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ORIGINAL ARTICLE

Demonstration lessons in mathematics education: teachers' observation foci and intended changes in practice

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Abstract As part of a teacher professional learning project in mathematics education, university mathematics educators taught demonstration lessons in project primary schools. These lessons were part of a "pre-brief, teaching, and debrief" process, in which up to eight teachers observed each lesson. Using brief questionnaires completed in advance of the lesson, during the lesson, following the debrief, and several

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weeks later, data were collected on teachers' intended and actual observation foci and any anticipated changes in their beliefs and practices arising from the experience. There were several common themes in teachers' intended observations, including a focus on questioning, catering for individual differences, and building student engagement. As evident in other research, teachers' intended and actual observations gave greater attention to teacher actions and decision making than to student learning and thinking. In this paper, we situate demonstration lessons within teacher professional learning models, describe the features of our model, summarise teacher data, and discuss issues arising from our work.

Keywords Teacher professional learning · Demonstration lessons · Observations · Teacher change · Pedagogical content knowledge · Student learning

Background

Demonstration lessons, when situated within a professional development or coaching program, have been shown to hold the potential to promote teacher change and raise the quality of the teaching and learning in a classroom (Grierson and Gallagher 2009; Joyce and Showers 1980; Saphier and West 2010). Many factors about professional development and coaching programs have been suggested to contribute to this change. These include the presentation of theory, professional support embedded in the workplace, the coach's or demonstration teacher's interpersonal skills and ongoing support, structured feedback, the examination of evidence of student learning, collaborative planning and reflection on practices, with demonstration lessons or modelling being a key component (see, e.g., Loucks-Horsley et al. 2003).

While most studies have not been designed to measure the impact of the separate components of a professional learning program, it is clear that in many studies the teachers indicated that "the modelling, observation, and debriefing were the most valuable components" (Butler et al. 2004, p. 447). Typically, demonstration lessons occur more than once and usually have a particular focus as determined by the program (e.g., particular content or teaching strategy) or by the teacher (e.g., identifying and addressing misconceptions, or questioning of students). The lessons usually include prelesson planning or discussion and post-lesson debriefing that reflect on the teaching and learning. Some programs (West and Curcio 2004; White and Southwell 2003) solely focus the observing teachers on the students' understanding through the child's actions and products.

Guskey (1986), in proposing a model of teacher change, suggested that when teachers try new approaches to teaching and learning, "significant change in teachers' beliefs and attitudes is likely to take place only *after* changes in student learning outcomes are evidenced" (p. 7). Clarke and Hollingworth (2002), while retaining the sequence of Guskey's linear elements, suggested that the model could be more usefully viewed as cyclic with multiple entry points, and broadened what they called the Domain of Consequence beyond changes in student learning outcomes to *changes in salient outcomes*, acknowledging that "individuals (teachers) value and consequently attend to different things (they consider different things salient)" (p. 954). They claimed that teacher change is personal and situated and "the support of teacher growth must offer teachers every opportunity to learn in a fashion that each teacher finds most useful" (p. 965).

Our research and practice in teacher professional development is grounded in a situated perspective—that teacher learning is "constructed through participation in the discourse and practices of a particular community" (Borko et al. 2008, p. 418). There is growing consensus regarding the value of creating opportunities for teachers to work together to develop their practice, and for these opportunities to be located in the practice of teaching, with a focus on teachers' everyday work (Higgins and Parsons 2009).

Various writers have discussed a range of teacher professional development models, their intended goals, the principles underpinning them, and their demonstrated impact on teacher learning (Clarke 1994; Loucks-Horsley, et al. 2003; Tytler et al. 2006). In what follows, lesson study models and demonstration lesson models will be discussed in some detail, given their common features and clear relevance to the study which forms the basis of this paper. Also, the research literature on demonstration lessons is far "sparser" than that of lesson study. It is hoped that this study will contribute to expanding research activity and discussion in this area.

An increasingly common model of teacher professional learning involves the use of *lesson study*, a model originating in Japan, but now in widespread use in several other countries, particularly Thailand, the USA, and Canada. During lesson study, teachers work collaboratively to formulate long-term goals for student learning and development; plan, conduct, and observe a "research lesson" designed to bring these long-term goals to life, as well as to teach particular content; observe carefully student learning, engagement, and behaviour during the lesson; and discuss and revise the lesson and approach to instruction based on these observations (Lewis 2002).

Takashashi and Yoshida (2004) elaborated further the process of lesson study as follows: "The research lesson occurs in the regular classroom, and participants observe as the lesson unfolds in the actual teaching-learning context. Debriefing following the lesson develops around the student-learning data collected during the observation. Through the lesson study process, participants are given opportunities to reflect on the teaching process as well as on the student-learning" (p. 437). Characteristics of lesson study that set it apart from typical professional development are that it occurs at the site where teaching and learning occurs and can provide a common context from which teachers can plan, reflect, and develop common understandings of teaching practices and student learning. Usually, there is an expectation that teachers who have observed the lesson will also try it with their own students and report back to the group.

There is a strong focus in lesson study on student thinking and learning. One member of the lesson study team teaches while the remaining members of the team collect specific data, "which generally include detailed narrative records of the learning of several students—what the students said and wrote, how the students used the materials, what specific supports encouraged understanding, and what obstacles to learning arose during the lesson" (Lewis et al. 2004, p. 20). Fernandez et al. (2003), in discussing a USA–Japanese lesson study collaboration, found that in order to benefit from lesson study, teachers needed to learn to apply three critical lenses to their examination of lessons, namely the researcher lens, the curriculum developer lens, and the student lens. Interestingly, they contrasted the actions of US teachers as observers who "took on the role of another set of *hands* in the classroom and worked with individual children" with those of Japanese observers who did not interfere with the lesson in any way, "rather they acted as the *eyes* in the classroom" (p. 175).

