

## Exploring relationships among teacher change and uses of contexts

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**Abstract** The three middle level mathematics teachers in this set of case studies were engaged in a longitudinal professional development program that sought to impact teaching practices through increasing participants' mathematical knowledge for teaching. This study investigates how teachers use the contexts in which their teaching practices are situated. Data collected include multiple classroom observations, videotapes, and interviews. The roles of contextual elements in the three teachers' teaching practices vary greatly, influenced by teachers' knowledge and beliefs. The realities of contexts were less important than how teachers chose to use those contexts. These cases specifically illuminate the complexities in teachers' uses of school structure, professional development, curriculum, testing policies, principal expectations, community expectations, and extra-curricular activities. For the three teachers in this study, the roles of contextual elements in their teaching practices varied greatly; such roles are influenced by teachers' knowledge and beliefs. While much of this analysis is specific to mathematics, some teaching practices transcend mathematics and thus be interesting to a wide audience.

**Keywords** Teacher change · Middle level · Mathematical knowledge for teaching

### Abbreviations

Math in the Middle M<sup>2</sup>

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## Teacher change

By most measures, typical ways of instruction are failing students in the United States (Augustine et al. 2010; Stigler and Hiebert 2004). In order to change the output of teaching, it seems logical to address inputs to learning: teacher knowledge, beliefs, and practices (Ball et al. 2001; Mishra and Koehler 2006). Yet, teaching is a cultural activity; teaching and learning are about negotiation of classroom norms between a teacher and his or her students (Nelson et al. 2001). Such negotiation occurs within the environment of the school and community (Swidler 2004). Change is a resource-intensive, often slow process, requiring time, reflection, and conversation—priorities too often lacking in American institutional culture.

Teaching practice, even when occurring in an isolated classroom, does not exist in a vacuum, but rather within interconnected layers of contexts, including those of students, colleagues, administration, schools, communities, resources, standards, curricula, and assessment. While it is expedient to talk about the culture of American schools (and even American culture), it is not the case that culture across the United States is uniform. Differences in school size, geographical location, and community size all contribute to the culture of the school, as does the socioeconomic base of the school. Many schools attempt to increase student achievement by changing some of the contexts of teaching, such as by adopting new content standards or textbooks. However, this study suggests how teachers use various contexts may be as important (or more important) than the specific context. This study analyzes teacher contexts and their interactions with knowledge, beliefs and professional development.

This study was guided by an overarching research question: How do middle-level mathematics teachers use their experiences participating in a professional development program? Specifically, this paper examines the following sub-questions:

- How do teachers' knowledge and practices change as a result of participating in a professional development program?
- How do teachers' contexts and uses of those contexts influence the nature and extent of these changes?

A closer look at teacher change through the lens of uses of contexts can answer the question of how particular teachers have overcome specific obstacles to change and what sort of obstacles still remain. While such a close look at a few individual teachers does not answer large scale questions of teacher change, change does not usually happen on a large scale. Rather it happens on a small scale, in a series of small, individual steps (Hargreaves 2005). Such research helps to elucidate the roles of contexts in the change process, this informing the content and design of future professional development and teacher support.

## The math in the middle institute

The three teachers in this study were participants in Math in the Middle (M<sup>2</sup>) Institute Partnership, a 7-year grant funded by the National Science Foundation which provided professional development to middle-level mathematics teachers in a Midwestern state. Teachers took twelve mathematics, statistics, and pedagogy graduate classes

to earn Master's Degrees from the [university blinded for review]. Summer courses were offered as intense, week-long on-campus experiences; academic year courses were offered as a blend of in-person (2 to 3 days) and distance education. An in-depth description of the M<sup>2</sup> program and its courses are found in Heaton et al. (2012a, b).

Students and teachers each follow individual learning trajectories (Simon 1997; Simon and Tzur 2004). Large-scale surveys or other quantitative measures of knowledge can be informative, but to see what the change process looks like in meaningful ways, one must focus more closely on individuals (Schifter 1996) and their contexts (Chaiklin and Lave 1993). These three case studies of teachers participating in a professional development program report the findings of an examination of teachers' contexts of beliefs and practices around mathematics, teaching, learning, and students. Additionally, the analysis led to interesting findings regarding how teachers were using various contexts in which their teaching practices were situated.

### **Complexities of context: definitions and uses**

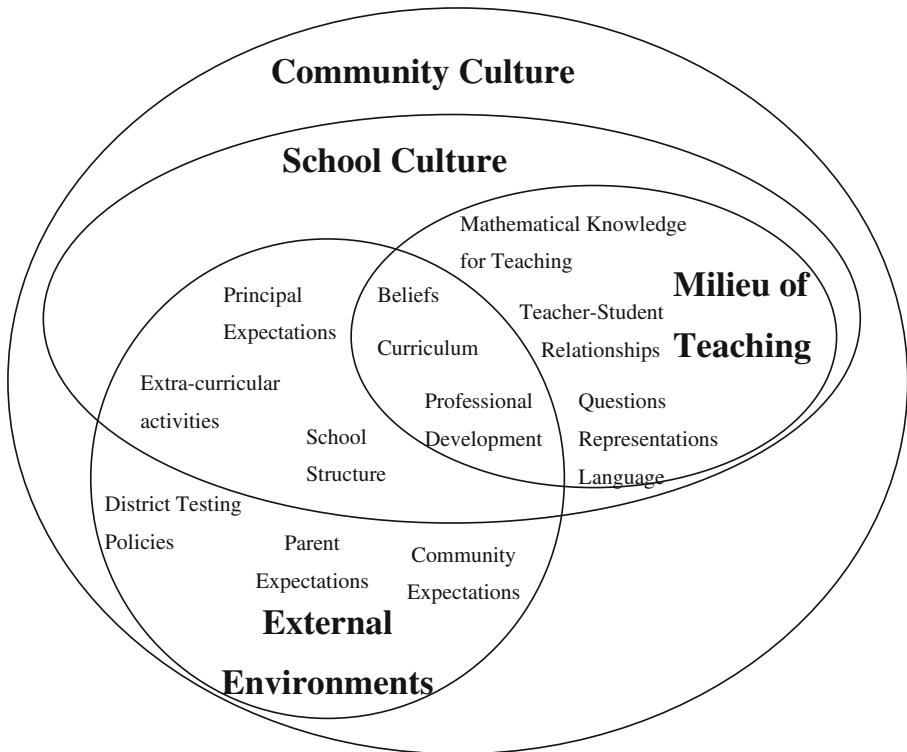
In looking across the three teachers' contexts, I quickly realized talking about "context" was overly simplistic (Chaiklin and Lave 1993). It is not enough to approach context as a checklist of features (such as textbook used, school structure, size of community) on which one can mark the presence or absence for each teacher, yet this is how most educational research treats context (Flores and Day 2006; Hadjioannou 2007). For these three teachers, knowing which textbook a teacher uses, if she teaches her students for multiple years, and if she participates in professional development are each insufficient to explain what is happening in her classroom.

