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ORIGINAL ARTICLE

Teachers' selection and enactment of mathematical problems from textbooks

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Abstract In order to investigate how teachers' use of textbooks creates different kinds of opportunities for student learning, this study focused on teachers' selection and enactment of problems and tasks from the textbooks and their influence on the cognitive demand placed on students. By drawing on data from three elementary teachers in the USA, two of which used a reform-oriented textbook—*Math Trailblazers* and one a commercially developed textbook—this study examined kinds of problems the teachers chose and ways in which they enacted those problems in relation to the cognitive demand of the problems. In particular, we attended to the kinds of questions the teachers asked in enacting the problems and ways in which those questions influenced the cognitive demand of the textbook problems. This study also identified critical issues involved in teacher decision-making on task selection and enactment, such as the match between teachers' goals and those of the textbooks, and teachers' perception of textbook problems. Based on the results of the study, we discuss implications for teacher education and professional development.

Keywords Task selection · Task enactment · Cognitive demand · Teacher questioning

Curriculum documents and materials have been a key agent to improve mathematics practice in ways that align instruction with the reformers' ideas (Askew et al. 2010; Cohen and Ball 1990; Kauffman et al. 2002; Pepin et al. 2001). For example, in the USA, the *Standards* documents by the National Council of Teachers of Mathematics (NCTM) (e.g., 1989, 2000) and curriculum materials developed with the support of the National Science Foundation (NSF) in the 1990s had greatly influenced teacher

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education and teacher development in changing the teaching and learning of mathematics in the classroom. However, the success of the reform efforts largely depends on how teachers actually use the ideas in the curriculum documents to design instruction. For this reason, previous studies have investigated teachers' curriculum use by examining various related aspects, such as types of textbooks used (e.g., Freeman and Porter 1989; Komoski 1977; Manouchehri and Goodman 1998), coverage of textbook content (e.g., Chval et al. 2006), and alignment between textbooks and instruction (e.g., Brown et al. 2009; Eisenmann and Even 2011; Kim and Atanga 2013). One common finding from these studies is that teachers often intentionally and unintentionally modify the tasks and activities from the textbook, and thereby, they can provide different learning opportunities from those intended by the authors.

In order to investigate how teachers' use of textbooks creates different kinds of opportunities for student learning, this study focused on teachers' selection and enactment of problems and tasks from the textbooks and their influence on the cognitive demand placed on students. The *Professional Standards for Teaching Mathematics* (NCTM 1991) emphasizes that opportunities for student learning are created by the level of thinking with which students engage with mathematics. Stein and her colleagues (e.g., 1996, 2000) articulated cognitive demand of mathematical tasks by distinguishing four different levels: memorization, procedures without connections, procedures with connections, and doing mathematics. Tasks that require students to analyze mathematics concepts or to solve complex problems can be considered cognitively demanding or high-level tasks. Such tasks offer opportunities for students to sharpen their thinking and reasoning in mathematics. In contrast, cognitively undemanding tasks (ones that may require little more than memorization and repetition) offer less opportunity to develop high-level cognitive processes. Thus, the kinds of tasks that entail different levels of cognitive demands determine to a great extent students' opportunities for learning.

A growing body of research studies has examined teachers' practices in terms of cognitive demand (e.g., Stein et al. 1996; Stein and Kaufman 2010; Stigler and Hiebert 2004) and reported teachers' tendency to decrease the cognitive demand of tasks during instruction. A commonly observed instructional pattern in USA classrooms is for teachers to begin tasks with a high-level of cognitive demand but then to decline the level of cognitive demand in which students actually engage in the tasks (Stein et al. 1996). Such teachers often lower the cognitive demands of a task by breaking it down into subtasks (Smith 2000), by adapting the tasks or teaching suggestions to be consistent with their personal notion of effective teaching and learning (Arbaugh et al. 2006; Lloyd and Wilson 1998; Remillard 1999), or by focusing only on correct answers to the exclusion of reasoning and explanation (Henningsen and Stein 1997). Prior research reported that the teachers' knowledge and their learning goals directly influence teachers' decisions about task selection (Stein et al. 1996). In particular, student factors, such as students' mathematics abilities and their learning disposition toward mathematics, were critical factors that led teachers to either maintain or decrease the challenge or rigor of tasks during instruction.

This study intended to extend the current literature on teachers' textbook use by investigating issues that are involved in teachers' decisions regarding task selection and implementation when student factors are controlled, in part, by teachers' perception. Although the findings from prior research are useful, there remain unanswered questions regarding teachers' textbook use. For example, when teachers assess their students' overall mathematical understanding at a comparable level, what issues are involved in teachers' decisions regarding task selection and enactment? The NCTM (2000) recommends teachers select tasks based on knowledge of students' understandings, interests, and experiences. Thus, teachers' perception on their students' current level of mathematical competence or understanding can play a crucial role in the selection and enactment of mathematical tasks (Archambault et al. 2012; Stein et al. 1996). In this study, we focused on three teachers who had evaluated their students' overall mathematical understanding at the medium level (in a low-medium-high scale) and intended to explore whether their task selection and implementation differed and what issues were involved in each teacher's decisions regarding task selection and enactment.

Not only the kinds of tasks teachers select but also how teachers enact tasks influence the kinds of student thinking required in the classroom. Therefore, this study examined how teachers selected tasks and problems from the textbooks[1] they used, how they implemented them during instruction, and how teachers' selection and enactment of tasks and problems affected the demand of students' thinking. The research questions that guided this study are the following:

1. What tasks do teachers select from their textbooks?
2. How do teachers implement the selected tasks during instruction?
3. How do the teachers' selection and implementation of tasks influence the cognitive demand placed on students?
4. What issues are involved in teachers' decisions regarding task selection and implementation?

In this study, we use *problems* and *tasks* interchangeably, although a mathematical task can be a set of problems with a particular goal. Whereas Smith and her colleagues (1996, 2000) explored the cognitive demand of tasks not individual problems, by focusing on reform-oriented curriculum materials, we examined the cognitive demand of individual problems and teacher questions that teachers selected for use from textbook lessons, including reform-oriented and traditional ones. In particular, as a way of examining the enactment of selected tasks and problems and its influence on cognitive demand, this study attended to the questions teachers asked and their reaction to student responses while they were using the tasks and problems in the classroom. This is because teachers' questions and the way they respond to student thinking play a critical role in guiding students' mathematical work (Stein et al. 2008).

Theoretical perspectives

Cognitive demands of mathematical problems and teacher questions

The questions and problems teachers pose are highly related to the quality of mathematics instruction. In particular, teachers' questions can significantly alter the cognitive

[1] By textbooks, we mean a set of curricular resources that teachers use for day-to-day teaching, which include student texts and workbooks and the teachers' guide.

demand of mathematical tasks and problems. Drawing on Stein and her colleagues' (e.g., 1996) work on mathematical tasks and Anderson and Krathwohl's (2001) work on the revised Bloom's taxonomy, we describe two levels of mathematical tasks and corresponding teacher questions.

According to Stein et al. (1996), mathematical tasks place different cognitive demands on students. Low-level cognitive demand problems (i.e., *memorization* and *procedures without connections*) ask students to perform a demonstrated procedure in a routinized way; hence, they place low-level, largely procedure-based demands on student learning. *Memorization tasks* involve exact reproductions of what students learned previously. There is little ambiguity about what needs to be done and how to do it. For instance, with the topic of fractions, this type of task could consist of problems requiring students to memorize the equivalent forms of specific fractions (e.g., $\frac{1}{2} = 0.5 = 50\%$). The other type of low-level problem is *procedures without connections*, which does not require students to make connections to the concepts that underlie the procedure being used. *Procedures without connection tasks* require limited cognitive demand for successful completion of the tasks. A typical example is a problem that asks students to convert fractions to percent using standard conversion algorithms in the absence of additional context or meaning (e.g., convert the fraction $\frac{3}{8}$ to a decimal by dividing the numerator by the denominator to get 0.375).

