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ORIGINAL ARTICLE

Adapting the academic motivation scale for use in pre-tertiary mathematics classrooms

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Abstract The Academic Motivation Scale (AMS) is a comprehensive and widely used instrument for assessing motivation based on the self-determination theory. Currently, no such comprehensive instrument exists to assess the different domains of motivation (stipulated by the self-determination theory) in mathematics education at the pre-tertiary level (grades 11 and 12) in Asia. This study adapted the AMS for this use and assessed the properties of the adapted instrument with 1610 students from Singapore. Exploratory and confirmatory factor analyses indicated a five-factor structure for the modified instrument (the three original AMS intrinsic subscales collapsed into a single factor). Additionally, the modified instrument exhibited good internal consistency (mean α=.88), and satisfactory test-retest reliability over a 1-month interval (mean r_{xx}=.73). The validity of the modified AMS was further demonstrated through correlational analyses among scores on its subscales, and with scores on other instruments measuring mathematics attitudes, anxiety and achievement.

Keywords Academic motivation scale · Mathematics motivation · Self-determination theory · Exploratory factor analysis · Confirmatory factor analysis

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The lack of academic motivation has been reported to be a useful predictor of negative education outcomes such as high withdrawal rates from school and students' aggression level (Barkoukis et al. 2008). Motivation has also been reported to correlate positively with desirable education outcomes such as high academic achievement and favorable subject-related attitudes (Gottfried et al. 2007). There is thus a clear need for valid instruments which measure motivation in various subject areas. However, various researchers (e.g., Ma and Kishor 1997; Palacios et al. 2014; Zan et al. 2006) have drawn attention to the lack of robust instruments to measure affective outcomes of mathematics education. In particular, Hannula (2002) and Mendick (2002) lamented that education researchers have given insufficient attention to the construct of mathematics motivation even though motivation has received the most attention among educational psychologists.

Among the most popular instruments used to measure mathematics motivation are the Fennema-Sherman Mathematics Attitudes Scales (Fennema and Sherman 1976) and the Attitudes Toward Mathematics Inventory (Chamberlin 2010, Tapia and Marsh 2004). However, these two instruments measure mathematics motivation as one of several domains that contribute to an overarching construct—attitudes toward mathematics. Seegers and Boekaerts (1993) argued that such general measurements of mathematics motivation (citing the Fennema-Sherman Attitudes Scales as an example) could only give a rough indication of students' motivation to do mathematics, and there is a need for an instrument that measures mathematics motivation more thoroughly.

The self-determination theory (Deci and Ryan 1985) is one of the most widely known theories on motivation (see the "Self-determination theory" section for details on the self-determination theory), and an instrument that measures mathematics motivation based on the self-determination theory would be greatly useful for use in research studies on mathematics motivation in schools. Among the many instruments designed to measure motivation based on the self-determination theory, the Academic Motivation Scale (AMS) (Vallerand et al. 1992) is one of the most frequently used instruments to measure motivation in a school's context (Grouzet et al. 2006). However, the AMS measures students' motivation to attend school and is not associated with specific academic subjects (Vallerand et al. 1992). Nevertheless, it is a potential instrument that could be adapted to measure motivation in mathematics comprehensively in accordance with the self-determination theory. The AMS comprises 28 items that measure motivation using seven subdomains of motivation, and hence would be a more comprehensive tool to measure mathematics motivation than the Attitudes Toward Mathematics Inventory and the Fennema-Sherman Mathematics Attitudes Scales which consist of only 5 and 12 items, respectively, that measure students' general motivation to do mathematics.

Self-determination theory

The self-determination theory specifies that academic motivation itself is made up of several subdomains, namely, amotivation (AMOT), extrinsic motivation (EMOT), and intrinsic motivation (IMOT). These domains exist on a self-determination continuum (see Fig. 1), according to individuals' perceived locus of causality that leads to their actions or behaviors (Baker 2004; Deci and Ryan 1985).

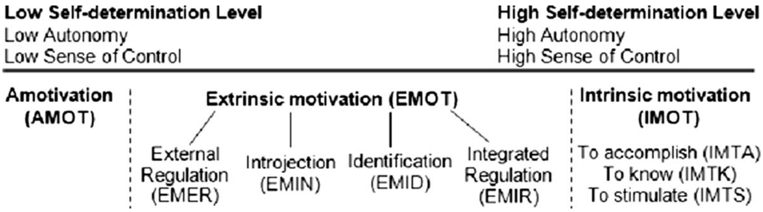


Fig. 1 Self-determination continuum (adapted from Ryan and Deci (2000b))

Amotivation lies on the extreme left of the self-determination continuum and represents the absence of both extrinsic and intrinsic motivation (Vallerand et al. 1992). It is associated with a perceived lack of control over one's actions, lack of value in a task, feelings of incompetence, or low expected returns from an activity (Ryan and Deci 2000b).

Extrinsic motivation lies in the middle of the continuum and stems from a desire to engage in an activity for an unrelated outcome (Ryan and Deci 2000b). Deci et al. (1991) stipulated that extrinsic motivation is further categorized, from lower to higher levels of self-determination (respectively), into external regulation (EMER), introjection (EMIN), identification (EMID) and integrated regulation (EMIR). External regulation is caused by rewards or punishments imposed by others (Vallerand et al. 1992). Introjection takes place when individuals internalize the reasons for their behaviors and impose their own rewards or constraints (Hayamizu 1997). It is associated with self-imposed controls such as guilt and ego-enhancement and takes place when individuals feel that they "ought to" participate in an activity. Identification results when an individual identifies with the reason for behaving in a particular manner. The behavior due to identification is valued by the individual and occurs because the individual "wants to" (Wang et al. 2009). Integrated regulation occurs when individuals internalize the reason for wanting to behave in a particular manner and accept that their actions are in line with their personal values and identities (Pelletier et al. 1997).

Intrinsic motivation lies on the extreme right of the continuum and refers to an inner desire to accomplish a task which results in feelings of pleasure and fulfillment for the individual (Hayamizu 1997).

Ryan and Deci (2000a) reasoned that high-quality learning and creativity are often the result of intrinsic motivation, and it is important that schools provide the factors and environment that engender intrinsic motivation. Ryan and Deci (2000a) further argued that intrinsic motivation could be undermined by the presence of extrinsic motivation that is associated with low level of autonomy (e.g., external regulation). However, more autonomous extrinsic motivation (e.g., integrated regulation and identification) are deemed to produce similar positive outcomes as intrinsic motivation. These outcomes include greater engagement in learning (Connell and Wellborn 1990), better performance (Miserandino 1996), and greater psychological well-being (Sheldon and Kasser 1995).

The Academic Motivation Scale (AMS)

The AMS is based on the self-determination theory described above. It is composed of 28 items measured on a seven-point scale with response options ranging from "does not correspond at all" to "corresponds exactly." The question common to all items is "Why do you go to college?"

In addition to the existing domains on the self-determination continuum, Vallerand et al. (1989) further categorized intrinsic motivation into three self-explanatory subtypes: intrinsic motivation to accomplish (IMTA), intrinsic motivation to know (IMTK), and intrinsic motivation to stimulate (IMTS) on the AMS. These three subtypes of intrinsic motivation are not differentiated on the self-determination continuum. Integrated regulation is excluded from the AMS, as very high correlations between integrated regulation, and its two adjacent subscales (i.e., identification and intrinsic motivation) have been found, and the authors of the AMS found no reason to measure integrated regulation as a separate subscale (Wang et al. 2009). In addition, integrated regulation is observed mainly in adults (Liu et al. 2009). The AMS thus measures seven factors: amotivation, external regulation, introjection, identification, and intrinsic motivation to accomplish, to know, and to stimulate. Given the underlying conceptual basis of the instrument, Vallerand et al. (1992) proposed that responses to the AMS should conform to a "simplex pattern," in which stronger positive relationships are expected between adjacent subscales, as compared to subscales that are further apart on the self-determination continuum. The strongest negative correlation is expected from subscales on opposite ends of the continuum (i.e., amotivation and intrinsic motivation).