Demonstration lessons or demonstration classrooms involve a professional learning strategy where teachers gather together to observe innovative instructional practice. The purpose of demonstration lessons as summarised by Loucks-Horsley et al. (2003) is to use a "prelesson, classroom demonstration lesson observation, and postlesson debrief cycle as a catalyst for in-depth reflection on mathematics teaching and learning" (p. 212). The prebrief involves the teacher for the day outlining the mathematical and/or pedagogical focus for the lesson, and participants discuss and take on observational roles for the classroom visit. During the lesson, observers make notes on student responses, teacher–student interactions, and other features of the lesson of interest. This is followed by a post-lesson debrief, during which observations are discussed and implications for future lessons and further individual action are considered. Some studies of demonstration lessons involved lessons that were modelled by an "expert" not from the school, while others involved lead teachers from the school who were prepared to "host collaboration classrooms—not 'model' classrooms—in which they non-defensively demonstrate risk taking, public teaching, and self examination of their own practice" (Saphier and West 2010, p. 49). In other contexts (e.g., Borko et al. 2008), videotapes of classroom practice are used to stimulate discussion of the teacher's role and student thinking. Other approaches involved colleagues observing each other's practice for the purpose of developing new pedagogical content knowledge, through jointly developing and examining classroom practice (Fernandez 2005).

There is variation across different models of demonstrations lessons in the extent to which the observation foci of teachers are prescribed. In the New Zealand Numeracy Development Project (NZDP, Higgins and Parsons 2009) facilitators modelled "complex pedagogies" and then observed primary teachers as they attempted to enact what they had observed. In the NZDP, facilitators explicitly guided teachers to notice certain "important things", on occasions even providing commentary on what the teachers were observing as the lesson was happening. Similarly, in the Supporting the Transition from Arithmetic to Algebraic Reasoning Project (Borko et al. 2008) where the demonstration lessons were videotaped and watched by groups of middle school teachers together, the required analytical frame for observations was classroom discourse. As part of the activities of the Center for Proficiency in Teaching Mathematics (Ghousseni and Sleep 2011), in which an experienced teacher educator worked with the same group of preservice students in a summer school setting for a week, 68 observers (mathematicians, mathematics educators, and school-based teacher developers) were free to nominate an observational focus early in the week, but as the week progressed, the observers were provided with more focused lenses for viewing lessons, and encouraged to attend to more subtle aspects of practice.

Bruce et al. (2009) examined the effect of two models of professional development in schools in Ontario, one based on lesson study principles and another involving demonstration lessons. This study articulated the similarities and differences between the two approaches, and offered advice on the contexts in which each might be most appropriately applied. The themes which were common across the two models included the use of an inquiry stance, the grounding in the classroom context, the focus being on a given lesson, and the attention drawn to specific teaching and learning strategies.

On the other hand, Bruce et al. (2009) noted differences between the two models such as intentional and careful listening to and observing students and their work in the case of lesson study compared to intentional observation of the overall classroom environment and teacher actions in the case of demonstration lessons. Demonstration lessons tended to involve a somewhat linear process over a relatively short period of time, while lesson study demanded longer term commitment and a more cyclical process. In determining the context in which each model might be better suited, the authors noted that demonstration lessons offered teachers "an entry point into the process of opening classroom doors to one another in collegial professional relationship building situations," while lesson study was described as "a much more elaborate process that requires significant participant commitment in terms of time, but also in terms of risk taking as the teachers are exploring specifically challenging areas of mathematics teaching and learning in one another's classrooms" (p. 530).

Putman (1985) collected data from teachers, principals, teacher educators, and teacher candidates to study the perceived benefits and limitations of five different approaches to demonstration lessons for preservice teachers, some involving video-taped lessons and some live lessons. She noted that live demonstrations of connected lessons that illustrated content cohesion and related teacher decision making were the most powerful. Putman also commented that the teacher educator's personal style may have contributed to the success or failure of the model. She reported that demonstration teaching assisted teacher educators in establishing credibility with classroom teachers, and providing a link between what was recommended in teacher education courses and classroom realities.

Grierson and Gallagher (2009) used a case study approach to describe and analyse the experiences of eight observing teachers, a demonstration teacher, and a program department consultant, who over a 9-month period participated in a demonstration classroom initiative. The focus was on literacy assessment and instructional practices. Protocols were setup so that there were no disruptions during the demonstration lesson by visitors or observing teachers. Observing teachers visited the classroom in groups of three or four, and used templates prepared by the consultant to focus and record their observations. Observing teachers were also interviewed regarding their observations as well as their experiences in modifying their own practices. Each observing teacher observed three lessons and had three full-day sessions including debriefing. The lessons were spaced several weeks apart, affording teachers time to reflect and enact targeted modifications to their teaching. Factors identified as supporting the observing teachers' change processes included: "the provision of 'believable' vicarious experiences in a local school context; the demonstration teacher's exemplary mentoring skills and respect for observing teachers' professional decision making; the cohesion created by tri-level alignment of programming goals; and the on-going support provided to observing teachers" (p. 567). (Tri-level here refers to the three levels of Ministry of Education, the School Board, and the primary consultant.)

Grierson and Gallagher (2009) noted that “to enhance potential for change, professional learning opportunities must be non-threatening, enhance teachers’ comfort taking risks with new practices, as well as support their abilities to be honest about the challenges and successes they encounter in doing so” (p. 569).

In the remainder of this paper, we outline the use of demonstration lessons as a catalyst for teacher professional learning, in the context of a large professional learning program with Catholic schools in the Archdiocese of Melbourne Australia, and discuss the data which emerged from the study of this process.