Problems with high cognitive demand (i.e., *doing mathematics* and *procedures with connections*) ask students to make conceptual connections and think about the mathematics in sustained and thoughtful ways. These problems can involve using procedures but must do so in a way that builds connections to underlying concepts and meaning. *Procedures with connection tasks* require students to make connections between ideas and procedures, which often includes using multiple representations. For instance, problems might ask students to use a diagram to explain how the fraction $\frac{3}{5}$ is equivalent to the decimal 0.6 or 60 %. The other type of task with high cognitive demand is *doing mathematics*, which would entail asking students to explore the relationships and invent ways to solve problems. In this type of task, students are challenged to apply their understanding of mathematical concepts in a novel situation.

As with the cognitive demand of tasks and problems, the c*ognitive demand of teacher questions* refers to the kind and level of student thinking required when students are engaged with "teacher questions." Six types of teacher questions based on the revised Bloom's taxonomy (Anderson and Krathwohl 2001) can be categorized into two groups: (1) low-level questions (i.e., *remember* and *knowing procedure*) and (2) high-level questions (i.e., *understanding*, *applying*, *reasoning*, and *evaluating*). Table 1 summarizes the characteristics of mathematical problems and teacher questions associated with two levels of cognitive demand with corresponding examples, modified from Stein et al. (1996). We used this framework in determining the cognitive demand of problems and questions presented in textbooks and in enacted lessons.

Research conducted in the past decade in a variety of classroom contexts has found that greater student learning occurs in classrooms where the high-level cognitive demands of mathematical tasks are consistently maintained throughout the instructional practices (Boaler and Staples 2008; Stein and Lane 1996; Tarr et al. 2008). For example, in a longitudinal comparison of three high schools over a 5-year period, Boaler and Staples (2008) determined that highest student achievement occurred in the

Table 1 Cognitive demand of tasks and questions in two levels

High level	Low level
• Explore complex and non-algorithmic thinking. • Require students to *explore* and *understand* mathematical concepts. • Require students to *analyze* the task and possible solution strategies. • Are usually *represent*ed in multiple ways (e.g., visual diagrams, manipulatives, symbols). • Require students to *make connections* among multiple representations. • Require engagement with the conceptual ideas that underlie the procedures. • Ask students to *evaluate* the task and possible solution strategies.	• Involve *reproduc*ing previously learned facts, rules, formulas, or definitions. • Require students to *use* a *procedure*, which is either specifically called for or is evident based on prior instruction or experience. • Do *not* require students to *make connections* to the concepts or meanings that underlie the procedure being used. • Are focused on *produc*ing correct answers. • Do *not* require students to *give explanations*. • Focus solely on describing the procedure that was used. • Ask students to *concentrate on factual information* that can be memorized.
Sample problem: Look for patterns in the number sentences, $\frac{1}{2}=\frac{2}{4}$, $\frac{1}{2}=\frac{4}{8}$, and $\frac{1}{2}=\frac{5}{10}$ and find another equivalent fractions	*Sample problem*: Complete. 1. $\frac{1}{2}=\frac{1\times 3}{2\times 3}=\frac{\square}{6}$ 2. $\frac{1}{3}=\frac{\square}{6}$
Sample question: What pattern did you notice? How do you know?	*Sample question*: What number did you multiply by 2 to get 6? Follow the rule to get an answer.

Italicized verbs represent the characteristics of each level

school in which students were supported to engage in high-level thinking and reasoning. Boaler and Staples attribute students' success to the ability of the teachers to maintain high-level cognitive demands during instruction through the teacher's use of questions that elicited and supported students' thinking. Stein and Lane (1996) and Tarr and his colleagues (2008) also found that classrooms in which teachers consistently encourage students to use multiple strategies to solve problems and also support students to make conjectures and explain their reasoning were related to higher students' mathematical performance. However, it is challenging for teachers to maintain the high cognitive demands of mathematical tasks/problems during instruction.

Teachers' enactment of textbook problems

We acknowledge that the use and meaning of textbooks vary in different countries, since textbooks typically represent how the societal visions and educational objectives presented in national policies and official documents are *potentially* implemented in classrooms (Schmidt et al. 1997; Valverde et al. 2002). Nevertheless, it is common that the cognitive demand of mathematical problems in textbooks as intended by the developers can be changed as teachers transform the content from textbook to teaching through two processes: selection and enactment of problems (see Fig. 1).

First, teachers may alter the cognitive demand put forth by the intended curriculum while planning the lesson, in particular, in selecting problems. This alteration of the intended cognitive demand represents the teacher's assumptions about the kinds of problems students should engage with and how students should learn. Next, cognitive demands can also be changed as teachers enact the problem during instruction. Teacher

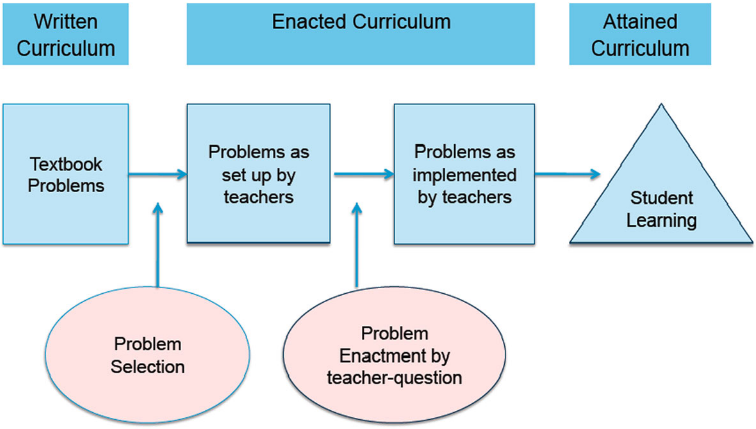

Fig. 1 Transformation processes of textbook problems (adapted from Stein et al. 1996)

questioning at this point in the lesson has the potential to transform low cognitive demand to high cognitive demand and vice versa (Brown and Campione 1994; Spillane and Zeuli 1999). Therefore, this study focuses on questions teachers ask, including follow-up questions as a reaction to student responses that may influence the enactment of tasks. Particularly, teacher questions as a reaction/feedback to student correct and incorrect responses are critical in giving further guidance in student thinking and learning through classroom discourse (Kim and Atanga 2014).

That teachers modify task difficulty is well documented. In the TIMSS 1999 video study, a random sample of 100 eight-grade mathematics classes was videotaped during the 1999 school year in the USA. Although 17 % of the tasks used by teachers were coded as high level, none of the tasks were implemented as intended. Instead, most of the high-level problems (e.g., making-connection problems) were transformed into procedural exercises (Stigler and Hiebert 2004). The TIMSS video study concluded that there was a minimal intellectual challenge in typical mathematical classroom instruction. Indeed, teachers face the tension between lowering the challenge of the task and appropriate scaffolding in the selection and implementation of the task due to various factors, including *teachers' knowledge* (Choppin 2011; Kauffman et al. 2002), *teachers' beliefs* about mathematics teaching and learning (Remillard 1999), and *teachers' views about textbooks* (Ball and Feiman-Nemser 1988; Freeman and Porter 1989). These findings call for more research that explores ways in which teachers alter the cognitive demand of the task when implementing the written curriculum and more research that takes into account the mediating factors between the written and the enacted curriculum. Some research has examined how such factors are related to teachers' use of textbooks from the perspective of the cognitive demand placed on students' thinking, in particular in the selection and enactment of mathematical problems. Sullivan and his colleagues (2009, 2014) and Choppin (2011), for example, highlighted the role of teacher knowledge play in implementing challenging tasks, and they especially emphasized teachers' knowledge of student thinking around the task and the mathematics embedded in it. In addition, student factors, such as students' limited knowledge and disposition towards mathematics, were reported as essential factors leading to the lowering of high cognitive demand tasks (Stein et al. 1996; Sullivan and Mornane 2014).

