Math Ed Res J (2014) 26:193–213
DOI 10.1007/s13394-013-0089-0

ORIGINAL ARTICLE

Exploring teachers' use of, and students' reactions to, challenging mathematics tasks

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Received: 19 September 2012 / Revised: 03 April 2013 / Accepted: 28 November 2013 /
Published online: 14 December 2013

Abstract This is the report of an investigation in partnership with a team of junior secondary classroom teachers of the potential of posing more challenging tasks to their students. A range of data sources are used to represent the experience of teachers and students in a unit of work that incorporated a range of challenging tasks. Students learned the intended mathematics and they reacted positively to the challenges. The teachers identified both opportunities and constraints in such teaching, especially those related to changes to their pedagogies and assessment of student work.

Keywords Mathematics teaching · Mathematics tasks · Mathematics pedagogies

Introduction

Teachers of mathematics plan experiences through which their students come to understand and use new mathematical ideas. A key decision for teachers is whether students can come to know and do new things by engaging in experiences that are challenging and which require some degree of risk taking by the student (and perhaps by the teacher), or whether it is enough that students step into the unknown in small, safe, carefully sequenced, and well supported micro steps. We are working on a project that is researching the former of these pathways, and the nature of classroom tasks that provide those challenging experiences. We are exploring whether students learn well through working on challenging tasks, through being willing to take risks and to persist sufficiently to engage with the challenge of the task. The impetus for the research is the common mention of challenge in descriptions of characteristics of systemic improvement (see, e.g., City et al. 2009), in general teaching advice (e.g., Education Queensland 2010), and in lists of the features of successful mathematics teaching (Clarke and Clarke 2004; Sullivan 2011).

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It can be anticipated that teachers maximise opportunities for students to learn from engaging with challenging tasks when they create or locate such tasks, when they identify whatever preliminary knowledge is necessary for students to engage with the task, when they structure their lessons to avoid reducing the challenge of the tasks, when they communicate with students about the benefits of persisting, and when they find ways to affirm such persistence. The research project, of which this report is a part, is based on a fundamental assumption that learning mathematics is about connecting ideas to each other, seeking insights into those ideas, and solving tasks that involve multiple steps. It is also based on an assumption that it is much more effective for students to make their own decisions on how to solve those tasks rather than to follow taught procedures. We are also assuming that, while it is possible for everyone to learn mathematics, it takes concentration and effort over an extended period of time to build the links between mathematical ideas, and to be able to transfer learning to practical contexts and new topics. Sullivan et al. (2011) argued that the types of actions in which students must engage include "connecting, representing, identifying, describing, interpreting, sorting, applying, designing, planning, checking, imagining, explaining, justifying, comparing, contrasting, inferring, deducing and proving" (p. 34). All of these require some persistence.

The following is a report of one aspect of a larger project[1] that is encouraging teachers to present students with challenging tasks, to explore teachers' interpretation of, and reaction to, the challenge; to examine what happens when teachers encourage their students to persist; to note changes in pedagogies required by such tasks; and to report on ways that students respond to the tasks.

We use the term *persistence* to describe the category of student actions that include concentrating, applying themselves, believing that they can succeed, and making an effort to learn. In summary, we identify persistence as occurring in classrooms when students:

- continue working on a task even if the solution pathway is not clear to them;
- continue working on a task even if the solution pathway has been identified or the problem is already solved;
- are willing to record their thinking and the steps in their solutions;
- explain their ideas and listen to the explanations of others.

In a report of an earlier iteration in this project, based on a series of task trials and classroom observations, it was proposed that challenging tasks allow students opportunities to:

– plan their approach, especially sequencing more than one step;
– process multiple pieces of information, with an expectation that they make connections between those pieces, and see concepts in new ways;
– engage with important mathematical ideas;
– choose their own strategies, goals, and level of accessing the task;

[1] The Encouraging Persistence Maintaining Challenge project is funded through an Australian Research Council Discovery Project (DP110101027) and is a collaboration between the authors and their institutions. The views expressed are those of the authors. We acknowledge the generous participation of the project schools. We are working with colleagues Jill Cheeseman, Doug Clarke, Jim Middleton, and Anne Roche on the project.

- spend time on the task;
- explain their strategies and justify their thinking to the teacher and other students; and
- extend their knowledge and thinking in new ways. (Sullivan et al. 2011, p.43)

It is stressed that the challenge is a result not of posing tasks requiring an understanding of content with which the students are unfamiliar, but of building connections between ideas with which the students are familiar. In other words, a challenging task for Year level X does not rely on content from the curriculum of Year X+2, but on students developing, for themselves, connections between ideas from the curriculum of Year level X.

In short, the project is exploring the relationship between teachers posing challenging tasks, student persistence, and student learning of mathematics.

Framework informing the research

The data collection for the research is informed by a framework that proposes that the teachers' intentions to act are informed by their knowledge, their disposition to task use, and the constraints they anticipate experiencing. Teachers then enact those intentions in their classrooms. The following explains each of these elements and how they connect to teachers posing challenging tasks, maintaining the challenge in those tasks, and encouraging students to persist.

