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ORIGINAL ARTICLE

An Early Mathematical Patterning Assessment: identifying young Australian Indigenous children's patterning skills

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Abstract This paper presents an Early Mathematical Patterning Assessment (EMPA) tool that provides early childhood educators with a valuable opportunity to identify young children's mathematical thinking and patterning skills through a series of hands-on and drawing tasks. EMPA was administered through one-to-one assessment interviews to children aged 4 to 5 years in the year prior to formal school. Two hundred and seventeen assessments indicated that the young low socioeconomic and predominantly Australian Indigenous children in the study group had varied patterning and counting skills. Three percent of the study group was able to consistently copy and draw an ABABAB pattern made with coloured blocks. Fifty percent could count to six by ones and count out six items with 4 % of the total group able to identify six items presented in regular formations without counting. The integration of patterning into early mathematics learning is critical to the abstraction of mathematical ideas and relationships and to the development of mathematical reasoning in young children. By using the insights into the children's thinking that the EMPA tool provides, early childhood educators can better inform mathematics teaching and learning and so help close the persistent gap in numeracy between Indigenous and non-Indigenous children.

Keywords Early mathematics assessment · Early numeracy · Indigenous education · Patterning

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Introduction

The Australian Government is committed to closing the gap in Indigenous disadvantage "to improve the lives of Indigenous Australians, and in particular provide a better future for Indigenous children" (Department of Education, Training and Employment [DETE] 2013). Although this commitment has seen an increase in the number of children accessing preschool education—in 2013, 85 % of Indigenous children in remote areas were enrolled in a preschool programme (Australian Government 2015)—statistics continue to show Indigenous children have the lowest educational outcomes in Australia. Results from the 2014 national literacy and numeracy tests (NAPLAN) for Year 3 (children aged 8–9 years) show that in all achievement domains (reading, persuasive writing, language conventions and numeracy) and for all jurisdictions, the mean scale score for Indigenous students was well below the mean scale score for non-Indigenous students (Australian Curriculum, Assessment and Reporting Authority [ACARA] 2014). In numeracy, in 2014, 95.7 % of non-Indigenous Australian children in Year 3 were at or above the national minimum standard, compared to 78.2 % of their Indigenous counterparts. Although the extent of this difference in numeracy outcomes varied year on year, from 12.8 to 22.2 percentage points, NAPLAN data from 2008 to 2014 shows little discernible recent progress towards closing the long-term gap. NAPLAN results also show that a similar or widening gap in numeracy outcomes persists across the school year (ACARA 2014).

Importance of mathematics in the early years

There is an increasing recognition of the importance of learning in the early years and specifically of early mathematical learning (Doig et al. 2003). Clements and Sarama (2007), for example, argue that early intervention in mathematics prevents later learning difficulties in school and beyond. High-quality, developmentally appropriate early years programmes produce both short- and long-term positive outcomes in children's cognitive and social development (Barnett 1995; Clements and Sarama 2007).

The quality and quantity of early mathematical experiences are the main factors in determining future mathematical success (Doig et al. 2003). As van Tuijl et al. (2001) found in their longitudinal study, the "long-lasting impact of an unfavourable start in formal education [is that] initial disadvantage seldom disappears, and there is evidence that gaps tend to widen" (p. 148). Aubrey et al. (2006) conducted a large-scale longitudinal study and concluded that "without active intervention it seems likely that children with little mathematical knowledge at the beginning of formal schooling will remain low achievers throughout their primary years and probably beyond" (p. 44).

Research on early mathematics learning shows that children from low socioeconomic backgrounds and other disadvantaged backgrounds have a lower level of achievement in mathematics than their peers when entering formal schooling (Tymms et al. 2004; Thomson et al. 2005). As Australia's Indigenous population continues to fare poorly on key socioeconomic measures, in comparison to non-Indigenous Australians, young Indigenous children are overwhelmingly disadvantaged on entering formal schooling (Australian Institute of Health and Welfare [AIHW] 2014).