In taking a closer look at context, I needed a careful definition: context in this study is taken to be the interrelated conditions in which something occurs (adapted from Jones 1997). I used a grounded theory approach (Glaser and Strauss 1967/2007) to code interview and field note data for contexts; Fig. 1 represents the codes that emerged from the data, using related literature (DuFour 2001; Flores and Day 2006; Hadjioannou 2007; Jones 1997; Shulman 1987) to organize relationships among contexts.

As I considered contexts surrounding each teacher's practice, I could see the situations were more complicated than the mere presence or absence of certain dimensions of context. Each teacher's context looked different, in part because of different aspects playing more prominent roles. Through interviews and observations, I identified the most important aspects of contexts for these teachers to be: school structure, curriculum, district testing policies, professional development (specifically, the M<sup>2</sup>), principal expectations, and parent and community expectations.

Contexts of teaching are multi-dimensional and multi-faceted. Milieus are the physical or social setting in which something occurs or develops. Environments are generally the aggregate of social and cultural conditions influencing the life of an individual or community. In this study, environments are taken to be the aggregate of conditions influencing the teaching practices of teachers.

All within the culture of the community, some of the environments surrounding teaching are also within the school culture: principal expectations, school structure, and extra-curricular activities. District testing policies and parent and community



**Fig. 1** Relationships among dimensions of context

expectations are part of the external community cultures that can be deeply connected to school cultures. Inside the milieu of teaching practices, one finds teacher-student relationships and mathematical knowledge for teaching—including teachers' choices and uses of questions, representations and precise mathematical language. The contexts of professional development, curriculum, and beliefs represent the blur between internal and external milieu.

### **Methodology and analytic framework**

This is a set of three case studies of teachers in the second cohort of the  $M^2$  Institute during their first year of participation. Two of the teachers taught in rural settings and the third taught in a city. The initial purpose of this research was to discover how teachers were using what they were learning in the  $M^2$  in their classroom practices. However, teachers' uses of contexts surrounding their teaching practices emerged as influential on the change process.

### **The Data**

I collected many forms of data related to this study from the three participating teachers and their students during the 2005–2006 school year. I conducted seven or

eight observations of each teacher's lessons throughout the year, and asked the teachers to videotape themselves an additional seven times; each observation included a pre- and post-observation conference. My fieldnotes reflect both the pre- and post-conferences as well as notes from my observations. I interviewed the three teachers three times each, conducted focus groups of their students twice, and interviewed their principals once. I transcribed all the interviews and videos (in the case of videos, also transcribing non-verbal actions). I also kept a personal journal, in which I recorded thoughts and observations about the study and my dual roles as researcher and graduate assistant for the M<sup>2</sup>. I kept selected copies of student classwork and teacher Institute work, as well as copies of teacher data from instruments administered as part of their participation in the M<sup>2</sup>. This study, in focusing on teacher change and uses of contexts, primarily draws upon the teacher data from observations, videotapes, and interviews. My claims about teachers' communities come from my interviews of the teachers, their principals, and focus groups of their students, as well as from my observations.

While not a limitation, something I wish to emphasize here is I collected data over teachers' first year of participation in a professional development program. It is to be expected teachers might make more and bigger changes during their second year of participation than their first (Fullan 1993), and more in the years that follow their participation in the Institute than while they are in the Institute. However, I deliberately chose to study teachers at the beginning of the program to better see the genesis of change. This study represents only one small slice of the teachers' stories. During teachers' second year in the M<sup>2</sup> Institute, they each designed and conducted action research projects based on changes each wanted to make in her teaching. However, data collection must be bounded (Stake 1995), and I chose to bound my data chronologically by a single year.<sup>1</sup>

## The analysis

There is a dearth of well-developed theories in mathematics education for researchers to draw upon (Silver and Herbst 2007). Shulman (1987) called for the development of coherent theoretical frameworks for the content knowledge needed for teaching. However, efforts have lacked coherence across subjects, and uneven progress has been made in different content areas (Ball et al. 2008; Niess 2005; Polly 2011; Voogt et al. 2012). In mathematics, the most coherent theory of knowledge needed for teaching is the theory of *mathematical knowledge for teaching* (Ball and Bass 2003; Ball et al. 2008). Mathematical knowledge for teaching focuses on teacher knowledge, and what teachers need to know in order to engage in the art of teaching mathematics. Mathematical knowledge for teaching seeks to organize what teachers need to know about mathematics, students, teaching, and curricula in order to help students learn mathematics effectively. Such knowledge also encompasses the types of questions teachers ask students (Boaler and Humphreys 2005).

The theory of mathematical knowledge for teaching was developed through investigating what teachers do as they teach (Ball et al. 2008). Ball and her colleagues

<sup>1</sup> For a research study of a Cohort 2 teacher in her second year, see Rolle (2008).

developed this grounded theory (Glaser and Strauss 1967/2007) with a “bottom-up” approach, beginning with teacher practice and focusing on the work of teaching. This theory is built on the premise general mathematical ability does not fully account for the knowledge and skills entailed in teaching mathematics (Hill et al. 2004).