The psychometric properties of the AMS have been established in numerous studies at both the tertiary level (Cokley 2000; Cokley et al. 2001; Fairchild et al. 2005; Smith et al. 2010; Vallerand et al. 1992) and the pre-tertiary level (Guimarães and Bzuneck 2008; Núñez et al. 2005; Núñez et al. 2006; Nuñez et al. 2010; Stover et al. 2012; Vallerand et al. 1993). Specifically, Vallerand et al. (1992) reported good temporal stability over a 1-month interval (n=57, mean r_{xx}=.79). Except for Cokley et al. (2001) who did not report internal consistency values, all other studies written in the English language reported satisfactory internal consistency levels ranging from .70 to .86 (n ranged from 217 to 2078, mean α=.82).

Additionally, Fairchild et al. (2005) and Vallerand et al. (1993) reported that the AMS displayed criterion-related validity with other instruments such as the Children's Academic Intrinsic Motivation Inventory (Gottfried 1986), the Task Orientation and Work Avoidance subscales (Nicholls et al. 1985), and the Work Preference Inventory (Amabile et al. 1994). Validity was supported by Cokley et al. (2001), Fairchild et al. (2005), and Vallerand et al. (1993) through correlational analyses between subscales of the AMS and other theoretically related constructs such as grades, perceived competence, self-concept, achievement goal orientation, and attitudes toward learning.

Despite these results, the simplex pattern had been found to be only partially supported, with some deviations from the introjection, identification, and intrinsic motivation subscales (Cokley 2000; Fairchild et al. 2005; Smith et al. 2010; Vallerand et al. 1993). Specifically, results from these studies suggest that introjection lies closer to intrinsic motivation, than to identification, which contradicts the self-determination continuum.

In addition, various studies that investigated the factor structure of the AMS reported only moderate support for the proposed seven-factor structure. Vallerand et al. (1992)

conducted a confirmatory factor analysis (CFA) that supported the seven-factor structure of the AMS, but only after 26 error covariances were added. The inclusion of so many error covariances requires cross validation with another sample (MacCallum 1986). However, this was not done. The rationales for adding the error covariances were also not discussed. Furthermore, the seven-factor structure of the AMS was only partially supported by a CFA conducted by Cokley et al. (2001), who suggested that the CFA might perform better if the intrinsic motivation subscales were collapsed. Cokley (2000) and Fairchild et al. (2005) reported that the intrinsic motivation factors were highly correlated and questioned the distinctiveness of these factors. Finally, Smith et al. (2010) reported that data collected from a group of masters students did not support the seven-factor structure, and further corroborated the instability of this structure. The above results suggested that further testing on the factor structure of the AMS was required.

Possible factor models for the adapted instrument

Nevertheless, the AMS is widely used by researchers who adopted the self-determination theory as the theoretical framework in their studies (Grouzet et al. 2006). In this study, a modified instrument from the AMS, termed the Academic Motivation Toward Mathematics Scale (AMTMS), was developed to create a tool to measure mathematics motivation based on the self-determination theory and validated using an Asian sample. Due to the mixed results obtained with respect to the factor structure of the AMS, one of the aims of this study is to assess the factor structure of the AMTMS, with first an exploratory factor analysis (EFA) to identify possible models. Subsequently, the results of the EFA, results from past research on the AMS, and theoretical reasoning (more details in the next paragraph) were used to identify the final models to be tested in a CFA.

Several researchers who had conducted studies on the AMS (Cokley 2000; Cokley et al. 2001; Fairchild et al. 2005; Vallerand and Bissonnette 1992) had suggested that other than the original seven-factor model, one other possibility was to collapse the three intrinsic motivation subconstructs (intrinsic motivation to accomplish, to know, and to stimulate) to form an overall intrinsic motivation construct. More importantly, the self-determination theory, which the AMS was based on, had advocated only a single factor for intrinsic motivation. Another possible model was to collapse the external regulation and identification subscales, since both sets of items refer to aspirations about the future, and results from past studies on the AMS (Cokley 2000; Fairchild et al. 2005; Smith et al. 2010; Vallerand et al. 1993) had suggested that the external regulation and identification subscales were closely related.

Predicted relationships with theoretically related constructs

An instrument that measures mathematics motivation as a multi-dimensional construct based on the self-determination theory will be useful for mathematics researchers and practitioners as various studies have reported that the various subdomains of mathematics motivation have varying degree of correlation with mathematics attitudes, anxiety, and achievement, which are important learning outcomes of mathematics education (e.g., Gottfried 1985; Ryan and Deci 2000b; Zakaria and Nordin 2008). This study aimed to make use of the close relationships between mathematics motivation and attitudes, between mathematics motivation and anxiety, and between mathematics motivation and achievement, to provide evidences on the concurrent validity of the AMTMS.

In this study, the short form of the Attitudes Toward Mathematics Inventory (short ATMI; Lim and Chapman 2013b), a revised version of the Fennema-Sherman Mathematics Anxiety Subscale (FSMAS-R; Lim and Chapman 2013a), and a pen-and-paper mathematics achievement test were used to measure mathematics attitudes, anxiety, and achievement, respectively. The short ATMI measures four domains of mathematics attitudes—enjoyment, motivation, self-confidence, and value—while the FSMAS-R measures two domains of mathematics anxiety—anxiety toward mathematics and ease with mathematics.

Table 1 summarizes the directions of the correlations predicted in this study, based on theoretical relations between the various subscales measured on the AMTMS and the external constructs measured in this study. The hypothesized pattern of correlations in this study may be described as an "external simplex pattern" (term adapted from Fairchild et al. 2005), in which favorable outcomes such as enjoyment of the subject matter, general motivation in learning the subject matter, and assignment of high value to the subject matter (i.e., three of the constructs measured by the short ATMI), as well as ease with the subject (measured in this study using the FSMAS-R), will demonstrate stronger positive correlations with the subscales of the AMTMS, as one moves from the left to the right of the self-determination continuum (i.e., amotivation→external regulation→introjection→identification→intrinsic motivation). This pattern was expected within the present study, as favorable outcomes should increase with higher level of self-determination (Ryan and Connell 1989). The reverse would be expected for perceived negative outcomes such as subject-related anxiety (measured in this study using the FSMAS-R). In other words, students who feel that they possess the highest level of self-determination should feel the least anxiety with mathematics. This would produce a pattern such that as one moves through the continuum from amotivation to intrinsic motivation, anxiety will decrease. This hypothesis is in line with results from other studies in mathematics education (e.g., Zakaria and Nordin 2008).

Two other outcomes would also be expected to demonstrate predictable relationships with the AMTMS. Various studies have suggested that amotivation and intrinsic motivation are (respectively) negatively and positively correlated with both self-confidence (e.g., Ryan and Deci 2000b) and achievement (e.g., Gottfried 1985).

The relationships between extrinsic motivation and self-confidence, as well as between extrinsic motivation and achievement, are less clear. Cognitive evaluation theory (Deci and Ryan 1980) suggests that the presence of an external reward or punishment may lead to lower intrinsic motivation which may result in lower levels of achievement and self-confidence. On the other hand, it is possible for an individual to put in more effort to achieve better results in the presence of an external reward or to

Table 1 Theoretical predictions for correlations with the short ATMI, FSMAS-R, and mathematics achievement

Construct	Enjoyment	General motivation	Self-confidence	Perceived value	Anxiety	Ease	Mathematics achievement
Instrument	Short ATMI enjoyment subscale	Short ATMI motivation subscale	Short ATMI self-confidence subscale	Short ATMI value subscale	FSMAS-R anxiety subscale	FSMAS-R ease subscale	Test score
Amotivation	–	–	–	–	+++	–	–
External regulation	0	0	0	0	++	0	0
Introjection	+	+	0	+	+	+	0
Identification	++	++	0	++	0	++	0
Intrinsic motivation	+++	+++	+	+++	–	+++	+

"+," "0," and "–" indicate positive, no, and weak correlations, respectively. Increasing number of "+" indicates increasing strength of positive correlation avoid punishment. Achieving better results may in turn boost the individual's level of confidence in performing the task.