Earlier studies by the author show that early and ongoing disparities in educational achievement can be overcome. Effective interventions in the early years can build children's mathematical thinking and reasoning, thereby presenting invaluable opportunities to help level the playing field before the critical start of formal schooling. In one study focused on the development of patterning strategies in young children ($n=53$) (Papic and Mulligan 2005), one preschool implemented a 6-month intervention focusing on repeating and spatial patterns. The second preschool, a comparison group, implemented their regular preschool programme and were not involved in the intervention. The intervention group outperformed the comparison group across a wide range of patterning tasks at post-intervention. Intervention children demonstrated greater understanding of patterns as repeated units and spatial relationships. In contrast, most of the comparison group treated repeating patterns only as alternating items without seeing the units of repeat. Children from the comparison group also rarely recognised simple geometrical patterns. Intervention children were able to justify various patterns and transfer patterns to different media. One year after the intervention, the intervention children continued to outperform the non-intervention children on patterning tasks, including growing patterns, and also outperformed the non-intervention children on a standard numeracy assessment at the end of the first year of formal schooling.

In a second study, 64 children and nine early childhood educators from two long day care centres (one Aboriginal community-controlled service and one mainstream service) participated in a 10-week intervention focused on patterning. The professional development component in this study concentrated on building early childhood educators' understanding of different types of patterns, early algebraic thinking and approaches to developing children's mathematical thinking including problem solving, reasoning and generalising (Papic 2013). Research from these two studies with both non-Indigenous and Indigenous children in early childhood settings suggests young children have the potential to benefit from appropriate early intervention in mathematics learning. Both studies provided empirical evidence that young children are capable of developing sophisticated pattern concepts and skills and that prior to formal schooling children can abstract, generalise and explain patterns and pattern structures. Children are capable of viewing patterns from different orientations and can use various materials to create complex linear, cyclic and 3D patterns.

These finding are broadly consistent with other research, such as a study by Starkey et al. (2004) of a mathematic intervention involving 163 pre-kindergarten children (3 years 9 months–4 years 9 months). The study included a control group who did not participate in the intervention. Children were assessed pre- and post-intervention "in order to examine the effectiveness of the curricular intervention in enhancing preschool children's mathematical knowledge" (p. 104). Intervention children's mathematical knowledge was assessed using the Child Math Assessment (Klein et al. 2000) which comprised 16 tasks that assessed informal mathematical knowledge (e.g. number, arithmetic, space, geometry, measurement, patterns and logical relations), numerical knowledge (e.g. counting, number comparison and ordinal numbers) and operations (e.g. addition and subtraction). A significant socioeconomic-related gap in mathematical knowledge was found at the beginning of the study. However, the intervention significantly enhanced the mathematical knowledge of children at both low and middle socioeconomic levels. "Low-income children acquired more knowledge, relative to their starting point, than middle-income children. The (resulting) extent of mathematical knowledge was similar in low-income intervention children and middle-income comparison children” (p. 99).

A study of Indigenous children yielded similar insights into the potential benefits of early education. The Young Australian Indigenous Students Literacy and Numeracy (YAILN) study involved 120 children attending prep (non-compulsory first year of school) and their teachers at five schools in the state of Queensland. One aspect of the study was pre- and post-one-on-one assessment interview (Warren et al. 2008). The interview schedule comprises three “tests”: (i) number, (ii) patterning and (iii) oral language test. The findings indicated that children who participated in this optional, additional year of early education “not only possessed a better understanding of numbers to 5 but also the associated mathematical language used to access this understanding” (Warren et al. 2008, p. 552).

Consequently, greater awareness of the importance of mathematics in the early years and the broader goal of closing the gap in numeracy achievement between Indigenous and non-Indigenous children prior to formal schooling suggest early intervention enabled by appropriate mathematics assessment tools and associated learning trajectories for young children has considerable potential. To intervene effectively, early childhood teachers need to be able to first determine individual children’s skills and to understand how they think mathematically. Such tools and approaches can ensure all children, regardless of socioeconomic status, receive the best possible start in mathematics and that mathematical learning in the early years provides the necessary foundation for life-long learning. However, given the urgent need to support young Australian Indigenous and/or disadvantaged children to make the best start to their formal schooling, and so help combat the persistent attainment gap across the school years, this group is the focus of our study.

Research on early mathematics assessment

In recent years, there has been an increase in the number of assessment tools available to identify the mathematical skills and competencies of young children to inform teaching and learning (Polignano and Hojnoski 2012). However, there are few tools specifically tailored to suit children from 3–5 years, and those available are limited in their scope (Clement et al. 2008). Assessment tools for young children also focus primarily on number knowledge and number sense rather than the broader domains of mathematical thinking and reasoning such as patterning and early algebra, for example: the Woodcock-Johnson III (Woodcock et al. 2001) and the Test of Early Mathematics Ability (Ginsburg and Baroody 2003).