Teacher knowledge

There are (at least) three aspects of the teacher's knowledge that influence the choice and use of challenging tasks: whether they know the relevant mathematics sufficiently well to allow them to be flexible; whether they are aware of pedagogical approaches that can facilitate learning based on students working on challenging tasks; and teachers' awareness of curriculum resources and sources of task suggestions.

In articulating different types of teacher knowledge, Hill et al. (2008) described diagrammatically components of two types of knowledge. The first type, *subject matter knowledge*, includes common content knowledge, specialised content knowledge, and knowledge at the mathematical horizon. Hill et al. (2008) used the term common content knowledge to include the mathematics that anyone needs to solve a challenging task, including connecting ideas across domains. The term specialised content knowledge is used to include:

> the knowledge that allows teachers to engage in particularly *teaching* tasks, including how to accurately represent mathematical ideas, provide mathematical explanations for common rules and procedures, and examine and understand unusual solution methods to problems. (Hill et al. 2008, p. 378)

We add that this includes recognising that a challenging task can be solved in different ways, and that students' emerging intuitive approaches are as much part of learning mathematics as algorithmic routines.

The second type of knowledge described by Hill et al. (2008), drawing on Shulman (1986), was *pedagogical content knowledge (PCK)*. Hill et al. delineated three sub-categories which include teachers knowing about appropriate sequencing tasks, being flexible about modes of representation, and being aware of the need to adapt tasks to suit circumstances. This knowledge also includes teachers anticipating students' cognitive and affective responses to challenging tasks and knowing ways to address those responses as well as awareness of resources, capacity to identify the purpose of tasks, and capacity to select tasks that have potential to be appropriately challenging. Part of the focus of the current research is to explore these different dimensions of teacher knowledge and their influence on ways challenging mathematics tasks are used.

Teacher disposition to task use

The second set of influences on teachers' intentions, which together we term disposition, includes beliefs that teachers hold about the nature of mathematics and processes by which students come to learn it (e.g., Thompson 1992), their attitudes to mathematics and its learning (e.g., Hannula 2004), and the extent to which they consider achievement to be amenable to effort (e.g., Dweck 2000). While these are all substantial influences on teachers respectively, important dispositions for this study include a willingness to allow students to struggle and to avoid the common tendency to over explain tasks or to intervene before students have had sufficient opportunity to engage with the challenge for themselves (see Middleton 1999). Various studies (e.g., Stein et al. 1996) have noted the tendency for teachers to over explain tasks, perhaps fearing negative student reactions to challenge.

While teachers' disposition to posing challenging tasks is evident at the planning phases, the extent to which they maintain challenge is a key aspect of their classroom pedagogy, and so is a focus of the project.

Constraints teachers might anticipate

Connected to teachers' dispositions are the constraints they anticipate they may experience in implementing challenging tasks in their classrooms. Some of these constraints are very real and can include the diversity of students' cultural backgrounds (Delpit 1988), and that skill and language levels of students might inhibit their willingness to engage with challenging tasks (see Stein and Lane 1996).

A major constraint, though, seems to be students' unwillingness to take risks and to persist. This has been identified by Doyle (1986) and Desforges and Cockburn (1987) who argued that students can resist challenging tasks by threatening classroom order. Dweck (2000) explained that those students who are more likely to avoid persisting have a performance orientation, meaning they seek social affirmation rather than understanding of the content. Elliot (1999) termed this "performance avoidance" and Dweck claimed that teachers can inadvertently encourage such responses by affirming easy successes and by failing to affirm effort. Such constraints are an integral part of data collection for the project.

Intentions to act in particular ways

One of the goals of the project is to explore ways for teachers to pose challenging tasks so that students appreciate the intention of the tasks and the relationship to their learning, and to consider lesson structures that support the use of challenging tasks effectively. Our common advice to teachers is to allow students to engage with the challenging task with limited instruction for some time so that students can consider possibilities and plan their approach. During this time we recommend that the teacher observe the individuals and groups, offering assistance for those who need it and posing challenges for those who are ready.

We agree with Marshall and Horton (2011) who argue that challenge can best be maintained if students have not already been told how to solve the task. This means that the key role for the teacher is managing the student discussion of the ideas and strategies they have developed after working on the task. Inoue (2010) explained that this review phase is so important for Japanese teachers that they have particular words to describe them. For example, the term *Neriage* refers to carefully managed whole-class discussion seeking the students' insights, and the term *Matome* refers to the teacher summary of the key ideas. Ruthven et al. (2011) described process for managing plenary reviews in a UK context. The whole-class discussion phase of lessons is a key aspect of the pedagogies associated with effective use of challenging tasks and so is part of the data collection for the project.

Actions when teaching

The overall project is also seeking to recommend particular actions to teachers to support the use of challenging tasks. Sullivan et al. (2011) present a detailed description of such actions, including that teachers should emphasise the nature of the task and student choice, and the ways of working and recording; promote the possibility of learning from errors; encourage persistence; invite students to be willing to explain their thinking; insist that students listen to the explanations of others; allow sufficient time, especially at the review phase; and connect the tasks to students' experience.

The context and process of data collection

In the overall project, as well as in this aspect, we are adopting a design research approach which "attempts to support arguments constructed around the results of active innovation and intervention in classrooms" (Kelly 2003, p. 3). The key elements for us are that we are *intervening* to prompt (possibly) different pedagogies from those used normally, our approach is *iterative* in that subsequent interventions are based on the findings from previous ones, and the intent is that outcomes address issues of *practice*.