The theory of mathematical knowledge for teaching includes categories of common content knowledge, horizon content knowledge, knowledge of content and students, knowledge of content and teaching, knowledge of content and curriculum, and specialized content knowledge. Specialized content knowledge is the mathematical knowledge and skill unique to teaching and involves uses of mathematics unlike those in other mathematical professions. Within specialized content knowledge, Ball et al. (2008) identify mathematical tasks of teaching, including,

responding to students’ ‘why’ questions, finding an example to make a specific mathematical point, recognizing what is involved in using a particular representation, linking representations to underlying ideas and to other representations, ...giving or evaluating mathematical explanations, choosing and developing usable definitions, using mathematical notation and language and critiquing its use, asking productive mathematical questions, and selecting representations for particular purposes. (p. 400)

These tasks of teaching require uses of mathematics unlike those of other professions.

I used the lens of mathematical knowledge of teaching to search for evidence of teacher change. However, the theory attempts to encompass the body of mathematical teaching practices, and this study needs a narrower focus. Thus, I analyzed a subset of the data (transcripts of five of the 23 videotaped lessons, including at least one tape from each of the three teachers’ classrooms), coding the transcripts to represent all domains of the model of mathematical knowledge for teaching and all subcategories of specialized content knowledge. I then examined the codes for both frequency and reliability of occurrences. I also examined dimensions of the analyses of other researchers in the area of analyzing teaching practice (Boaler and Humphreys 2005; Janvier 1987; Lampert and Blunk 1998).

The coding and resulting analysis led me to focus the specialized content knowledge related to questions, choosing and using representations, and precise mathematical language to focus the analysis of the rest of my data. Within the category of questions, I coded all instances of teachers and students asking and answering questions, using sub-categories for types of questions to delineate lower- and higher- order thinking questions (Boaler and Humphreys 2005), and also coding answers (who answered, type of answer). Within representations, I coded all instances along with the source and type of representation, and degree of connection to other representations. For precise mathematical language, I coded for oral and written terminology from both teachers and students, looking for both precision and lack of precision, as well as the critique of precision.

### **The cases: investigating teacher change**

This section focuses on investigating the first research question: How do teachers’ knowledge and practices change as a result of participating in a professional

development program? With her secondary mathematics endorsement, Ms. Thompson<sup>2</sup> had taken the most college-level mathematics courses before beginning her participation in the M<sup>2</sup> Institute. While Mrs. Zatechka and Ms. Lamb both have elementary endorsements, the simple variable of time since last math course taken gave Ms. Lamb (2 years) an advantage over Mrs. Zatechka (19 years) in this area. All three did well in their first year Institute math courses, earning high grades. Table 1 gives some demographic information about each teacher's school.

Ms. Thompson was in her fifth year teaching at Cather Middle School, and her tenth year overall. Her teaching assignment was eighth grade: teaching Math 8, algebra, differentiated (gifted) algebra, and math intervention. Math intervention was a second daily math class for students in Math 8 who were achieving below grade level.

Ms. Lamb was in her third year of teaching overall, all at Morris Elementary School. Ms. Lamb's assignment was to teach fourth, fifth, and sixth grade mathematics in the morning, and to help teach kindergarten in the afternoon. After the 2005–2006 school year, due to another teacher retiring, Ms. Lamb continued to teach math in the mornings, but then taught the science in the afternoons.

Mrs. Zatechka was in her first year teaching seventh through ninth grade mathematics, but her fourteenth year at Cherry River School, and seventeenth year teaching overall. She taught two sections each of seventh, eighth, and ninth grade mathematics, as well as one section of ninth grade physical education. Since Ms. Zatechka switched from being Cherry River's fourth through sixth grade math teacher to the seventh through ninth grade math teacher, Mrs. Zatechka was in her fourth consecutive year teaching the seventh graders I observed, as a "super" looping<sup>3</sup> situation.

When I asked teachers to videotape themselves teaching at least seven times during the school year, I specifically asked them to videotape lessons which represented a change of some type to their teaching practices. The following analyses are based specifically on the set of three to four lessons each teacher videotaped to represent change to her teaching practices, but generally on the body of data collected. The changes teachers identified take various forms, including teaching new mathematical topics and incorporating new pedagogies.

### **The case of Nora Thompson: effective explaining**

Ms. Thompson's teaching looked very typical for a U.S. classroom: reviewing previous homework, lecture over new material, then time for guided and independent practice (Stigler and Hiebert 1999). A typical day's lesson opened with a "warm up" (review) exercise for students to complete, then moved to correcting the previous night's homework by reading the answers aloud, followed by a lecture over new material which included some individual guided practice, and concluded with time for students to begin the new homework assignment, working individually. Due to

<sup>2</sup> All names are pseudonyms.

<sup>3</sup> Looping in this context means the teacher teaches the same students across multiple years. In Mrs. Zatechka's school, teachers typically looped with students in a subject (e.g., mathematics) across 3 years. By changing from teaching 4th–6th grades to 7th–9th grades, Mrs. Zatechka was thus in a 6-year loop with her mathematics students.

**Table 1** School demographics for case study teachers 2005–2006

	Carrie Lamb (6 <sup>th</sup> grade math)	Daria Zatechka (7 <sup>th</sup> grade math)	Nora Thompson (8 <sup>th</sup> grade math)
School	Morris Elementary School	Cherry River School	Cather Middle School
School Size	K-6 126 (capped at 20 per grade)	K-12 400 (two classes per grade level)	6 <sup>th</sup> –8 <sup>th</sup> 900 students (capacity 750)
School Structure	K-3 self-contained <sup>a</sup>  4 <sup>th</sup> –6 <sup>th</sup> departmentalized <sup>b</sup>	K-6 and 7–12 operate as separate schools sharing a common cafeteria and gym  K-3 self-contained; 4–6, 7–9, 10–12 departmentalized with one teacher per subject	6 <sup>th</sup> grade mainly self-contained  7 <sup>th</sup> & 8 <sup>th</sup> grades departmentalized (5 math teachers)  Students take core classes with their team (approx 150 students per team)
Student Population	Mostly white (98 %) Middle to high SES (but only 6 % free/reduced lunch) Low mobility (3 %)	Mostly white (98 %) Low to middle SES (31 % free/reduced lunch) Low mobility (5 %)	Mostly white (95 %) High SES (less than 7 % free/reduced lunch) Low mobility (4 %)
Location	Not in town; near a mid-size city (approx 20,000)	Small town (approx 850)	Large city (approx 250,000)