Research aims of current study

The current study aims to answer the following questions: Using data from a predominantly Chinese sample from Singapore,

1. What is the factor structure of the AMTMS?
2. Did each of the AMTMS subscales show satisfactory internal consistency?
3. Did the AMTMS show satisfactory test-retest reliability over a 1-month period?
4. Did correlations among the AMTMS subscales conform to the simplex pattern?
5. What were the relationships between scores on the AMTMS subscales and scores from measures of theoretically related constructs—mathematics attitudes, anxiety, and achievement?

Method

This study was conducted in three phases. Phase 1 (n=12) was a pilot study that preliminarily assessed the suitability of the items in the AMTMS using an open-ended survey form followed by a discussion (see the "Procedures" section for details of the pilot study). Phase 2 (n=1610) was the main study that assessed the instrument's psychometric properties. Finally, phase 3 (n=208) assessed the instrument's test-retest reliability over a 1-month period.

Participants

All participants were grade 11 and 12 mathematics students enrolled in 6 out of the 21 pre-tertiary institutions in Singapore. All pre-tertiary institutions in Singapore prepare grade 11 and 12 students for the General Certificate of Education Advanced Level (GCE "A" level) Mathematics Higher 2 (Syllabus 9740) examination administered by the University of Cambridge-London Examination Syndicate (UCLES) in the English language which is the participants' first language. In terms of academic achievement at a national mathematics examination conducted annually, these six institutions consisted of two top, two average, and two bottom institutions. Phase 1 participants had an average age of 17.9 years. Males made up 41.7 % of the sample. All the participants were Chinese. Phase 2 participants had an average age of 17.8 years and were composed of 42.2 % males, 49.3 % females, and 8.5 % who did not indicate their gender. Additionally, 98.9 % were Chinese, 0.62 % were Indians, and the rest were Malays.

Phase 3 participants had an average age of 17.7, and were composed 51.4 % males, with the ethnic composition being 99.5 % Chinese and 0.5 % Indians.

Instruments

Academic Motivation Toward Mathematics Scale (AMTMS) The common question to the original Academic Motivation Scale (AMS) for all items was changed from "Why do you go to college?" to "Why do you spend time studying mathematics?" Specific individual statements were also adapted. Table 2 shows the modified and original items, and four new items that were created to replace items that could not be made specific to mathematics. The omitted items consist of one item each from the original amotivation, introjection, identification, and intrinsic motivation to accomplish subscales. The new items were created after referring to other established scales that measure academic motivation such as the Children's Academic Intrinsic Motivation Inventory (Gottfried 1985) and the Academic Self-Regulation Questionnaire (Ryan and Connell 1989).

Mathematics attitudes The ATMI is one of the most recent instruments developed to measure students' attitudes toward mathematics (Chamberlin 2010). As participants had to complete three affective instruments in a single session, the short form of the ATMI (Lim and Chapman 2013b) was used to avoid survey fatigue. The short ATMI comprises five enjoyment, four motivation, five self-confidence, and five value items. The four subscales respectively measures "the degree to which students enjoy working (on) mathematics," "(students') interest in mathematics and (their) desire to pursue further studies in mathematics," "(students') confidence and self-concept of (their) performance in mathematics," and "students' beliefs on the usefulness, relevance, and worth of mathematics to their lives" (Tapia and Marsh 2004, p. 17). Each item is measured on a five-point Likert-type response format with response options ranging from "strongly disagree" to "strongly agree."

Its factor structure, reliability, and validity were assessed with grade 11 and 12 students in Singapore (Lim and Chapman 2013b). Specifically, confirmatory factor analyses supported the original four-factor structure. The short ATMI exhibited strong correlations with the original scale ($r \geq .96$, $p < .01$ for all subscales), good overall internal consistencies, both for the full short version ($\alpha = .93$) and for the individual subscales ($\alpha \geq .85$), and satisfactory test-retest reliability over a 1-month period ($r_{xx} \geq .73$, $p < .01$). Further, Lim and Chapman (2013b) reported that the correlations between the short ATMI subscales, and their correlations with mathematics anxiety and with achievement concurred with the results of numerous other empirical studies and theoretical reasoning. These results supported the construct validity of the short ATMI.

Mathematics anxiety The Fennema-Sherman Mathematics Anxiety Scale (FSMAS) (Fennema and Sherman 1976) is one of the most commonly used mathematics anxiety scales in research studies (Evans 2001). In this study, its revised version (FSMAS-R; Lim and Chapman 2013a), which comprises two subscales measuring anxiety and ease, was used to measure mathematics anxiety. Items from the anxiety subscale and the ease subscale are measured using the same five-point Likert-type response format used in the short ATMI. The anxiety subscale contains four items and measures "feelings of anxiety, dread, nervousness, and associated bodily symptoms related to doing mathematics" (Fennema and Sherman 1976, p. 326), while the ease subscale contains five items and measures participants' comfort level when they are engaging in mathematics activities (Lim and Chapman 2013a).

Table 2 Modified and original items from the AMS

Label	Modified	Original
Question	Why do you spend time studying mathematics?	Why do you go to college?
AMOT1	Honestly, I don't know; I feel that it is a waste of time studying mathematics.	Honestly, I don't know; I really feel that I am wasting my time in school.
AMOT2	I can't see why I study mathematics and frankly, I couldn't care less.	I can't see why I go to college and frankly, I couldn't care less.
AMOT3	I don't know; I can't understand what I am doing in mathematics.	I don't know; I can't understand what I am doing in school.
AMOT4	(Replacement item) I am not sure; I don't see how mathematics is of value to me.	(Removed) I once had good reasons for going to college; however, now I wonder whether I should continue.
EMER1	Because without a good grade in mathematics, I will not be able to find a high-paying job later on.	Because with only a high-school degree I would not find a high-paying job later on.
EMER2	(No change) In order to obtain a more prestigious job later on.	
EMER3	(No change) Because I want to have "the good life" later on.	
EMER4	(No change) In order to have a better salary later on.	
EMIN1	Because of the fact that when I do well in mathematics, I feel important.	Because of the fact that when I succeed in college I feel important.
EMIN2	(Replacement item) Because I want to show to others (e.g., teachers, family, friends) that I can do mathematics.	(Removed) To prove to myself that I am capable of completing my college degree.
EMIN3	(No change) To show myself that I am an intelligent person.	
EMIN4	Because I want to show myself that I can do well in mathematics.	Because I want to show myself that I can succeed in my studies.
EMID1	Because I think that mathematics will help me better prepare for my future career.	Because I think that a college education will help me better prepare for the career I have chosen.
EMID2	Because studying mathematics will be useful for me in the future.	Because eventually it will enable me to enter the job market in a field that I like.
EMID3	Because I believe that mathematics will improve my work competence.	Because I believe that a few additional years of education will improve my competence as a worker.
EMID4	(Replacement item) Because what I learn in mathematics now will be useful for the course of my choice in university.	(Removed) Because this will help me make a better choice regarding my career orientation.
IMTA1	For the pleasure I experience while surpassing myself in mathematics.	For the pleasure I experience while surpassing myself in my studies.
IMTA2	For the satisfaction I feel when I can solve challenging mathematics questions.	For the satisfaction I feel when I am in the process of accomplishing difficult academic activities.
IMTA3	(Replacement item) Because I want to understand mathematics.	(Removed) For the pleasure that I experience while I am surpassing myself in one of my personal accomplishments.
IMTA4	Because I want to feel the personal satisfaction of understanding mathematics.	Because college allows me to experience a personal satisfaction in my quest for excellence in my studies.
IMTK1	Because I experience pleasure and satisfaction while learning new things in mathematics.	Because I experience pleasure and satisfaction while learning new things.