Building on previous research, this study intends to extend the current literature on teachers' textbook use by investigating issues that are involved in teachers' decisions regarding task selection and implementation when student factors are controlled by teachers' perception in part. Although a growing body of research studies addressed mediating factors from the perspective of the cognitive demand placed on students' thinking, more studies are needed to explore what critical issues are related to teacher decision-making in using textbook problems and how the various issues support or hinder teachers' productive use of textbook problems. It would be preferable for teachers to select tasks based on knowledge of students' understandings, interests, and experiences (NCTM 2000). Teachers' perception on their students' current level of mathematical competence or understanding can play a crucial role in the selection and enactment of mathematical tasks (Archambault et al. 2012; Stein et al. 1996). Therefore, when teachers evaluate their students' overall mathematical understanding at a similar level, do they use their textbooks in similar ways in the selection and implementation of tasks, and if so, what issues arise in their selection and enactment of the tasks? Based on the analysis of classroom observations, interviews, and teachers' survey responses, this study intends to address these questions and account for the issues critical to three elementary teachers in the selection and enactment of problems from their textbook with respect to the cognitive demand on student thinking.

Methods

Since fractions typically represent a serious excursion into abstract mathematics and understanding of fractions is difficult for some teachers as well as many students (Kilpatrick et al. 2001; Son and Crespo 2009), the topic of fractions was chosen for the study.

Participants and textbooks

Participants were recruited from a larger study in which the first researcher examined 178 teachers' reports on their textbook use and influential factors in the USA. Three criteria were used to select participants—textbook type (reform-oriented vs. commercially developed textbooks), teacher's perceptions on their textbook, and a level of perception towards students' mathematical competence. In the USA, there are two major formats of textbooks with differing pedagogical approaches, referred to here as traditional (commercial) and reform-oriented curricular materials. Reform materials are those that adopt the recommendations of the NCTM (1991, 2000) with the support from the National Science Foundation to include a classroom pedagogy that fosters the understanding of discrete concepts through communication and problem solving (Schoenfeld 2004; Son and Senk 2010). Traditional textbooks tend to utilize direct instructional methods and reinforce concepts through individual practice (Improving Curriculum Use for Better Teaching (ICUBit) Project 2011). While many traditional textbooks cite the content recommendations of the NCTM, ideological and political disputes have allowed them to retain their traditional pedagogy (Schoenfeld 2004). In the USA, the choice of a mathematics textbook often occurs at the school level, and school districts have had a choice between traditional and reform curriculum materials (Reys et al. 2003).

Three teachers, two fourth-grade teachers (Brad[2] and Karen) and one fifth-grade teacher (John), were selected for the current study, because these teachers used different types of textbooks (reform-oriented vs. commercially developed textbooks) and revealed varying perceptions about their textbooks and textbook use. In addition, our study used the level of perceived student competence as the criteria to select teachers to partially control the effects of perceived ability level on task selection. These teachers all rated their students' overall mathematical competence at the medium level in a low-medium-high scale.

Brad, with 7 years of teaching experience, and Karen, 4 years, taught at the same elementary school, using *Math Trailblazers* (Wagreich et al. 2004), which is an elementary mathematics textbook series developed with the funding by the NSF in the USA. Brad and Karen were chosen for the study because they used *Math Trailblazers* differently. The mathematical tasks in this series were designed to develop elementary students' mathematical thinking, reasoning, and problem-solving skills based on the reform movement in mathematics education. Prior research that examined *Math Trailblazers* (e.g., Kim and Atanga 2013; Stein and Kaufman 2010) reported that the vast majority of the mathematical problems presented in *Math Trailblazers* require students to use *procedures with connections* to meaning or concepts and *doing mathematics*, that is, a high level of cognitive demand. The same finding was noted in our analysis (see Table 2). For example, our analysis revealed that *Math Trailblazers* presents the following problem for students to explore equivalent fractions: "Look for patterns in the number sentences (e.g., $\frac{1}{2}=\frac{2}{4}=\frac{3}{6}=\frac{4}{8}=\frac{5}{10}$) and find other equivalent fractions" (Grade 4, Wagerich, Goldberg, and TIMS Project Staff 2004, p. 922). The formal exploration of equivalent fractions is first addressed in grade 4 in the USA (NCTM 2000), in which students are expected to recognize and generate simple equivalent fractions. While commercially oriented textbooks typically present an algorithm for finding equivalent fractions—multiply the numerator and denominator by the same number and ask for the application of the algorithm to find equivalent fractions—the aforementioned problem requires students to focus on the relationship between numerators and denominators, search for the patterns, and use them to find equivalent fractions. The problem was thus coded as *procedures with connections*.

The other teacher, John, with 33 years of teaching experience, used *Scott Foresman-Addison Wesley Mathematics* (Randall et al. 2005), a commercially developed textbook series. In contrast to NSF-funded textbook series, such as *Math Trailblazers*, the majority of the problems and questions presented in commercially developed textbook series are categorized as low-level problems and questions that require students to use *procedures without connection* (Atanga 2014; ICUBiT Project 2011), such as "Find the sum. Simplify $3\frac{1}{2}+2\frac{1}{6}$" (Grade 5, Randall et al. 2005, p. 372). This problem was presented after a procedure was being introduced. It was coded as *procedures without connection* because it does not require students to make connections to the concepts or

[2] All names are pseudonyms.

Table 2 Enacted patterns, cognitive demand, and related issues

	Textbook (overall level)	Enacted pattern			Related issues in teacher decision
		Selected problems	Enacted problems	Enacted questions	
Brad	*Math Trailblazers* (higher level)	Higher level	Higher level	Higher level	• State framework • Understanding-oriented teaching goal • Constructivist view on teaching, learning, the role of teacher and student, and curriculum • Satisfaction with text
Karen	*Math Trailblazers* (higher level)	Higher level	Lower level	Lower level	• State framework • Procedure-oriented goal • Constructivist view on learning and curriculum • Traditional view on the role of teacher and student • Dissatisfaction with text
Randy	*Scott-Foresman* (lower level)	Lower level	Lower level	Lower level	• State framework • Assessment • Procedure-oriented goal • Traditional view on teaching, learning, the role of teacher and student, and curriculum • Time • Student ability

meanings that underlie the procedure being used or does not require students to give explanations. Instead, it involves producing a correct answer by asking students to produce the sum in simplest reduced form (e.g., $5\frac{1}{2}$ not $5\frac{3}{6}$ or $\frac{33}{6}$). In addition, *Scott Foresman-Addison Wesley Mathematics* rarely provides questions that teachers can use as they enact mathematical problems, whereas *Math Trailblazers* provides teacher questions that accompany the mathematical problems. A detailed analysis of each textbook on the topic of fractions was conducted by the researchers and is presented later.

Data sources and collection process

This study employed largely qualitative research methods (e.g., observations and comparative case studies) along with teacher self-report on a Likert-scale survey. Survey data were used to explore the teachers' overall use of textbooks in selecting and enacting problems. Observations and resulting cases were used as a major data source for the analysis of patterns in the selection and enactment of problems. Since the three teachers taught fraction units at different times of the school year, data were collected over a 1-year period. The participant teachers completed a survey and were observed four lessons teaching fractions and interviewed after the observations. Each teacher was observed two consecutive lessons in the fall and spring semesters, respectively, to make accurate observations as recommended by Marshall (2009). In doing so, we ensured that a similar phase of the chapters taught by the teachers was observed.

