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ORIGINAL ARTICLE

Developing learning environments which support early algebraic reasoning: a case from a New Zealand primary classroom

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Abstract Current reforms in mathematics education advocate the development of mathematical learning communities in which students have opportunities to engage in mathematical discourse and classroom practices which underlie algebraic reasoning. This article specifically addresses the pedagogical actions teachers take which structure student engagement in dialogical discourse and activity which facilitates early algebraic reasoning. Using videotaped recordings of classroom observations, the teacher and researcher collaboratively examined the classroom practices and modified the participatory practices to develop a learning environment which supported early algebraic reasoning. Facilitating change in the classroom environment was a lengthy process which required consistent and ongoing attention initially to the social norms and then to the socio-mathematical norms. Specific pedagogical actions such as the use of specifically designed tasks, materials and representations and a constant press for justification and generalisation were required to support students to link their numerical understandings to algebraic reasoning.

Keywords Mathematics education · Primary education · Early algebra · Classroom communication

Introduction

Significant changes have been proposed for mathematics classrooms of the twenty first century in order to meet the needs of a 'knowledge society'. An increased focus in both national and international research and curricula reforms has led to the need for teachers to develop learning communities where all students have opportunities to engage in mathematical discourse and practices which underlie algebraic reasoning (e.g. Blanton & Kaput, 2005; Department for Education and Employment DfEE 1999; National Council of Teachers of Mathematics 2000). This has arisen from growing

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acknowledgment of the insufficient algebraic understandings that students develop during schooling and the way in which this denies them access to potential educational and employment prospects (Kaput, 1999; Stacey & Chick, 2004). As a result, teachers are charged with providing their students with opportunities to learn algebra with deep understanding embedded in rich conceptual talk.

Opportunities to engage in rich conceptually embedded mathematical talk are a critical component of any mathematics classroom. From a wide range of empirical and theoretical research (e.g. Goos, 2004; Hunter, 2009; Lampert & Cobb, 2003), we learn of the positive outcomes for students when classroom practices focus on communication, interaction and conceptual understanding of deeper mathematical ideas. These studies provide us with a view of teachers in reform classrooms facilitating student engagement in mathematical inquiry in learning communities. The aim in these communities is for all participants to have opportunities to engage in mathematical discourse using proficient mathematical practices. These practices include providing mathematical explanation, justification, representation and generalisations—a key to students' development of early algebraic reasoning. Recent national and international policy documents follow the same trend. For example, in New Zealand, the most recent curriculum document requires that teachers develop classrooms as 'learning environments that foster learning conversations and learning partnerships and where challenges, feedback and support are readily available' (Ministry of Education, 2007, p. 24). Similarly, the Principles and Standards document in the USA (NCTM, 2000) promotes the centrality of teachers supporting student communication of mathematical ideas and reasoning.

The teacher's role is pivotal in establishing learning environments that focus on both development of mathematical discourse in a community of inquiry and early algebraic reasoning. There are a range of studies which illustrate how students construct early algebraic concepts from numerical context (e.g. Blanton & Kaput, 2005; Carpenter, Franke & Levi, 2003; Carraher, Schliemann, Brizuela & Earnest, 2006) and potential difficulties for students when developing algebraic reasoning (e.g. Kieran, 1981; Kuchemann, 1981, MacGregor & Stacey, 1997). These include the authors' own papers (Anthony & Hunter, 2008; Hunter, 2010; Hunter & Anthony, 2008). In these research papers, the author illustrated student construction of aspects of early algebraic reasoning (for example, relational thinking, the use of variables and generalisation of functional patterns). There are also the many studies (some of which are described in the previous paragraph) which describe classroom practices that support students developing inquiry discourse within communities of learners. In these papers, we are made aware of the many difficulties teachers encounter in constructing these environments. However, few studies specifically address the explicit pedagogical actions teachers take which structure student engagement in dialogical discourse and activity which facilitates early algebraic reasoning.

Research studies note the difficulties that teachers have in their changed roles and identities teaching in inquiry communities (e.g. Hunter, 2009; Stein, 2007), whilst others note the problems caused by the teachers' own lack of experience teaching or learning in inquiry environments (e.g. Sherin, 2002). Teachers' inexperience in teaching extends to environments which explicitly promote the discourse and mathematical practices which support student construction of early algebraic reasoning (Blanton 2008). A number of researchers (e.g. Kaput, 1999; Stacey & Chick, 2004) note that teachers face many challenges to find ways to make algebra accessible to all students of all ages and to create an environment in which students learn with conceptual understanding. These researchers draw our attention to the need for multiple models for how teachers might enact mathematical discourse and practices which promote student construction of early algebraic reasoning within classroom communities. This article will provide a picture of the pedagogical actions a teacher took to gradually construct an environment in which her students were offered multiple opportunities to engage in the mathematical practices and discourse through which they accessed early algebraic reasoning.

The role of the teacher in constructing the social and socio-mathematical norms of inquiry classrooms

The role of the teacher in facilitating the norms which support development of an inquiry learning environment is significant. In an established inquiry classroom, the locus of responsibility is shared by all participants within the classroom community. Both students and teachers share responsibility to develop a community of learners who are able to communicate their own thinking, listen and learn from others and reflect both on their own and others' thinking (McCrone, 2005). Initially, it is the teacher who guides the development of the ways of working within the classroom and ensures that all students actively engage in mathematical inquiry. Through deliberate teacher actions, a range of social and socio-mathematical norms are co-constructed with the students. For example, Hunter (2009) in a study with primary students worked with four teachers to construct communities of mathematical inquiry where students engaged in the discourse of inquiry and argumentation. In this study, Hunter illustrated how the gradual co-construction of a range of social and socio-mathematical norms supported student engagement in increasingly more proficient mathematical practices, practices which supported student generalisations. Through the co-construction of the social and socio-mathematical norms, the students learnt about valued ways of working both socially and mathematically.