Table 2 (continued)

Label	Modified	Original
IMTK2	For the pleasure I experience when I discover new things in mathematics that I have never learnt before.	For the pleasure I experience when I discover new things never seen before.
IMTK3	For the pleasure that I experience in broadening my knowledge about mathematics.	For the pleasure that I experience in broadening my knowledge about subjects which appeal to me.
IMTK4	Because studying mathematics allows me to continue to learn about many things in mathematics.	Because my studies allow me to continue to learn about many things that interest me.
IMTS1	For the excitement I experience when I am communicating mathematics ideas to others.	For the intense feelings I experience when I am communicating my own ideas to others.
IMTS2	For the pleasure that I experience when I learn how things in life work, because of mathematics.	For the pleasure that I experience when I read interesting authors.
IMTS3	For the pleasure that I experience when I feel completely absorbed by what mathematicians have come up with.	For the pleasure that I experience when I feel completely absorbed by what certain authors have written.
IMTS4	For the "high" feeling that I experience while reading about various interesting mathematics materials.	For the "high" feeling that I experience while reading about various interesting subjects.

AMOT amotivation, *ERID* combination of external regulation and identification subcales, *EMIN* introjection, *IMOT* intrinsic motivation, *EMER* external regulation, *EMID* identification, *IMTA* intrinsic motivation to accomplish, *IMTK* intrinsic motivation to know, *IMTS* intrinsic motivation to stimulate

The factor structure, reliability, and validity of the FSMAS-R were assessed with grade 11 and 12 students in Singapore (Lim and Chapman 2013a). Exploratory and confirmatory factor analyses supported a two-factor model. Good internal consistencies (full scale α=.91; αs=.92 and .82 for the anxiety and ease subscales, respectively) and 1-month test-retest reliabilities ($r_{xx}\geq.77$, $p<.01$) were obtained. Lim and Chapman (2013a) also reported that the correlations between the FSMAS-R subscales, and their correlations with self-confidence in mathematics and with mathematics achievement, were all in line with theoretical justifications and past empirical research. These results supported the construct validity of the FSMAS-R.

Mathematics achievement A 3-h pen-and-paper test was used to assess mathematics achievement. The test contains 14 questions with three to six parts each. The questions were similar to past years' GCE A level mathematics questions. Marks were allocated based on students' working and final answers. Figure 2 shows a sample question from the achievement test. The marking scheme for this question can be found in Appendix 1. Five teachers, each with at least 5 years of mathematics teaching experience in a local pre-tertiary institution, set the test items and marking scheme. Additionally, a setter of the GCE A level mathematics paper from UCLES reviewed the test items and marking scheme and provided feedback on their validity. Each test item was marked by the same teacher for all scripts using the same marking scheme.

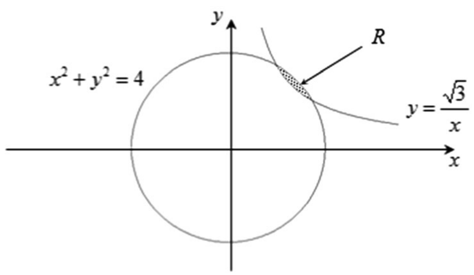
Procedures

In phase 1, 12 participants responded to the AMTMS. They were also asked to respond to open-ended questions, adapted from Bell (1987), on whether the instructions and items were clear, and how the scale could be improved. Besides obtaining written responses to these open-ended questions, a discussion based on the same questions was held with every participant to solicit feedback that participants might otherwise fail to report from the written survey. Results from this pilot phase indicated that the instructions and items in the modified scale were clear and appropriate. However, four participants suggested the use of a five-point Likert-type response format instead of the original seven-point, as they felt frustrated when choosing options from the smaller intervals of the seven-point response format. Furthermore, the seven-point scale discouraged them from choosing the two extreme options. As Dawes (2008) has shown that five- and seven-point scales produced the same mean score when rescaled to a comparable mean score out of 10, a five-point Likert-type response format was chosen for the AMTMS. The participants in phase 1 took at most 15 min to complete the AMTMS.

In phase 2, the AMTMS, measured on a five-point Likert-type response format with the following response options—1, "does not correspond at all"; 2, "corresponds a little"; 3, "corresponds moderately"; 4, "corresponds a lot"; and 5, "corresponds exactly"—was administered to 1618 participants by nine teachers in classroom or lecture settings. All participating teachers attended a briefing before they administered the scale. Participants were told explicitly that the results of the survey would not affect their school grades in any way and that they could choose to remain anonymous or opt out of the survey. Out of these 1618 participants, 1593 also completed the short ATMI and the FSMAS-R. In addition, a code was given to every participant to trace the mathematics test scores of the achievement test described above. This allowed the

(i) Using the substitution $x = 2\sin\theta$, show that $\int_1^{\sqrt{3}} \sqrt{4-x^2}\,dx = \frac{\pi}{3}$.

(ii) The diagram shows the shaded region R which is bounded by the circle with equation $x^2 + y^2 = 4$ and part of the curve with equation $y = \frac{\sqrt{3}}{x}$, $x > 0$. Verify that the two curves intersect at the points $(1, \sqrt{3})$ and $(\sqrt{3}, 1)$. Hence find the exact area of region R.



(iii) Express the volume of the solid generated when R is rotated through 2π radians about the y-axis as an integral and find this value using a graphic calculator.

Fig. 2 Sample question used in achievement test researcher to trace the test scores of 1027 participants. The missing data for the test scores appear to be random oversights (rather than systematic exclusions).

In phase 3, the AMTMS was administered two times over a 1-month period to assess test-retest reliability. In the first and second administrations of the AMTMS, respectively 232 and 215 participants completed all items in the survey forms. After matching participants who completed all items in both administrations, data from 208 survey forms were used.

Results and discussion

Prior to analysis, the data were inspected for missing cases. All the missing responses appeared to be random oversights of individual items within the survey. Any completed AMTMS form, which had more than five missing responses, was excluded from the final data pool. This reduced the initial sample of 1618 participants to 1610. Out of the remaining 1610 AMTMS forms, 13 forms had one missing response and these missing data were handled using the expectation-maximization (EM) or full information maximum likelihood (FIML) methods, which are two estimation approaches to impute missing data. Data imputation was chosen as this method had been widely recognized to be a better approach than other methods such as pairwise or listwise deletion (Vogt et al. 2014). The remainder of this section is organized according to the five aims stipulated for this study.

Factor structure of the AMTMS

The sample was split randomly (stratifying for gender and school) into two subsamples (n=805 each). Each subsample had 340 males, 397 females, and 68 participants with unknown gender. No statistically significant differences on the variables studied in this research were observed between the two subsamples (see Table 3). An EFA and CFA were performed on one subsample each using SPSS 19 and LISREL 8.71, respectively.

Prior to conducting these analyses, screening tests for conformity to underlying EFA and CFA assumptions were conducted. These tests generally produced satisfactory results. Mahalanobis distances and z-scores (using a conservative α level owing to the large sample size) indicated no notable multivariate or univariate outliers within the dataset. There was also no evidence of multicollinearity among all the item scores. For subsample 1, the skew indices ranged from −.77 to −35, and kurtosis indices ranged from −.12 to .54. For subsample 2, the skew indices ranged from −.74 to −35, and kurtosis indices ranged from −.70 to .59. Following Kline's (2010) recommendations that the skew and kurtosis indices should be within ±3 and ±10, respectively, the data for both subsamples were considered normal.