In particular, we are intervening to prompt the use of tasks that are more challenging than those commonly used by many teachers. In the iteration that is reported below, we worked with a group of five teachers of Year 8 (age 13) students from a school in suburban Melbourne. This school was chosen because they had expressed interest in participation in a research project with our university as part of their interest in teacher professional learning. They were not aware of the nature of the project when they indicated this interest. In this school, two “accelerated” groups are separated (around one quarter of the cohort), with the rest of the students in mixed-ability groups. We are working with teachers of the students in the mixed-ability groups. The classes can be taken as broadly representative of students at this level elsewhere.

We met with the teachers early in the academic year to ascertain their topic sequence and their approach to planning. The teachers informed us of the topic that they planned to teach. Then, we administered an online pre-test for all the students using a mix of items suggested by the teachers and some suggested by us, and met with the teachers again to report the pre-test results, to revise plans for the unit, and to talk through the suggestions for challenging tasks. In these discussions, it was clear that the teachers had no difficulty with the mathematical demands of the tasks, and the focus of the discussion was on clarifying the purpose of the suggestions and associated pedagogies.

In an earlier iteration of the project it appeared that the style of teaching we recommended was unfamiliar to teachers and students, and the teachers specifically requested the project team to teach a lesson to one of the classes to model the particular pedagogies. To do this, the first author taught a lesson based on challenging tasks that was observed by the participating teachers, collected work samples and other data from the students, and led a post-observation meeting with the teachers. This is referred to as *modelling the pedagogies* in the report below and is part of the intervention.

Subsequently, the teachers planned and then taught the unit incorporating some of the tasks suggested, and the students completed an online post-test including some attitudinal questions. The results were reported to the teachers at a meeting and their reflections on the students’ reactions to the challenging tasks were recorded. The second author interviewed students who were involved in the modelled lesson to ascertain more detailed reactions from the students about working on challenging tasks. The teachers gathered samples of student responses to an extended problem-solving task that was completed by all students and scored using a collaboratively developed rubric. We conducted a further written post-assessment on the student learning. The results of these data collection events are reported in the following.

The research questions informing this aspect of the project were:

i. How do teachers respond to and use suggestions of challenging tasks?
ii. What pedagogical constraints do they experience?
iii. What is the nature of students’ learning from and response to the tasks?

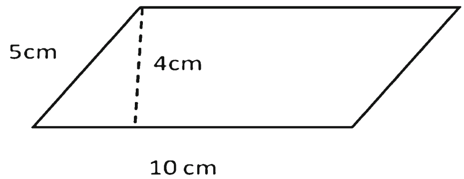
The intention is that the results inform future support for, and advice to, mathematics teachers.

The tasks

This section describes characteristics of tasks that are used to elaborate the data reported below. Many more tasks than these were suggested to, and used by, the teachers, but these form the basis of the subsequent discussion. The first of these tasks is not intended to be challenging, but the others are representative of the type of task that we are recommending teachers use as part of the project.

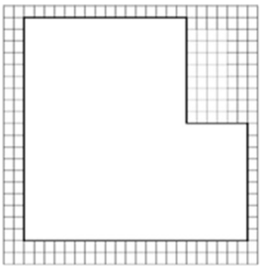
Parallelogram

The students were asked to calculate the perimeter and area of this parallelogram. This task was used as part of the pre- and post-test, has only one correct answer for each question, and is a straightforward version of the textbook exercises that the students completed.



Hidden square units

The students were asked to use different strategies for calculating the number of square units covered by the white L shape. In particular, the intention is to explore why multiplying length and width gives area. This task was suggested to the teachers for inclusion in the sequence and was also used as part of the modelling-the-pedagogies lesson.



L-shaped hectare

Students were asked to draw some paddocks in the shape of an L that have an area of 1 ha, and to work out the perimeter of each of their paddocks. They were given the definition of a hectare and asked to show how they got their answer. The task is about exploring composite shapes, and developing the relationship between dimensions and area. This task was part of the sequence suggested to the teachers, and was also included as part of the demonstration lesson.

Training track

The students were told that the area of the shape was 1 ha, and asked "What might be the distance around the outside?" This task was part of the sequence suggested to the teachers, and was posed as an extended investigation by the teachers.

The italicised titles are used below to refer to these tasks. The latter three of these tasks are examples of the type of challenge we are seeking. They address content from the curriculum at this level, they connect different ideas (such as perimeter and area), and it is assumed that the students have not been shown how to solve such tasks by the teacher and will devise their method of solution and recording that solution.

Results

There are five aspects of the results presented in the following:

- a report of the modelling-the-pedagogies lesson;
- reactions of teachers to that lesson;
- teacher reflections on the unit of work overall;
- measures of student achievement; and
- some affective student responses to both the demonstration lesson and to the challenging tasks generally.

Each aspect communicates part of the experience of the iteration, and it would be misleading to present any aspect in isolation, so selected excerpts that address each of the aspects are presented below. The results collectively are an attempt to communicate the experience of teachers and students, and therefore of the impact of and their reactions to the challenging tasks.