<sup>a</sup> Self-contained in this context means one teacher per grade level teaching all subjects

<sup>b</sup> Departmentalized in this context means one teacher teaches one or two subjects to multiple classes of students

her participation in the M<sup>2</sup>, Ms. Thompson experienced firsthand the power of working in collaborative groups on difficult mathematics problems. Thus, Ms. Thompson began incorporating more problem solving and group work into her eighth grade classes. While the group work was very teacher-directed, it represented a large change to her teaching practices, since she previously did not allow students to work together.

Ms. Thompson's videotapes of change in her teaching included a three-lesson sequence about surface area and volume. For these lessons, Ms. Thompson incorporated manipulatives and group work (neither of which she would normally use) and also attempted to teach the topics more conceptually than procedurally. Ms. Thompson introduced surface area by having students sum areas of the faces of various geometrical solids. On the second day, Ms. Thompson introduced some of the standard formulas for surface area, which led to apparent confusion among students. So, the third day, she returned to a more conceptual approach.

Over these three lessons, 85 % of the questions Ms. Thompson asked led students through a procedure or requested them to recall facts. Overall, Ms. Thompson herself answered 10 % of the questions she posed, and posed questions without pausing for



answers 20 % of the time. Her students grew accustomed to this pattern of rhetorical questions and Ms. Thompson answering her own questions. Students typically waited for Ms. Thompson to ask for volunteers before the same group of about six students would raise their hands to answer; the exception to this was when the question seemed easy, in which case about half the class would say the answer together. Ms. Thompson only called on volunteers to answer questions, and typically accepted brief answers without probing student understanding.

The following videotape excerpt is from this surface area lesson. A cube is drawn on the board (Fig. 2) to represent a rectangular prism, and Ms. Thompson is now leading students through a process to find the surface area of this polyhedron. Ms. Thompson's students were used to figures not always being drawn to scale on the board.

NT: What's the formula for area of a rectangle?

Several students: Base times height.

NT: Base times height. So the surface area of just the front is 12. Which other part has the same area? The back. The front and the back are the same. So I know the surface area of the front and the back is 24. That's only two of the six sides. Anyone know what to do next? I got you started now. What other parts of the box do we have?

Several students talk at once, calling out top, bottom, left, right.

NT: Top and bottom. Let's do the top next. The top of like a shoe box is what shape?

Several students: Rectangle.

NT: It's also a rectangle. What are the dimensions of the top, it's a little bit tougher? What are the dimensions of the top or the bottom?

Students call out various numbers, including six and four.

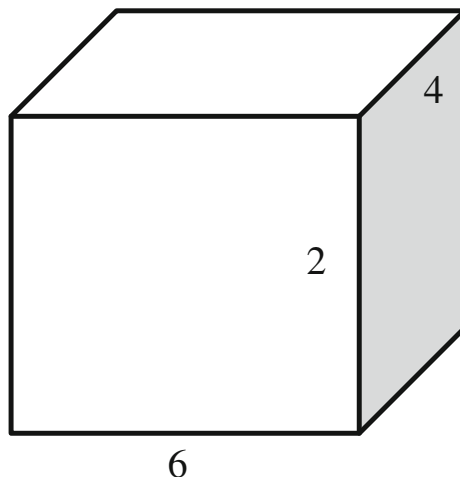
NT: Six and four... So what's the area of the top?

Several students: 24.

NT: 24. So what's the area of the bottom, them?

Several students: 24.

**Fig. 2** Rectangular prism for surface area example



NT: On a box, the top and bottom are the same, the front and back are the same.

What are we missing?

Student: Left and right.

NT: Good. The left and right of the box are also rectangles. What are their dimensions?

Students: Four. Two.

NT: Four by two. So what is the total area of the left side?

Many students: Eight.

NT: Eight. (Videotape April 10, 2006)

Ms. Thompson believed her eighth grade students needed concrete representations to build mathematical understanding. "I learned that most of the kids who are in that Math 8 group are pretty concrete thinkers and if they can see it or if they can think about it in a certain way, they can get it" (Interview 2006). This was evident in her repeated referral to a shoebox when discussing rectangular prisms and a soup can when discussing cylinders. She saw her participation in the PD as having expanded her repertoire of representations. Ms. Thompson now also thought more about common student misconceptions when planning lessons, and thought about how various well-chosen examples and representations could be used in the lesson to help counter these misconceptions. Ms. Thompson saw the use of representations as an area of her teaching experiencing ongoing change, as she tried to refine her existing lesson plans.

The use of precise language in Ms. Thompson's classroom was notable through its absence. Ms. Thompson focused on developing usable definitions with her students, and did not prompt students to use precise mathematical language. For instance, Ms. Thompson and her students talked about sides of a box rather than faces of a rectangular prism in the surface area lesson. There was a tension for Ms. Thompson between trying to create a safe environment in which students feel comfortable sharing their mathematical reasoning and trying to critique such reasoning. For instance, when Mr. Thompson asked, "What's surface area, then, do you think?" one student offered a volume definition. Ms. Thompson responded, "That is close but that would be volume also" (Videotape April 7, 2006). Since making student reasoning public was something new for Ms. Thompson, she tended to err on the side of creating a safe environment, and let pass some incorrect mathematics and mathematical language.

