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ORIGINAL ARTICLE

The effects of Polya's heuristic and diary writing on children's problem solving

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Abstract This paper presents the results of a study that aimed at increasing students' problem-solving skills. Polya's (1985) heuristic for problem solving was used and students were required to articulate their thought processes through the use of a structured diary. The diary prompted students to answer questions designed to engage them in the phases of Polya's (1985) heuristic. While it appeared as though most students did not internalise the diary questions, further analysis of students' responses indicated that most students showed improvement in their solution strategies. These results indicate that having students write about their thinking may be beneficial for developing their problem-solving skills.

Keywords Problem solving · Polya · Heuristic · Elementary · Diary writing

Teaching mathematics through problem solving has long been suggested as a way to help all students learn (National Council of Teachers of Mathematics 2000). It can allow for deep understanding to develop (Hiebert et al. 1997; Lesh and Zawojewski 2007; NCTM 2000; Schoenfeld 1987), connects school mathematics to that of the "real world" (Romberg 1994) and provides opportunities for all students to become engaged in and successful at mathematics (Hiebert et al. 1997; Lambdin 2003; National Research Council 1989). These potential benefits of problem solving and NCTM's (2000) insistence that problem solving is both an essential skill and means of learning mathematics led teachers and researchers alike to seek ways to support students' problem-solving skills. George Polya (1985), through his four phases, provides one heuristic for solving problems that may help students become more successful problem solvers. This paper presents the results of a study that attempted

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to increase students' problem-solving skills by having them follow Polya's phases and keep a diary about the process.

Review of the literature

Problem solving

The term *problem solving* is often interpreted differently from one person to the next. A problem, by its very nature, causes confusion and uncertainty (Lambdin 2003). Rather than being solvable by a prescribed, often recently taught method or rule, problem-solving situations have no clear solution method (NCTM 2000; Van de Walle 2003). Thus, problem solving in a mathematics classroom can be described as a situation in which students struggle to find a solution to a given question (Hiebert 2003). Without a clear path, students must engage in "iterative cycles of expressing, testing and revising mathematical interpretations" (Lesh and Zawojewski 2007, p. 782). A well-chosen problem should be challenging yet attainable (Hiebert 2003) and designed to allow students in the same class to develop different solution strategies (Roberts and Tayeh 2007). Developing these different solution strategies is one of the main goals of problem solving (Hiebert 2003; Hiebert et al. 1997; Polya 1985; Schoenfeld 1987). As such, teaching mathematics through problem solving does not end with the student stating their solution. In traditional mathematics classrooms, students are not typically encouraged to reflect on the problem solving strategies they have used; instead, expectations are that they will finish a problem, check its correctness and move on to the next problem or task. In contrast, a major part of the teaching in a problem-solving situation focusses on students explaining and justifying their solution strategies (Lesh and Zawojewski 2007; Polya 1985; Schoenfeld 1987). This reflective part of the problem-solving process is crucial. Discussing solutions and how they have arrived at them allows students to explain their thinking, learn from other students, and develop better, more efficient methods for solving problems (Hiebert 2003; Schoenfeld 1987). Additionally, being asked to explain *why* a particular strategy was chosen allows students to develop "systems for interpreting problem situations" (Lesh and Zawojewski 2007, p. 768), which may be more important than the development of ways to implement solution strategies. Therefore, in a problem-solving situation, the teacher's role is non-traditional. In a traditional setting, the teacher tends to instruct students on a particular method or rule, present examples of how to solve problems using that method, and then provide students with many similar problems to practice. In contrast, when engaging students in problem solving, the teacher might begin by posing a question, allow students to develop their own methods for finding the solution, and conclude with a class discussion about those methods, their validity, and how those methods could be improved.

The value of problem solving

The benefits of teaching mathematics through problem solving are many. For example, problem solving aids in connecting the mathematics learned to the real world, improves understanding, and can help to make mathematics accessible to struggling learners.