Prior to the observations, we collected lesson plans that included mathematical problems and teacher questions they planned to use from the teachers' guide and, if any, other curricular resources. Rather than creating a detailed lesson plan, the teachers circled problems and teacher questions they planned to use in the teachers' guide. If the teachers supplemented or added any problems or questions from other resources, they also gave a copy to us. With these lesson plans from the teachers' guide, we observed each teacher's lessons, which helped us see the main activities that would take place during the lesson and allowed us to examine similarities and differences between the written lessons and the lessons planned and enacted by teachers. Each teacher's lessons were observed and videotaped four lessons to fully capture teachers' questions and their reaction to students' responses along with problems and activities they used. The classroom observations and field notes accordingly focused on examining how selected problems were enacted along with teacher questions during instruction.

Adopted from interview protocol of Kauffman et al. (2002), the post-observation interviews focused on teacher decisions regarding *what* to use from the teachers' guide and *how* to use it to teach mathematics, as well as what teachers recognized as constraints or affordances of the textbook series that they used. The interview protocol consisted of two parts—(1) use of their textbooks in planning and teaching fractions, and (2) aspects influencing their textbook use. Three questions were posed to help the teachers to articulate the process of their transformation of textbook lessons to planned lessons and enacted lessons. Teachers were asked to explain resources they used in planning lessons and similarities and differences among the textbook lessons, their planned lessons, and the lessons taught with respect to lesson objectives, classroom activities, problems, and teacher questions. They were also asked to provide a rationale for why they added or deleted particular activities, problems, and questions from the textbook lessons.

Furthermore, they were asked to explain their plan for the next lesson. To identify potential aspects related to their textbook use, questions were asked regarding teacher knowledge, teacher learning goals, and teachers' view of textbooks. For example, to examine teacher knowledge about fractions, they were asked to describe the big ideas in the fraction unit. The participants were also asked to describe goals for the lessons the researchers observed and any parts of the teachers' guide or other resources they found particularly helpful. Lastly, as a way of exploring what they recognized as constraints or support in their textbooks, teachers were asked to explain what influenced their selection and enactment of problems when teaching fractions.

In order to triangulate the patterns identified from observations and interviews, the survey data were obtained from the three teachers. The survey was developed by employing items from *2000 Horizon Research Survey* (Weiss et al. 2003; Ravitz et al. 2000) and TIMSS 1999 questionnaires[3] to document teachers' background (e.g., education and teacher experience), their views on their textbooks (i.e., satisfaction) and textbook use, teacher knowledge, and teacher beliefs about teaching and learning. For example, teachers were asked to indicate their agreement on the five statements regarding their use of textbook as a primary resource in planning and teaching mathematics on a 5-point scale.

They were also asked to indicate their views on textbook problems as well as their views on mathematical problems and teacher questions used in practices. For example,

[3] The survey was validated by experts, piloted in multiple ways, and modified. See Son (2008) for the details.

teachers were asked to indicate the frequency of various types of high-level problems (e.g., problems that allow students to explain and justify their ideas) and low-level problems (e.g., problems that require students to use rules and procedures) presented in their textbooks, as well as the frequency of corresponding types of high/low-level mathematical problems used in teaching. They were also asked to indicate the frequency of various types of high/low-level teacher questions used in their instruction. Furthermore, they indicated their beliefs about teaching, learning, curriculum, and the role of teachers and students based on given contexts or statements. Five items drawn from Ravitz et al. (2000) were used on which teachers checked the box that best shows their teaching philosophies, which were later categorized into either the *constructivist* view or the traditional *transmissionist* view (e.g., teachers as facilitators and students as meaning-makers vs. teachers as explainers and students as listeners).

Data analysis

As mentioned before, the units of analysis are mathematical problems and teacher questions presented in textbooks and used in teaching. From the three teachers' responses on the survey, descriptive statistics (e.g., frequencies) were obtained to generate an overall picture of their use of textbooks. Next, analytic case study narratives were written for each teacher (within-case analysis) (Miles and Huberman 1994) by analyzing each teacher's textbook lessons, lesson plans obtained, observed field notes, and interview data. First, each problem presented in the textbook lessons was classified as either high-level or low-level based on the framework shown in Table 1. Next, based on the observation data, each problem used by the teachers was classified as either high-level or low-level. The two authors independently coded each problem in the textbooks and the lessons and then checked inter-rater reliability. The percent agreement of the two raters was between 95 and 98 %. We resolved the disagreed items through consensus.

Then, the overall level of cognitive demand in the textbook lessons and the overall cognitive demand of problems used by teachers in enacted lessons were categorized as either high- or low-level, using the ratio of $\frac{1}{3}$. For example, if more than $\frac{1}{3}$ of the total problems in a textbook require high-level cognitive demand, then the overall cognitive demand was considered higher-level. Conversely, if less than $\frac{1}{3}$ of the total problems in a textbook require high-level cognitive demand, then the overall cognitive demand on student thinking was considered lower-level. The ratio of $\frac{1}{3}$ was determined based on previous studies. For example, Sanders (1966) highlighted that teachers in both instruction and evaluation should devote "a minimum of one-third of the time allotted to questioning to levels above memory" (p. 156). Referring to the ratio of cognitive domains in the TIMSS 1999 video study, Kadijević (2002) used the following ratios to measure expected students' mathematics learning outcome in the TIMSS 2003:

> The chosen target percentage of the TIMSS 2003 framework devoted to the cognitive domains in grade 8—knowing facts and procedures (15 %), using concepts (20 %), solving routine problems (40 %) and reasoning (25 %)—are quite appropriate and well balanced (p. 98).

According to Kadijević, these ratios were drawn from Pólya (1981) and the well-known Bloom's taxonomy. In the TIMSS 2003 cognitive domains, only *reasoning* is matched with the high cognitive levels in problems and teachers' questions, and its percentage (25 %) is less than the ratio of "one third" that Sanders articulated. Thus, the ratio of $\frac{1}{3}$ is considered an appropriate criterion to decide the overall cognitive demand of problems and questions in textbook lessons and in instruction. Then, the level of cognitive demand of the textbook problems and questions and the level of cognitive demand in enacted problems and questions were compared to see the coherence or discrepancy of the cognitive demands in the process of teachers' transformation of textbook lessons to enacted lessons.

Once analytic case study narratives were written for each teacher (within-case analysis), cases were compared to one another based on the enactment patterns of each teacher in order to explore possible reasons why teachers used the same textbook differently or used different textbooks in a similar way (cross-case analysis). In particular, the interview and survey data were analyzed in order to explore possible reasons why one teacher maintained the high cognitive demand in their teaching, whereas the others decreased the level of demand. Interview data were triangulated for each case with the survey, the textbook lessons, and the observations (and field notes). To ensure that the narrative of each case and the claims made therein were accurate and trustworthy, descriptions of each case and lesson tapes selected were reviewed again and compared with the narratives.

Results

We found that although all three teachers evaluated their students' overall mathematical competence at a same level, these three teachers' selection and enactment of textbook problems and questions varied, along with the cognitive demand of the problems they enacted. Analyses of teachers' report on the survey revealed patterns similar to those in classroom observations, interviews, and resulting cases. Table 2 summarizes these results for an overall comparison among the three teachers. Table 3 shows three distinctive curriculum use patterns resulting in variation in the selection and enactment of mathematical problems with respect to cognitive demand on student thinking.