Pedagogical actions which promote productive discourse

Collaborative interaction and classroom mathematical discourse are inter-linked. Discourse fosters a learning community, and at the same time, learning communities have the possibility of generating useful dialogue between learners (Manoucheri & St John, 2006). Zack (1999) illustrated how fifth grade students used the norms of the learning community to work together to prove or refute arguments and counterarguments when solving an algebraic problem. Engagement in collaborative interaction requires a shift away from the more traditional role of students as passive receivers of instruction to active and constructively critical participants. Whilst not easy to achieve, we do have some insights from researchers and their studies about effective pedagogical practices that support such a shift (e.g. Goos, 2004; McCrone, 2005; O'Connor & Michaels, 1996). McCrone illustrated how a teacher shifted fifth grade students' participation in discourse from parallel conversations characterised by a lack of active listening to that of critical active participants. The teacher used specific pedagogical actions such as modelling active listening and reflecting on the ideas of others. She also initiated explicit discussions to emphasise the importance of active reflection and participation in mathematical discussions. Subsequently, the teacher modified her role to become a facilitator during discussions by 'interpreting students' solutions and encouraging them to respond to each other (redirecting and suggesting)' (p. 130) using revoicing. Researchers (e.g. O'Connor & Michaels; Stein, 2007) explain that specific teacher actions including rephrasing, repeating and revoicing are key pedagogical tools used by teachers to position students in interactive dialogue. O'Connor & Michaels showed how through teacher revoicing, students learnt to take a specific stance in the dialogue and develop the skills of inquiry and mathematical argumentation as they defended or challenged ideas. Other researchers (e.g. Lampert & Cobb, 2003; Stein) illustrated how revoicing can be used to build on student thinking, clarify reasoning, highlight specific aspects of the mathematical thinking or extend, rephrase and further develop it.

Structuring small and large group discussion activity

Many reform mathematics environments feature the use of small group (2–4 children) mathematical activity followed by whole class discussions. During small group work, learning opportunities arise from collaborative dialogue and the resolution of differing points of view (Whitenack & Yackel, 2002). Learning to not always agree but to be able to resolve disagreement is an important aspect of small group activity. However, Weingrad (1998) maintains that the students' concept of mathematical disagreement or argumentation needs to be directly addressed because many students interpret disagreement as inappropriate unless they are able to understand ways to do it 'politely'. Working in small group settings also provides less confident or less able students with opportunities to learn to explain, question, agree and disagree and test their thinking in a less threatening context before engaging in a larger class discussion (Hunter, 2009). Hunter illustrated the importance of small group discussions as a means for students to rehearse their explanations, justification and analysis of their solution strategies as the students prepared for questioning and challenge from the larger group. This study and others (e.g. Monaghan, 2005; Rojas-Drummond & Zapata, 2004) outlined specific teacher scaffolding in the structuring of small group interactions. In these studies, careful guidance was provided so that the students learnt appropriate ways to disagree which were mathematically productive and socially acceptable. These researchers built on the important work of Mercer (2000). Mercer outlined how young students use three different forms of talk—exploratory, disputational and cumulative talk during small group interaction. The three forms of talk involve different levels of engagement in the reasoning of peers. Mercer described disputational talk as characterised by students focusing on self-defence and holding control rather than trying to reach joint agreement. In using cumulative talk, students avoid questions and argument which results in a lack of evaluative examination of reasoning. These are both unproductive forms of talk. Exploratory talk in contrast is a productive form of talk in which the students explore and critically examine the shared reasoning. In many different studies, Mercer and his colleagues (e.g. Littleton et al., 2005; Mercer, Littleton, & Wegerif, 2004; Mercer & Sams, 2006) showed that constructing group interactions which use exploratory talk requires specific teacher attention, intervention and scaffolding of group talk.

The use of whole class discussions extends mathematical reasoning beyond that of individual small groups. In a reform setting, students are positioned to listen actively and make sense of a range of mathematical explanations. Again, the teacher takes a central position in orchestrating and facilitating productive whole class discussions (McCrone, 2005). In the environment of reform, the teacher participates in the discourse as a facilitator whilst also leading shifts in the discussion to ensure that it is conceptually focused and reflective. In this way, student thinking can be advanced as the rationale for specific actions becomes an explicit topic of conversation (Lampert & Cobb, 2003). Kazemi (1998) illustrated how discourse promoting conceptual reasoning was achieved through the use of specific pedagogical actions. These included questioning in sustained exchanges, pressing students to provide conceptually focused justification for mathematical actions and facilitating student examination of similarities and differences across multiple strategies. Teachers also take a central role in guiding the questions and prompts used to probe mathematical thinking. Wood and McNeal (2003) illustrated the significant role the teacher played in shifting students from explaining mathematical solution strategies to justifying and defending solution strategies within collaborative dialogue. Teacher-led questions and prompts were central to extending the social norms so that socio-mathematical norms were constructed (as an example the students learnt what made a proficient mathematical explanation or justification). Although the teachers in both Kazemi and Wood and McNeal's study pressed the students to justify their reasoning which often included generalised thinking, this was not the focus of their studies. In contrast, this study directly examines and explores what happens when teachers employ the many different pedagogical actions described by researchers in reform literature so that students' explanatory arguments draw on early algebraic reasoning to provide justification.