Real-world mathematics There is a vast and growing need for students to be able to use mathematics in their daily lives and in their future careers (Friedman 2005; NCTM 2000). However, in U.S. society, which is focussed on accountability and scores, there is a large discrepancy between the “test-driven curriculum materials and the kind of understanding and abilities that are needed for success beyond school” (Lesh and Zawojewski 2007, p. 764). The mathematics and thinking taught in the traditional classroom are not the type of mathematics and thinking required for success outside of school (Lesh and Zawojewski 2007). But what good is learned mathematics if it cannot be applied (Lambdin 2003)? Additionally, our increasingly globalised society needs citizens who are able to solve complex problems, yet U.S. schools are not adequately preparing students with the skills they need to do so (Friedman 2005). Learning mathematics through problem solving may afford a solution to this dilemma. According to Polya (1985), problem solving serves two purposes. The first, of course, is to solve a particular mathematics problem. The second purpose, however, is to develop students’ thinking and abilities so that they may solve future problems on their own, including those that they may encounter outside of school. In a review of relevant literature, Lesh and Zawojewski (2007) conclude that problem solving makes it possible for thinking to be developed. Further, careful choosing of problems can allow mathematics to be connected to other subjects and students’ daily lives (Romberg 1994).

Increased understanding One of the greatest arguments for teaching mathematics through problem solving is that it aids in understanding (Hiebert et al. 1997; Lesh and Zawojewski 2007; NCTM 2000; Schoenfeld 1987). NCTM (2000) states, “problem solving is a hallmark of mathematical activity and a major means of developing mathematical knowledge” (p. 116). Through working with peers to solve problems and discussing solution strategies, students’ understanding of the mathematics involved is developed (Hiebert et al. 1997; NCTM 2000; Romberg 1994), and they tend to perform better on measures of mathematics achievement (Charles and Lester 1984; Dees 1991; Ginsburg-Block and Fantuzzo 1998). Consequently, understanding and problem solving are intimately intertwined, and should thus be the primary goals of mathematics education (Lambdin 2003).

Accessibility Problem solving, by its very nature, requires students to struggle. It is a struggle, however, that is structured by the teacher in such a way as to be beneficial to students, rather than frustrating them to the point of despair (Hiebert 2003). Students expected to engage in problem solving on a regular basis “view making sense of mathematics as a challenge rather than see difficult problems as a signal to give up and consider oneself a failure” (Lambdin 2003, p. 11). Some authors (NCTM 2000; Van de Walle 2003; Van de Walle and Lovin 2006) thus argue that problem solving can provide for students who were once unsuccessful at mathematics a reason to persist rather than shut down, and may allow every student to contribute and build upon their existing knowledge, making mathematics accessible to all types of learners.

In addition to helping make mathematics understandable, accessible, and relevant, problem solving is believed to optimise learning and allow mathematics to become enjoyable for students (Van de Walle 2003). It may also aid in memory and recall, enhance transfer, support positive and productive attitudes and beliefs about mathematics, and encourage the development of autonomous learners (Hiebert et al. 1997; Lambdin 2003; NRC 1989; Van de Walle et al. 2010). Research indicates that problem solving increases student motivation (Charles and Lester 1984; Ginsburg-Block and Fantuzzo 1998) and achievement (Boaler 1998, 2002; Charles and Lester 1984; Ginsburg-Block and Fantuzzo 1998; Silver et al. 1995; Verschaffel et al. 1999). As will be described in more detail in a later section, this study seeks to explore some of the proposed benefits of engaging students in problem solving, particularly, whether engaging students in problem solving coupled with diary writing affects their ability to solve other problems.

Solving problems using Polya's heuristic

Having established the "What?" and "Why?" of problem solving, the question now becomes, "How?". How do mathematics teachers engage their students in problem solving, and how do they improve students' problem-solving skills? George Polya (1985) suggests one answer to that question in *How to Solve It*, first published in 1945, based on his experiences as both a mathematician and mathematics educator. Polya suggests that there are four phases through which all students must progress in order to successfully solve a problem: (a) understanding the problem; (b) devising a plan; (c) carrying out the plan; and (d) looking back. These four phases (described in more detail below) are not intended to be part of a linear, step-by-step process. Instead, they are part of a dynamic process during which problem solvers cycle back and forth between phases (Lesh and Zawojewski 2007; Polya 1985). Polya's work is not based on formal research, but stems from his experiences as a former student, problem solver, mathematician, and teacher. Even so, *How to Solve it* is a common text used in teacher education courses about problem solving (e.g., Suh 2005), is well known and respected in the mathematics education community, and serves as the foundation for much of the research conducted on problem solving (Schoenfeld 1992).