There has been much debate on the use of parametric analysis for Likert data. However, Norman (2010) suggested that Likert data could be analyzed using parametric tests without "fear of coming to the wrong conclusion." Jamieson (2004), as well as Lubke and Muthen (2004), also recommend that Likert-type response format surveys could justifiably be treated as interval data if the following conditions are met: the sample size is sufficiently large, there are at least five categories of response, and the

Table 3 Descriptive statistics and T-statistics for differences between subsamples 1 and 2

Label	Subsample 1 (n=805)		Subsample 2 (n=805)		Differences between subsamples	
	M (SD)	Skew (Kurtosis)	M	Skew (Kurtosis)	t	p
AMOT1	2.04 (1.14)	−.76 (.51)	2.04 (1.13)	−.73 (.49)	−.04	.97
AMOT2	1.96 (1.08)	−.72 (.54)	1.98 (1.1)	−.70 (.59)	−.43	.67
AMOT3	2.02 (1.13)	−.76 (.51)	2.01 (1.08)	−.70 (.51)	.16	.87
AMOT4	2.00 (1.09)	−.71 (.52)	1.99 (1.10)	−.70 (.55)	.14	.89
EMER1	3.22 (1.04)	−.35 (−.05)	3.20 (1.03)	−.35 (−.05)	.43	.67
EMER2	3.16 (1.05)	−.37 (−.04)	3.18 (1.05)	−.39 (−.05)	−.55	.59
EMER3	2.92 (1.13)	−.53 (.03)	2.91 (1.15)	−.57 (.04)	.28	.78
EMER4	3.00 (1.07)	−.42 (−.01)	2.99 (1.10)	−.47 (.00)	.28	.78
EMIN1	2.55 (1.13)	−.64 (.17)	2.50 (1.15)	−.67 (.20)	.81	.42
EMIN2	2.70 (1.13)	−.58 (.11)	2.64 (1.13)	−.60 (.13)	1.10	.27
EMIN3	2.73 (1.18)	−.67 (.11)	2.67 (1.17)	−.64 (.13)	1.04	.30
EMIN4	3.07 (1.16)	−.59 (−.02)	2.99 (1.15)	−.57 (.01)	1.40	.16
EMID1	3.26 (1.05)	−.40 (−.07)	3.32 (1.03)	−.39 (−.09)	−1.30	.20
EMID2	3.37 (1.03)	−.41 (−.10)	3.45 (1.01)	−.42 (−.12)	−1.66	.10
EMID3	3.14 (1.05)	−.38 (−.04)	3.20 (1.05)	−.38 (−.05)	−1.10	.27
EMID4	3.30 (1.16)	−.62 (−.10)	3.36 (1.15)	−.60 (−.12)	−1.01	.31
IMTA1	3.37 (1.05)	−.44 (−.10)	3.36 (1.04)	−.41 (−.10)	.33	.74
IMTA2	3.33 (1.22)	−.73 (−.12)	3.29 (1.22)	−.72 (−.11)	.71	.48
IMTA3	2.95 (1.10)	−.48 (.02)	2.93 (1.10)	−.47 (.02)	.41	.68
IMTA4	3.01 (1.20)	−.65 (.01)	2.94 (1.17)	−.61 (.02)	1.16	.25
IMTK1	3.06 (1.13)	−.53 (−.01)	3.02 (1.08)	−.44 (−.01)	.72	.47
IMTK2	2.96 (1.10)	−.47 (.01)	2.93 (1.11)	−.50 (.03)	.61	.54
IMTK3	2.93 (1.07)	−.41 (.02)	2.89 (1.07)	−.42 (.03)	.68	.50
IMTK4	2.95 (1.08)	−.43 (.02)	2.89 (1.04)	−.37 (.03)	1.06	.29
IMTS1	2.44 (1.16)	−.69 (.22)	2.41 (1.09)	−.62 (.22)	.44	.66
IMTS2	2.83 (1.10)	−.49 (.05)	2.83 (1.06)	−.42 (.04)	−.02	.98
IMTS3	2.56 (1.21)	−.75 (.20)	2.48 (1.16)	−.69 (.21)	1.31	.19
IMTS4	2.34 (1.18)	−.77 (.30)	2.29 (1.15)	−.74 (.31)	.88	.38

AMOT amotivation, *ERID* combination of external regulation and identification subcales, *EMIN* introjection, *IMOT* intrinsic motivation, *EMER* external regulation, *EMID* identification, *IMTA* intrinsic motivation to accomplish, *IMTK* intrinsic motivation to know, *IMTS* intrinsic motivation to stimulate

distribution is normal. The data collected in this study satisfies all the above mentioned conditions, and thus, parametric analyses which are more powerful and provide more information than nonparametric alternatives were used in this study.

As the amotivation items are negatively worded, a low mean score would mean that students had high level of motivation. For all the other items, a low mean score would mean that students had low level of motivation. Most of the mean scores were about the midpoint of 3. Students scored below 3 for most of the items in the AMTMS. This is not surprising given that past studies on students' attitudes toward mathematics such as the fourth cycle of the Trends in Mathematics and Science Study (TIMSS) (Mullis et al. 2008) have shown that East Asian students tend to rate their own attitudes toward mathematics lowly despite doing better than their western counterparts in mathematics. This could be due to the East Asian countries' cultures to appear humble and tendency to underrate one's ability in front of others.

EFA on subsample 1 Out of the 805 cases for subsample 1, 6 had missing responses. One of the most commonly used factor analysis procedures, maximum likelihood estimation (Brown 2006; Conway and Huffcutt 2003), was used in the initial extraction of factors from responses to the 28 items. As the subscales were expected to correlate, an oblique (promax) rotation that permits correlation among factors was used (see Table 4 for factor loadings and communalities, and Table 5 for correlations among factors). Based on Kaiser's criterion and Cattell's scree test, the items were loaded onto four factors, which together accounted for 66.78 % of the total score variance. Using a loading cutoff of .40 (Hair et al. 2006), cross-loading of items was not detected. This relatively simple structure contrasts with the original seven-factor structure of the AMS. In the present study, the intrinsic motivation to accomplish, to know, and to stimulate subscales from the original AMS collapsed into a single factor (labeled IMOT). The external regulation (EMER) and identification (EMID) subscales also combined into a single factor (labeled ERID).

CFA on subsample 2 Four alternative models were tested in the CFA due to reasons given in the first section of this paper: (1) a four-factor model based on the EFA results, which collapsed the three intrinsic motivation subscales to form IMOT and combined external regulation (EMER) and identification (EMID) to form ERID; (2) a five-factor model which collapsed the three intrinsic motivation subscales to form an overall intrinsic motivation subscale (IMOT); (3) a six-factor model which retained the three intrinsic motivation subscales but combined external regulation (EMER) and identification (EMID) to form ERID; and (4) the original seven-factor model.

Several indices were used to assess model fit (see Table 6). Since the chi-square statistic (χ^2) is strongly dependent on sample size and it is almost always statistically significant in models with large sample sizes (Brown 2006; Gatignon 2010; Hu and Bentler 1999), χ^2/df ratios instead of probability values are presented for each model. In general, χ^2/df ratios ranging from 2 to 5 are considered to represent adequate model fit (Byrne 1994). The chi-square change ($\Delta\chi^2$) statistic (Hu and Bentler 1999) was used to test for differences in fit between the models, since the various models tested were nested. Three absolute fit indices (the root-mean-square error of approximation with confidence interval, RMSEA, the standardized root mean residual, SRMR, and the goodness-of-fit index, GFI) and two incremental fit indices (the non-normed fit index, NNFI, and the comparative fit index, CFI) are also presented. Various authors (e.g., Browne and Cudeck 1993; Byrne 1994) suggest that good model fit is indicated by indices of less than .08 for SRMR, greater than .90 for the GFI and NNFI, and greater than .93 for CFI. Browne and Cudeck (1993) recommended that the RMSEA index should be less than .05 and .08 for close and reasonable model fits, respectively. All the χ^2 values were significant due to the large sample size (Brown 2006; Gatignon 2010; Hu and Bentler 1999). The five- and seven-factor models fit the data best with acceptable χ^2/df ratios and RMSEA index, and good SRMR, NNFI, and CFI indices.