Howell and Kemp (2010) in their study with 176 children in preschools and childcare centres assessed young children’s number sense prior to school entry. Assessment items included the following: counting components (e.g. rote counting, ordinal value and addition and subtraction stories), number principles (e.g. order irrelevance, inversion and commutative addition) and number magnitude components (e.g. subitising and ordering groups). Howell and Kemp concluded that most Australian children enter formal schooling at least being able to count to 10. However, they also acknowledge that children have varied mathematical language and skills in the early years.

While number knowledge and number sense are important, the integration of patterning into early mathematics learning is critical to the abstraction of mathematical ideas and relationships and to the development of mathematical reasoning in young children (Papic et al. 2011). In the earlier study of 53 children in two preschools (Papic and Mulligan 2005), an Early Mathematical Patterning Assessment (EMPA) tool implemented prior to and directly after the 6-month intervention was successful in assessing children's proficiency with repeating, spatial and growing patterns. However, as the tool consisted of 25 individual tasks, it required a revision to include simpler and shorter interview-based tasks. This revised EMPA was designed to be a "fun" series of hands-on tasks implemented within 15 min. This time frame and approach to assessment attempted to alleviate unnecessary stress on young children. The revised EMPA, containing 13 repeating and spatial pattern tasks, was trialled in the second study reported earlier with 64 preschoolers across two early childhood settings and was then utilised in the larger study reported in this article. This revised tool is aligned with research-based learning trajectories focused on repeating and spatial patterns. It recognises that to build successful learning trajectories, assessment tools that emphasise "assessment for learning" are needed that can identify the starting points from which individual children can progress, rather than the more familiar "assessment of learning" that may only serve to situate individual children within a group or on an achievement scale.

The Patterns and Early Algebra Preschool (PEAP) Professional Development (PD) project

Based on emerging research on young children's patterning and early algebraic thinking (e.g. Blanton and Kaput 2005), a mathematical patterning and early algebra programme was developed for young children (4–5 year olds) who were enrolled in their final year of non-formal education. The Patterns and Early Algebra Preschool (PEAP) Professional Development (PD) project (Papic 2013), focused on "developing young children's awareness of pattern and structure in order to promote structural development, relational understanding and generalisation, albeit emergent, from an early age with a view to laying the foundation for mathematical thinking" (Papic et al. 2015). The project provided early childhood educators with intensive professional development in their childcare centre to enhance their mathematical pedagogical and content knowledge. The professional development also trained educators in the use of the assessment tool, EMPA. Two hundred and fifty-five children were assessed at the start of the project using the EMPA. Educators were supported to analyse children's responses in order to inform their teaching and learning.

The use of the EMPA aimed to establish the effectiveness of the tool in supporting interventions to enhance young children's mathematical development, to consequently:

1. Advance young Indigenous and/or disadvantaged children's early algebraic thinking and mathematical reasoning and
2. Close the gap in numeracy achievement for Indigenous children

Theoretical framework

Pattern recognition is considered a sign of one's ability to generalise and abstract ideas (Papic et al. 2011), and research has shown that it is integral to the development of mathematical cognition: the processes of representation, symbolisation, abstraction and generalisation (e.g. Liljedahl 2004; Mulligan and Mitchelmore 2009). Pattern formation is also considered essential in developing and understanding structural relationships in mathematics, and an awareness of mathematical structure is critical to mathematical reasoning and problem solving. Recent studies show that the early development of pattern and structure influences positively mathematical achievement overall and enables a stronger foundation for algebraic thinking (Mulligan et al. 2006; Warren 2005). However, as most contemporary studies focus on the 6- to 8-year age range, many unanswered questions remain as to how we assess younger children's patterning skills and how learning activities can be designed and implemented to enable the development of early algebraic thinking in the years prior to formal schooling.

The definition of a pattern used in the instrument design was that a pattern is any replicable regularity. In the early years, prior to formal schooling, children are predominantly exposed to two types of patterns: (i) repeating patterns and (ii) spatial structure patterns.