The Contemporary Teaching and Learning of Mathematics Project

The Contemporary Teaching and Learning of Mathematics Project (CTLM)[1] is a professional learning and research project that has involved 82 Catholic primary schools in Victoria (Australia) between 2008 and 2012. Each school participated in a 2-year program with Australian Catholic University (ACU), consisting of 10 and 12 full days of teacher professional learning (including workshops, professional reading, and between-session activities), and in-classroom support from the research team. Mathematics consultants, called School Advisers Mathematics (SAMs) from the Catholic central office provided regular support within schools, particularly around planning issues. Each year, a new cohort of schools began their first year of the project with ACU (i.e., intake 1 in 2008, intake 2 in 2009, intake 3 in 2010, and intake 4 in 2011). This cycle of professional learning continued until 2012 when the final intake completes their second year. The major project aim was to enhance teacher pedagogical content knowledge (see Shulman 1986). The ACU team also conducted parent information evenings in every school in the first year of its involvement, recognising the importance of parent involvement in student mathematics learning (Feuerstein 2000; Yan and Lin 2005).

Given the large numbers of schools and teachers with whom the ACU team was working each year (38 schools and approximately 680 teachers in 2011, for example), the team needed to develop strategies for maximising the impact of school visits. It was agreed that given the team’s strong research background in effective teaching in mathematics classrooms (see, e.g., Clarke and Clarke 2004; McDonough and Clarke 2003) and reasonable confidence and expertise with teaching mathematics in schools, school visits built around demonstration lessons would be appropriate. Given the large numbers of teachers, a lesson study model was considered prohibitive in terms of time and the lack of readiness of project teachers for this approach, particularly in light of the research of Bruce et al. (2009) above. Demonstration lessons had the potential to impact approximately 20 teachers in one morning. This pragmatic approach also has the advantage that it is more likely to be funded by

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departments of education on a broader level than a very intensive lesson study model, which is usually only funded for small groups at a time.

Methodology

In this section, we outline the demonstration lesson process used in each school, and then discuss the data collection methods and how these data were analysed.

The demonstration lesson model

The ACU team planned to visit each school between two and four times during the school year. Each visit, around 6 h in total, involved a similar format. We used the following model:

- We taught three lessons in a day at the school, with grade levels and content negotiated with teachers via the School Mathematics Leader (SML). The ACU team worked in pairs, with one ACU staff member usually taking the lead in each lesson, while the other ACU person observed individuals and groups as they worked, and had incidental conversations with observing teachers about what they were noticing.
- We met with all teachers who were going to be observing one or more lessons before school for a 20-min prebrief. During this time, we outlined what we planned to teach, the mathematical focus of the lesson, and the major tasks and activities in which the students would be engaged. We then asked the teachers to complete the first question on our provided proforma, which asked, "What are you planning to focus on in your observations with regard to teaching? … student learning?"
- We taught the lessons with typically four to eight teachers observing. If the lesson was a grade 1/2 lesson for example, we might have had all the grade 1 and 2 teachers observing, as well as the SAMs, SMLs, and sometimes the principal. Teachers were encouraged strongly to make notes on their observations for later sharing, to observe the teacher and students closely and to ask students questions about their mathematical thinking related to the tasks at hand. They were discouraged however from active teaching as they observed. Photographs were also taken of the demonstration teacher in action, students working, and of student work samples. These photographs were sometimes used to prompt discussion at subsequent teacher meetings at the school or conversations with the parent community about the changing classroom environment in mathematics over time.
- After each lesson, we had a 15-min debrief on each lesson, where teachers reported on what they had observed in relation to both their chosen observation focus and other aspects which they had noticed. These debriefs were facilitated by the SAMs or one of the ACU staff members.
- Later on in the day, the SML and other members of the school mathematics leadership team met with the SAMs to discuss issues that had arisen during the day, particularly those which were likely to lead to productive follow-up. The

SAMs were given a copy of the teachers' completed proformas for their future use.

Research questions

Our main research question was the following:

> What do teachers attend to when observing a demonstration lesson in mathematics and what change(s) in practice do teachers report might occur as a result of this experience?

A subquestion of particular relevance to this article was the following:

> Are teachers more likely to focus their observations and subsequent discussion on the teacher actions or the student actions/thinking?

Data collection

Data have been collected from four sources: proformas completed on the day of the classroom observations; written responses to several items included in a questionnaire completed on the last day of professional learning for the year; individual semistructured interviews with a sample of teachers whose response to the second questionnaire indicated positive experiences with the demonstration lesson process, and those teachers' principals; and semistructured, individual interviews with ACU staff who were involved in teaching the demonstration lessons. Given the large numbers of teachers with whom we were working, questionnaires were considered to be the most functional data collection instrument for the majority of participants.

As mentioned previously, all teachers completed a three-stage proforma to focus their classroom observations. The proforma evolved over time and the latest version of this is presented in Appendix 1. Our experience to date is that the temptation on the part of teachers is to focus their observations largely on teacher actions and decision making, and dispositional aspects and engagement of individual students, to the exclusion of student thinking, products and learning (as noted by Bruce, et al. 2009). In responding to this and in light of the strong message from the coaching literature on the importance of a focus on student thinking in the process of teacher professional learning, we asked teachers specifically to nominate an observation focus under two headings: teaching and student learning, respectively. This part was completed before the lesson, during the prebrief session.

During the lesson and at the start of the debrief session, the teachers completed part 2 of the proforma. This part of the questionnaire has been revised, as teachers initially tended to record the sequence of *activities* for the lesson when a more open-ended format was used. The prompt on the latest version of the proforma encouraged teachers to nominate two "significant events": one where the words or actions of the demonstration teacher prompted a particularly interesting response from a student or group of students; and one where the words or actions of a student or students prompted a particularly interesting response from the teacher. This amendment appears to have been more helpful in eliciting insights and issues for productive discussion during the debrief session.

At the conclusion of the debrief session, teachers were asked to write their responses to the prompt, "Is there anything that occurred today that you believe might contribute to a change in your teaching? If so, can you please describe the intended change?"

On the final workshop day of the professional learning program for the year in November 2011, as part of a questionnaire seeking information on the overall effect of the year's program, teachers were also asked to answer the questions shown in Fig. 1.