Table 4 Factor loadings and communalities for items in the AMTMS based on subsample 1

Item labels	Items collapsed to form	Pattern matrix					
		Factor				Communalities	
		1	2	3	4	Initial	Extraction
IMTK3	IMOT	**.91**	−.02	.03	−.01	.76	.78
IMTK2		**.88**	−.01	.02	−.03	.72	.73
IMTS1		**.85**	−.02	.05	−.06	.65	.63
IMTS4		**.84**	−.03	.13	.03	.62	.60
IMTS3		**.82**	.00	.08	.04	.63	.63
IMTS2		**.81**	.11	.06	−.13	.60	.62
IMTK1		**.80**	−.08	−.11	−.04	.68	.68
IMTA4		**.78**	−.06	.00	.10	.65	.63
IMTA3		**.76**	−.03	−.02	.05	.63	.60
IMTK4		**.72**	.16	.00	−.03	.62	.60
IMTA2		**.61**	−.15	−.09	.21	.55	.51
IMTA1		**.59**	−.07	−.05	.19	.52	.46
EMER2	ERID	−.07	**.84**	.02	.05	.67	.70
EMER4		−.16	**.82**	.10	.19	.73	.76
EMER1		−.15	**.78**	.08	−.05	.53	.53
EMER3		−.14	**.76**	.07	.22	.68	.69
EMID2		.23	**.60**	−.15	−.13	.62	.52
EMID1		.23	**.59**	−.14	−.17	.60	.50
EMID4		.17	**.56**	−.10	−.10	.43	.41
EMID3		.34	**.44**	−.07	.04	.54	.48
AMOT3	AMOT	.04	.02	**.88**	−.02	.66	.72
AMOT2		.05	.01	**.86**	−.01	.64	.70
AMOT4		.05	.00	**.86**	.02	.64	.70
AMOT1		−.04	−.02	**.82**	−.07	.66	.72
EMIN4	EMIN	.02	−.07	−.07	**.80**	.54	.61
EMIN3		.03	.05	−.01	**.77**	.58	.65
EMIN2		.05	.06	.02	**.71**	.54	.57
EMIN1		.17	.12	.00	**.55**	.50	.48

Bold values signifies α = .05

AMOT amotivation, *ERID* combination of external regulation and identification subcales, *EMIN* introjection, *IMOT* intrinsic motivation, *EMER* external regulation, *EMID* identification, *IMTA* intrinsic motivation to accomplish, *IMTK* intrinsic motivation to know, *IMTS* intrinsic motivation to stimulate

However, the GFI fell short of acceptable cutoff for both models. Modifications were then made as explained later.

The intrinsic motivation (IMOT) subscale in the five-factor model has a very high Cronbach's alpha of .95. Nunnally and Bernstein (1994) recommend internal consistency scores between .70 and .91 and suggest a high level of item redundancy, if the internal consistency score is above .91. Moreover, the IMOT subscale contained many

Table 5 Correlations among factors extracted by EFA

	IMOT	ERID	AMOT	EMIN
IMOT	–	.39**	−.49**	−.42**
ERID		–	.19**	.45**
AMOT			–	.06
EMIN				–

AMOT amotivation, *ERID* combination of external regulation and identification subcales, *EMIN* introjection, *IMOT* intrinsic motivation

**$p<.01$

items after combining the intrinsic motivation to accomplish (IMTA), intrinsic motivation to know (IMTK), and intrinsic motivation to stimulate (IMTS) subscales. Hence, some items in the IMOT subscale were deleted to give a modified five-factor model. Three criteria were used to delete items: (1) item loadings, (2) improvement in fit indices, and (3) internal consistency. Specifically, items that had low factor loadings or high cross-factor loadings were shortlisted and removed one by one. Fit indices and internal consistencies were examined after the removal of each item. The removed item was returned to the scale, if fit indices worsened or internal consistencies fell outside the recommended range of .70 to .91. Subsequently, only five items (i.e., IMTA4, IMTK2, IMTK3, IMTS2, and IMTS3) were retained in IMOT. The resulting modified five-factor model without further post hoc modifications produced better fit indices (RMSEA=.061, 90 % CI [.057–.066]; SRMR=.047; GFI=.92; NNFI=.97; CFI=.98) than the seven-factor model. In particular, the GFI increased to .92. Hence, the more parsimonious modified five-factor model was chosen as the final model for the AMTMS. Figure 3 shows standardized path coefficients and standard errors (in parentheses) in

Table 6 Fit Indices for four CFA models

Model	χ^2	df	χ^2/df	RMSEA (90 % CI)	SRMR	GFI	NNFI	CFI
Four-factor (AMOT, ERID, EMIN, IMOT)	2808.07**	344	8.16	.094 (.091–.098)	.088	.80	.95	.96
Five-factor (AMOT, EMER, EMIN, EMID, IMOT)	1671.02**	340	4.91	.070 (.066–.073)	.052	.87	.97	.97
Six-factor (AMOT, ERID, EMIN, IMTA, IMTK, IMTS)	2599.29**	335	7.76	.092 (.088–.095)	.087	.81	.96	.96
Seven-factor (AMOT, EMER, EMIN, EMID, IMTA, IMTK, IMTS)	1453.77**	329	4.42	.065 (.062–.069)	.050	.89	.97	.98

RMSEA root-mean-square error of approximation, *CI* confidence interval, *SRMR* standardized root mean residual, *GOF* goodness-of-fit index, *NNFI* non-normed fit index, *CFI* comparative fit index, *AMOT* amotivation, *ERID* combination of external regulation and identification subcales, *EMIN* introjection, *IMOT* intrinsic motivation, *EMER* external regulation, *EMID* identification, *IMTA* intrinsic motivation to accomplish, *IMTK* intrinsic motivation to know, *IMTS* intrinsic motivation to stimulate

**Significant at α=.001 level

the final model. All paths in the model were significant. Amotivation correlated positively with external regulation, but negatively with all the other factors. The most negative correlation was with intrinsic motivation. This result is expected as amotivation is defined to represent the absence of intrinsic motivation (Vallerand et al. 1992). The decreasing correlation between amotivation and other factors as one moves across the self-determination continuum could be explained by the simplex pattern (more discussion later). The final instrument can be found in Appendix 2.

The EFA and CFA results suggested the collapse of the three original intrinsic motivation subscales of the AMS. This could be due to the modifications made to the original items, and the replacement of items that could not be modified. However, even without these modifications, this result was foreseeable as the distinctiveness of the intrinsic motivation items in the original AMS had been questioned by Cokley (2000) and Fairchild et al. (2005). Moreover, a single factor for intrinsic motivation corroborated the self-determination theory (Deci and Ryan 1985) which the AMS is based on, and as measured by other instruments that measure academic intrinsic motivation such as the Children's Academic Intrinsic Motivation Inventory (Gottfried 1985). Seven

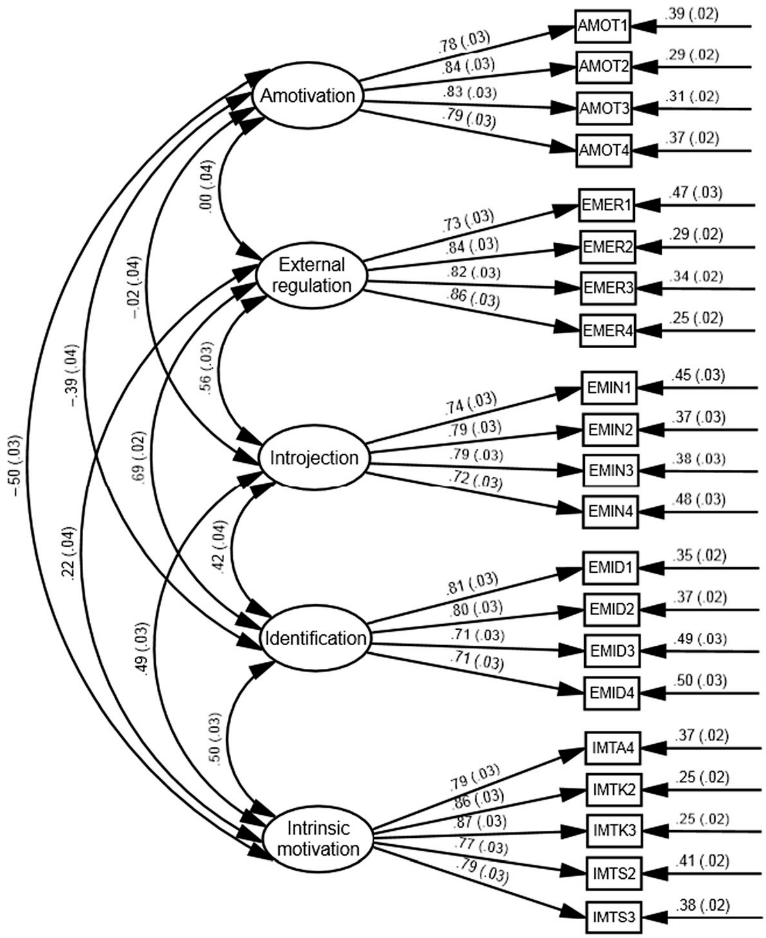


Fig. 3 Factor structure of the AMTMS

redundant intrinsic motivation items were removed to give a more parsimonious five-factor structure using CFA. This produced better fit indices than the original seven-factor structure. This result is supported by the results of other studies (Cokley 2000; Cokley et al. 2001; Fairchild et al. 2005; Smith et al. 2010; Vallerand et al. 1992) which had found only partial support for the seven-factor structure in the AMS.