Repeating patterns

Repeating patterns contain a discernible unit of repeat (Threlfall 1999); that is, "the pattern has a cyclic structure that can be generated by the repeated application of a smaller portion of the pattern" (Liljedahl 2004, p. 27). A unit of repeat is often described as comprising a number of "elements" sometimes referred to as "repeating elements". For example, Δ O is a repeating element of a pattern containing Δ O Δ O Δ O. This "smaller portion" can vary in the number and complexity of items depending on attributes such as size, shape, dimension and direction and is commonly referred to as the pattern unit, segment or part.

A critical aspect of proficiency with repeating patterns is being able to identify the unit of repeat (Papic et al. 2011). The understanding of a pattern as a "unit of repeat" is linked to the development of multiplicative concepts, such as the development of equal groups and multiple counting (skip counting). For example, knowing how many units of repeat there are in a pattern when the total number of items and the size of the pattern element are known. Similarly, the unit of repeat in measurement contexts such as with length and area units lies in the application of equal-sized units. Fraction concepts also necessitate partitioning and repetition of equal parts. In order to identify the unit of repeat, children need to be able to recognise structural similarities and differences between patterns. According to Mason et al. (2007), "becoming aware of similarities and differences results in stressing or fore-grounding and consequently ignoring or back-grounding" (p. 55); as a result, children can focus on underlying concepts such as unit of repeat and spatial structure.

Spatial patterns

Another dimension that is integral to the concept of pattern is the notion of "spatial structure". "Spatial structure patterns are invariant relations between various features of geometrical shapes. Examples of shapes are triangles, squares, blocks, arrays, and grids; examples of features are the number, size, collinearity, and spacing of the elements of these shapes" (Papic et al. 2011, p. 239). For example, array patterns of four and six dots and triangular patterns of three and six dots.

Being able to recognise and discriminate between numbers of objects and instantly identify how many there are (known as subitising) is a developmental prerequisite to counting. According to Clements (1999), the benefit of subitising tasks is "that different arrangements suggest different views of that number" (p. 404). Identifying the number of items in different spatial patterns and partitioning a spatial arrangement of dots into its composite parts and recognising the whole in this way is a more advanced form of subitisation known as "conceptual subitisation" (Bobis 2008, p. 6). Conceptual subitisation assists in the development of children's understanding of number concepts such as conservation and compensation, the development of counting strategies (e.g. unitising and counting on) and the "understanding of arithmetic and place value" (Clements 1999, p. 404).

Method

Settings and participants

Fourteen Aboriginal-controlled early childhood services and two privately operated services with a high percentage of enrolments from Indigenous families were invited to participate in the study. One Aboriginal-controlled service did not agree to participate. The services who did participate were spread across 12 rural, regional and inner city communities across NSW and one remote area of the ACT. All services, supported by Gowrie Indigenous Professional Support Unit (IPSU), had engaged in culturally appropriate half-day numeracy workshops led by the author over a number of years, prior to the study. Gowrie IPSU is an Australian Government funded organisation established to support staff to ensure that all Indigenous children attending eligible Indigenous childcare services have access to high-quality care. IPSU provides professional development through mentoring, advice, support, referral and training. A collaborative partnership with Gowrie IPSU, over a period of 6 years, enabled a relationship to be established with these services and provided the opportunity for participation of these services in the study.

All children from the participating services who were in their final year of preschool/childcare, and were enrolling in formal schooling the year after the study, were invited to participate. Two hundred and fifty-five children aged between 4 and 5 years consented to the study and comprised two cohorts: 125 in 2011 and 130 in 2012. Eighty-five percent of staff and children from the Aboriginal-controlled services identified as Indigenous, as did 50–60 % of the staff and families of children enrolled at the private centres.

Due to the high number of Indigenous children in the Aboriginal community-controlled services, these 13 centres are the focus of this article and comprised 217 children. As the study did not individually identify any child, Indigenous children were not separated from non-Indigenous children when analysing the data. Those non-Indigenous children in the study, however, experienced comparable levels of socioeconomic disadvantage as their Indigenous peers and could also be expected to be at risk of beginning their formal schooling with a lower level of mathematical knowledge than children from higher socioeconomic backgrounds (Tymms et al. 2004; Thomson et al. 2005). The study participants that are the focus of this article were, therefore, predominantly Indigenous, low socioeconomic young children.