Generally, Ms. Thompson is a teacher who "owned" the knowledge in her classroom. Students looked to her to tell them when answers were right or wrong. When talking about her daily routine, Ms. Thompson mentioned, "I like having homework time during class so I can check to see how they're doing; if they have any questions they can ask me" (Interview 2005). Students knew when they had questions, the teacher was the one with the knowledge to answer the questions. When a student gave an answer, Ms. Thompson almost always repeated back correct answers, or called on another student to answer if the given answer was incorrect. When asked about teaching in the follow up interview (2007), Ms. Thompson replied that she sees herself "as a good explainer." The role of students was to listen to Ms. Thompson, thinking about information she was presenting, and participating in the guided practice problems; this pattern of lecture and guided practice predominated my observations, and were how Ms. Thompson described her typical day in my

interviews with her. Ms. Thompson said of her typical day, “Usually it’s the same routine every day unless we have a test” (Interview 2006).

Ms. Thompson felt her instruction was constrained by needing to follow her district’s objective card for Math 8 (Interview 2005, 2006); the objective card included a list of textbook lessons teachers must cover each semester. Ms. Thompson saw the objective card as non-negotiable. Thus, when the  $M^2$  instructors asked Ms. Thompson to conduct a Learning and Teaching Project, or when I asked Ms. Thompson to videotape lessons revealing changes she has made as a result of her Institute participation, Ms. Thompson saw these as additions to her already tight schedule. Ms. Thompson’s situation was similar to that of Cathy Swift in Peterson’s (1990) case, in which Swift was asked to do more and more in the same amount of time. Time pressures on teachers are very real; for Ms. Thompson, the district objective card magnified those time pressures. The pressures Ms. Thompson and Cathy Swift felt to maintain pacing preclude them from engaging students in the types of in-depth problem solving and exploration the National Council of Teachers of Mathematics (2000) would characterize as “reform” mathematics.

Yet, through all of this, Ms. Thompson did make changes to her teaching. During data collection, she was taking the first steps toward giving her students mathematical experiences closer to those she was experiencing through her participation in the  $M^2$ . While Ms. Thompson’s lecture-based, teacher-centered teaching at the beginning of data collection looked similar to that described by Stigler and Hiebert (1999) as typical of middle-school mathematics teachers in the United States, her teaching practices had begun to change. Ms. Thompson was paying closer attention to the types of questions she asked: this is a step toward making student reasoning more public. Ms. Thompson was choosing mathematical representations to better support student learning, and considering more carefully the mathematical implications of representation choices. Sustainable change is best created by changing small things, one at a time, in order to gradually construct larger changes (Hargreaves 2005).

### **The case of Carrie Lamb: meeting expectations**

Ms. Lamb’s teaching also looked very typical (Stigler and Hiebert 1999). Her principal was closely involved in choosing the same textbooks for core subjects for the four K-6 schools in the district, so that students would be on equal footing entering the 7–12 consolidated county school. With 126 students, Morris Elementary was the largest feeder school for the district’s consolidated county school (the other three schools had 12–62 students each). The school district recognized the chosen textbook series were a good match for the state standards and district assessments. Thus, Ms. Lamb was strongly encouraged to follow the textbook very closely as she taught math. It quickly became obvious, even after a single visit to Ms. Lamb’s classroom, that she and her students had internalized the textbook routine: everyone knew what to expect at all times, and students made transitions between sections of the lesson without teacher direction. I observed Ms. Lamb’s sixth grade math class. Since she taught math to the fourth, fifth, and sixth graders, I was seeing a class in their third year together with the same teacher (and seventh year together as a class using the textbooks, since usage began in kindergarten).

Unlike typical textbooks organized around conceptual units, the textbook series chosen by the district is one that takes all of the mathematical topics for a year then distributes and mixes related lessons across the year. Thus, Ms. Lamb videotaped four related but non-consecutive geometry lessons (area and perimeter of polygons) to represent change to her teaching. For Ms. Lamb, the biggest change she saw to her teaching related to her experiences in the  $M^2$  was that she no longer skipped the Investigations sections of the textbook, since she now felt more comfortable allowing students to tackle these less-routine problems. This comfort came from deeper mathematical knowledge; Ms. Lamb felt better prepared to answer students' questions. Prior to this year, she had skipped over all of the Investigation sections in part due to her mathematical insecurities and in part due to the added time the lessons took: it was quicker for her to tell students information than to have them work with manipulatives to construct solutions to problems.

Since Ms. Lamb followed the textbook routine so closely, she mainly asked questions to lead students through a procedure or have them recall a fact. Over these four geometry lessons, only 10 of 140 questions asked could be classified as something other than these low level questions. Yet, asking these types of questions fit the primary purpose of the textbook of having students reach automaticity in performing procedures and recalling facts. Thus, Ms. Lamb was following the intent of her textbook in asking students to recall facts and step through procedures.

The representations Ms. Lamb used were the ones suggested by the textbook. She did not deviate from the representations provided in the textbook for each lesson. The change Ms. Lamb made with representations was that she now did fewer demonstrations with manipulatives, instead allowing students to use the manipulatives directly during the lessons. Ms. Lamb credited this change to her participation in the  $M^2$ , as she learned how much more powerful manipulatives were for her own learning when she could use them herself, rather than watching an instructor demonstrate.

Missing from Ms. Lamb's representations was the embedded mathematics: she seemed to follow the suggested textbook representations without considering the mathematical appropriateness of those representations. An example of this was when Ms. Lamb proceeded with the textbook's example of a square with 4 in sides in demonstrating area and perimeter; in this case, the area is  $16 \text{ in}^2$  and the perimeter is 16 in. Ms. Lamb's explanation did not help students differentiate between area and perimeter very well: "[For] perimeter, I'm going to take four times four. To find the area, I'm going to take four squared" (Videotape March 22, 2006). Representations become more powerful when connected to each other. Since the textbook did not explicitly draw such connections among algebraic, geometric, and other representations, neither did Ms. Lamb.

The textbook did include precise mathematical language and (mostly) correct definitions. While Ms. Lamb did use mathematical terminology and expected her students to follow suit, such terminology was not always used correctly. When reading through a lesson in the textbook during class, Ms. Lamb usually asked students to paraphrase definitions provided. While this is an excellent strategy, Ms. Lamb often accepted incorrect or partially correct paraphrases. For instance, in this

rare example of Ms. Lamb probing a student's thinking, she accepts Natalee's incorrect paraphrase of the definition of polyhedra:

CL: So someone paraphrase that to me. What did Aaron just read? Tell me what that little paragraph was about?