The modelling-the-pedagogy lesson from the perspective of the "teacher"

As part of this iteration of the project, a lesson based on two of the suggested tasks was modelled as a way of stimulating discussion among teachers about the pedagogies associated with lessons that incorporate challenging tasks. The lesson was taught by the first author, and the following includes extracts from the post-lesson reflection written immediately after the lesson, reported in the third person. Analysis of videotaped extracts of student responses also contributed to the insights presented. The lesson was observed by six teachers and three researchers, and their reactions are reported in the subsequent section.

In a handout, the focus of the pedagogy modelling was described to the teachers:

> The intention is for you to observe a class, and for us to discuss what you see in the context of the (challenging tasks) project.
>
> I will ask you to comment later on the way that students persist, show how they get their answers, explain their reasoning, and listen to each other. Also comment on the nature and quantity of teacher "talk".

At the beginning of the lesson, the teacher emphasised to the students that they should persevere, record their work, be willing to explain their thinking, and listen when others are explaining their thinking. The teacher also described the learning intention, which was that students should explore perimeter and area of composite shapes. In the post-lesson review, the observers commented that the expectations were made clear.

The worksheet for the first task in the lesson, *Hidden square units*, had two examples of the task and students were asked to use different strategies for calculating the number of square units covered by the white L shape. Other than that, there was no explanation of how to do the task. Many students seemed puzzled at first, but then started to engage with it, working individually. The observers commented on the ease with which students got to work, and the way they continued to engage with the task.

There was a review of the task in which the teacher drew the diagram on the board, and selected students explained their strategies. There were four solution types presented by different students, including the solution that involves subtracting from the overall rectangle enclosing the L shape. One student gave a clear explanation for why the length and width are *multiplied* to give find area, although this obvious learning opportunity for others was not used effectively by the teacher.

The students were invited to go onto the second task, the *L-shaped hectare.* Note that the class had not had prior experience with such tasks, although the definition of the hectare was presented on the worksheet. Again the students were invited to work on the task without any instruction on how to solve it. The students were clearly puzzled and most students were making limited progress. The teacher invited students to stop working, and asked a student identified as making progress to explain her thinking. The student explained that she created two parts to the "L": 6,000 and 4,000 sq. m, and said she would find lengths to produce those areas. The teacher invited questions for this student and, when there were none, asked the class to continue working on the task.

There were two alternate tasks prepared for students who might have experienced difficulty:

> Draw (on squared dot paper) some paddocks in the shape of an L that have an area of 16 square centimetres. Work out the perimeter of each of your paddocks.
>
> Draw some paddocks in the shape of an L that have an area of 100 square metres. Work out the perimeter of each of your paddocks.

Neither task was used, mainly because most students appeared to be progressing well. It is possible that there were students experiencing difficulty but not identified, which may have been due to the unfamiliarity of the students to the "teacher."

There was also an extension task planned for students who might have finished quickly.

> Draw some paddocks in the shape of an M that have an area of 1 ha. What would you need to know to work out the perimeter of each of your paddocks?

This task was not used, mainly because the suggestion of finding different solutions seem to provide sufficient challenge for all students. One of the observing teachers later suggested that such students could be asked to find the smallest perimeter of such a paddock with side lengths that are integers. This is a much better extension question.

The students worked individually (and silently) on the main task, but after a while were invited to talk to the student next to them. This created positive discussions, but it was clear that the individual thinking time was critical in enabling this discussion.

With just 7 min to go, the teacher initiated a discussion of solutions, having selected some students to contribute. The girl who had made early progress gave a clear explanation of a solution. Emphasising the complexity of such class reviews, the teacher made her reporting more difficult by representing her solution inaccurately. We have subsequently found that it is preferable to use technology to present students' actual solutions rather than having them rewritten by the teacher or the student.

Another student had created an L shape from ten rectangles 100 m × 10 m and explained his solution, including the process for calculating the perimeter. His answer was correct, although when explaining to the class he drew 11 of the smaller rectangles, but gave the correct perimeter. The teacher invited students to comment but no one did. This further highlights the complexity of such reviews in that it appeared that neither the students nor the teacher was checking this solution.

Another student gave an interesting analytic solution, working out the dimensions before drawing the diagram. A further student explained that he had four squares, 2,500 (50 × 50), and even had a short cut for calculating the perimeter (double height + length). This is an important and significant generalisation that was not elaborated by the teacher.

In summary, from the perspective of the "teacher," the tasks were appropriately challenging and the students did persist with that challenge. The lesson did model some of the pedagogies associated with using challenging tasks, including emphasising some of the constraints associated with using such tasks, such as monitoring the quality of the student reports on their work, taking advantage of incidental opportunities, and the timely identification of students who may need support.

Reactions of the teachers to the lesson modelling the pedagogies

The purpose of the modelled lesson was to prompt consideration by the project teachers of aspects of the pedagogies that we see as maintaining the challenge in tasks and encouraging the students to persist. While the teachers noted the artificiality of the situation of having multiple observers in the classroom, there were comments on three aspects of relevant pedagogies: the posing of the task; the strategy of individual work on the task followed by small-group discussion; and issues arising from students reporting on their strategies. There were also comments on student engagement in the lesson. The following presents representative extracts from the project teachers' comments that were recorded after the lesson, subsequently transcribed, read, coded and categorised.