The theoretical context of the study

The theoretical stance of this study draws on the emergent perspective of Cobb (1995). From this perspective, Piagetian and Vygotskian notions of cognitive development connect the person as an individual, the cultural and the social factors. Within this view, the learning of mathematics is perceived as both an individual constructive process and a social process involving the social negotiation of meaning within interaction with others. Language within this frame is considered to hold both a communicative/cultural and psychological function providing a tool for both thinking together and jointly creating knowledge and understanding. In the social and cultural setting of the classroom, Cobb and his colleagues (1992) describe how 'taken-as-shared' knowledge is created and social norms constructed. According to these researchers, taken-as-shared suggests that all participants gain an individual sense of aspects of the shared knowledge within a collective interpretative framework. Although each individual holds their own view, the shared knowledge provides the basis for communication and interaction among all participants.

Concepts of social norms suggest shared agreement of all participants (including the teacher) of the expectations they hold of themselves and others of what it means to practice mathematics in the community they have co-constructed. Sociocultural norms extend the social norms in specifically mathematical ways. Through dialogical, language-based activity, all participants co-construct a set of socio-mathematical norms of which mathematical explanations, representations, justifications and generalisations are acceptable within the taken-as-shared understandings of the classroom community.

Methodology

This research reports on episodes drawn from a larger study which involved a 3-month design experiment undertaken at the beginning of a new school year. The larger study focused on building on numerical understandings to develop algebraic reasoning with young students (Hunter 2007). This paper specifically focuses on the pedagogical actions the teacher used to co-construct with the students social and socio-mathematical norms which supported them engaging in collaborative discourse and constructing early algebraic reasoning. The question addressed in this paper is what pedagogical actions can teachers use which support students to engage in early algebraic reasoning?

The research was conducted at a New Zealand urban primary school and involved 25 students aged 9–11 years. The students were from predominantly middle socio-economic home environments and represented a range of ethnic backgrounds. The teacher was an experienced teacher who was interested in strengthening her ability to develop early algebraic reasoning within her classroom. This teacher was selected as a case because the results from the pre- and post-task-based interviews with her students demonstrated significant learning gains in a range of algebraic areas. A selection of the results from the pre- and post-task-based interviews is shown on Tables 1, 2 and 3 below.

Table 1 Percentage of students (n=25) successfully solving open number sentences using relational or computational strategies

	Relational strategy	Computational strategy	Error or no response
23+15=_+17	68 % (20 %)	28 % (12 %)	4 % (68 %)
81+_=83+26	84 % (24 %)	12 % (8 %)	4 % (68 %)
76−27=78−_	76 % (20 %)	12 % (16 %)	12 % (64 %)

Initial interview results are in parentheses

Table 2 Percentage of students (n=25) using forms of notation for an unknown quantity

	Correct notation e.g. (A+5)×2	Non-standard/incorrect notation e.g. A+5=B B+B =	Number as notation e.g. 5+5=10	No response
Situation A	92 % (24 %)	0 % (4 %)	4 % (60 %)	4 % (12 %)
Situation B	92 % (28 %)		4 % (60 %)	4 % (12 %)
Situation C	44 % (16 %)	44 % (12 %)	8 % (60 %)	4 % (12 %)

Initial interview results are in parentheses

What is a mathematical statement or sentence to represent each of the following situations: (A) I have some lollies and then get five more; (B) I have some lollies, then I get five more and then I get three more; (C) I have some lollies then I get five more and then I double the number of lollies I have

Table 3 Percentage of students (n=25) correctly using the functional relationship

	Correct use of functional relationship	Incorrect response—directly modelled the equation erroneously adding two and three	Other incorrect response	No response
Part A	88 % (40 %)	8 % (32 %)	4 % (20 %)	0 % (8 %)
Part B	84 % (28 %)	4 % (32 %)	8 % (20 %)	4 % (20 %)

Initial interview results are in parentheses

To make copies of a poster, a store charges a setup fee and an additional amount per poster. Use the information to answer the questions. To make copies of a poster, a store charges $3 as a setup fee and an additional $2 for each copy. (A) What is the cost to make 10 copies of a poster?; (B) What is the cost to make 21 copies of a poster?; (C) What is a mathematical equation that you could use to find the cost to make copies of a poster if you know the number of copies you want?

There were a total of 17 video-recorded classroom observations during the study. Each lesson followed a similar approach. They began with a short whole class introduction, and then the students worked in pairs or small groups. During this group work, the video camera focused on one group of students working. In each lesson, this was a different group of students. The lesson concluded with a lengthy whole class discussion which was also video recorded.

Collaborative teaching design experiment (Cobb, 2000) was used, and this supported a teacher-researcher partnership and the development of a trajectory which focused on developing algebraic reasoning. Although algebraic reasoning is not the focus of this paper, the development of social and socio-mathematical norms within an inquiry environment were integral to supporting student development of early algebraic concepts. During the design, experiment data was generated and collected through classroom artefacts, participant observations, video-recorded observations and tape-recorded reflective discussions with the teacher participant.

The findings of the classroom case study were developed through ongoing and retrospective collaborative teacher-researcher data analysis. In the first instance, data analysis shaped the study as the researcher and teacher collaboratively examined the classroom practices and modified the participatory practices to develop an inquiry learning climate. At completion of the classroom observations, the video records were wholly transcribed, and through iterative analysis, using Nvivo, patterns and themes were identified. The developing participation patterns of individuals and small groups of students were analysed in direct relationship to their responses to the classroom mathematical activity. The first level of analysis examined the types of algebraic reasoning which was a focus of the lesson. This included the tasks which were used and student responses to these tasks. The next level of analysis examined the classroom climate including both the pedagogical actions of the teacher and student actions. The identified categories for teacher actions included facilitating discussion as well as providing space to self-correct, re-think and reflect, developing group norms, positioning students to take a stance and validate logic, pressing students to agree/disagree and convince others. The identified categories for student actions included the types of talk and questions, their use of disagreeing and agreeing, social norms and socio-mathematical norms. These categories were used to develop a descriptive narrative of classroom activity and the pedagogical actions of the teacher to support it.

