also appear to have had a positive impact. For some

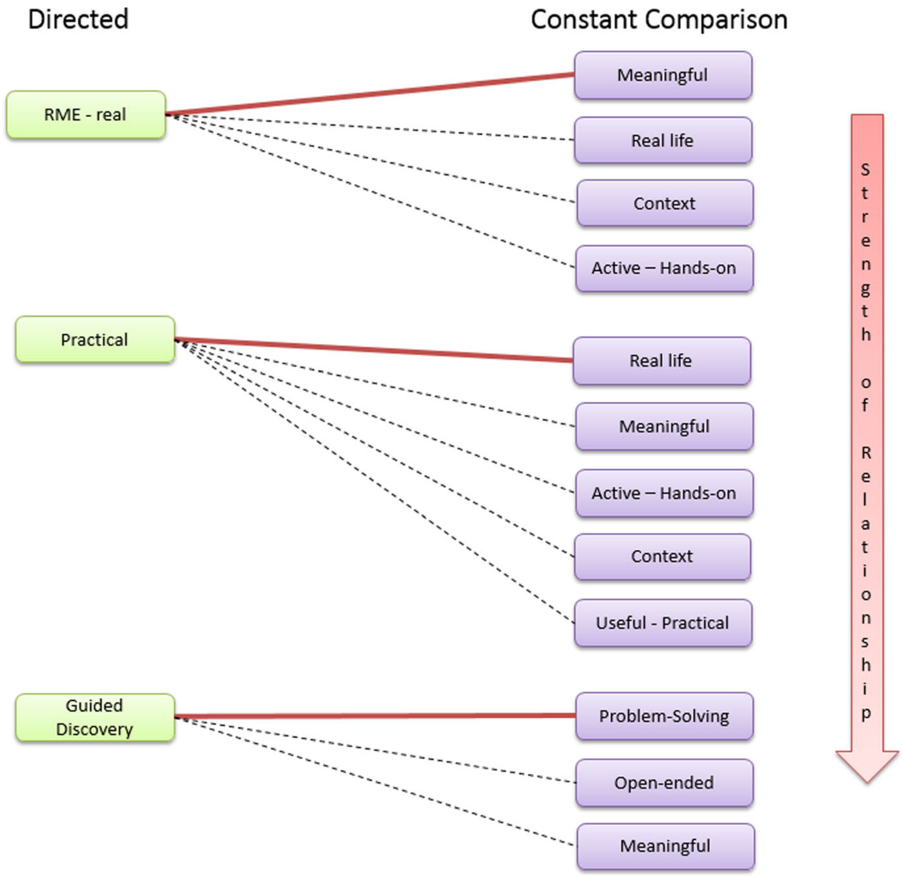

Fig. 8 Overlap of task design codes

students, engaging in the activities changed their outlook on mathematics completely:

Student A, School 1: It changed the way I look at maths.

Student B, School 1: It was a life changing experience.

Comparison of the analyses illustrates that the positive increase in AE is most significantly correlated with the sense of meaningfulness experienced by the students. This is not only strongly linked to the RME aspect of task design but is also connected to the idea of guided discovery—participants were not given all of the instructions at the beginning, but instead were scaffolded through their own discovery of the mathematics. The hands-on nature of the tasks and the open-ended, high-ceiling attributes of the activities also appear to have a positive effect on AE:

> Student A, School 3: You're looking at a piece of paper and thinking "this is impossible, like, how is this possible??" I was just like, "I can't do this." Then you gave us kind of hints and stuff, like there are different ways… I started thinking of shapes and everything you used to do in maths …and different ways you can do [things].

Behavioural engagement

The most significant motivator for BE appears to be a desire for understanding, with ownership, being part of a team and curiosity also significant. The team-based environment also seems to contribute directly to peer learning, encouraging weaker students to participate positively in the class. The practical and conceptual aspects of the learning outcomes are also recognised as motivating factors.

> Student B, School 3: We learnt much more. Because we learned by what we did. It was me and not just what someone said.