The last two items were asked, not necessarily with the intention of facilitating such a process in the future, but to gain an awareness of teachers' willingness to participate in the ways described. Item #5 provides an indication of whether the demonstration lesson process has the potential to continue when the support of ACU is withdrawn at the end of a school's second and final year working with the ACU team. The final item (#6) provides an indication of whether or not an extended and more potentially useful model, in which teachers observe an ACU staff member who subsequently observes the teacher teach a similar or related lesson, is feasible in the future.

Drawing upon the responses from teachers on their demonstration lesson proformas and final questionnaire, a purposeful sample of 10 teachers and their principals was selected subsequently for telephone interviews, in order to elaborate their responses to the items. Teachers who had indicated a likely change in their practice as a result of the demonstration lesson process were asked to elaborate on this change and any developments since the day of observation. The sample was also chosen to represent a range of themes in terms of their intended shifts.

In addition, all ACU staff members who had taught at least three demonstration lessons during the year were interviewed for their perspectives on the demonstration lesson process and any insights about their own teaching of mathematics which had emerged.

1. Did you observe a demonstration lesson by ACU staff in 2011? ……Yes / No *(If "no" go to Q.5)*

2. If "yes", what was the most helpful insight which emerged from your observation and the discussion which followed?

3. Has your teaching practice changed in any way as a result of the demonstration lesson(s) you observed and/or the related discussion? ………………………………………………Yes / No

4. If "yes", please describe this change?

Your answers to the next two questions are just hypotheticals and do not imply any commitment on your part:

5. Would you be willing to be observed by colleagues at your school while teaching a mathematics lesson, for the purpose of teacher professional growth and reflection? ....Yes / No

6. Would you be willing to be observed by ACU staff while teaching a mathematics lesson at your school, for the purpose of teacher professional growth and reflection? ….......Yes / No

Fig. 1 Questionnaire items completed on the final day of professional learning for the school year

In this paper, we report only on the data emerging from the proformas completed on the day of observations and the questionnaire data from the end of the year. Our intention is to report the interview data at a later time.

Data analysis

One aspect of the data analysis involved a team of ACU staff analysing the written responses made by a random sample of 200 teachers to parts 1 and 3 on the proforma used to focus their observations during demonstration lessons. Teachers had completed part 1 on the proforma during the prebrief session and this included two written responses: their planned observation focus with regard to teaching (item 1a), and their planned focus with regard to student learning (item 1b). Teachers had completed part 3 at the conclusion of the debrief session and this related to their perceived contributions of the demonstration lesson to a change in their teaching. These three datasets of written responses were each transcribed and analysed using an iterative process of inductive coding and collaborative revision by teams of coders (Creswell 2007; Miles and Huberman 1994).

The computer analysis software program *NVivo* (Bazeley 2007) was utilised for progressive cycles of line-by-line coding of these datasets and subsequent reconfiguring of codes and grouping into conceptual "buckets" (Corbin and Strauss 2008). It documented the process of analysis by forming an audit trail of the coding undertaken by each coder and allowed for comparison of coding and frequencies, and repeated revision of codes and conceptual sets.

The following process of analysis was undertaken for each of the three sets of transcribed data from the teachers' proformas:

- A team of four or five ACU staff members each inductively and manually coded several responses.
- The team then discussed their lists of conceptual labels and specific coding examples, and developed an agreed-upon initial coding scheme with operational definitions for each code.
- The team then recoded the same responses using the initial coding scheme and collaborated on further revisions.
- Three researchers from the team (Wilkie, Wright and Roche) subsequently undertook line-by-line coding of the first 50 responses from the sample of teachers using NVivo and compared their coding frequencies for each code. Differences were resolved and consensus was reached on the coding scheme and the coding of the first 50 responses.
- Two of the three researchers then coded the next 150 responses and check-coded their two complete sets of 200 responses to ensure consistency between coders for the whole sample.

In the following section, we report on the key themes which emerged from the data analysis in terms of observation foci (for teaching and learning, respectively) and perceived changes in future practice.

Results and discussion

Prior to observing the demonstration lesson, teachers participated in a briefing session of approximately 20 min. During that session, the ACU staff member described the lesson they intended to teach, including the mathematical focus and tasks the students would be given. At the end of the briefing, teachers were given time to record their proposed foci (item 1).

What are you planning to focus on in your observations with regard to:

- Teaching?
- Student learning?

After their observation of the demonstration lesson, teachers were asked to reflect on their experiences and how these might affect a change in their teaching practice (item 3). Teachers' responses to these three questions (items 1a, 1b, and 3) are each discussed in turn in the following subsections.

Intended lesson focus—teaching

The categories which emerged from the data analysis on teachers' intended observation foci with regard to *teaching* are presented in Table 1. The categories are presented in order of decreasing frequency, with the exception of the categories of mismatch and missing data being placed at the foot of the table. Several teachers responded with more than one intended focus and some responses were coded in more than one category. For example, the response "how to include all in question and answer and those who might opt out or get it easily" was categorised as relating to "Questioning" and to "Differentiation".

We have chosen here to indicate the percentages of teachers who made statements within a given category out of the 200 teachers, thus indicating the proportion of teachers who mentioned a proposed category of subsequent action. We could also have provided these data in terms of the percentage of responses which fell within a given category (out of 445 responses), but these are easily converted, by dividing the percentages in Table 1 by 2.225.

The largest category for this item was found to be "Questioning". Of the 200 teachers included in the sample, nearly 36 % made some reference to their interest in questioning: how the demonstrating teacher would use questioning to probe students' thinking, draw out their understanding, and direct their attention to the key mathematical ideas of the lesson. Some teachers referred to the type of questioning to be used at different stages of the lesson, such as during the introduction to draw students "into the lesson", throughout the lesson to "extend children's mathematics during tasks" and at the conclusion to "gauge student strategies and understandings." Several teachers referred to questioning techniques and others mentioned the various purposes for questioning, such as "prompting", "probing", and "extending".