Findings

The initial interaction patterns

In the first instance, initial observations of the students in the large group discussion showed that many of them viewed their role in the mathematics discussions as passive listeners and receivers of knowledge. The discussions were characterised by unproductive silence in which only a few members of the class spoke or asked questions and the others passively listened. For example, in the first classroom episode, there were no instances where students agreed or disagreed with each other's reasoning. Following the initial classroom episode, there were two instances in the second lesson and one instance in the third and fourth lesson where a student agreed or disagreed with a peer's mathematical reasoning. However, in each of these instances, it was the same student, Steve.

The student's response to how they perceived the expectations of their role as members of a discussion is illustrated by a student when the teacher asked him to describe what he thought the role of class members was in the discussions:

> Mike: Sit quietly and listen to what they are saying and don't interrupt.

The students also did not voluntarily question what was being explained unless asked to by the teacher. Analysis of the data shows that in the first lesson, one question to clarify an explanation was asked by a student, and in the second lesson, two clarifying questions were asked by students. However, the teacher's press as outlined in the following section resulted in a shift in the student's use of questioning to clarify an explanation. In the third lesson, six instances were evident.

In the first six lessons, many of the student-initiated questions were more often limited to questions the teacher modelled as examples of types of questions they could use. This use of teacher-modelled questions was copied in a way which often did not relate to their need to sense-make. For example, a student listened to an explanation and then asked:

> Hamish: Could you have done it any other way?

The teacher during a reflective discussion noted the student use of teacher-modelled questions. She stated:

> Teacher: The kids are using questions like “can you convince me?” or “can you explain that in another way?” and part of it is for some now that they have got that vocab, they are practising it so they kept saying “can you convince me?” just because they knew that was a good thing to say but I think that's good for this time of year as well to be doing that.

Close observation of the students engaging in small group work revealed that the social norms the students had constructed were those of either cumulative or disputational talk (Mercer, 2000). Table 4 shows the coded percentage of time that students engaged in either disputational or cumulative talk during small group work.

Table 4 Disputational or cumulative talk during small group work

Lesson number	Percentage of time of disputational talk	Percentage of time of cumulative talk
1	10	8
2	18	24
3	27	0
4	2	21
5	0	15
6	5	4
7	0	7
8	0	0
9	0	6
10	2	0

As evident in the table, disputational talk was a common feature of the first four lessons and also occurred during the sixth and tenth lesson. Cumulative talk was also a feature of many of the first nine lessons.

Neither form supported the students to work collaboratively nor engage analytically in the reasoning under consideration. The interactions when engaged in small group activity showed that many students focused on self-defence rather than collaborating to reach joint agreement. They focused on holding control of the discussion and their own explanation with little consideration for the sense-making of others. For example, when a student questioned another about his choice of solution strategy, he responded with:

Peter: Because I felt like it.

In another instance, a group examining true and false number sentences failed to reach joint understandings because they did not engage with each other's reasoning:

Rani: If you plus 3 to equal that.

Matthew: No you can't do that.

Rachel: Why?

Matthew: Because if you do then it's changing the whole thing.

Zhou: I'm getting even more confused.

Through collaborative discussion and reflection on current existing interaction patterns, the teacher and I constructed a set of teacher actions to change the nature of students' participation so they became critical active participants who engaged in productive discourse. During a reflective discussion with me, the teacher outlined the next focus:

> Teacher: We need to look at the ways that you ask questions if you want someone to explain something so "can you show me that?" or "explain just that part?" type of thing. It was really obvious today that people can't just sit there when someone is explaining.

The next section will outline the interactive strategies the teacher used to co-construct with the students a set of social norms which supported them to collaboratively construct and question mathematical explanations during small and large group activity.

Structuring the norms for collaborative interaction

Immediate focus was placed on guiding how the students worked together during small group work. To guide the development of social norms in which they engaged respectfully in the reasoning of all members of the group, the teacher maintained ongoing discussions related to their responsibilities whilst working in small groups. She introduced as a central focus the requirement that they collaboratively constructed a shared mathematical explanation which they all understood and could explain. The students were introduced to a code in which they first discussed their ideas, and then using only one pen and one piece of paper, a recorder represented an agreed solution strategy. The teacher maintained a focus on group norms which required that they remain aware of their accountability within the group. They were consistently reminded of their responsibility to each other as well as their accountability to themselves to actively engage, question and individually sense-make:

> Teacher: You have to help and you have to understand, everyone in your group needs to understand the strategy. It is not good enough if it is only one person you need to try and help the rest of your group understand it.
>
> Ruby: You have to ask questions if you don't understand.
>
> Teacher: Exactly you don't just sit there and hope that others will explain it to you. You need to ask questions yourself.

As she engaged the students in discussions which emphasised individual and group accountability, she reinforced the importance of active listening and questioning of the reasoning:

> Teacher: Your job in maths is to actually think about what other people are saying and whether or not you are agreeing. Think about is there a question I need to ask as she goes along.

She used models of appropriate student behaviour to make explicit the social norms being enacted. For example, as Zhou was representing a group solution strategy, he made a recording error. When Josie (a member of Zhou's group) stepped in to progress his explanation, the teacher drew attention to the specific behaviour which affirmed the ways in which Josie had modelled positive aspects of group responsibility:

> Teacher: Thank you Josie for helping to clarify there. Can you see what she did then? That's what I mean, get the help, the support from your group.