Natalee: Prisms and pyramids are polyhedrons.

CL: Why?

Natalee: Because they do not have curved faces.

CL: What is special about polyhedron—what's a polyhedron? What's the definition?

Natalee: If every face is a solid figure, then it's a polygon. Polygons are solid figures.

CL: Good. (Videotape March 17, 2006)

It seemed Ms. Lamb either needed a deeper or more usable knowledge of mathematics (Ball and Bass 2003) in order to better evaluate and critique student language and definitions.

Ms. Lamb was meeting the expectations of her principal and the parents of her students in taking the textbook as her mathematics curriculum. Within Ms. Lamb's un-changing math routine, the lessons Ms. Lamb videotaped do represent change. As Cohen (1990) points out, "Changes that seem large to teachers who are in the midst of struggles to accommodate new ideas often seem modest or invisible to observers who scan practice for evidence that new policies have been implemented" (p. 312). For the first time, Ms. Lamb asked students to complete textbook activities such as cutting out a pair of triangles and arranging them to form a parallelogram and cutting out nets of Platonic solids to glue together. In the past, Ms. Lamb had not involved students directly in such activities, instead demonstrating the activity herself, simply telling students about an activity, or skipping the activity completely. Additionally, in two instances during this school year, Ms. Lamb departed from her daily requirements of students to work silently and independently, and allowed them to work in small groups. Ms. Lamb's routines were slowly beginning to change, based on her experiences in the  $M^2$ , and quite possibly, more are to come.

### **The case of Daria Zatechka: a stance of inquiry**

Ms. Zatechka's teaching looked the least typical (Stigler and Hiebert 1999) of the three teachers involved in this study. During class discussions of mathematical concepts, students often talked directly to each other, and did not speak only in response to teacher questions. A common practice was for students to pass a white-board marker around, as one student would begin to share some reasoning at the board, then would pass the marker on for another student to show a different method or critique or expand on the first method. Students were actively involved in learning mathematics in Mrs. Zatechka's classroom, and worked frequently in cooperative groups.

Mrs. Zatechka's school also used the same textbooks as Ms. Lamb's district. However, Mrs. Zatechka saw a textbook as one of many curricular resources she had for teaching mathematics. She also looped with students for 3 years, and saw this opportunity as one of having 3 years to get students from a seventh grade level to a ninth grade

level. She did not worry about “covering” topics at certain times, and varied the pacing of lessons based on her understanding of her students’ strengths and weaknesses.

The four lessons Mrs. Zatechka chose to videotape were from a three-week unit related to patterns and functions, and represented new mathematical material not normally a part of the seventh grade curriculum. Mrs. Zatechka chose to add this unit after being asked to have her eighth grade students work with the Fibonacci sequence as part of a Learning and Teaching Project from her current semester’s  $M^2$  course. Mrs. Zatechka saw her eighth graders struggle with this project, and realized that nothing in the previous 8 years of mathematics had prepared the students for this type of non-routine problem. Thus, she decided to add a unit to her seventh grade curriculum to have students learn how to represent patterns in multiple ways, and to connect those representations to each other.

I observed Mrs. Zatechka and her students engage in a dialog of questions. Mrs. Zatechka almost never directly answered student questions, instead posing a question back to students, to help them reason out their own answers. In this way, questions served a much different purpose in Mrs. Zatechka’s classroom than Ms. Lamb or Ms. Thompson leading students through procedures. Seventy percent of the questions Mrs. Zatechka asked were higher-order questions, often probing student thinking and orienting students to problem solving. Mrs. Zatechka also had a high tolerance for public incorrect answers as part of class discussions. When a student provided some reasoning, Mrs. Zatechka always asked other students to agree or disagree; in this fashion, students were expected to evaluate each others’ reasoning, and Mrs. Zatechka rarely had to correct errors once students engaged in a critique.

A short example of this type of exchange is seen in the following excerpt of a lesson in which students were trying to develop an algebraic representation of the pattern in Fig. 3. After four students put three different formulas on the board, the class has a discussion that leads to the determination the last formula is correct (the  $n^{\text{th}}$  figure in the pattern needs  $6n+4$  one-unit stamps to cover its surface area).

DZ: Is that the only formula? Is there another way to do it? Gavin: Start with ten and then plus six and plus six and plus six.

David:  $n4$  plus six.

DZ: Will that work?

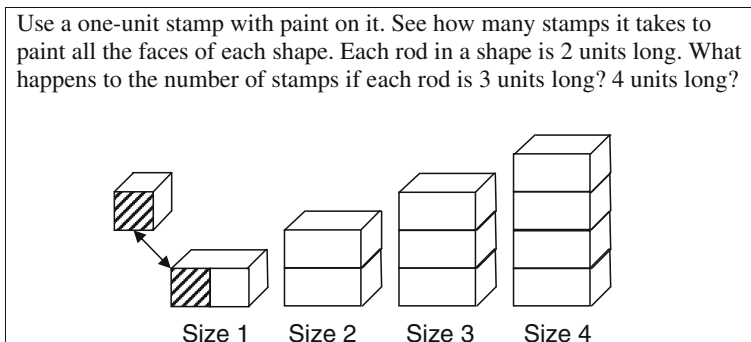


Fig. 3 Rod Stamping Pattern (Friel et al. 2001, p. 10)

There is a pause as students talk among themselves.

Gavin: No, but it worked for some of them.

DZ: And you need to say what  $n$  is.

David:  $n$  is the size number.

There is a little further conversation before the class decides Anna's formula is the only one that will work for all of the numbers. (Fieldnotes April 26, 2006)

Mrs. Zatechka saw a big change to her teaching based on her  $M^2$  experiences in the way she now used questions to direct student thinking. Since she had two sections of seventh grade math, she took time after the first section to reflect on what her students learned that day, and refined her planned questions to help the second section learn even more. Mrs. Zatechka's primary goal in teaching is to teach students to think; her questioning techniques align well with this goal.