Similar to AE, the meaningful aspect of the activity design, linked to guided discovery and RME, has the most significantly positive effect on BE. Not surprisingly, the fact that the activities were active had an effect on student behaviour within the classroom, but so too did the open-ended nature of the tasks, and the fact that they were team-based. Other aspects associated with positive BE were the problem-solving, contextual task design and the use of technology. In particular, the use of mobile technology facilitated the transformative nature of the activities, permitting learning through social as well as content interactions, in a variety of contexts (Crompton 2013).

> Student A, School 2: It was kind of like maths through computers and things. And ways - different ways - to learn maths. Basically, more exciting and involving ways for the people.

Mathematical confidence

The most significant indicator of positive MC in the students was easily identified as, once again, a desire for understanding:

> Student B, School 2: And this kind of explained it a bit more. And better, like.

The confidence that the students felt seems to stem from a real, personal understanding of the mathematical concepts that underlie the activities in which they participated:

> Student C, School 2: You can have your own idea, even if the teacher is explaining it wrong, or, well not wrong, but like in a different way. It's like

you have your own idea about it and you can add to what they are telling you to do.

Student B, School 2: It's like you're adaptive to it. It's something you've never seen before and you get someone just to show you how to do it and then… you might not be able to do it yourself, but you'll be able to figure out a way to do it and you'll eventually get there.

Evidently, their confidence is bolstered by a sense of ownership, but also the practical nature of the learning outcomes and the developing understanding of new content are all indicative of increased MC.

Associating the elements of the task design with the positive MC indicates that the contextual aspect of the Design Heuristics, along with the active nature of the tasks, has the most impact. In addition to this, the realistic, meaningful and guided elements of the design also appear to have had a positive effect.

Student A, School 2: You're actually seeing it happening in front of you and you have to figure out what's happening for yourselves.

For less able students, the peer learning that emerged in the mixed-ability teams led to an increased level of confidence.

Student C, School 1: We had one person in ordinary level, and I think she didn't know what functions were, and I think [student x] explained it to her… and so it made more sense to her then.

The use of technology also appears to have a very positive impact on some of the students' confidence with the mathematics. The potential to outsource some of the calculation was particularly impactful, but the ability to represent, relate and manipulate things using technology also led to increased confidence.

Student D, School 1: Although it was stuff that I understood, actually, that should be "understood", in quotation marks! Stuff that I had been taught, but didn't really fully get - because it was on a computer, it was a lot easier for me to deal with.

Technological confidence

TC appears to be the simplest MTAS subscale. It is motivated by a desire to use technology and is strengthened through learning outcomes related to technology. Unsurprisingly, the aspect of the Design Heuristics which is most significantly associated with TC is technology-mediated tasks. However, TC showed the least significant change across pre- and post-tests. One possible reason is that the students were already confident using technology and thus engaging with it in a mathematical context did not have a strong impact on the level of student confidence. This suggestion is supported by the fact that mean pre-test scores were highest in this category and that the majority of students felt they were able to adapt quickly to the technology and would not require prior instruction to be able to use it in a meaningful way:

> Student D, School 2: It only took us about 5 minutes to get used to it.

> Student B, School 2: Yeah, it was pretty simple to use.

That is not to say that use of technology did not have any impact on the students. For some, using the software permitted them to be more playful with the mathematics.

> Student D, School 1: The software was the push for me, using the computers was really handy, because it meant that I could understand it and have fun with it, without having to stress about getting it wrong. Because, as long as I typed in the right numbers, it was going to be okay.

Attitude to using technology for learning mathematics

Quantitative analysis of pre-/post-test MT scores indicates a statistically significant improvement in student attitudes. A positive attitude to using technology for learning mathematics can be identified with motivation to learn technology, and learning outcomes associated with technology, but also with a desire for understanding that can be realised through the use of digital tools.

> Student D, School 2: It's just instead of like, having to measure it and hold it up against a giant ruler, and then drop it down, you can just use that software that we were using. It's a lot easier. Saves a lot of time.