The teacher constantly drew on her careful observations of small and large group interactions to model patterns of behaviour which supported collaborative group work. The teacher during a reflective discussion following a lesson noted the importance of developing collaborative interaction:

> Teacher: The good thing was that I could start off with kids who hadn't, that had grasped some of the problem, but not all of it and they were able to share back and then choose some different people to build on what they had said. It is working quite well to have them stop and talk to someone else and predict what is going to happen next or what the person might write down next.

Although tasks which supported construction of early algebraic reasoning were used, the focus of discussion was placed on construction of shared explanatory arguments which all group members agreed with and could explain.

Strategies to develop exploratory talk

Reviewing group observation, video records showed that the focus on collaborative behaviour meant that the students generally no longer used disputational talk. However, they still often employed cumulative talk as illustrated in Table 4. An illustration of this is apparent in lesson four as a group of students worked together to solve an algebraic word problem (see Fig. 1).

If you had \$9 in your bank and wanted to buy a t-shirt for \$17, how much do you need to save? What about if the t-shirt cost \$20 or \$26 or \$40? Have a go at solving the problem and see what changes and what stays the same. See if you can find a way to write a number sentence algebraically so someone could use your number sentence to work out how they need to save no matter what the cost of the t-shirt.

Fig. 1 The T-shirt problem

To develop a solution strategy, the students drew on their earlier experiences with number computations and focused on the use of a range of different solution strategies as the goal of the activity:

> Heath: Because it's nine you could just add that up to ten and then you'll have to plus seven and then you'll minus the one again and plus on to the seven to make the eight.
>
> Sangeeta: Another way you could do it is… you could take one off the seven and add on and that will make ten a tidy number and then…
>
> Ruby: And see how much it is to get to the seventeen

In the shared discussion, the students showed little attempt to analyse each contributing idea as each contribution was merely accepted and built on by the different individuals. Generally, as the students worked together, questions and argument were avoided which meant that each strategy was not explored or examined. As a result, the algebraic nature of the task remained unexamined or addressed by the students. In the reflective discussion following the lesson, the teacher noted that she regarded this as related to their previous mathematical experiences:

> Teacher: They were doing these odd addition and subtraction strategies. I think they have some expectation of that's what maths is about, adding things a billion different ways.

It was evident that the teacher recognised that in order for the students to engage in algebraic reasoning, she needed to attend on how they engaged in talk which better supported the students in examination of the algebraic task.

The teacher directly addressed how the students were to disagree within small and large group activities. She initiated discussions about the need for the students to critically examine each other's reasoning and as needed to disagree with the reasoning. She modelled explicit statements that the students could use and intervened when she saw opportunities to engage the students in the possibility of them not agreeing with mathematical argument being presented:

> Teacher: What if you don't agree?
>
> Mike: If you don't agree ask them why…why did you do that?
>
> Teacher: You can say I'm not sure about that, I'm not convinced by that part there. Can you convince me?

At other times, she sought opportunities to affirm the students' right to disagree. She made the students aware of the inherent risks involved in assertive questioning or challenging but at the same time confirmed this behaviour as socially appropriate for what it meant to do mathematics in the classroom:

> Teacher: Good on you Bridget, that takes someone brave to say they are not entirely convinced.

The use of think time as a pedagogical tool

In large group discussions, in order to shift student attention to close examination and analysis of each other's reasoning, the teacher introduced the use of thinking time. She would halt discussion to provide space for the students to reflect on their reasoning and the reasoning of others. At regular intervals in an explanation, she would intercede to provide space for questioning:

> Teacher: Does anyone need to ask a question?

Space for thinking was also provided because the teacher had observed that accessing algebraic reasoning within a numerical context was not always easy for many of the young students. She introduced a requirement that the students used spaces in the discussion to reflect on their own reasoning in relationship to what was being explained. She exerted a press on the students to validate the reasoning used by others' as they reflected during the explanations:

> Teacher: I just want everyone to reflect on their own learning and I want you to think are you convinced by what Susan said there? That 471 take away 382 equals 474 take away 385 because you add three to both those numbers 471 and 382 so you are going to end up with the same number. If you're not convinced that's fine you can just tell us. Anyone not convinced?

Errors coupled with think time were also tools used to deepen conceptual reasoning. In the evolving social learning climate, the teacher explicitly modelled that errors were opportunities to explore and extend the algebraic reasoning within the think time provided. The students were provided with prompts to facilitate them to reflect on and validate their reasoning as both listeners and explainers. For example, during a class discussion, a student provided a correct representation but then provided an incorrect explanation for a problem (see Fig. 1 on page 11);

> Heath: [writes Z – 9=X] The first one is how much we needed to save, the next one is how much is in the bank, that's the nine and the last one is the cost of the t-shirt.

The teacher intervened and without commenting on the explanation directed the listening students to analyse Heath's explanation and construct further questions:

> Teacher: Talk to the person next to you about a question you might need to ask Heath.

She then used revoicing to reposition Heath to provide him with an opportunity to validate or re-explain his argument:

Teacher: I'm just going to ask Heath now that you've had a little opportunity to rethink that… you said the amount that needs to be saved takeaway nine equals the cost of the t-shirt. Can you rethink that a little bit?

Heath: I actually meant that that [points to the Z] is the cost of the t-shirt then we minus nine off that to get how much we need.

When the teacher observed that the students often rethought their explanations in the act of presenting explanations to the class, she stopped prompting them to rethink. She still provided space for others to reflect on the reasoning, but she no longer guided their direction. For example, during a class discussion about the properties of zero, the students were asked to generate number sentences to represent the conjectures they had developed (e.g. any number multiplied by zero equals zero or a number added by zero will be equal to the same number). One student made a conjecture:

Gareth: H times zero plus Z equals X.