Although Mrs. Zatechka did plan which representations may be most appropriate for a given mathematical topic, nearly all representations in Mrs. Zatechka's mathematics lessons are generated by students. While students worked in groups, Mrs. Zatechka circulated and made decisions of which groups would share their reasoning based in part of her desire for certain representations to be shared. Mrs. Zatechka credited her  $M^2$  experience with helping her better evaluate and choose which representations to make public and discuss. In some cases, Mrs. Zatechka picked representations which embodied a misconception, in order to have a class discussion about that misconception.

Mrs. Zatechka's classroom featured frequent usage of precise mathematical language by Mrs. Zatechka and her students. Mrs. Zatechka critiqued students' language and required them to look up and explain definitions of mathematical terms under discussion. The students knew no "math slang" was allowed in Mrs. Zatechka's room; any time students could not provide an accurate definition for a term used, they knew Mrs. Zatechka would make them turn to their textbook glossary, read the definition, then explain what the term meant. While Ms. Thompson did not publicly critique student language in an effort to build a safe environment, Mrs. Zatechka's seventh graders had pretty much known each other from birth, so did not need to engage in community-building activities during mathematics. The environment was safe and supportive, and errors were seen as useful learning opportunities.

Overall, Mrs. Zatechka engaged in a high level of reflection-in-action (Schön 1987); she constantly asked herself questions designed to increase student learning. This stance of inquiry (Cochran-Smith and Lytle 2009) enabled Mrs. Zatechka to reflect on both her practices and student reasoning in order to support the development of student conceptual understanding of mathematics. Tubbs (2000) discussed some of the paradoxes teachers may encounter in trying to become reflective practitioners. He notes in order to have reflection move teachers in the direction of change, teachers need to "accept uncertainty" (p. 174). Mrs. Zatechka did seem to accept uncertainty. She was willing to engage in mathematical content with students when her own understanding was fragile (Smith 2008). Closely related to uncertainty is the classroom locus of control. In typical classrooms (including Ms. Thompson's and Ms. Lamb's), the teacher "owns" the knowledge and dispenses knowledge to students (Goldsmith and Schifter 1997). However, as the focus in a classroom moves toward student reasoning, the locus of control moves toward a joint dialog between the

teacher and students (Thompson 1984). Sharing responsibility for the mathematics with students is a highly uncertain proposition, but one which can lead to greater student learning (Goldsmith and Schifter 1997).

### Unpacking complexities of context and uses of context

This section explores the research question: How do teachers' contexts and uses of those contexts influence the nature and extent of these changes? While these cases of teaching and teacher change are unsurprising to those familiar with related literature (e.g. Ried and Zack 2010a, b), I was very intrigued to see the classrooms that looked the most similar were those with widely differing contexts. Why did classes look similar when they were situated in widely differing communities with very different school structures? How was it two teachers using the same textbooks series, in similar communities with similar school structures, had classrooms that looked so different? How could one teacher see applications of her  $M^2$  experiences permeating her teaching, while another saw nearly no connections? It seemed teachers' beliefs about teaching, learning and students interacted in complex ways with various contexts, leading to differing uses of those contexts.

The small-school structure of both Cherry River and Morris foster different types of student relationships than does the larger Cather. While all three schools have low mobility rates (at most half the state average), Cather students do not experience the same opportunities for getting to know their peers. Morris and Cherry River students are each in the same classes beginning in kindergarten. Although there are two classes per grade level in Cherry River, by the time I observed students in seventh grade, they all knew each other very well. Eighth graders at Cather Middle School certainly have close friends within their classes, but do not know everyone in their class as well as the students in smaller schools do. Student community seemed important to mathematical discourse, and critiquing mathematical language, as well as the productive use of student errors.

For Ms. Thompson, her district's curriculum (objective cards) and testing policies had the biggest influence on her teaching. She felt driven by the district's objective cards and by the district's eighth grade semester criterion-referenced tests. Ms. Thompson's district and principal placed her in several leadership roles due to her participation in the  $M^2$  Institute. Principal expectations played a moderate role in Ms. Thompson's teaching practices, as she strove to make changes to her teaching commensurate to the expectations placed on her. The changes Ms. Thompson was making were all related to her  $M^2$  experience, thus it also played a moderate role as in influencing context to Ms. Thompson's teaching practices. Ms. Thompson's school structure was the third context to have a moderate influence on her teaching practices, as she planned with the other eighth grade teacher and led mathematics department meetings.

Probably because Ms. Thompson taught in an urban setting, community expectations were largely invisible as a context for her teaching practices. While Ms. Thompson knows her students' parents expect students to do well on district assessments, Ms. Thompson does not believe this leads her to teach any differently than she otherwise would (Interview 2005).



The biggest contextual influence on Ms. Lamb's teaching practices, dwarfing all other influences, was her use of the district textbook as her mathematics curriculum. Following the textbook accounted for most instructional decisions Ms. Lamb made on a daily basis. Morris school structure had Ms. Lamb loop with her students, providing Ms. Lamb with the opportunity to get to know her students both personally and mathematically over time. However, Ms. Lamb did not appear to explicitly use these relationships to plan her mathematics instruction for these specific learners, choosing instead to follow the scope and sequence of her textbooks, assigning all problems from each lesson. She mentioned to me the fifth grade textbook is almost all a review of fourth grade mathematics, but material is new again in sixth grade. Rather than re-organize how she presents mathematical topics, she noted instead her students struggle in fourth grade, breeze through fifth grade, then struggle again in sixth grade (Fieldnotes October 17, 2005). Thus, while Ms. Lamb knew a great deal about her sixth graders' strengths and weaknesses in mathematics class, she did not visibly use this information to alter how she approached teaching from the textbook.