> Student A, School 2: Yeah, and it kinda shows you the way, like how you get all your measurements and all, if you get me?

In addition to the increased speed and accuracy that the use of technology can foster, an increased level of interest was also identified as a factor relating to positive MT

> Student C, School 2: You're going to have much more interest when you can use computers and other physical things, instead of just thinking.

In addition to the technology-mediated aspects of the task design, analysis indicates significant association with the categories "Enabled by technology" and "adaptable", illustrating that students' positive attitude was also linked to the flexibility offered by the technology.

Discussion

This study set out to answer two questions. The first related to effects on student engagement and confidence that participation in activities designed in accordance with a particular set of heuristics may have. The second focused on the identification of primary factors that may have impacted on the recorded changes in engagement and confidence.

The findings of this research provide a compelling story of the positive effects that activities designed in accordance with the Design Heuristics can have. Exploration of the relationship between the positive aspects of the MTAS subcategories and the Design Heuristics has led to identification of the activity attributes that appear to have the most significantly constructive impact and has also underlined some of the possible rationales for the associations. In particular, the RME realistic activities provide students with tasks that are situated in contexts that they perceive as meaningful. They are interested in solving the problems and challenges and want to understand the mathematics in order to be able to achieve this. This attribute of the tasks is particularly positively associated with affective and behavioural engagement, and mathematical confidence. The practical, as well as the hands-on, nature of the task design also emerges as strongly correlated with AE, BE and MC.

The guided discovery approach, which requires problem-solving of open-ended tasks, also appears to be positively associated with AE, BE and MC. This seems to be related to a sense of ownership and autonomy in the students over their own learning processes and appears to lead to an increase in conceptual understanding of the mathematics involved in the activities.

The impact of teamwork on AE, BE and MC is predominantly positive. Most students seem to like working in teams, which leads to increased enjoyment of mathematics and improved participation in the class. In addition, the mixed-ability groups facilitate peer learning in a supportive and exploratory environment. Associated with this is the low-floor/high-ceiling aspect of the tasks, which permits all of the students to meaningfully engage with the activities. Unsurprisingly, MT and TC are most associated with the technological aspects of the heuristics. The students generally find the technology helpful and straightforward to use, with some evidence of it supporting their development and understanding of mathematical concepts, connections and representations.

The small number of negative associations with the interventions relate primarily to unsuccessful teamwork and technological failures. This emphasises the importance of careful selection of the team members in order to ensure that they can work well together. Technological problems can be difficult to anticipate, but careful planning and practice have proven to be somewhat effective in alleviating problems.

Conclusions

While these initial results are promising, the relative novelty of the approach may be a contributing factor. Although the interventions were conducted in conventional school environments, they did so in a year in the Irish school system that allows for flexible approaches to curriculum and timetabling. If, however, the findings can be replicated—both for repeated use with students and also for classes following syllabi leading to state examinations—it would augur well for addressing some of the issues surrounding student engagement and confidence identified in the literature regarding mathematics.

In addition to the modified school environment, differences between the interventions themselves are also not examined. It would be interesting to explore whether the various activities are received differently or whether the duration and setup of the activities have an impact. These are all areas that can be addressed in future research.

This study set out to identify whether activities designed within a technology-mediated, socially constructivist, RME setting could increase student engagement with, and confidence in, mathematics, and if so, what were the most significant factors that caused the shift. As detailed in the Discussion section above, positive effects are evident, particularly in relation to students' affective engagement and attitudes to using technology for mathematics learning. It thus appears likely that the Design Heuristics described in this paper adequately outline an approach to the design and implementation of RME/Bridge21-style mathematics learning activities that have the potential to increase student engagement and confidence with the subject. Students are motivated by problems that appeal to their interests, set in meaningful contexts. They develop a sense of ownership and autonomy over their learning, all of which results in an improvement in student attitudes, behaviour and confidence.

Compliance with ethical standards Ethical clearance was obtained, including permission to use the student images.

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