In a review of relevant literature, Lesh and Zawojewski (2007) argue that Polya's problem-solving process should not be treated as a list of strategies to be used when students are stuck on a problem, as is often done. Instead, "the heuristics are intended to help students go *beyond* current ways of thinking about a problem…Polya's heuristics can be thought of as providing a language to help problem solvers think back about their problem-solving experiences" (Lesh and Zawojewski 2007, pp. 769–770). The heuristics thus allow students to describe their processes, reflect on them, and eventually develop flexible thinking and skills that can be drawn upon in future problem-solving situations. Polya's heuristics are for helping students develop their thinking and reflecting abilities and to aid in their interpretations of problems, and should thus be used throughout the problem-solving process (Lesh and Zawojewski 2007). If properly engaged in all four phases, students' problem-solving skills may benefit (Polya 1985), and instances of carelessness and misunderstandings may decrease (Jacobbe 2008). Despite these findings, there is little research that specifically examines the effects of engaging students in the four phases of Polya’s heuristics. This study seeks to add to the literature on Polya’s problem-solving process. First, a description of each phase is warranted. While they are presented linearly for the purposes of clarity, the authors remind the reader that the expert problem solver typically cycles back and forth between these phases as they work.

Understanding the problem The first phase, understanding the problem, begins with motivating students. Understanding a problem is not enough on its own if students do not care to solve it. Thus, Polya (1985) suggests the chosen problem should be challenging yet attainable, natural, interesting, and relevant to students’ lives.

This first phase also involves the student fully comprehending both the given information in a problem and the ultimate goal (Lesh and Zawojewski 2007). Unfortunately, this phase is often neglected, which may lead to difficulties for students. Skilled problem solvers are persistent, in that they may not understand a problem immediately, but will read a problem carefully or ask questions until they understand it completely (NCTM 2000).

Devising and carrying out the plan According to Polya (1985), fully understanding a problem and having a desire to solve it will naturally lead to the development and implementation of a plan. It should be clear to both the teacher and student, before the student begins implementing their plan, that the plan, or strategy, has the potential to lead to the solution. As one works, however, it may become apparent that the chosen strategy needs some revision or needs to be discarded entirely. Carrying out the plan thus includes computation as well as pausing after each step to check one’s work and ensuring that the chosen plan is still the best choice (Polya 1985). This may mean students have to go back periodically and attempt a different strategy before reaching the final solution.

Looking back The final phase of Polya’s heuristic is often the most neglected of all (Jacobbe 2008). Looking back on a solution means examining it, checking that the result makes sense, finding an alternative solution strategy, and applying the solution to a different problem (Polya 1985). This phase requires students to consolidate their knowledge through reflection on their work and discussion about their solution, and occurs both at the end of the problem-solving process as well as throughout. Reflecting on the process is a fundamental part of developing students’ problem-solving abilities (Polya 1985; Schoenfeld 1987), and requires students to be able to monitor their work and reflect on the conclusions reached and the strategies used.