Another significant observation focus raised by 32 % of teachers was related to "Differentiation": ways to provide engaging learning experiences for all students at their particular level of ability or understanding. Several teachers used the terms

Table 1 Categories of responses to item 1a, with descriptions and illustrative examples of each

Category	Description of category	Percentage of teachers out of 200 (%)	Illustrative examples
Questioning	How questioning is used to elicit student responses and understanding	35.5	"The types of questions asked to probe students' thinking" "Questioning - drawing out children's understanding"
Differentiation	Ways that a range of abilities and learning styles are catered for, types of grouping used, ways to challenge students	32.0	"How to engage all students who are at different levels in the whole class instruction" "Dealing with "at risk" students/ high achievers"
Lesson structure or features	Features of particular stages of the lesson (e.g., tuning in, pulling the lesson together), how the mathematical focus is emphasised	25.5	"How they explain their focus and how they maintain it throughout lesson" "Delivery - how much input prior to activity, length of time spent on each part of the lesson"
Other pedagogy	Strategies, methodology, creativity	20.0	"The mathematics being taught 'creatively'" "How they get around to help everyone"
Specific content	Interest in a particular mathematics concept, links between concepts	13.0	"Strategies to engage children with fractions - a difficult area to teach" "Prompting children's learning in regards to shape/ transformation"
Language	Use of mathematical language and terminology with students	11.0	"How are key words developed?" "How a shared language was built during the lesson"
Communication	How concepts are explained or demonstrated, how students are instructed, how they are encouraged to participate in discussion	7.0	"Instructions given" "Discussion with students about their learning (discussion that aims to take them further) "
Assessment	How students are listened to, observed, and monitored (including their language)	4.0	"Assessing children's understandings" "Gauging student understanding of topic through questions"
Materials and representations	Use of resources, models and representations	2.5	"Use of materials/resources" "Equipment used"
Tasks	Choice of tasks	2.5	"Activities taught" "The different activities"
Classroom organisation	Arrangement or setup of the classroom	1.0	"Setting up/organizing e.g. seating etc. " "Classroom organisation"

Table 1 (continued)

Category	Description of category	Percentage of teachers out of 200 (%)	Illustrative examples
Mismatch	Where the response does not relate to the question	6.0	"Student involvement in activities" "Visualising the number activity will be more effective to practice"
Missing data		4.5	

"enabling" and "extending" to describe their interest in how the demonstrating teacher would achieve this in the lesson.

Some referred to how the "at risk students" and "more capable students" would be supported. One teacher wrote "How do I help those who are not keeping up without making them feel inadequate?" Another was concerned about "dealing with high achievers". The issue of managing the various "levels" within the class and catering for different abilities in the one lesson was highlighted as of particular interest to many teachers.

Just over one quarter of teachers indicated that they intended to focus on the delivery and format of the lesson, or on its features, such as the introduction, body, and conclusion; the timing or pace of each stage; the sequence of activities; and the maintenance of student focus at each stage of the lesson.

One fifth of teachers made reference to aspects of teaching which were categorised as "Other pedagogy". These included several responses in which teachers simply wrote "strategies". Some teachers mentioned a particular teaching methodology, such as "teacher talk compared to child discovery", and "leading children to make connections", or a classroom management issue, such as ensuring children "are all keeping on task".

Thirteen percent of teachers responded with an observation focus related to specific mathematics content to be taught that lesson, as explained beforehand by the ACU staff member during the prebrief session. Strategies for introducing, demonstrating, or teaching these particular concepts, or making connections between concepts, seemed to be the main interest for these teachers.

Just over one tenth of teachers described their intention to focus on the demonstrating teacher's use of "Language": "key words" used, "terminology", "language used to explain instructions", and "language to use with students to help them through issues". A closely related category "Communication" was used for responses that describe particular types of communicating, rather than the use of mathematical language as such, for example, explaining, discussing, and instructing.

In item 1a, a small number of teachers (6 %) referred to an intended focus that was categorised as a "Mismatch", for the most part because these related to observing *student actions* rather than *teaching* (requested in the second part of item 1). Nearly 5 % of teachers did not make any response to this item.

Intended lesson focus—student learning

Table 2 provides a summary of the categories that emerged from the data on teachers' intended focus in respect of student learning. The comments tended to be highly situated in that there was often commonality in the comments made by different teachers observing the same lesson. This may reflect the impact of the briefing meeting and lesson description in influencing the teachers' choices, the collective focus of the teachers aligned to their school aims, or uncertainty about what to look for, resulting in one teacher taking on another's idea.

Table 2 Categories of responses to Item 1b, with descriptions and illustrative examples of each

Category	Description of category	Percentage of teachers out of 200 (%)	Illustrative examples
Student knowledge, understandings	The knowledge and understanding of specific mathematical concepts, or understanding in general	33.5	"Connections made between division and multiplication" "Improvement in map/ identification of features"
Student communication	Student sharing ideas through questioning, responding, and explaining with a focus on the use of mathematical language	27.5	"Children can verbalise their understanding of concepts" "How the children will explain what they have learnt"
Student affect	Engagement in the task by students, considering diverse needs, and attitude/confidence	25.5	"Engagement of all students in lesson" "How are students going to work through a challenging task they haven't learnt about before?"
Student action	Activity that students demonstrate, including applying problem solving strategies, recording, and interpreting	21.0	"The way children adjust their strategies (if they try more complex ones as they lesson goes on)" "How they use different models/concrete material to demonstrate learning?"
Student interaction	Collaboration and teamwork	2.5	"How the children learn from each other?" " Interaction to task & learning"
Mismatch	Not relevant to focus or unclear	11.5	"What value open ended questions have in the learning process?" "Drawing out prior knowledge."
Missing data	No response	10.5	

Twenty percent of the teachers either provided no comment or made comments that were either ambiguous or unrelated to the question. Nearly 12 % of comments were classified as mismatches mainly because they referred to observing teacher actions rather than the thinking or actions of the students. This relatively high incidence of irrelevant comments may indicate that for some teachers their main focus during demonstration lessons was on the actions of the teacher and that adopting a student focus was difficult for them. Other studies have indicated a growing comfort with a student observation focus over time (e.g., Davis and Walker 2005; Goldsmith et al. 2009), and anecdotal comments from our research team are reflecting this in the second year of a school's involvement.