Contexts of principal and community expectations are fairly invisible to one observing most classrooms, though they may be a strong influence in why teachers teach in particular ways. The influence became more visible for Ms. Lamb when she changed an aspect of her teaching.<sup>4</sup> One factor in the way this influence became visible was that Ms. Lamb was a community outsider; she did not live in the school district, and this was only her third year teaching at Morris. Test scores were high for fourth graders on the state standards, and Morris students did well when they moved on to Beattie Consolidated Junior/Senior High. In the face of these successes, neither Ms. Lamb's principal nor the community saw a need for change. This attitude is also present in Swidler's (2004) case of a teacher whose recitation style of teaching drew high praise from the community for its effectiveness in achieving high student test scores. Fullan (1993, 2001) has found repeatedly change is unlikely to be sustained when the community is satisfied with the status quo. Thus, expectations teachers continue in the same paths that have led to success did form an important context for Ms. Lamb's teaching.

For Ms. Lamb, the contexts which played the smallest roles were the district testing policies and her participation in the  $M^2$ . The influence of district testing policies was invisible in observing Ms. Lamb teach sixth grade, because her district at the time assessed students in fourth, eighth, and twelfth grades. Thus, Ms. Lamb did not see any influence of district testing on her sixth grade teaching, but only on her fourth grade teaching. The eighth grade test was not "close" enough for Ms. Lamb to consider its influence on teaching sixth grade.

Ms. Lamb's PD experience played a very small role as a context for her teaching practices because she saw little relevance between what she was learning as a student and her role as a teacher teaching sixth grade math. This attitude was likely due to several factors. The mathematics Ms. Lamb was learning did not neatly fit into any textbook topic increments, so she did not see a place for it in her curriculum. Ms. Lamb found the mathematics she was learning though the  $M^2$  to be incredibly difficult and challenging;

<sup>4</sup> During the 2006–2007 school year, when Ms. Lamb implemented an action research project in her classroom as part of the requirements of the  $M^2$ , she was met with vocal parent protests over the changes she tried to make to homework.

it is reasonable for a teacher to believe mathematics with which she struggles is too difficult to teach to sixth graders. The  $M^2$  vision of teaching students to develop the habits of mind of a mathematical thinker does not align well with the textbook goal of developing procedural fluency to the point of automaticity in students. Thus, the mismatch between the  $M^2$  and Ms. Lamb's adherence to the textbook led to the  $M^2$  playing a very small influence as a contextual factor in Ms. Lamb's teaching practices.

In looking at Mrs. Zatechka's teaching contexts, the dimension of context with the biggest influence was her school structure, including looping and study hall. Mrs. Zatechka knew her students both from a mathematical and personal perspective. Mrs. Zatechka used looping to inform her instruction: she skipped over some textbook lessons, spent more time on others, and supplemented with other curricular materials based on her knowledge of what her students knew mathematically. Additionally, Mrs. Zatechka was seen as a member of the Cherry River community (although she lived in an adjacent town), so she knew her students' families.

Mrs. Zatechka had an open period immediately after the period of seventh grade math I observed. During each of my seven observations, at least three students (usually more) dropped by during Mrs. Zatechka's open period, either to talk about sports, to ask a question about math, or just to hang out. Students at Cherry River had a study hall, but apparently were allowed to leave study hall to go ask other teachers questions. It was during these times I was able to see the close relationships Mrs. Zatechka has formed with students. Most students who would drop by were high school students, who no longer had Mrs. Zatechka as their math teacher, but still came back daily to talk with her. Mrs. Zatechka elaborated on students' coming into her classroom during the times she was not teaching:

Because they come in and they want somebody to talk to or ask questions or it's about sports. You know, so I think that's way different in a small school, and you're expected to be there for that kind of thing. You just do it. You know. I could probably lock my door maybe in a bigger school and get away with it. But they rely on you for a lot more things than just a teacher, I think, in a smaller place. (Interview 2006)

Mrs. Zatechka's personal relationships with students were a large part of the context of her teaching. The students knew she cared about their lives in ways extending far beyond the walls of the mathematics classroom. Such teacher interest and care typically translate into highly motivated and achieving students (Deci and Ryan 2000).

After school structure, curriculum and the  $M^2$  Institute played moderately influencing roles on Mrs. Zatechka's teaching practices. Mrs. Zatechka was a teacher who "owned" her curriculum and saw curriculum longitudinally. She saw what she learned in the  $M^2$  Institute as greatly increasing her mathematical knowledge, and thus better supporting her use of textbooks and other materials in developing a curriculum to meet her students' mathematical needs.

The contextual factors with the smallest influence were district testing policies and principal and community expectations. The minimally visible role of principal and community expectations was likely due to Mrs. Zatechka being a community insider: she shared those same expectations, so it was not obvious where her expectations

ended and community or principal expectations began. Since Mrs. Zatechka believed her main job as a classroom teacher was to teach students to think, she did not see a large role for district testing policies to influence her teaching practices: what was important was teaching the mathematics and teaching students to think.

### Conclusions: looking beyond context

This analysis suggests one cannot simply speak of a teacher's "context." Context is not a static entity, but rather is composed of multiple aspects playing differing roles to different degrees for different teachers. Contexts are multi-layered and complex; different teachers may well use the same context in differing fashions. In these three cases, the multi-dimensional roles of context look different for each of three teachers. These cases specifically illuminate the complexities in teachers' uses of school structure, professional development, curriculum, testing policies, principal expectations, and community expectations.

Teachers' uses of contexts are grounded in a complex interaction among beliefs, practices and resources. Those beliefs include beliefs about students, teaching, learning, and the nature of mathematics and knowing. Individual features of a teacher's surroundings are not singularly predictive of teaching practices. In investigating teaching practices, one must inquire about the contexts surrounding teaching, as well as how teachers are using such contexts. While teacher professional development is often designed to cause changes in teaching practices, how teachers choose to use professional development experiences is key to the impact of the professional development on a teacher's practices.

While much of this analysis is specific to mathematics, some teaching practices transcend mathematics. As teachers begin to make changes to their teaching practices, the early stages of change are often invisible to observers; teachers may begin to think more about anticipating student misconceptions, or make more deliberate choices about particular examples or representations. Such invisible changes are changes nonetheless, and researchers need to carefully probe for such changes when investigating teacher change.

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