The Early Mathematical Patterning Assessment (EMPA)

The EMPA (Papic 2013) was administered by the educators with the support of researchers to the 217 children due to enrol in formal schooling the year after the project. The assessment was used in the 13 participating centres in the week prior to the commencement of a 12-week numeracy project, The Patterns and Early Algebra Preschool Professional Development Project (Papic 2013). The EMPA was administered as a one-on-one interview, over approximately 15 min. The EMPA comprised 13 tasks which assess the children's facility with simple repetition (8 tasks) and spatial patterns (5 tasks).

The repeating pattern tasks investigated the children's ability to identify, copy, represent through drawing and continue a pattern presented as a tower made of coloured blocks. The repeating pattern tasks were initially inspired by Maher's longitudinal study (2002) in which children solved combinatorial thinking tasks using coloured blocks made into towers. The spatial pattern tasks investigated whether there were differences in pattern recognition and representations using a spatial structure of equal-sized units and spaces. It explored children's ability to subitise various spatial arrangements. The design of these tasks was informed from studies on visual memory, pattern, structure and unitising with older students (Battista 1999; Bobis 1996; Mulligan et al. 2004). The subitising tasks were adapted from the Schedule for Number Assessment 1 (SENA 1) (NSW Department of Education and Training 2001). Children were initially given an assembled six-block tower. The blocks were arranged in a red-blue repeating pattern. Children were asked to look at the tower and to make a tower exactly the same as the one provided—the same colour and same number of blocks in the same pattern. Children were then asked to use coloured markers to draw the tower—"using the same colour and number of blocks in the same positions". Children were then asked to continue the tower to make it taller but to still keep the pattern the same.

In the next series of tasks, children were given another assembled six-block tower (orange-green repeating pattern); however, in this tower, the third and fourth blocks were screened. Children were asked to identify what colour they thought the screened blocks were and then to draw the whole tower representing the true colours of the screened blocks. Children were asked to justify their response. The next task also had a screened block; however, only the fifth block in the six-block tower (yellow-black repeating pattern) was screened. Again the children were asked to identify the screened block and explain their strategy for solving the task.

In the final series of repeating pattern tower tasks, the children were shown a six-block tower (pink-brown repeating pattern). After viewing the tower for 5 s, the tower was removed from the children's view and they were asked to copy it from memory and then explain how they remembered the tower. After being shown the tower again for 5 s, the task was repeated, but this time, the child was asked to draw the tower from memory and once again explain his/her solution strategies. Memory tasks were inspired by Mulligan and Prescott (2003) who in their study with first graders found that some children "do not develop structured images of critical mathematical concepts early" (p. 545), and teachers need to assist children to "attend to structure in early mathematical situations…to focus on all aspects of developing mathematical and spatial structure. This may be a simple as enabling students to visualise and record a simple pattern accurately" (p. 545).

The spatial pattern tasks first began with simple counting tasks to explore children's proficiency with the number sequence and counting items to six. Children were first asked to count to six. Counters were then placed in front of children in a horizontal row, and children were asked to identify how many counters there were. The number of counters were presented in the following sequence—three, five, one, four and two.

Spatial tasks then explored the children's subitising skills beginning with regular dot patterns (dice dot patterns) presented on cards. Cards were shown in the following order: three, five, four and six. Children were then shown dots on a grid (refer to Fig. 1) and a staircase made from five then three blocks (refer to Fig. 2). Finally, children were shown dot patterns in irregular formations in the following order: five, four, six and three.

With all the spatial patterns, the representations were shown to children for 3 s and then removed from their view so they were unable to count the dots or blocks.

The training of educators covered the purpose, implementation and analysis of the EMPA. At least one member of the research team acted as participant observer for every interview and assisted the educators in the implementation and interpretation of the assessment. Procedures were consistent with those of Papic and colleagues in an earlier study (Papic et al. 2011).

Data collection and analysis

As children worked through the series of tasks, the educator coded the children's responses and recorded any dialogue. The responses were initially coded for accuracy. The responses and associated notes were then analysed to identify whether the response (concrete, verbal or drawn) (i) used or identified the correct colours, (ii) had the correct number of blocks and (iii) represented the pattern or unit of repeat.