Eighty percent of teachers made comments related to students' understanding, student affect, and student actions. Nearly 34 % of the teachers gave their intended focus as student knowledge, skills, and understanding. Of these, approximately one third were very specific to the mathematical concepts being taught, such as subitising of set patterns, grid references, and tessellation. Other comments were more general and referred to conceptual development, overcoming misconceptions, and use of representations. The focus on student thinking was also evident in the comments about student communication made by nearly 28 % of the teachers. These teachers appeared to understand that students' use of language through questioning and responding was significant for conceptual growth and also useful as a window to thinking.

There were also some references to learning being socially mediated, such as "How the children learn from each other," "How children explain their learnings/strategies," and "How they use different models/concrete material to demonstrate learning."

The comments that were classified under "Student affect" were dominated by two related themes, engagement and diversity. Over one quarter of the teachers indicated a prelesson focus on how their students would engage with the task in a sustained manner and how the task would meet the needs of the diverse range of students' needs and abilities in their class. Some teachers particularly mentioned able students and those with significant learning needs. There was no indication that the comments were prompted by cynicism about the viability of the whole-class, open-ended task approach often being used. Rather there seemed to be a clear acceptance that catering for diversity was an important feature of successful classroom environments. The category of "Student action" encompassed a range of comments about activity. However, most comments were about generic problem solving strategies and use of representations, especially recording.

The data indicated that adopting a student focus was not easy for a considerable proportion of teachers. Those teachers who did focus on students appeared to be able to identify important observable features of student behaviour. Some were very specific about the demonstration of mathematical knowledge, communication, engagement, and action they would look for, while others were more open and general.

Teachers' perceived future change in practice

Following written responses on proposed observations foci (item 1) and actual observation notes (item 2, not discussed in this article), teachers were asked to reflect on the observation experience and respond to the following open prompt:

Is there anything that occurred today that you believe might contribute to a change in your teaching? If so, please describe the intended change.

The categories which emerged from data analysis for item 3 are shown in Table 3. In each case, a description of each category is given, as are one or more illustrative examples of the kinds of responses within this category.

Not surprisingly, there was overlap between the responses to item 1a (intended teaching observation foci) and item 3 (perceived changes in teaching practice), but some important differences also. Some statements were coded in more than one category. For example, a statement about future teaching practice such as "I will include a picture story book to motivate children at the beginning of the lesson" picks up on three of the categories in the table: student affect, materials and representations, and lesson structure or features.

Nearly 14 % of teachers made a comment about future action which did not relate to the question. These were typically *evaluative comments* (86 % of all mismatches), which appeared to offer a "verdict" on the teaching they had seen (more appropriately a response to item 2, if at all) or *emotive comments*, indicating their excitement at what they had experienced. These were generally positive, although some criticised the way the lesson had panned out, or the lack of engagement or understanding of some students. Nearly 8 % of teachers did not indicate an area of intended action.

Teachers therefore made 445 comments, an average of 2.3 comments per teacher, indicating that teachers generally were able to see themselves building upon what they had seen and learned from the experience. In broad terms, we see that teachers were looking to raise the level of engagement of their students in worthwhile mathematics with a more active approach, differentiate according to student needs with a greater emphasis on pair or small group work, make greater use of appropriate materials and representations, increase their level of use of appropriate mathematical language and questioning, and consider more carefully the way in which they structured a given lesson with particular focus on allowing appropriate time for each part.

Certainly, the strong interest in questioning alluded to in responses to item 1a was evident in teachers' perceived future action, and so the demonstration lessons appeared to be helpful to teachers in this respect. If the percentage of teachers mentioning aspects of questioning was combined with those who discussed aspects of time (increased wait time, holding back from telling, etc.), this would total nearly 36 %, indicating it to be the largest broad area of both interest and subsequent action.

Probably the greatest differences between intended actions and the proposed observation foci stated in item 1a were in the comments relating to materials and representations, where 28 % of teachers mentioned this as an area of future attention, while less than 3 % had intended to focus on this prior to the lesson. This discrepancy is probably not too surprising given that many teachers were presumably about to witness the use of a particular resource or model which was not in their personal teaching repertoire when they completed item 1a.

Questionnaire data

As reported earlier, at the end of one school year of demonstration lessons, teachers completed a questionnaire which included questions about whether they observed at

Table 3 Categories of responses to item 3, with descriptions and illustrative examples of each

Category	Description of category	Percentage of teachers out of 200 (%)	Illustrative examples
Materials and representations	Using particular resources, models and representations, more "hands-on" approach, emphasis on student recording	30.0	"Using unifix, especially to calculate difference" "Engaging the students with more concrete activities"
Time	Increasing wait time, changing pace of lesson, holding back from telling,	24.5	"Give them more thinking time" "Giving students time to explain themselves without butting in or finishing their ideas"
Communication	Encouraging students to explain and share their thinking and reasoning	22.5	"Allowing students to discuss their answers freely" "Lots more talk in my maths lessons"
Questioning	Improving questioning, using more open-ended questions, asking students how and why	18.5	"Use more open ended questions" "Asking the students the how and why"
Lesson structure or features	Refining particular stages of the lesson (e.g., tuning in, pulling the lesson together), making mathematical focus more explicit	16.5	"Keeping clear explicit teaching focus" "Whole group focus relate to lesson rather than number fluency and counting patterns"
Specific content	Giving greater emphasis to particular content, more planning in advance	14.5	"Counting on a fraction number line" "Linking number to quantity"
Differentiation	Catering for a range of abilities and learning styles, pair and small group work, challenging students, and peer teaching	13.5	"Allow the students to work in small groups on different tasks" "More challenging options available"
Language	Using mathematical language and terminology appropriately or more often	13.5	"Allowing time to really unpack the language" "Importance of maths language – understanding terms ,e.g., congruency"
Tasks	Choosing tasks which engage, contextual tasks, open-ended tasks, relating to students' experiences, using games	10.0	"Real life situations are so important" "More open ended activities"
Other pedagogy		9.0	"Guide children to a deeper understanding"
Assessment	Listening to, observing, and monitoring students (including their language) to guide instruction	8.0	"Observation to drive teaching" "I think I would listen to the children and move onto