All three teachers reported using the assigned textbooks as their main instructional resources. However, they revealed varied satisfaction with and different use of the textbooks, different perceptions on the presence of high-level textbook problems, and different patterns both in the survey and observations despite the fact that their perception towards students' mathematical competence is similar. Brad and Karen taught fourth grade and used the same textbook, *Math Trailblazers*. In the survey, Brad expressed that he was pleased with *Math Trailblazers* and that the textbook matched what he thought was important in mathematics. In contrast, Karen and John took a neutral position on their satisfaction with the textbooks they used. These different levels of satisfaction with the textbooks seem to be mirrored by their use and enactment of the textbooks. For example, Brad claimed that he "followed" *Math Trailblazers*, and we observed that his instruction was closely aligned with the textbook, using high-level problems and questions. In contrast, Karen claimed to be a modifier of *Math Trailblazers* and lowered the cognitive demand of textbook problems by using a lower level of teacher questions. Similar to Karen, John, using a commercially developed textbook, *Scott Foresman-Addison Wesley Mathematics*, claimed to be a modifier of his textbook and was observed to provide low-level problems and

Table 3 Teachers' enactment of the textbook problems

	Planning	Enactment			
		Problem	Question	Reaction to incorrect student response	Reaction to correct student response
High level (Brad)	• Reads, evaluates, and uses textbook problems that require high-level thinking • Reads, evaluates, and modifies low-level problems into high-level one	• Does not change the problem and maintains the cognitive demand of the problem	• Asks students to defend their answers • Uses questions that maintain high cognitive demand • Asks students to justify • Asks student to explain their reasoning • Asks students to find patterns and applies the patterns to other problems • Asks students to verify the patterns with manipulatives	• Redirects students without taking over student responsibilities • Asks other students to clarify the question, • Asks students to explain their reasoning and invites other students to agree or disagree with the ideas.	• Asks students to explain their thinking. • Selects students to agree or disagree • Connects and sequences student ideas
Low-level teaching (Karen/Randy)	• Reads, evaluates, and selects textbook problems that require high-level thinking but often supple ment it with problems that require low-level thinking • Selects low-level problems • Deletes problem contexts or delete the use of representation	• Changes high-level problems into low-ones by telling the patterns and asking students to apply the rule to solve the problems. • Does not change the low level problems and maintains the low cognitive demand.	• Uses questions that require low level thinking • Asks students to focus on the rule and procedures • Starts with high level questions but use low level questions as students struggle or as students make mistakes.	• Tells students correct answers, • Ignores a student who gave incorrect response by asking other students to give a correct answer • Does not provide any opportunities to discuss the idea.	• Accepts student answers and does not probe student thinking. • Asks students to explain their solution strategies, but the focus is on how to get the answer • Does not connect student ideas and does not sequence student ideas

questions in his teaching. The following sections describe each teacher in detail in accordance with the analysis of the cognitive demand of selected problems, enacted problems, and teacher questions during instruction, as well as possible rationale for the three teachers' decisions. The examples were drawn from the specific instances that best represent each teacher's selection and enactment of textbook problems in regard to the cognitive demand on student thinking.

Selected problems

Overall, most of the problems presented in the textbook and used by Brad require a high level of cognitive demand from students. In one of the observed lessons, Brad taught the topic of equivalent fractions using *Math Trailblazers*. His lesson plan came directly from the teachers' guide of his textbook. He mentioned that he generally read the teachers' guide before teaching the lesson. He evaluated but rarely omitted or modified the textbook problems. In this lesson, the mathematical problems Brad used included the following: (a) Find all of the fractions from a fraction chart that are equivalent to $\frac{1}{2}$, (b) look for patterns by comparing the numerators and the denominators of fractions equivalent to $\frac{1}{2}$, (c) determine a rule for finding a fraction equivalent to $\frac{1}{2}$ by looking for patterns in the number sentences involving fractions equivalent to $\frac{1}{2}$, and (d) find fractions equivalent to $\frac{3}{4}$, $\frac{1}{3}$, and $\frac{2}{5}$ using the rule (multiplying or dividing the numerator and the denominator by the same number). According to Stein et al. (1996), these problems are categorized as requiring *procedures with connections* to concepts, which demand complex thinking and a considerable amount of cognitive effort.

In *Math Trailblazers*, there were also problems that required recall of a previously learned rule for finding equivalent fractions in a "student exercise," for example, "complete the number sentence, $\frac{3}{4} = \frac{?}{8}$ and check your work using your fraction chart" (Wagreich et al. 2004, p. 224). Without observing Brad's class, this textbook problem can be categorized as requiring a low-level cognitive demand. Yet, from Brad's viewpoint, this problem is a very meaningful to his students, as he commented during the interview:

> Looking at the fraction chart and seeing that [equivalent fractions], because I think that's the part they are going to thinking, that's going to give them most understanding as opposed to just knowing the rule.

Through the discussion in Brad's class, students attempted first to find patterns, then to derive the rule, and finally to verify the rule from a fraction chart. Therefore, the above problem, seemingly requiring just using the rule for equivalent fractions, *instead* was enacted with justification and verification of procedures to find equivalent fractions, which demanded students to think conceptually about equivalent fractions rather than simply applying the rule.

Karen used the same set of *Math Trailblazers* problems that Brad used in one of her observed lessons. While these problems were designed to focus students' attention to the discovery and use of patterns for finding equivalent fractions, Karen changed the cognitive demand of the problems as she interacted with students during instruction.

For example, when students failed to provide her expected answer, she changed the problem to make it less ambiguous and provided students with specific procedures to use. This tendency was more apparent when students generated incorrect responses. Karen then *concentrated on factual information* that could be memorized or she replicated previously learned rules.

John presented problems from *Scott Foresman-Addison Wesley Mathematics* (Randall et al. 2005) that placed low cognitive demand on students. Many problems presented in this textbook consisted of, "Find each sum or difference. Simplify, $3\frac{1}{3}+2\frac{1}{6}$" (Randall et al. 2005, p. 372), which focuses on procedures or the recall of previously learned procedures. Some problems were set in "real-world" contexts and/or required students to explain their solution. However, John often omitted such problems or lowered the cognitive demand of those problems. Even when John selected the textbook problems that represented fractions using pictures, he changed the problems to focus on computational procedures without using the representations.

Enacted problems

Brad closely followed the teachers' guide by using most of the examples, problems, and teacher questions from *Math Trailblazers*. Brad began the "teacher-led" activity suggested in the teachers' guide. He asked students to find fractions equivalent to $\frac{1}{2}$ using a fraction chart. As students found all fractions equivalent to $\frac{1}{2}$, Brad listed them on the board: $\frac{1}{2}=\frac{2}{4}=\frac{3}{6}=\frac{4}{8}=\frac{5}{10}$. He then asked students to explore the patterns among denominators. Students found that "they are all even, skip counting, multiples of two." Brad continued to have students explore the relationship between the numerator and denominator of each equivalent fraction, maintaining the high level of cognitive demand of the textbook problems. As students found the relationship between the numerator and denominator of each equivalent fraction (e.g., double the numerator and you get the denominator), Brad put a fraction $\frac{20}{40}$ and $\frac{15}{30}$ on the board and asked students to see whether these fractions were equal to $\frac{1}{2}$.

> Brad: You know me, right? If you say "Yes", I am going to say, "How do you know"? If you say "No", [I say] "Why not?." Student 1.
>
> Student 1: Because if we take away the zeros it's just the same as two-fourths.
>
> Student 2: Because twenty plus twenty equals forty and that (20) is half of 40.

In watching Brad interact with his students, it was obvious that a teacher-led activity in his class involved a discussion with students rather than recitation or lecture in which only the teacher talks. In particular, by using teacher questions that require reasoning and explanation, Brad seemed to try to elicit opinions and ideas, not just "right" answers, from students. Brad recorded several number sentences, for example, $\frac{1}{2}=\frac{2}{4}$, $\frac{1}{2}=\frac{4}{8}$, and $\frac{1}{2}=\frac{5}{10}$, and asked students to explore the relationship in the number sentences *by comparing the numerators of the fractions and then the denominators*. Most of the students seemed to recognize that in each pair of equivalent fractions, the numerator and denominator were multiplied by the same number.