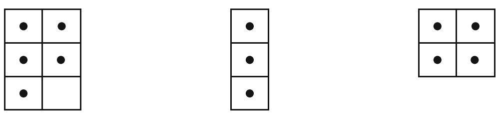

Fig. 1 EMPA grid dot patterns

Fig. 2 EMPA staircase patterns

Responses to repeating pattern tasks were then classified into one of four increasing levels of sophistication "focusing on the structure of the representation and the use of a unit of repeat" (Papic et al. 2011, p. 247). Children were placed on the:

i. *Pre-structural* (*PR*) *level* if they had difficulty completing the tasks. Children either gave no response or their solution strategies for drawn tower representations were markings where no clear units were evident.
ii. *Emergent* (*E*) *level* if their solution strategies for tower representations were predominantly not represented in a row or column or, if they were presented in a row or column, the incorrect number and/or colour of blocks were evident. While children attempted to respond to the tasks, their solution strategies identified at this level were frequently inconsistent and did not show repetition of a unit of repeat.
iii. *Structural* (*S*) *level* if their solution strategies for tower representations were frequently presented in a row or column with at least one property (colour, number or unit of repeat) in most of their representation.
iv. *Advanced structural 1* level (AS1) if their solution strategies were predominantly accurate with the correct colour, number and pattern element represented in a row or column.

Responses to the spatial pattern tasks were classified into one of four increasing levels of sophistication based on Wright's (1996) Learning Framework in Number: (i) pre-emergent, (ii) emergent; (iii) perceptual and (iv) conceptual. Children were placed on the *pre-emergent* (*PE*) level if they could not count to six by ones or count out six items. At the *emergent* (*E*) level, children could recognise regular dot patterns for very small numbers such as two or three but used unitary counting for larger numbers. In contrast, the children who could recognise regular dot patterns to six but only small numbers for irregular patterns, using unitary counting of the irregular dot pattern for larger numbers, were placed on level 3, *perceptual* (*P*). Children who instantly recognised regular and irregular dot patterns one to six were placed on level 4, *conceptual* (*C*).

At least 90 % of the interviews were co-analysed by a member of the research team to increase reliability of data collection and analysis (Papic et al. 2015).

Results

Two hundred and seventeen children were assessed on the repeating pattern and spatial pattern tasks prior to commencing a 12-week numeracy project (refer to Papic et al. 2015 for details). Table 1 shows the classification of responses for the two types of patterns.

Table 1 Classification of children's responses to the EMPA

Repeating pattern classification levels	No. of children	Spatial pattern classification levels	No. of children
Level 1—PR	83	Level 1—PE	108
Level 2—E	83	Level 2—E	101
Level 3—S	45	Level 3—P	8
Level 4—AS1	6	Level 4—C	0

Repeating patterns

Of the 217 children assessed at the start of the implementation of the numeracy project, 76 % (n=166) fell into the first two categories (pre-structural and emergent) for the repeating patterns tasks. Half of these (n=83) responded that they could not complete the tasks or their responses predominantly did not represent one or more of the properties of the tower pattern (colour, size or pattern). Drawn representations, while attempting to represent the tower, did not characterise the distinct units of the blocks in the tower.

The responses that were emergent, level 2 (n=83), predominantly showed distinct units and often represented the tower vertically. In these responses, the units were not always the correct colour and the tower was frequently an incorrect size. For example, when asked to copy the six-block tower (red-blue repeating pattern), Oscar made a tower with seven blocks: blue, blue, blue, orange, blue, red and orange. This form of representation was consistent when drawing the tower. Figure 3 shows Talihas' attempt

Fig. 3 Tahlia's drawn representation of a six-block tower (red-blue repeating pattern)

to draw the same tower. While red and blue blocks are clearly evident in the tower, there are additional colours included and more than six blocks represented. In comparison, Bob used the correct colours blue and red, and the drawing includes units to represent the blocks; however, these units were not represented in a row or column (refer to Fig. 4).

Twenty-one percent (n=45) of the responses to repeating pattern tasks represented the units in rows or columns demonstrating some structure in the representation, and they contained one or more of the properties evident (colour, number or pattern) (level 3, structural). When one property was evident, it was usually the property of colour rather than size or pattern. For example, Chris's drawn representation of a six-block tower (orange-green repeating pattern) was orange, green, green, orange, orange and green. While not accurately representing an AB pattern of orange and green, his drawn representation did contain the correct colours of orange and green and the correct number of blocks. In contrast, 3 % of responses (n=6) reflected the correct colour, number and pattern in most copied and drawn representations (level 4, advanced structural 1).