Table 3 (continued)

Category	Description of category	Percentage of teachers out of 200 (%)	Illustrative examples
			the next step according to their understanding"
Student affect	Motivating students, building confidence, positive attitudes	5.0	"Get students away from fearing maths"
			"Continue to use positive encouragement, praise in maths"
Physical involvement	Using movement, acting out, being outdoors	4.5	"Get children physically moving- acting out problems"
			"Change of environment – indoor/ outdoor"
Mismatch or unclear	Where the response does not relate to the question	14.5	"I thought this was a great lesson"
			"I was surprised at how quickly students can adapt to new situations"
Missing data		7.5	

least one demonstration lesson in the year, the most helpful insight which emerged from this, whether their teaching practice had changed as a result and the nature of this change, and whether they would be prepared to be observed at school while teaching mathematics, by their colleagues and the ACU team, respectively.

Table 4 provides the data on teachers' answers to three of these questions. The percentages in each case refer to the percentages of teachers who answered "yes" out of all those who answered the given question.

There was a fair deal of consistency in the data across the grade levels, with the extent of observation of demonstration lessons being 87, 94, and 93 %, for teachers of grades prep to 2, grades 3 to 4, and 5 to 6, respectively. The willingness to be observed by colleagues varied across the grade levels very

Table 4 Teachers' extent of observation and willingness to be observed while teaching mathematics

Response	"Did you observe a demonstration lesson?"	"… Willing to be observed by colleagues …?"	"… Willing to be observed by ACU staff …?"
Yes	372	264	218
No	40	88	130
Blank	73	16	16
Total	485	368	364
Percentage "yes"	90 %	75 %	63 %

little, being 77, 77 and 70 %, respectively. Willingness to be observed by ACU staff varied across the grade levels little also, being 61, 64, and 64 %, respectively.

Conclusion

Van Driel and Berry (2012), in discussing the development of pedagogical content knowledge, noted that such development is “a complex process that is highly specific to the context, situation, and person.” They argued further that professional development programs “should be organised in ways that closely align to teachers’ professional practice, including opportunities to enact certain (innovative) instructional strategies and materials and to reflect, individually and collectively, on their experiences” (p. 27).

We claim that the process described in this paper recognises these issues, and by being situated in the classrooms and schools of individual teachers, has the potential to support careful reflection on observed practice, while offering choices for teachers in their observation foci. We admit readily that the process we have described in the CTLM project involves just initial steps in an appropriate professional learning sequence, and further study into the actions teachers *choose* to take and *do* take (and not just what they *claim* they will take or have taken) will be important. It will also be important to study the nature of the support needed to assist teachers in making greatest subsequent use of what has been learned through observations and subsequent discussions.

Having said that, much has been learned from this study. After the major tasks and activities of a given lesson had been outlined to them, teachers identified readily observation foci for a given lesson. Around 95 % of teachers documented a teaching observational focus when requested, while around 90 % did so for a student learning focus. In addition, around 78 % identified an intended change in their teaching as a result of the demonstration lesson experience. We note of course the potential limitations of teacher self-report in relation to this latter aspect (Boyle et al. 2005).

As indicated earlier, some demonstration lesson facilitators in other projects nominate the observational focus for teachers, rather than inviting them to choose. The advantage of offering a choice is that we are able to learn about those aspects upon which teachers really wish to focus, as well as the empowerment of enabling teachers to make choices for their own professional learning.

Coding teachers’ written responses was difficult given the individualised and situated nature of the comments, and the categories being inter-related in many cases. We noted that some of the language which formed part of the professional learning days was evident in their written responses. For example, the use of *extending prompts* and *enabling prompts* (see Sullivan 2011) are cases where teachers had a language to describe what they were intending to observe or what they actually noticed.

It was of interest to us that aspects of questioning accounted for over one third of teachers’ interests in relation to teaching observations. This observation focus translated through into their intended change in teaching, with many comments around a greater emphasis on students articulating their thinking. There was also an emphasis in intended practice on increasing student thinking time, and allowing greater wait time when asking questions. The research of Mary Budd Rowe (1974) many years ago on wait time in 300 science lessons found that, on average, a teacher would speak again if a student did not respond within 1 s, and that a teacher normally reacted or asked another question within an average of 0.9 s of a student response. Clearly, many of the CTLM teachers saw a need to extend wait time. A related aspect was the desire of teachers to "hold back from telling students" in the future, thus giving them more time to reason things through for themselves.

Teacher strategies for differentiation were also of interest in observations of teaching, with just under one third of all teachers listing this as an intended observation focus. Interestingly, around half of that proportion chose aspects of differentiation as their intended change of practice. How the demonstration teacher structured the lesson was also of interest to many teachers, and many teachers identified strategies which they intended to implement, particularly in relation to engaging the students early in the lesson, and "pulling" the lesson together.