However, one common approach was comparing the denominator of the first fraction with the numerator of the second fraction. For example, in the second number sentence $\frac{1}{2}=\frac{4}{8}$, one student answered "If you take 4 and times it by 2, you get 8." This gave an opportunity to see how Brad reacted to unexpected student responses, one of the challenges that many teachers face when they try to conduct lessons that build on students' thinking.

Brad: Let's try this one. The numerator in one-half is one and then numerator in four-eighths is four. Any relationship between one and four?

S 1: If you take four and times it two you get eight.

Brad: Okay but you're looking at the numerator and denominator in four-eighths. I want you to compare the *numerator* of one-half to the *numerator* of four-eighths. Over here $\left[\frac{1}{2}=\frac{2}{4}\right]$ we said if you multiple the numerator times two you get this numerator. If we multiple the denominator of one-half times two we would get the denominator here which is four. Student 2.

S 2: You multiply it, one times four equals four.

Brad: Even if we multiple the numerator one times four we get the denominator four. With that same thinking, how is the denominator? Two and eight.

S 3: If you times it by four, two times four equals eight.

When presented with unexpected responses, many teachers take over students' thinking and reasoning or tell students how to do the problem, thereby reducing the cognitive demand of the task (Smith and Stein 1998; Stein et al. 1996). However, in this instance and other observed instances, Brad asked other students what they thought of the incorrect response. He refocused students to find the *relation* between the equivalent fractions, not within each fraction.

After students shared patterns they found in the equivalent fractions, Brad asked students to generate the rule for finding equivalent fractions. Using examples, Brad and his students tested the rule and then checked the results with the fraction chart. Brad seemed to alter the "teacher-centered" portion of the lesson, facilitating students' discovery of patterns concerning a rule for finding equivalent fractions.

Karen used the same problems from *Math Trailblazers* as Brad. However, the ways in which students worked on the problems differed in the four classes. Like Brad, Karen used the activities suggested in the teachers' guide, asking students to look at the fraction chart and find all of the equivalent fractions to $\frac{1}{2}$. Karen then asked students to find patterns by comparing the denominators in $\frac{1}{2}=\frac{2}{4}=\frac{3}{6}=\frac{4}{8}=\frac{5}{10}$. Students noticed two patterns, "counting by twos" and "even," but they did not come up with the answer that Karen expected, which was "multiples of two." In this instance, Karen interacted with her students differently from Brad. Whereas Brad guided students to think about numerator-numerator and denominator-denominator and had students explain their ideas, Karen told the expected answer. This reaction demonstrates how teachers can decrease the cognitive demand by taking over student reasoning and telling students how to do the problem (Henningsen and Stein 1997).

This pattern of teacher-student interaction was observed in the rest of the lesson. For example, Karen asked students to look at the numerators and denominators of equivalent fractions and find patterns. Several students responded, but none provided her expected answer. Karen again said, "The numerator is half of the denominator, isn't it?" Karen's telling was more obvious when students did not figure out patterns in the equivalent fractions, as she wrote four fraction equivalents on the board and said,

> I have on the board four number sentences. Look at that pattern in these number sentences, first by looking at the numerators and then by looking at the denominators. What do you see?

$$\frac{1}{2} = \frac{2}{4}, \quad \frac{1}{2} = \frac{4}{8}, \quad \frac{1}{2} = \frac{5}{10}, \quad \frac{1}{2} = \frac{6}{12}$$

As in Brad's classes, Karen's students tended to compare the denominator of the first fraction and the numerator of the second fraction, and find the relationship, which was not what Karen expected. For example, one student said, "If you take two and times it by two you get four." Karen responded to this student as follows: "Is this what you were saying? One times two is?" Karen paused and students said, "Two". Karen again said, "Two times two is?" and paused. Students said, "Four."

Although Karen set up problems that encouraged finding relationships that would lead to the rule for finding equivalent fractions, as students became confused, Karen gave the rule. Karen said, "Here is the rule. Get your pens ready. If you multiply both the numerator and the denominator of a fraction by the same number, the result will be an equivalent fraction." During interviews, Karen said, "Students need to memorize it [the rule] to apply it," in contrast with Brad, who said, "Students need to make a discovery for themselves." By using the rule, Karen tried to have students see patterns in the rest of the fraction equivalents. Karen asked students to take a look at the third equation, $\frac{1}{2} = \frac{5}{10}$, and see whether the rule works. Some students nodded their heads to say yes and some kids shook their heads for no. During the follow-up interview, she said,

> That's what the rule says. Multiply both the numerator, that's my numerator and the denominator by the same number. The result will be an equivalent fraction and I said over here that one half equals five tenths.

Karen led the students through four number sentences (equations) involving equivalent fractions, while focusing on the procedure. For example, Karen repeatedly asked questions such as, "Three times what equals six?" pausing for students to say, "Two," and "Four times what equals twelfths?" pausing, as students said, "Three." These questions were procedure-oriented. Karen ended the lesson by asking students to create number sentences involving equivalent fractions, writing them as $\frac{1\times2}{6\times2} = \frac{2}{12}$. Karen called on one student to complete a similar number sentence. When the student said, "I don't know how to do this," Karen then called on other students. Indeed, it was noticeable during observation that a majority of students struggled with finding equivalent fractions when working on exercise problems. On the following day, Karen kept emphasizing procedures and repeatedly said, "That's what the rule said. Apply the rule" to find equivalent fractions.

John's class was procedure-oriented, with no use of drawings or manipulatives. During the interview, John described his class as "very much traditional lessons from the textbook." He perceived his fifth-grade students as mostly average achieving, similar to Brad and Karen. In his district, teachers could choose between *Investigations in Number*, *Data*, *and Space* (NSF-funded, Wagreich et al. 2004) and *Scott Foresman-Addison Wesley Mathematics* (Randall et al. 2005), the latter of which John chose as his main textbook. Whereas the textbook lessons had three parts—(1) Introduce, (2) Teach, and (3) Close and Assess—all four lessons observed in John's class were organized into two parts: an introductory "teacher-led" activity and "student exercises." However, different from Brad, a "teacher-led" activity in John's class consisted of lecture and demonstration rather than discussion or student involvement. John guided the students through example problems, breaking each problem into simple steps. Then, the students were given practice problems to solve on their own. He admitted that he not only generally read the teachers' guide before teaching a lesson but also considered redesigning curriculum as his major work. John mentioned that he often omitted problems or activities since textbooks included too many activities and problems to complete in one lesson. He also modified difficult problems into easier ones for students. Once he planned activities and problems, he rarely changed them during instruction.

John started his lesson by reviewing adding and subtracting fractions with unlike denominators and then addressed the topic for the lesson by saying, "We are going to be adding and subtracting today." John used the "example problem" below suggested in the teachers' guide. This problem was presented as a word problem in a real-world context in the textbook. However, without using the problem context, John asked students to tell him how to solve the problem. He first wrote it on the board and then asked students, "What do you think we need to do?"

$$\begin{array}{r} 3\frac{1}{2} \\ -2\frac{1}{8} \\ \hline \end{array}$$

Although the research revealed mixed results on the effect of word problems on students' mathematical learning (e.g., Nathan et al. 1992), it is often reported that word problems allow students to connect the problem to a familiar context in their own lives, motivate students to understand the importance of mathematics concepts, and help students develop their creative and critical thinking, and problem solving abilities (Verschaffel et al. 2000). Yet, John dismissed the context that could help students see the connection of subtraction with fraction to the real-world context and provided the following rationale for studying this topic: "It's on our assessment and that's why we're doing it."

During the observations, it was noticeable that, like Karen, John barely paused, moving quickly on to the correct answer. He did not even raise follow-up questions when students did not produce the correct answer. If students' responses reflected the correct answer, he paraphrased the answer, changing it slightly to make it more accurate and explicit. For example, this discussion took place for the question presented above:

John: What do you think is the first thing we have to do?