The first set of reactions related to the posing of the task, particularly the minimal introduction by the teacher. For example:

> It surprised me that they were just happy to go straight into calculating perimeters and areas when I haven't been through that with them so it was I guess pleasant to see that they're happy to just get in there and give it a go

> I love the way you didn't give them too much detail, you sort of just set them off without too much introduction and I think that's what got the variety of the different approaches.

Such reactions are relevant since it is at this stage that teachers can inadvertently reduce the challenge of tasks by over explaining them.

The second set of reactions related to encouraging the students to engage with the task individually after which there was discussion with peers. For example:

> I thought that was good the way you didn't let them (talk) at first and then with the harder task you said to them well now you can talk to your next-door neighbour and some really good conversations came out of that. I thought it was gold, sort of first giving them a task where they've got to do it on their own so they get into the work ethic rather than letting somebody else do it for them, a lot of kids will sit back and just cruise if they're sitting next to a pretty talented kid.

> I think it also meant that their conversations were richer because they'd been working on the task and it meant they actually then had kind of invested some time to find a solution whereas if you said "talk about this" they might talk about it for a minute and then start going "What are you doing after school?" And the conversation goes off topic, whereas that way they actually wanted to get was a kind of a solution out of the conversation of what you and the person did.

It seems that the individual student work prior to discussion was noted by the teachers and had potential to inform their pedagogies.

The third set of reactions was those associated the constraints when students are explaining their work. One teacher connected his comment to one of the difficulties observed in the modelled lesson, in that one of the students who had something interesting to say faced the whiteboard when speaking:

> I had a student explaining something on the board yesterday and he stood in front of it and he talked to board so the students couldn't see what he was writing, they could hear what he was saying and they realised they've got to actually train the students how to use a white board such as the class can see what they're explaining.

> …it could be a little different if they had to just listen to someone and then react to that or produce some work as a result of it or write a summary of what the

> person's said or to compare the difference between what was said and what they think because it's very common for students if someone's talking at the front of the room for them to just disengage, turn their brain off or there's nothing, I don't feel as if I'm here to learn.

Teachers do need to find ways to educate students about the processes of reporting and the importance of listening to other students when they are reporting.

There were also comments that indicated the observing teachers noted some of the benefits:

> I think the ideal is that they come to the front because then they've got to take responsibility for the whole thing…

> … if you've got an open-ended problem that might have different solutions you start the kids on the task and the goal is simply to go around and see, find several students who have simpler solutions and several students with more advanced, and just make a note of them and then call the students up in that order because if it's an open-ended enough problem then even the weakest maths kid in the class could probably present the first obvious answer and they feel like they've contributed in class rather than getting the best maths kid to put in the simplest answer because that might be the only one the weaker kid came up with and he has nothing else to add on after that.

This is illustrative of the ways that the modelling of the pedagogies focused the teachers' attention onto relevant pedagogies and options.

There were also comments on student engagement, the first from the teacher of the class involved in the modelling:

> Most of the students who were kind of contributing were the higher ability students that I would probably expect that from but it was good to see that all of the students were producing work even if they weren't necessarily the ones talking a lot about it.

> … getting different kids up to give their different solutions was a good way of kind of, and after their own thinking time was a good way of kind of cementing their own learning abilities within the individual students, not just the students who went up but also the ones who were sitting there.

It seems that the intent of the lesson demonstration, to model aspects of pedagogies associated with challenging tasks in ways that teachers might consider incorporating those pedagogies into their own practice, was achieved.

Reactions of teachers after teaching the unit

The third aspect of the data relates to the reactions of teachers to the unit as a whole. We met with the teachers after they had taught the unit of work in which they had incorporated at least some of the challenging tasks that we had suggested. The meeting was audio recorded, transcribed, and then analysed to identify themes in their reactions. The teachers talked through their students' reactions to each of the suggested tasks. The following are representative extracts of their comments on aspects of their own teaching, although only comments that are reactions to tasks already discussed are presented. The teachers did make similar comments on other challenging tasks that indicated insights on the pedagogies on those tasks, so the following discussion is just a subset of the overall unit reviews. There were four themes: endorsement of the strategy of students starting work on the task with minimal introduction but with the intention of class discussion subsequently; the notion of using an introductory task as a bridge to the challenging task; using tasks with possibility of solutions at different levels of sophistication; and constraints experienced when conducting whole-class reviews.

In commenting on the strategy of students starting on the task with minimal introduction but stopping for class discussion subsequently, referring to particular experiences, teachers commented:

> They spent 5 or 10 min on this and a lot of them were really struggling and then I had one group explain to the class how it is still two shapes with a rectangle and then they found one solution and then I suggested to the class they find more solutions and they were happy then.

> If you left (the discussion) to the end after they hadn't done anything or if you did it at the start they might not be as interested either but they've kind of, you've switched them into a little bit because they struggled with it. But I think it is a balance between how long you let them struggle before they become disinterested and they're not even interested in kind of a hint to how to get it because they already feel like it is out of their reach.

These teachers experienced benefits from at least some student persistence on the tasks.

There were also comments on using an introductory task as a bridge to the challenge task:

> And also give them a few questions like here is a paddock with an area of half a hectare. It is rectangular. What are the possible perimeters? Then you go off and find some half hectare rectangles and they put those out and then you give them this one and a fair few of them will jump straight away and we just need to put half a hectare next to half a hectare and we've got some L shapes.