The large proportion of mismatches to the question or missing data in the case of intended observation focus for student learning (item 1b) may suggest that identifying a clear focus for noticing was problematic for some teachers. In relation to intended observations with regard to student learning, 22 % of teachers either did not nominate a focus or nominated one which was not judged by the research team to be related to student learning. This confirmed the findings of others that teachers more readily identify and focus on aspects of teaching in demonstration lessons than on aspects of student learning. It is reasonable to be a little concerned about the limited extent to which these teachers may be noticing important student thinking, actions, strategies and learning, as alluded to by Fernandez et al. (2003) in their study. However, there is evidence from other projects, that with sufficient support and experience, teachers can move progressively to a greater focus on students (van Es and Sherin 2008).

Research in the last 10 years has demonstrated the complexity of *mathematical knowledge for teaching* (Ball et al. 2008). This term encompassed not only the subject matter knowledge and pedagogical content knowledge of Shulman (1986), but also additional categories as well. The difficulty for some teachers in nominating a focus on student thinking reflects the complexity of connecting knowledge about students with knowledge about mathematics, termed *knowledge of content and students* by Ball and colleagues in their framework. Such connections involve anticipating students' thinking about the mathematical concept(s) involved, including potential confusions or misconceptions, the motivational effect of different contexts, and the difficulty level of tasks. A significant implication is that teachers need to act "in the moment," integrating different categories of knowledge to enable their interactions with students (Ball and Bass 2000). A reasonable hypothesis is that teachers who find nominating a student focus difficult are also likely to find spontaneous interactions with their students about learning even more so.

Strategies for increasing the focus on student learning are important, given the findings from research on the power of attention to this, in enhancing teacher professional learning (Clarke and Hollingworth 2002; Guskey 1986; Takashashi and Yoshida 2004). The template in the proforma for Question 2 (Appendix 1) appeared to be somewhat successful in generating observations with a clear focus on the thinking and actions of students.

Of those teachers who did nominate an aspect of student learning for their observational focus, the knowledge and understanding of specific mathematical concepts was the most frequently identified. Teachers were also highly interested in student oral communication, including their explanations of their strategies and understanding, and their use of appropriate mathematical language. Student affect also rated highly as a topic of interest, with teachers looking out for levels of engagement, particularly from those who were not normally very engaged during mathematics lessons.

Interestingly, the category most frequently mentioned in relation to subsequent changes in teaching practice (materials and representations) was one that was rarely anticipated by teachers in their choice of observation foci. This category included the use of particular resources or models, a more "hands-on" approach, and a greater emphasis on student recording. When we refer to representations, most of the examples teachers gave involved what have been described as external representations, rather than internal representations, which can be thought of as mental models (Goldin and Shteingold 2001). It is not surprising that this category of materials and representations was not generally anticipated, given that it is only in observing the use of a particular resource or model that a teacher sees its potential and considers the potential use of it in the future. Although not mentioned frequently, the category "physical involvement" was also one which emerged after observations, but was not mentioned before.

In a recent review of research in mathematics education, Sullivan (2011) offered six principles for effective teaching of mathematics. They were articulating goals, making connections, fostering engagement, differentiating challenges, structuring lessons, and promoting fluency and transfer. With the possible exception of the last one, these principles find considerable synergy with the observational interests and intended teaching practices nominated by teachers in this study.

Considering the future possibility of teachers being observed by their colleagues, around 75 % of teachers claimed to be open to this possibility, with a lower proportion (63 %) being prepared to teach with university staff watching. Of course, we have no idea whether these numbers are greater or less than what they might have been prior to the demonstration lesson process they experienced, but they are encouraging figures just the same.

In conclusion, we acknowledge once more that the demonstration lesson model described above is limited, and there are many ways in which follow up support and study of teachers would be helpful in enhancing professional learning. The present research however has provided considerable insights into the intended observation focus of teachers and evidence that the model does prompt teachers to consider seriously new approaches to the teaching of mathematics in their classrooms.

Appendix

Appendix 1: demonstration lesson teacher proforma

Date:	Grade level of Demo class:	School:
The ACU teacher who led the lesson:		
Observing teacher's name:		
Observing teacher's role (e.g., Grade 1/2, SML, ACU - PST, etc):		
Are the children being observed in this lesson currently taught by you? Yes or No		
Lesson "title"(provided during prebrief by the teacher leading the lesson):		
Mathematical focus:		

1. BEFORE THE LESSON:

What are you planning to focus on in your observations with regard to:

- ❖ teaching?

- ❖ student learning?

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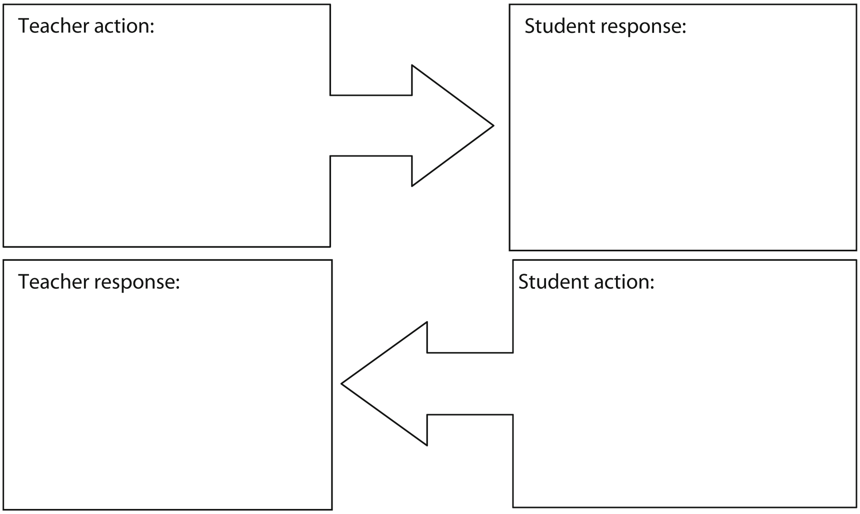
2. DURING/AFTER THE LESSON: (Please focus on one scenario in each starting box)



Other comments:

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3. AFTER THE DEBRIEF:

Is there anything that occurred today that you believe might contribute to a change in your teaching? If so, please describe the intended change.

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