When drawing towers by copying, children used three main strategies to solve the task (either incorrectly or correctly). These were evident across classification levels and included:

1. Tracing around the given tower and then colouring inside the traced shape
2. Counting blocks in the given tower and then counting blocks drawn
3. Checking each block drawn with the given tower (one-to-one correspondence)

Fig. 4 Bob's drawn representation of a six-block tower (red-blue repeating pattern)

Those children who accurately copied the tower with blocks frequently used the given tower as a reference and copied one block at a time using the skills of one-to-one correspondence and then measuring up against the given tower to check the size. Vanessa justified her solution strategy: "cause it red, then blue, then red, then blue, then red, then blue, cause I was copying it". No child at the assessment made individual units of repeat and joined them together. Children who were successful with various tasks relied on an "alternating colour" strategy rather than seeing and creating units of repeat, as was evident with Vanessa.

Spatial patterns

Of the 217 children assessed at the start of the implementation of the numeracy project, 50 % (n=109) could count to six by ones and count out six items. Of these 109 children, 101 could recognise regular dot patterns for very small numbers such as two or three; however, they needed to count for items larger than three (level 2, emergent), and 8 could subitise up to six objects if they were presented in regular formation such as the dice pattern, however, found it difficult to subitise four to six items that were presented as irregular dots, staircases or in a grid formation (level 3, perceptual). At this assessment, no child could subitise six items presented in both regular and irregular formations (level 4, conceptual).

Discussion and conclusion

The EMPA was utilised to assess the ability of Indigenous and/or low socioeconomic children to solve problems based on patterns: repeating and spatial. The activities, while including tasks that required children to "copy" a visible pattern, also required children to replicate patterns through visual memory. The results of the assessments indicated that the predominantly Indigenous young children in the study had varied patterning, number and counting skills.

The assessment tool, after initial training, was easily implemented by the early childhood educators. The training gave educators an understanding of what the assessment is for, that is, it is to support children's learning. Further, by co-analysing the interviews, the educators with the support of the researchers were able to establish the various levels of mathematical understanding of individual children for the first time. Just as significantly, this analysis provided an insight into the children's thinking when solving tasks and identified the aspects of the patterns they focused on. This information allows educators to identify appropriate learning trajectories for individual children. "Most content knowledge is acquired along developmental progressions of levels of thinking. These progressions play a special role in children's cognition and learning because they are particularly consistent with children's intuitive knowledge and patterns of thinking and learning at various levels of development" (Clements and Sarama 2007, p. 464).

Assessment of preschool children's skills and understandings is a controversial topic. Assessments can assist in identifying which programmes or approaches to pedagogy are effective in increasing children's mathematical knowledge. Assessments can also assist in identifying children's specific skills and strategies for solving problems. However, researchers and professionals alike also highlight that the outcomes of assessments on young children, particularly prior to formal schooling, are not reliable and may cause unnecessary stress to young children. Consequently, the EMPA was designed to be an engaging series of hands-on tasks implemented within 15 min. In addition, its purpose is "assessment for learning". It is not designed for use in ranking young children in comparison to their peers.

The EMPA results identify the considerable opportunity for active intervention to support mathematical learning in the early years, particularly to ensure the most disadvantaged young children do not start schooling behind and, subsequently, stay behind. The results of the EMPA also highlight the importance of assessment tools to supplement current assessment methods that focus primarily on "observations or evidence to make judgements about children's learning" (Australian Government, Department of Education, Employment and Workplace Relations, 2009, p. 37) by early childhood educators. The results of this study raise the question: Is it time to move beyond observing children engaged in everyday tasks and instead explicitly assess what children can do mathematically and how they are thinking mathematically in order to build on this learning more effectively?

By using the insight, into the children's thinking that the EMPA tool provides, early childhood educators can better inform mathematics teaching and learning. Effective mathematics teaching and learning in the early years has the potential to close the persistent gap in numeracy between Indigenous and non-Indigenous children. If we are to close the gap in numeracy achievement between Indigenous and non-Indigenous children—and between low socioeconomic and higher socioeconomic children—we need to consider varied approaches to mathematics assessment, teaching and learning in the years prior to formal schooling. Our current approach is not meeting the needs of Indigenous and other particularly disadvantaged children. EMPA is an important tool that can both inform effective early intervention programmes and measure their short- and long-term successes.

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