The teacher intervened, and without indicating her own position or validating the conjecture as correct, she directed all participants to think carefully about what was being explained. This provided space both for the explainer and the other students to examine what had been presented, re-conceptualise the reasoning and develop mathematical backing for agreeing or disagreeing with the conjecture:

Teacher: Talk to the person next to you. Do you agree with this statement H times zero plus Z equals X? You need to convince us why you agree or disagree.

She then facilitated extended discussion within the group before returning to Gareth and providing him with the opportunity to present his reconceptualised conjecture:

Gareth: We worked out there had to be two Z's, one after the equal sign because H times zero equals zero… the Z should be on both sides of the equal sign. H times zero plus Z equals Z.

In extended discussion around any conjectures, the students made combined with think time, and an expectation that students would use mathematical means to validate their own position increasingly became a central feature of the classroom climate.

Developing connections and making links with numerical concepts as algebraic reasoning

Analysis of the data illustrates shifts in the way the students engaged and participated in the classroom in response to the teacher actions detailed in the following section. Table 5 shows the percentage of time the students engaged in exploratory talk during small group work over the 17 lessons.

Table 5 Percentage of time using exploratory talk during small group work

Lesson number	Percentage of time of exploratory talk
1	0
2	0
3	0
4	0
5	0
6	0
7	8
8	35
9	0
10	17
11	24
12	24
13	19
14	20
15	35
16	38
17	26

As the table illustrates in the first six lessons, students did not use any form of exploratory talk. However, from lesson seven onwards, disputational and cumulative talk was replaced by exploratory talk when the students were examining each other's mathematical explanations. Table 6 shows the way in which the students used a range of actions to access the mathematical reasoning of their peers and to participate in the interactive dialogue.

As the table illustrates over the course of the lessons, the initial focus of student actions was to question for further clarification of mathematical explanations. In lessons one to eight, the students increasingly began to actively agree or disagree with the reasoning provided by their peers. However, in lessons five to eight, the focus of student questioning included the need for justification. In this phase, the increased activity by the students reflected their heightened awareness of the new teacher expectations. As they learnt to provide

Table 6 Student communicative actions during mathematical discussions

	Lessons 1–4	Lessons 5–8	Lessons 9–12	Lessons 13–17
Agree/disagree with reasoning	5	14	10	9
Question for clarification	9	11	5	3
Question for justification	2	13	1	4
Provide justification	1	10	6	9
Development of a generalisation			8	6

clearer explanations and justification in the latter lessons, the need for other students to question for clarification and justification decreased. In lessons nine to 17, whilst the students continued to actively agree and disagree with the reasoning, the press had shifted towards an expectation for justification and generalisation. This was in response to specific pedagogical actions the teacher took to engage them in a range of mathematical practices.

To scaffold the students to make connections across explanations, the teacher began to ask the students to explore and analyse similarities and differences between mathematical explanations. At times, she provided a think time as a reflective space to allow the students to provide other examples or to compare the reasoning. For example, a student presented an algebraic number sentence as a solution strategy for a problem (see Fig. 2):

Rachel: [writes 9+■=A] Nine plus square equals A.

The teacher halted her explanation, and after a long pause, she invited the students to provide alternative thinking:

Teacher: Does anyone else have a different way of representing that problem? Okay Matthew.

Matthew: [writes 9+●=B] Nine plus circle equals B.

The students were then asked to examine and compare the algebraic number sentences:

Teacher: I want everyone to look at that and I want you to think has Matthew shown us a different way or is it similar to a way that is already there?

She then returned to the original explainer and giving him intellectual ownership of his explanation asked him to analyse the similarities or differences.

Teacher: Matthew do you think that is similar or different to the one that is already there?

Matthew: Similar.

But she increased the press for explanatory justification by setting an expectation that he would validate his response with mathematical reasons:

Teacher: Why is it similar?

Matthew: Because that [points to 9+●=B] is just another way of doing that [points to 9+■=A].

Her actions shifted student focus from providing mathematical explanations to making connections, analysing and critiquing their own and others' arguments.

You have $9 in your wallet and want to buy a CD. How much money do you need? See if you can find a way to write a number sentence algebraically so someone could use your number sentence to work out how much money they need no matter what

Fig. 2 The CD problem

Increasingly, the students drew on generalised reasoning which they represented symbolically as part of their justification. The explicit focus the teacher placed on the students recognising how the concepts connected resulted in them making links across their numerical properties and relationships.

Scaffolding the students to use a range of different questions was a powerful pedagogical tool that the teacher used to develop richer mathematical argumentation and generalisations. As she listened to how they questioned each other in small and large group discussions, she explicitly pressed them to ask questions which shifted the focus beyond providing more information for an explanation to those which drew generalised reasoning:

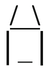
> Teacher: What I have noticed often is that people are asking the question can you explain that in a different way? Now that isn't always helpful and sometimes we just use it because we don't know what else to ask so what are some other questions that we might have to ask during this session?
>
> Bridget: We had to convince people that it would work for any number including zero.
>
> Teacher: Great so you can use words like convince us that it would work for any number?

The introduction of questions which drew justification established a social climate in which the students knew that they needed to explore and examine conjectures in depth. This was because they knew that in class discussions, their reasoning was always going to be subject to challenge, and therefore, they needed multiple ways to elaborate on their reasoning. However, there were a group of students who needed additional scaffolding to engage in dialogue premised in inquiry and argument. The teacher would carefully observe student engagement, and when needed, reposition these students so that they had to take a stance. For example, as the students discussed the commutative law, one stated:

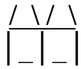
> Rachel: It didn't work with everything.
>
> Teacher: So what did it work with?
>
> Rachel: Pluses
>
> The teacher expanded her statement and in doing so positioned her to take a stance:
>
> Teacher: So you're saying it only worked with the plus or it worked with plus?
>
> Rachel: It worked with plus.