Although the EFA performed on subsample 1 gave only four factors, where the external regulation and identification subscales collapsed to form a single factor, this was probably due to the similarity between the items in both subscales—both refer to aspirations about the future. Results from the CFA suggested that these two subscales should still be separated.

Internal consistency and test-retest reliability

The internal consistency scores for both subsamples were within the recommended range for both subsamples (see Table 7). According to Cohen's (1988) guideline of at least .70 for good test-retest reliability, test-retest reliability over a 1-month period for all subscales ranged from .71 to .75 and was deemed satisfactory (mean r_{xx} across all subscales=.73).

Presence of simplex pattern

To determine whether correlations among the five subscales of the AMTMS conformed to the simplex pattern proposed by Vallerand et al. (1992), the distributions of all summed variables were examined prior to calculating the correlation coefficients. No notable deviations from normality were detected. Pearson's product–moment correlation coefficients (see Table 8) generally conformed to the proposed simplex pattern. Significantly stronger relationships (p<.05) were found between adjacent subscales, relative to subscales that are further apart on the self-determination continuum. This was true with the exception of external regulation, which correlated significantly more positively with identification (EMID; r=.58) than with introjection (EMIN; r=.47), and introjection which correlated more positively (but not significantly different) with intrinsic motivation (r=.42) than with identification (r=.38). Similar results were reported by Cokley (2000), Fairchild et al. (2005), and Smith et al. (2010). This result

Table 7 Internal consistency scores for both subsamples and test-retest reliability

	Amotivation	External regulation	Introjection	Identification	Intrinsic motivation	Mean
Subsample 1 (n=805)	.91	.89	.84	.83	.91	α=.88
Subsample 2 (n=805)	.89	.89	.85	.84	.91	α=.88
Test-retest reliability (n=208)	.71**	.73**	.73**	.73**	.75**	r_{xx}=.73

**p<0.01

Table 8 Correlations among subscales of the AMTMS

	Amotivation	External regulation	Introjection	Identification	Intrinsic motivation
Amotivation	–	.01	−.06*	−.34**	−.49**
External regulation		–	.47**	.58**	.16**
Introjection			–	.38**	.42**
Identification				–	.46**
Intrinsic motivation					–

**p<.01, *p<.05

could be due to the similarity between items in the external regulation and identification subscales. This result should not indicate that introjection represents more self-determined behavior than stipulated by the self-determination continuum, since the simplex pattern was observed with the other subscales.

Correlation with theoretically related constructs

The last research question in this study aims to investigate the pattern of correlations between subscales of the AMTMS and theoretically related constructs—attitudes, anxiety, and achievement (see Table 9). As expected, the external simplex pattern was observed with the enjoyment, motivation, and ease subscales (i.e., enjoyment, motivation, and ease demonstrated a stronger positive correlation from amotivation to intrinsic motivation). Interestingly, the value subscale did not conform entirely to the external simplex pattern: Value had the strongest positive correlation with identification (r=.69), followed by intrinsic motivation (r=.57), introjection (r=.31), external regulation (r=.31), and then amotivation (r=−.54). To explain this discrepancy, it is necessary to examine the items within the identification subscale. These items refer to a desire to engage in mathematics because it will better prepare the student for a future career, be useful in the future, improve work competence, or be useful for the student's desired course in university. Thus, these items clearly relate to perceived value of the subject in terms of its usefulness. Utility is a component of most definitions of value. Indeed, the definition of the value subscale provided by Tapia and Marsh (2004) indicates that it is designed to measure "students' beliefs on the usefulness, relevance, and worth of mathematics in

Table 9 Correlations between subscales of the AMTMS and other constructs

	Enjoyment	Motivation	Self-confidence	Value	Anxiety	Ease	Achievement
Amotivation	−.60**	−.56**	−.56**	−.54**	.58**	−.40**	−.35**
External regulation	.02	.08*	−.08*	.31**	.06*	.02	−.04
Introjection	.27**	.25**	.06	.31**	−.06*	.17**	.04
Identification	.36**	.39**	.22**	.69**	−.26**	.28**	.15**
Intrinsic motivation	.75**	.75**	.42**	.57**	−.45**	.54**	.33**

**p<.01, * p<.05

their life now and in the future" (p. 17). Therefore, given the way in which the AMTMS operationalizes identification, the reasons for the strong relationship between this subscale and the value subscale of the ATMI are clear. This does not in any way compromise the validity of the AMTMS.

As predicted, the reversed external simplex pattern was observed with anxiety. On the other hand, self-confidence followed the external simplex pattern and showed stronger positive correlations with the subscales of the AMTMS as one moves from the left to the right of the self-determination continuum (i.e., amotivation→external regulation→introjection→identification→intrinsic motivation). This result is unexpected given that there is neither theoretical basis nor prior empirical research to explain the presence of the external simplex pattern with self-confidence. However, self-confidence in mathematics and mathematics anxiety are very strongly correlated (Tapia and Marsh 2004). Hence, the external simplex pattern observed with self-confidence could be due to the strong correlation between self-confidence in mathematics and mathematics anxiety.

The results of this study showed that the external simplex pattern was observed with mathematics achievement. Similar results were reported by Vallerand et al. (1993). Fairchild et al. (2005) and Vallerand et al. (1993) also reported that achievement was not significantly correlated with external regulation and introjection. This result attests to Ryan's (1982) suggestion that there is no clear relationship between achievement and extrinsic motivation. However, identification correlated positively and significantly with achievement, providing support for identification's proximity to intrinsic motivation on the self-determination continuum. Other studies (e.g., O'Dwyer 2005; Shen 2002) supported the strong positive correlation between intrinsic motivation and achievement shown in this study.

Conclusion

Compared with existing studies on the original AMS, the results of this study, based on the AMTMS, were better aligned with the hypothesized internal and external simplex patterns. Overall, based on the results presented above, this exploratory study on the AMTMS shows considerable promise for measuring mathematics motivation as a multi-faceted construct that is based on the self-determination theory at the pre-tertiary level in Asia.

However, the results of this study are based only on a predominantly Chinese sample from Singapore. Further studies with students from other cultures and schooling levels are necessary before it can be used with other samples. In addition, the responses collected for this study were in the form of self-reports which were susceptible to problems such as the inability to comprehend items and to map judgments onto a response scale (Tourangeau et al. 2000). Great care was taken to minimize these problems. For instance, a pilot phase was administered to discern possible problems that participants might have with the survey questions, and all affective instruments used to check the validity of the AMTMS had been assessed with participants with similar characteristics to those in the present study. The quality of the achievement measure used in this study could be better improved with the use of standardized tests such as the SAT. This could be explored in future research.

Future research could also explore the addition of items to address the integrated regulation subdomain on the self-determination theory which forms the theoretical framework for the Academic Motivation Scale. Past studies on the AMS have found no reason to measure integrated regulation as a separate domain as it had very high correlations with two adjacent subscales. However, it is possible that samples from different cultures and background would give different findings.