While there is a clear tension between reducing the challenge of the task, and appropriate scaffolding, it seems that these teachers were conscious of maintaining the challenge.

Other teachers commented on the strategy of having tasks with solutions at different levels of sophistication. For example, when talking about a particular task:

> … there was one solution but there were different ways of getting at it. So the students got to the end so they could kind of feel they had something rather than being uncertain. They had something concrete that they were trying to achieve but there was multiple ways they could get it. So even the weaker students could just do what one of the kids did and just count all of the squares, rule up their own grid and count them up or another kid could see a couple of rectangles or subtract something away or add two together. So I think that worked really well because there was a lot of different angles to get to the same solution but at the end of the day there was that solution.

As identified above, there were also comments on the constraint that students may be reluctant to listen to the explanations of others:

> One thing I found, which probably tells us more about where kids have been and where they might have resistance to these more challenging tasks is that once they've got an answer out using a method, … one of my classes, they were very self-satisfied and they weren't a captive audience to anyone else who got up and explained how they did it. In other words, if they did the vertical cut to get the two areas well they were very happy to see someone else do that. When someone was saying the horizontal cut I was very disappointed that they weren't captive and say "I didn't do it that way. It does work that way." … The kids didn't want to hear it. They … were calling him down but he said "No, but I got it right."

There were many similar comments that refer to the constraints arising from students reporting on their solutions.

There was also substantial discussion of their decision to use the *Training track* task as an extended problem-solving assessment, using a collaboratively developed rubric. For this assessment the task was too difficult. While some students completed the task, and many others used appropriate approaches, in general most of the students could not find and answer and so were disappointed. The teachers took steps to pose an alternate assessment and to assure students they would not be disadvantaged by their responses to this task, but it highlights the fact that tasks need to be appropriately, and not too, challenging.

It seems that the modelling the pedagogies lesson followed by implementation of selected task suggestions did engage the teachers with the relevant pedagogies. The teachers used the strategy of setting students to work with minimal introduction, they used tasks that have multiple pathways to solution, and they encouraged students to report on their work with the intention that other students would learn from it. The issue of the bridging task is interesting and connects to Dooley (2012), who proposed the notion of consolidation tasks that follow on from tasks that facilitate construction.

Inferences about the learning of students and their reactions, drawing on written responses

We sought data on student learning and their reaction to challenge in two ways: from written responses to tests and surveys; and from individual interviews. The interviews are reported in the subsequent section. This section offers insights into the nature of student learning from the unit of work. It describes affective student responses both through their reactions to the lesson modelling the pedagogies and also through a survey completed at the time of the post-unit assessment. It also provides evidence of student learning through responses to items on the pre- and post-test completed online, and, most importantly, coding of student responses to a subsequent written assessment.

There were two sets of written data gathered to gain insights into the ways that students reacted to the challenging tasks: a survey based on the *Impact Procedure* (Clarke 1988) after the demonstration lesson, and responses to prompts as part of the post-test.

After the lesson modelling the pedagogies, the students responded in writing to some prompts.

- On a scale ranging from 1 to 10, with 1 being "I prefer to work things out for myself" to a score of 10 labelled "I prefer to be told what to do," the mean was 5.3 with all scores but 1 between 3 and 7 (inclusive).
- On a scale ranging from "I hated this lesson" to "I loved this lesson," the mean was 5.7, with nearly all scores between 3 and 7.
- On a scale ranging from "The work was too hard" to "the work was too easy," the mean was 5.6, with all scores between 3 and 8.

Asked to circle words that represented how they felt in the lesson, the following were the words chosen, with the number of students in brackets: challenged (16); interest (15); confused (11); relaxed (10); bored (10).

The students were invited to suggest their own word, with most responses indicating interest or enjoyment of the lesson.

The students were also invited, on the online post-test, to offer a reaction to the tasks. Table 1 presents the responses to two of the tasks.

The most interesting result in this is that it seems that most students see such challenging tasks as "same as usual," indicating that teachers should not avoid such tasks from fear of student reactions. There were some who responded positively to them, and a similar number who reacted negatively. Pedagogies associated with the use of such tasks need to accommodate these varied student reactions, and find ways to encourage those students who might react negatively. It does appear that teachers' possible fears of widespread negative student reaction to challenge are unfounded.

We were interested in the learning of the students from the challenging tasks. We analysed the student worksheets in the modelled lesson: 13 students gave two correct methods to the *Hidden square units* task, eight gave one correct response, and three more made a careless error. Only one student had none correct. For the *L-shaped hectare* task, 18 students gave one correct answer, five others confused the notion of area and perimeter, and three made little progress. While care is needed in interpreting this, since students may have merely reproduced what they saw when other students reported, this does indicate that many students made substantial progress towards learning from the experience of working on these tasks.