The classroom climate had shifted as illustrated in Table 6, and the students now had a clear set of criteria for what they expected as competent explanatory justification. The teacher actively guided the development of the norms including the requirement that explanations consist of mathematical arguments. She also required that consensus be reached by the students through mathematical argument both in constructing their solutions in small group activity and in large group discussions. She expected that all students take a stance to validate their reasoning and she also stepped in as a participant

Jasmine and Cameron are playing "Happy houses". They have to build a house and add onto it. The first one looks like this…..



The second building project looks like this….



How many sticks would you need to build four houses?
How many sticks would you need to build eight houses?
Can you find a pattern and a rule?

Fig. 3 Happy houses problem

to model how to validate it. For example, during a large group discussion, a student offered the following generalisation for the task (see Fig. 3):

> Mike: You would times it by five and then you would minus one because of the six… it would be one over so you would have to minus this to make it fair.
>
> The teacher extended the explanation adding the problem context to make the explanation experientially real for the other students:
>
> Teacher: Mike said the number of houses times five minus one because of the six you get at the start. Does everybody agree? Could somebody show us why or why not you agree?

The teacher's actions allowed the students to access what was being explained and supported them to develop their own thinking using a range of ways to think about the problem. This included the use of materials or pictures to represent their thinking. One student argued her position through drawing a house that acted as a referent for her generalisation:

> Ruby: [draws a house] That is one house and if you added another one, that is always going to be a six so when you times it by five you would actually add one because you have timesed that by five and it's still a six so you would add it on.

The teacher used this as an avenue to step in as a participant and offer a counter argument that modelled how to justify a position using equipment:

> Teacher: [builds representation of two houses] I could show you another way why it doesn't work. Now I have to times by five and two times five is ten, now if I take it away I am going to have an incomplete house. I have to add one so that is my two times five, to make it complete I need to add one.

Through these pedagogical actions, the students learnt ways to engage analytically in what others explained. Conjectures would be proposed, and they would examine and explore each aspect of the argument until they reached consensus. They used exploratory talk to investigate and critically examine their shared reasoning. For example, as a group examined a functional relationship problem (see Fig. 4), they interacted and explored ideas.

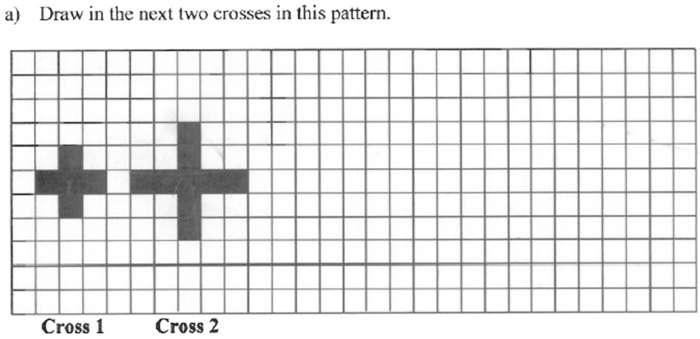


Fig. 4 Cross problem

Initially, a student made an error:

Josie: It's fourteen.

Another student, Steve, disagreed and provided mathematical reasoning based on his understanding of odd and even numbers for his disagreement:

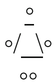
Steve: No it is thirteen because you are adding two each time. It doesn't work because if you are adding two on each time and it is odd numbers it can't be fourteen because it's an even number.

The other student listened carefully to Steve's argument and then built on his reasoning to explain how her explicit generalisation was linked to the geometric model:

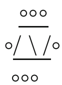
Josie: [points to the vertical line] There is always one in the middle. It is always an uneven number because there is always one in the middle for that line there.

The students appropriated the teacher's many models of questioning and challenging. They had adopted her requirement that other students provide multiple models including those which used early algebraic reasoning. They used this to engage in productive exploratory dialogue as illustrated in the following example during a large group discussion of a problem (see Fig. 5).

At the table 5 people can sit like this ……



When another table is joined this many people can sit around it…



Can you find a pattern? How many people could sit at 3 tables or 5 tables or 10 tables? See if your group can come up with a rule and make sure you can explain why your rule works.

Fig. 5 Table problem

Heath: You add three to each table then the plusing two bit.

A student asked a question focused on eliciting justification:

Josie: Why isn't it two fives added together?

At this point, other students stepped in to collaboratively develop the explanation and provide justification whilst Josie continued to press for further justification:

Matthew: [points to the model] Because you couldn't put one there.

Josie: But each table is meant to have five.

Heath: Yeah but on one table it's five, it starts off with five but then you…

Hayden: You can't sit someone right in the middle of the table. They can't sit here [points to the middle of the model] because they'd be on the table… the people on the edge always had to move out again.

The shift towards algebraic reasoning and student development of generalisations in the latter lessons was recognised by the teacher. For example, in a reflective discussion after a lesson in which students had been asked to solve H+B=30, she described their use of algebraic reasoning:

Teacher: The neat thing was that a lot of them were doing relational thinking in that they started at 15 and 15 and then were adjusting it to 16 and 14 and so on. The kids who have previously struggled with relational thinking like Bridget were able to capture that quite clearly.