The results of this study showed that amotivation, identification, and intrinsic motivation had a significant relationship with mathematics achievement. Future studies may consider the use of multiple regression to further investigate the relationships among these variables.

Overall, the results of this study showed that the AMTMS can potentially be used to better understand students' motivation toward mathematics. In addition, the AMTMS can possibly be used to evaluate the effectiveness of intervention programs designed to improve students' motivation in studying mathematics. The instrument can also be easily modified to measure motivation in other academic subjects.

Appendix 1

Marking scheme for sample item in achievement test

Method 1 for (i)	B1 – differentiate substitution correctly
$x = 2\sin\theta$	
$\frac{dx}{d\theta} = 2\cos\theta$	
$x = 2\sin\theta = \sqrt{3} \Rightarrow \theta = \sin^{-1}\frac{\sqrt{3}}{2} = \frac{\pi}{3}$	B1 – correct limits
$x = 2\sin\theta = 1 \Rightarrow \theta = \sin^{-1}\frac{1}{2} = \frac{\pi}{6}$	B1
$\int_1^{\sqrt{3}} \sqrt{4-x^2}\,dx = \int_{\pi/6}^{\pi/3} \sqrt{4-4\sin^2\theta}\,2\cos\theta\,d\theta$	
$= \int_{\pi/6}^{\pi/3} 2\sqrt{1-\sin^2\theta}\,2\cos\theta\,d\theta$	
$= \int_{\pi/6}^{\pi/3} 4\cos^2\theta\,d\theta$	M1– substituting for x and dx
	A1 – $k\cos^2\theta$
$= \int_{\pi/6}^{\pi/3} 2(\cos 2\theta + 1)\,d\theta$	B1 – 2 $\cos^2\theta$ = cos 2θ + 1
$= [\sin 2\theta + 2\theta]_{\pi/6}^{\pi/3}$	B1– integrate $\cos^2\theta$ correctly to obtain $\frac{\sin 2\theta}{2} + \theta$
$= \left(\frac{\sqrt{3}}{2} + \frac{2\pi}{3}\right) - \left(\frac{\sqrt{3}}{2} + \frac{\pi}{3}\right) = \frac{\pi}{3}$	A1 – correct answer (AG)
Method 2 for (i)	B1 – differentiate substitution correctly
$x = 2\sin\theta$	
$\frac{dx}{d\theta} = 2\cos\theta$	
$\int \sqrt{4-x^2}\,dx = \int \sqrt{4-4\sin^2\theta}\,2\cos\theta\,d\theta$	M1
$= \int 2\sqrt{1-\sin^2\theta}\,2\cos\theta\,d\theta$	
$= \int 4\cos^2\theta\,d\theta$	
$= \int 2(\cos 2\theta + 1)\,d\theta$	A1 – $k\cos^2\theta$
$= \sin 2\theta + 2\theta$	B1 – 2 $\cos^2\theta$ = cos 2θ + 1
	B1– integrate $\cos^2\theta$ correctly to obtain $\frac{\sin 2\theta}{2} + \theta$

$= 2\sin\theta\cos\theta + 2\theta$ $= 2\left(\frac{x}{2}\right)\sqrt{1-\left(\frac{x}{2}\right)^2} + 2\sin^{-1}\left(\frac{x}{2}\right)$	M1 – convert back from θ to x correctly A1 – $2\left(\frac{x}{2}\right)\sqrt{1-\left(\frac{x}{2}\right)^2} + 2\sin^{-1}\left(\frac{x}{2}\right)$
Hence, $\int_1^{\sqrt{3}}\sqrt{4-x^2}\,dx = \left[2\left(\frac{x}{2}\right)\sqrt{1-\left(\frac{x}{2}\right)^2} + 2\sin^{-1}\left(\frac{x}{2}\right)\right]_1^{\sqrt{3}}$ $= \left(\frac{\sqrt{3}}{2}+\frac{2\pi}{3}\right) - \left(\frac{\sqrt{3}}{2}+\frac{2\pi}{6}\right) = \frac{\pi}{3}$	A1 – correct answer (AG)
(ii) When $x=1, (1)^2+y^2=4$ $\Rightarrow y = \sqrt{3}$ (since y is positive) When $x = 1, y = \frac{\sqrt{3}}{1} = \sqrt{3}$ When $x = \sqrt{3}, \left(\sqrt{3}\right)^2 + y^2 = 4$ $\Rightarrow y=1$ (since y is positive) When $x = \sqrt{3}, y = \frac{\sqrt{3}}{\sqrt{3}} = 1$ Thus, two curves intersect at the points $\left(1, \sqrt{3}\right)$ and $\left(\sqrt{3}, 1\right)$.(Verified)	B1 – able to verify one of the points correctly B1 – verify other point correctly
Area $= \int_1^{\sqrt{3}}\sqrt{4-x^2}\,dx - \int_1^{\sqrt{3}}\frac{\sqrt{3}}{x}\,dx$	B3 – correct expressions, correct limits B2 – correct expressions, incorrect limits
$= \frac{\pi}{3} - \left[\sqrt{3}\ln x\right]_1^{\sqrt{3}}$ $= \frac{\pi}{3} - \left(\sqrt{3}\ln\sqrt{3} - 0\right)$ $= \frac{\pi}{3} - \sqrt{3}\ln\sqrt{3}$	√ M1– use (i) B1 – $\sqrt{3}\ln x$ √A1 – $\sqrt{3}\ln\sqrt{3}$ answer from previous part A1 – correct answer
(iii) Volume = $\pi\int_1^{\sqrt{3}}\left(4-y^2-\frac{3}{y^2}\right)dy$	B1 – correct expression $\int(4-y^2)\,dy$ B1 – correct expression $\int\frac{3}{y^2}\,dy$ B1 – correct limits B1 – $\int_1^{\sqrt{3}}\left(4-y^2-\frac{3}{y^2}\right)dy$
= 0.262 π or 0.822	B2 – 0.262 π [B1 if π omitted]

Notes. M=mark allocated for a correct method applied to appropriate numbers, A= mark allocated for accuracy and depends on M marks; M0 A1 is not possible, B= independent accuracy marks; A fully correct final answer may receive full marks without the need to check for method.

Appendix 2

Academic motivation scale

Directions:

In the answer sheet and using the scale below, shade ● the number that best indicates the extent to which each of the following items presently corresponds to one of the reasons why you spend time studying mathematics.

Why do you spend time studying mathematics?

Does not Correspond at all	Corresponds a little	Corresponds moderately	Corresponds a lot	Corresponds exactly
1	2	3	4	5

1. Honestly, I don't know; I feel that it is a waste of time studying mathematics.
2. Because I want to show to others (e.g., teachers, family, friends) that I can do mathematics.
3. Because I want to show myself that I can do well in mathematics.
4. I am not sure; I don't see how mathematics is of value to me.
5. Because without a good grade in mathematics, I will not be able to find a high-paying job later on.
6. Because I believe that mathematics will improve my work competence.
7. For the pleasure I experience when I discover new things in mathematics that I have never learnt before.
8. In order to obtain a more prestigious job later on.
9. To show myself that I am an intelligent person.
10. In order to have a better salary later on.
11. For the pleasure that I experience when I feel completely absorbed by what mathematicians have come up with.
12. Because what I learn in mathematics now will be useful for the course of my choice in university.
13. Because I want to feel the personal satisfaction of understanding mathematics.
14. Because studying mathematics will be useful for me in the future.
15. For the pleasure that I experience in broadening my knowledge about mathematics.
16. Because I want to have "the good life" later on.
17. I don't know; I can't understand what I am doing in mathematics.
18. Because I think that mathematics will help me better prepare for my future career.
19. I can't see why I study mathematics and frankly, I couldn't care less.
20. Because of the fact that when I do well in mathematics, I feel important.
21. For the pleasure that I experience when I learn how things in life work, because of mathematics.

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