S 1: We have to simplify the numbers.

John: Not yet. At the end yes, we will have to simplify at the end if we get to the end.

S 2: I just want to say that we change the denominator and numerator.

John: Which means we are finding what?

Sts: (silence)

John: The equivalent fractions. We're finding equivalent fractions. We have to find equivalent fractions here at least for one of them before we can add.

S 1: Do we need to have them have the same denominator?

John: Yes, they have to have the same denominator.

After demonstrating how to solve this example problem, John asked students to work on the rest of the textbook "practice" problems, which focused on the procedure of finding sum or difference and then simplifying the answer. While students were working on these practice problems, John answered individual questions, prompted the next step in the procedure, and pointed out mistakes. As students finished the given task, John demonstrated the following example in the same manner he did in the first example.

Teacher questioning

As shown above, the questions the teachers asked during the observation of their lessons varied in terms of the cognitive demand they required from students. The question, "What is the pattern between numerators and the denominators?" most frequently used in Brad's class was suggested in the teachers' guide of *Math Trailblazers*. The verbs used in his questions, such as asking students to "compare" fractions and "look for patterns," require a high level of cognitive demand from *students*. Brad used these questions frequently, thus maintaining the cognitive demand suggested by the textbook. In addition, he also frequently used questions that require students to explain, justify, and verify their thinking, such as, "How do you know?" "Is it true?" "Prove it." In an interview, he provided his rationale for using these types of questions:

> One point I said, “You know me. I need you to explain *why*.” Because that’s never enough for them to just say “here is the answer.” I always want them to explain why. I need them to explain their thinking and give them a deeper understanding.

Even though the same lesson was observed in Karen’s classroom, her questions were quite different from Brad’s. The question most frequently used in Karen’s class was “what number do we multiply?” Karen confirmed it in her interview:

> I think probably the question that I asked over and over again and maybe not in this exact word is “What number should I multiply both the numerator and the denominator by to find the equivalent fraction?” I probably said that a hundred times.

This question focused on procedures without connections, and Karen was aware of her use of procedure-oriented problems and how frequently she used this sort of questions. Even though *Math Trailblazers* provides many suggestions for how teachers should question or discuss concepts, Karen did not use those suggestions. Like Brad, Karen did ask, “How did you solve the problem?” but the nature of “explaining” in her class meant restating the process or procedure of “how to do it,” which reduced the level of cognitive demand required by the textbook to the use of procedure without connections.

John’s questions also focused on procedures without connections. Further discussion regarding why this procedure made sense was rare. In an interview, John said “I’ll pose questions that have detailed answers and sometimes students will do part of that answer.” This emphasis on “detailed answers” implies a lower level of student thinking. He did not encourage students to explain how or why a procedure worked. Indeed, John’s questions were consistent with textbook questions requiring lower cognitive demand.

Issues critical to teachers’ selection and enactment of textbook problems

Table 2 summarizes related issues critical to each teacher’ selection and enactment of textbook problems. A cross analysis of teacher self-reports on the survey and interviews and classroom observations revealed four particular aspects that are related to teachers’ decisions on selecting and enacting textbook problems: (1) match between teacher beliefs and goals and those of the textbooks, (2) teachers’ views on their textbooks, (3) teacher interpretation of state curriculum framework and assessment, and (4) teacher knowledge or orientation toward student thinking.

Teacher goals and those of textbooks Teachers use their own beliefs and views to interpret the guidance in the curricular resources (Remillard 2005). In this study, the three teachers’ goals of teaching, pedagogical approaches, and teaching philosophy made them see things differently in the textbooks they used. Brad, using *Math Trailblazers*, set students’ understanding as an important teaching goal as indicated in the textbook. As a result, he selected and enacted textbook problems as intended by textbook designers. While enacting the selected problems, he used questions to support students’ understanding and thinking of the patterns in equivalent fractions, maintaining the high cognitive demand of the problems. In contrast, Karen, using the same textbook, did not value the guidance in helping students find the patterns in equivalent fractions; instead, she focused on the application of the rule to determine equivalent fractions. Therefore, despite the fact that she selected the same problem that Brad did, Karen enacted the problem in a way that decreased the cognitive demand of the problem. In fact, it was interesting to see that Karen exhibited a mixed view on student learning. Although in the survey and interviews, she mentioned the importance of students' understanding and their thinking about mathematics, Karen placed a greater emphasis on procedural aspects of student learning in instruction and stated during an interview that she considered "the role of teacher as an explainer," which did not match the intention of the textbook and made her lower the cognitive demand of the textbook problems.

As in the case of Brad, John's goals of teaching matched those of his textbook. Thus, he selected and used the textbook problems with a traditional view of mathematics teaching and learning as presented in the textbook (e.g., teachers as explainers and students as listeners; learning through reception of facts and repetitive practice of discrete skills). He did not see the need to alter the problems he selected. The match between teacher views on teaching and those of the textbooks led to Brad's and John's satisfaction with textbook problems they chose to use in instruction. On the other hand, the mismatch between Karen's goals and the textbook's and the mismatch between her view on the role of teacher and the textbook's made her ignore students' exploration of equivalent fractions during instruction that was specified in the teachers' guide.

Teachers' views on their textbooks In addition to the alignment between teacher beliefs and goals and those of the textbooks, teachers' perceptions of textbook problems seem to influence teachers' decisions in selecting and enacting textbook problems. We observed some discrepancies in Karen's and Brad's thinking about the textbook problems (see Table 4). Brad rated the presence of textbook problems in *Math Trailblazers* higher than Karen in the areas of explanation, communication, relationships, and real-life application. Indeed, Karen's view on her textbook problems was similar to John's, although John rated the presence of textbook problems differently in the areas of recall and communication. In conjunction with Karen's procedure-oriented teaching goals, her perception of textbook problems, in particular, problems that require recall, may explain why she added more practice-oriented problems in her teaching.

State curriculum framework and assessment It was found that all three of the teachers considered state curriculum documents and assessment when selecting and enacting textbook problems. Although they were all teaching the same grade in the same state, their interpretation of state curriculum documents and assessment was quite different. Brad used the state curriculum documents to make sure he covered all the recommended content in his teaching. Although Karen and John did so, they acknowledged more of procedural emphasis on student learning in the state documents and highlighted procedural components of state assessment. Both Karen and John reported that they supplemented a lot of practice problems to teach mathematics toward state assessment. They claimed to be "modifier" of the textbook by reporting that, in total, 50 % problems were from other sources. John's selection was primarily based on his determination of whether such a problem would appear in the state assessment. Karen

Table 4 Teachers' report on the presence of textbook problems

How often the following types of problems are presented in textbook?				
	Types of problems	Brad	Karen	John
High level	Problems requiring explanation and justification	4	3	3
	Problems involving communication	5	3	2
	Problems requiring developing own methods	3	3	3
	Problems emphasizing the relationships	4	3	3
	Problems requiring the use of representations	2	3	3
	Problems requiring real world application.	4	3	3
Low level	Problems requiring recalling facts and formulas	2	2	3

1=never, 2=rarely (once a unit), 3=sometimes (two to three times a unit), 4=often (four to five times a unit), 5=all most all lessons

said that she supplemented the practice problems to meet the state content expectations. Indeed, Karen's interpretation of state curriculum document was based on her notion of what and how students should learn, as evidenced by the focus of her problems and questions on *how* rather than *why.*

Student thinking All three teachers were found to consider student need in selecting and enacting textbook problems, yet they identified it differently, as evidenced in their interviews. Karen and John assigned a lot of practice problems because they thought that their students needed "procedural fluency" and "mastery of procedure." With the problem on equivalent fractions, Karen taught students to be "proficient in writing equivalent fractions" and "master the concept and apply it." Similarly, John attended "to get students to understand this one specific algorithm [algorithm for adding two mixed numbers] in adding and subtracting mixed numerals." When asked to explain what he meant by "understanding," he said, "know how to *apply* the algorithm in adding and subtracting mixed numbers." Even though he used the verb "understand," for John, "understand" means memorizing rules and applying them in procedural tasks. Although these three teachers' perception toward students' mathematical competence was similar, John only reported students' mathematical competence as one of the reasons he emphasized rules and procedures. In addition, both Karen and John could not tolerate seeing students have difficulty with problems and thereby provided specific procedures to follow to complete the problems, or simply skipped such problems. Karen said, "It's my job to explain, to show students how to do the work, and to assign specific practices." In contrast, Brad assigned challenging problems and supported students to persevere in solving them by asking questions rather than immediately providing a specific step to follow.