However, she was aware that the understanding they were developing was fragile and would need to be consistently revisited. For example, in the final lesson when students were asked to solve problems such as 149+__=146+65 or 45−37=43−__, she noted:

Teacher: It did prove to me today that that knowledge is fragile and it does need to keep on being revisited because when it came to the subtraction problem some did feel that they could take a number away and then add it on the other side when in actual fact it is adding it on both sides.

However, she also noted the generalisation a student had formed in response to this problem:

Teacher: One of the kids came up with bigger number take away bigger number equals smaller number take away smaller number and that seemed to help clarify them for some of the other children.

Discussion and implications

Clearly, learning environments in which students engage collaboratively in productive mathematical discourse can be constructed. In the classroom, under focus in this paper excerpts use the teacher voice to outline the key pedagogical strategies she used to construct an inquiry community. Similarly, excerpts of student voice illustrate their development of rich forms of early algebraic thinking. Also illustrated are the students' growth in responsibility and accountability for their learning and that of the classroom community, as the classroom learning situation changed and evolved. To explain the significant shifts in the classroom environment, we return to the earlier theorising of a number of researchers (e.g. Hunter, 2009; Kazemi, 1998; Manoucheri & St John, 2006; McCrone, 2005; O'Connor & Michaels, 1996; Mercer, 2000; Stein, 2007) and draw on different key aspects of their work as a framework to discuss the findings of the project classroom. These include the social and socio-mathematical norms, small group discourse and activity, attention to the development of mathematical practices and teacher facilitation strategies.

Evident in this study was the need for teachers to use an integrated pedagogical approach. In this study, this began with an immediate focus on the construction of appropriate social norms to support how the students engaged in the mathematical activity. Attending to how students engaged in small group discussions was an essential part of establishing social norms which support inquiry discourse. Initially, the use of disputational or cumulative talk (Mercer, 2000) was a consistent factor in how the students engaged with each other and the mathematics. As described by Mercer engaging in this type of talk did not support students to collaboratively examine the different reasoning used by their classmates in both small group and large group situations. Furthermore, this form of talk corresponded with their passivity in teacher-led discussion or activity. They assumed that it was the teacher's role to explain and question and their role to listen. The teacher's careful attention to a gradual induction into the discourse of inquiry using prolonged discussion and observed models of desired behaviours and repositioning provided them with time and space to construct their own understandings of what was required as a learner in this changing classroom environment. The enactment of community-accepted social norms not only provided the foundation for communally agreed socio-mathematical norms but also began to shift student responsibility and accountability for their own learning and the learning of others. Important teacher actions which the teacher took to support individual and collective responsibility included the requirement that students construct a shared mathematical explanation that they used a representation to do so and that they ensured that all members could explain it. Foundations were laid through these expectations for later requirements that the students justify their mathematical statements through generalised reasoning.

Teachers hold an important role in developing student use of mathematical argumentation—a key practice to engage students in early algebraic reasoning. In this study, the teacher held a significant role in shifting the students towards greater use and facility with argumentation and justification, activity which is critical in the development of early algebraic reasoning. Kazemi (1998) showed that teacher press on student thinking is what makes students engage at higher cognitive levels. In this project, the teachers explicit focus on questioning and on positioning students to agree or disagree with the mathematical reasoning were key elements which facilitated student engagement in mathematical discourse which drew generalised justification. She skillfully drew the students' attention to examples of desired behaviour and talk and used them as models which promoted a more intellectual community. This included drawing on examples of risk taking, actions which many of these students the data illustrates had not considered important. She scaffolded a set of questions as learning tools the students could use which extended them beyond clarifying explanations to requiring justification. She actively provided space for thinking and reasoning and celebrated errors as useful thinking tools. As a result, explanations including errors became reflective tools which provided the students with opportunities to interrogate and reconceptualise their own and others' reasoning.

For all students to engage in early algebraic reasoning, specific attention needs to be paid to the types of talk used within the classroom. Evident in this study were the changing participation patterns which began with what Mercer (2000) termed disputational or non-evaluative cumulative talk. The reflective stance the teacher and researcher took supported the teacher to observe and manage group changes. Mercer contends that teachers need to attend to how students work together cooperatively in small group activity and this was confirmed in this study. The pedagogical actions the teacher took to shape the group norms were evident in the positive outcomes achieved as the talk shifted towards justification and argumentation. As a result, the students increasingly worked together to construct generalisations related to numerical patterns and relationships. Then, when she was confident, the students were able to listen to each other and explain and question each other she carefully scaffolded the use of exploratory talk. Important to the development of this discursive form of talk was the explicit focus she placed on agreement and disagreement based within mathematical reasoning. Weingrad (1998) maintained that attention to students' concept of agreement and disagreement were important and this was illustrated in this study. Until the teacher explicitly discussed and explored this concept, the students either disputed the reasoning or passively accepted it rather than analysing what was being claimed. However, when the teacher focused on it, they then began to use it as an analytical tool.

Developing classroom contexts in which students engage in early algebraic reasoning take considerable time and attention. In this study, facilitating change in the students' interaction was a lengthy process which required consistent and ongoing attention initially to the social norms and then to the socio-mathematical norms. Both the students and the teachers co-constructed what they accepted as acceptable explanations, representations and justifications. However, specific pedagogical actions were required to extend these so that the students linked the numerical concepts to algebraic reasoning. These included the use of specifically designed tasks, the use of materials and a constant press towards the development of generalisations. Clearly, the different stances the teacher took in the classroom activity were important as she acted as a role model, participant and guide. Many students come from classrooms where the teacher's role is static, they impart the knowledge and the students receive it. In this study, the lengthy time the teacher took to change the classroom climate supported all the students to learn to take active and shared roles in the classroom community, but more importantly, their understandings of algebraic reasoning were deepened.

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