Student responses to the online pre- and post-test were also analysed to get further insights into their learning. Table 2 compares the pre- and post-test results to the parallelogram question (see above). Even though there was a challenging task

Table 1 Students' reactions to particular tasks (n=187)

Question	Prompt	less than usual	same as usual	more than usual
White covered squares	l liked working on this task	28	125	33
	I learnt from doing this task	34	110	43
	I prefer this task to the usual questions	52	89	45
L-shaped hectare	l liked working on this task	37	79	34
	I learnt from doing this task	33	78	39
	I prefer this task to the usual questions	46	65	39

Table 2 Percentage of students correct on the *Parallelogram* question (n=211)

	Pre-test	Post-test
Perimeter	50 %	78 %
Area	9 %	56 %

suggested on this topic, the focus of student work in the unit was on working through conventional textbook exercises on exactly this content.

Although for both perimeter and area there was substantial improvement, given the simplicity of the items, the results indicate that learning from work on textbook items that offer practice of a formula may have assisted only some of the students. In other words, despite the widespread reliance on textbooks for teaching at this level, it seems that student learning from working through textbook exercises might not be ideal for all students.

The pre- and post-test also included the *L-shaped hectare* task. There were very few responses to this item on the pre-test. On the post-test, many more students responded, although the results were not encouraging. For example, only 7 % of the students' responses were scored as correct. An example of a correct response, presented here as it was typed, is as follows:

> … because an area of a hectare is 10 000 m^2 you have to find the area of both of the shapes (you split the shape to get two rectangles) one of the shapes should equal 8,000 m^2 and the other 2,000 m^2. The perimeter for the one with 8,000 m^2 would be 80 m and 100 m the perimeter for the one with 2,000 m^2 would be 20 m and 100 m the total perimeter would be 560 m

A further 21 % gave responses that were detailed and scored as close to correct, and 22 % gave responses that had some aspects correct. At the post-unit meeting, the teachers claimed that this result underestimated what the students could do, and this was also an inference from the results of the pedagogic modelling lesson. The teachers also commented that the students had mentioned that it was hard to respond to such questions in an online format, especially one without a drawing option. As an aside, we conducted a short survey of students on this, and around 75 % of the students expressed a preference for pen-and-paper assessments.

To provide an alternate insight into student learning, one teacher who taught two classes agreed to do a further assessment, and posed the following related item under test conditions, although written.

> Draw some paddocks in the shape of a T that have an area of 1 ha. Work out the perimeter of your paddocks

An example of a correct response on the written assessment was as follows (Fig. 1):

Even though there is an issue with the way the units are written, this is a clear coherent solution that explains and elaborates the steps taken that, in a sense, prove that the response is correct. There were many such responses to this written assessment. The responses overall to this written assessment were coded, and are presented in Table 3.

In other words, to this challenging task, 78 % of the students presented sophisticated responses indicating that they had learned the important ideas of connecting perimeter and area from their experience of the unit. This is the same percentage that were correct

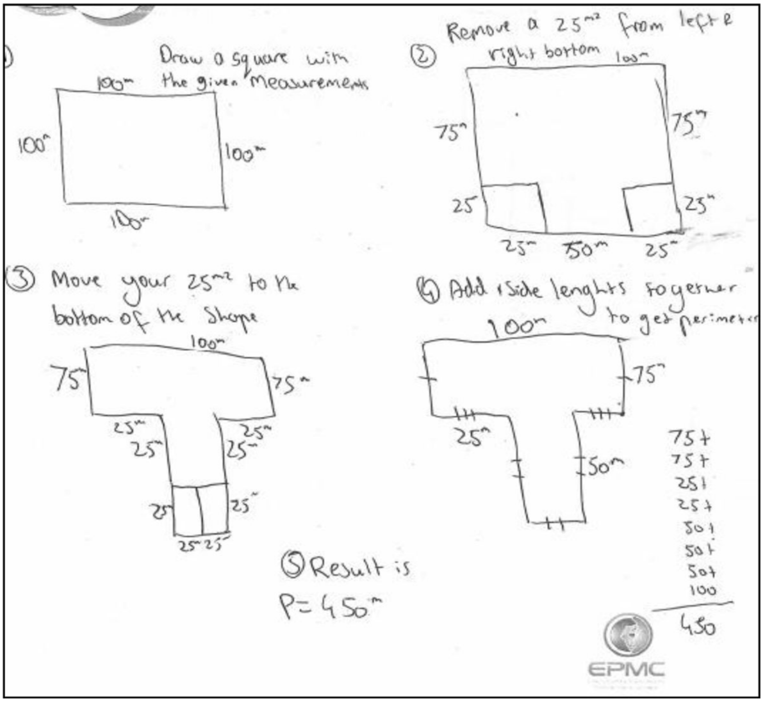


Fig. 1 Sample student response to the supplementary assessment task

with the simple parallelogram perimeter item. This task is substantially more difficult than is common for students at this level (noting that the top 25 % of the cohort were not assessed) and so this can be taken as a strong indication that it is possible for students to learn from engaging with challenging tasks.

Reactions to the challenging tasks based on students interview responses

We also sought more detailed responses from some students through interviews that were audio record, transcribed, and examined to identify additional perspectives to the written responses. Eight students from the class that participated in the lesson modelling the pedagogy were interviewed. The students were withdrawn from class individually, and were given a series of prompts that encouraged them to recall the experience of the modelled lesson as well as the other tasks they experienced. The interviews were recorded, transcribed, and coded.