Discussion and implications

This study intended to contribute to the current literature on teachers' textbook use by exploring what tasks and problems teachers select, ways in which they enact the tasks and problems, and aspects or factors related to such teacher decisions. It was found that, although Brad and Karen used the same written lessons to design instruction, they posed different questions, which could lead to different levels of student thinking in terms of cognitive demand. Brad posed questions to develop the underlying ideas in equivalent fractions and generate the rule to find equivalent fractions, whereas Karen posed questions to guide students to apply the rule to find equivalent fractions. Despite some limitations, such as using one reform-oriented textbook series and focusing only on the topic of fractions, this study has implications for teacher education and professional development.

Despite the efforts in teacher education with NCTM's *Standards* documents and innovative curriculum materials, Karen and John still had limited view of mathematics and mathematics teaching and learning, and their teaching practice seemed far from reform in mathematics education. Though beliefs about teaching and learning are not always directly translated into teaching practices (Hativa et al. 2001), as evidenced in Karen's case, we think that challenging teachers' beliefs about teaching and learning is fundamental in helping them enact tasks with high cognitive demand. Teacher education and professional development can challenge teacher beliefs by generating opportunities for teachers to reexamine fundamental issues, such as what it means to learn mathematics and how students learn mathematics, and how teachers should create opportunities for students to learn.

Curriculum documents and textbooks are based on a particular pedagogical orientation, which is implicitly or explicitly addressed in them, and teachers interpret the guidance provided in the textbooks in a way that makes sense to them (Remillard 2005). This means that depending on the lens they use to interpret the textbooks, teachers can enact the textbook tasks and problems in ways different from those intended. For example, Karen valued the procedure and rule-based aspects of the textbook problems and ignored the meaning or understanding that was put forth by the textbook.

Understanding the essence and intention of the curriculum documents and materials is critical in order for teachers to select and enact tasks and problems as desired. It seemed that Karen did not understand the underlying philosophy of the textbook she used. Her use of textbook problems was mainly characterized as misuse or mechanical use (Kong and Shi 2009). Even when teachers choose tasks with high cognitive demand, there is a lower degree of maintaining high cognitive demand of the tasks in using reform-oriented textbooks (see Nie et al. 2013): Teachers set up high-level learning goals and end up accomplishing lower-level ones. It does not seem to be trivial for teachers to understand teaching goals of curriculum documents and textbooks.

Some professional development for teachers in the USA is provided with a particular textbook series they use (see, for example, http://www.mimathandscience.org/math/professionaldevelopment). We think that it is also important to use various types of textbooks as a context for teachers to learn. Ways to use textbooks in teacher learning should include examining affordances and limitations of various types of textbooks and investigating ways teachers can maximize the affordances and overcome the limitations in order to use the textbooks productively. Choppin (2011) elaborated on teacher knowledge of resources that facilitate student thinking, suggesting that teachers need to recognize the affordances of resources to help students learn the content.

Affordances and constraints that teachers perceive may not match the actual affordances and constraints of a textbook (Sullivan and Mornane 2014). For example, Karen thought that since her textbooks did not include sufficient practice problems, she needed more for her instruction. Professional development opportunities for teachers to discuss and analyze the actual affordances and constraints of a textbook in connection with current reform efforts (e.g., increasing the level of students' thinking) may address this issue. In addition, anticipating student thinking, such as how students might solve the problems and what struggles they may have, is one critical component that teacher education and professional development should include.

Teacher knowledge has been used to explain the nature of classroom practice and effectiveness of teaching. In this study, the kind of teacher knowledge most critical in the three teachers' selection and enactment of textbook problems is knowledge of student thinking (Carpenter et al. 1989; Choppin 2011). Although all three teachers revealed competence in the content of fractions during the interviews, it did not seem that Karen and John paid close attention to student thinking (what students actually think and the level of their thinking). Rather, they focused on explaining the rule that they wanted students to use. It seems that the biggest weakness of Karen and John is that they did not know what to do with student misconceptions. It is not a content problem, per se. It is that they did not know how to work from misunderstandings. Choppin (2011) particularly explained teacher knowledge of student thinking and the process of its development as an important aspect for using challenging tasks. Often, teachers mention students' need as a reason for selecting particular tasks (e.g., Nie et al. 2013), as did the teachers in this study. The question is whether teachers are well aware of actual students' need and their thinking. Thus, teachers like Karen and John may benefit from professional development programs whose content focuses on the curriculum, on "in-depth study" of student misconceptions, or on how to deal with student thinking. This will also help teachers evaluate the actual affordances and constraints of the textbook problems.

One may argue that this is already included in many professional development programs. Yet, Birman et al. (2007) show that few teachers receive intensive, sustained, and content-focused professional development in mathematics. A research synthesis also confirms the difficulty of translating professional development into teacher knowledge gains and student achievement gains despite the intuitive and logical connection (Yoon et al. 2007). Some studies have indicated that teachers in the USA typically do not analyze tasks in terms of cognitive demands, that is, in terms of the level of thinking that the task elicits from students (Arbaugh and Brown 2002; Stein et al. 1990). Instead, they tend to categorize tasks with respect to similarities in mathematical content or surface-level features such as "word problems" or "use of manipulatives." Other studies suggest that the teachers' selection of instructional tasks may be largely determined by lists of skills and concepts that they believe they are mandated to cover (Hiebert et al. 1997) or by their strict adherence to the tasks found in their textbooks (Remillard 1999). In fact, this study shows such cases. It seemed that Karen placed her emphasis on application of the rule to determine equivalent fractions without much consideration of student thinking and incorrectly recognizing current reform efforts. The reasons why teachers such as John and Karen did not work from such a premise seem much more complicated than supply changing mathematics teacher education. We emphasize the importance of teachers' learning opportunities of student thinking in connection with the affordances and constraints of their textbook and the significance of current reform efforts.

Nie and her colleagues (2013) reported that teachers using traditional textbooks, as opposed to those using reform-oriented textbooks, tend to follow the guidance in the textbooks in selecting and using tasks. As we observed in John, who followed the traditional teaching approach of the textbook, it is hard to increase the cognitive demand of the tasks and problems in textbooks with low cognitive demand in general, which requires a lot of demand on the teacher in altering a low-level task to a higher-level one. Comparing various textbooks and examining a range of tasks and problems that demand different levels of student thinking can help teachers using traditional textbooks to increase the cognitive demand of tasks by altering them to require a higher level of student thinking.

Furthermore, we suggest that discussion on "ill-defined substances" (Groth 2007), such as basic skills or procedural fluency, should be an important element of professional development. Karen mentioned "proficiency and "mastery" to explain her choice and use of tasks and problems. In her case, "proficiency" and "mastery" seemed only application of rules, although mastery of the grade-level expectations in state curriculum documents requires both conceptual understanding and computational skills (Wiggins and McTighe 2005; Wormeli 2005). Teachers need more opportunities to reexamine the real meaning of such fundamental substances.

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