Table 3 Coding of student responses to the subsequent assessment

Code	Written (n=50)
Response fully correct with steps well explained	44 %
Response mainly correct, but with minor errors	34 %
Some indication of connecting perimeter and area	20 %
Inaccurate response or no answer	2 %

Students commented on their responses to the challenging tasks, the connection between a supportive classroom culture and risk taking, relationships with teachers, coming to understand the task, and the role of teacher feedback.

One of the interview prompts invited students to comment on their reactions when engaging in challenging tasks. There were some common themes that emerged. Although the data above suggests that there were mixed responses, all students interviewed indicated a preference for challenging tasks over working through the text book or the teacher writing questions on the board. One student remarked:

> I prefer the maths where we have to work out the problem. I think because they are a bit more challenging and the ones in the book are pretty easy and they only require one formula, whereas the (challenging tasks) require a number of different formulas.

Some students commented that, when the teacher explained that there could be a number of ways of working, they were more prepared to attempt and persist with the task as they felt less pressure in terms of making a mistake in front of their peers. Some students indicated that when tasks had multiple ways of working they were able to "… show their thinking and that could show the class another way to solve that problem." Some students even indicated they were willing to risk making errors in front of their peers in such circumstances.

It seems that among the students interviewed the classroom culture was an important factor in how they responded to the challenging tasks. When asked what helped them to try hard in mathematics, each interviewed student reported that the teacher was the most influential factor. One student explained why the relationship was so critical to their learning.

> Because you'd learn more, you'd be more comfortable asking more questions but if I didn't get on, I wouldn't ask as many questions because I'd be nervous. But I have a good relationship with my teacher.

Some students also commented on the teacher's feedback. For example:

> We had an assessment task. We had a test first and (the teacher) gave us heaps of feedback about it before we did the assessment task so we knew what we had to improve for the task. If you have a better relationship with the teacher you can go and ask her for help anytime. You know she's always going to be there to help you.

It seems that the teacher's feedback in this instance was important for two reasons. First, the student was able to understand the areas she needed to work on and improve. Second, she continued to persist with a difficult task because she was comfortable with the teacher/student relationship and with the knowledge that the teacher would assist her if she found she was struggling with the task.

Another theme that emerged was that understanding what the task was asking was vital to students when beginning the lesson. Some students specifically asked that, at the initial stages, key terms could be explained and connections made to what they already know. This is an important insight to incorporate into the recommended pedagogies.

Overall, the interviewed students were willing and in some cases eager to work on more challenging tasks. They were aware of the subtleties in the classroom that affected their learning in mathematics and the factors that either assisted or constrained their ability to maintain persistence when facing challenging tasks.

Conclusion

This is the report of an iteration of a project working with teachers on using challenging tasks. Data included a reflection on a pedagogy-modelling experience, reactions of project teachers to that teaching and the unit overall, some quantitative data on student learning, and both qualitative and quantitative responses from students on the tasks.

As with all classroom-based research, there were considerable ambiguities in the data. In terms of the research questions, the teachers were willing and able to implement the challenging tasks, and their reports on task use indicated they had tried many of the pedagogies that had been modeled for them. It seemed from the meetings that they had the subject matter knowledge required for these tasks, but not necessarily the pedagogical content knowledge. The teachers commented positively on the posing of tasks with limited introduction so as to maintain the challenge, the notion of individual work prior to discussions with peers, and the strategy of an interim or scaffolding task as a lead in to the challenging task.

They also noted constraints, especially those around reviewing student work, and the challenge of ensuring that other students listen to and learn from the reports of others. The teacher reflections also noted that opportunities for learning are sometimes missed. Another challenge identified was that the extended problem-solving task, the *Training track*, was too difficult for the students and provoked negative student reactions, highlighting that finding the right level of challenge is important. The online assessment of student learning from the tasks that required students to work with diagrams was inappropriate.

It seemed that students did learn the content, since a substantial subset of the students was able to complete a challenging task under test conditions. Both in the modelled lesson and from reports of teachers, the students were willing to persist sufficiently to engage with such tasks. The students' responses to surveys indicated that they did not see challenging tasks such as these to be exceptional, and in interviews expressed a preference for such tasks. There were students who did not endorse the idea of challenging tasks, and teachers need to be aware of such responses, and use a range of task types in their teaching. Interestingly, the students expressed a desire for more information about relevant terms and the ways of working on the tasks.

The intention is that the results inform not only future iterations of the project, but also future support for, and advice to, mathematics teachers. It seems that teachers appreciated the suggestions of challenging tasks that aligned with the content of their teaching, and also that the modelling following by trialling of relevant pedagogies engaged teachers in substantive reflection on those pedagogies. The requirement of using different methods to solve the tasks was not only mathematically effective, in that it encouraged flexible thinking, but also pedagogically effective, in that it extended those students who had completed the task one way.

Even though the students learned and engaged well with the tasks, it was also clear that specific actions by teachers to encourage and affirm persistence, along with information about the processes of explaining their thinking in writing and verbally to other class member, and the importance of active listening, would be helpful. Indeed, as students move to mathematics study at more senior levels, an orientation to persist and a growth mindset will be useful assets for them

The results also have implications for teacher education. Many discussions around presentation of mathematics concepts to children are focused on avoiding the complexities. Yet it may be that coming to grips with the complexities is exactly what children need. It seems important that teacher educators include at least some consideration of challenging tasks and student persistence in their presentations to prospective and practising teachers.

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