It is possible that these phases are not as absolute as Polya (1985) describes. For instance, it may be possible that a student has only partial understanding of a problem, but as he or she begins to work, he or she comes to understand it more fully. For example, a child may focus on one factor and as he or she proceeds, may realise that he or she needs to factor in other information. Additionally, it may not be clear whether a plan will work until one begins to solve the problem, at which point one may decide to modify that plan. Schoenfeld (1987) supports this last point in his classic article about metacognition, in which he argues for the importance of understanding a problem, planning, and reflection. He describes a study in which he analysed videotapes of students and a mathematician solving problems. He found that students did not take time to understand the problem, devise a strategy, and pause to reflect upon and revise that strategy as they worked. Instead, they quickly chose one (incorrect and cumbersome) strategy and continued with it for over 20 min, unable to find the solution. This is what Schoenfeld calls a “wild goose chase” and says can lead to “an extremely high failure rate” (Schoenfeld 1987, p. 193). On the other hand, the mathematician devised many plans for solving his given task, rejected some before even starting them, and rejected many others within a few moments of beginning them. Unlike the students, the mathematician spent much more of his time *thinking* rather than working on the problem and was successful in solving it. This study highlights the importance of fully understanding a problem, devising a plan, and reflecting and revising that plan as one works; without doing so, the students were unsuccessful in their problem-solving attempts. The metacognitive skills (e.g., knowledge about thought processes and self-regulation) required to engage in these phases are thus crucial for problem-solving success (Schoenfeld 1987). Teachers therefore need to help students develop their metacognitive skills in order for them to become better problem solvers. The following section details research that examines how to support students to improve their metacognitive and problem-solving skills.

Supporting problem solving and metacognition

Interventions aimed at improving students’ metacognitive skills, and specifically their ability to self-monitor and self-regulate, can improve problem-solving skills and strategy use (Cardelle-Elawar 1992; Desoete et al. 2003; Montague 2007). Consider, for example, two studies by Kramarski and Mizrachi (2006a, b). In both studies, seventh-grade Israeli students in the treatment group engaged in online discussions with peers while receiving metacognitive guidance. This guidance entailed a series of self-questions printed on index cards and provided to students, which they were prompted to answer to themselves while they worked and as they interacted with peers in the online setting. The questions focussed on:

a. Problem comprehension (e.g., What is the problem about? What is the question?);
b. Connecting to previous tasks (e.g., How is this problem different from what you already solved?);
c. Strategy use (e.g., What strategy can be used to solve this task?); and
d. Reflection (e.g., Does the solution make sense? What difficulties/feelings do I have? Kramarski and Mizrachi 2006a, b).

While Polya was not referenced in these two studies, it is interesting to note that these question types correspond to his heuristic for problem solving: the first and second relate to understanding the problem, the second and third correspond to devising a plan, and the fourth relates to the looking back phase. Those students who received the metacognitive instruction performed better on measures of computational skills, problem solving, and mathematical reasoning and used more problem-solving strategies than their control-group peers (Kramarski and Mizrachi 2006a). Additionally, they demonstrated improvements in mathematical reasoning, self-regulation strategy use, and mathematical abilities (Kramarski and Mizrachi 2006b).

One issue that arises with the Kramarski and Mizrachi's (2006a, b) studies is that students were not required to explicitly answer the question prompts they were given. It is unclear whether students answered the questions at all and, if they did, *how* they answered them, leading to concerns about whether the results were caused by the metacognitive self-questioning, the online discussions, or some other unidentified factor. One solution to this would have been to ask students to write their answers to the self-questions as they answered them. Such writing would have provided the researchers with a written record of the students' thinking, which can be useful for the purposes of data analysis (Schmitz and Weise 2006).

Benefits of writing for mathematical understanding

Research indicates that writing holds various benefits for students. In fact, writing, and specifically journaling, may benefit learners in terms of intellectual growth and development, and self-expression, and may even increase their abilities to reflect and think critically (Hiemstra 2001). Unfortunately, the research on the subject of journaling and mathematics is somewhat sparse and inconclusive (Herrick 2005). One study examined the effects of four training conditions (self-regulation training, problem-solving training, combined self-regulation and problem-solving training, and a control group) on students' self-regulatory skills and problem-solving ability (Perels et al. 2005). In this study of 249 students (ages 13–15), two subgroups in each training condition were asked to answer questions in a diary before and after their homework for 6 weeks. The authors (Perels et al. 2005) found that self-regulatory skills were optimised by the combined treatment, though the other two treatment groups also showed improvement. Additionally, students' problem-solving skills improved following all three treatments. However, while the authors hypothesised that the reflection required in diary writing would support the training they received, the authors did not address this hypothesis in their results or otherwise provide any details regarding the effects of the diary homework assignment. This leaves the reader wondering what role (if any) the diaries played in supporting the researchers' goals of improved self-regulatory and problem-solving competence.

In another study that examined the effect of writing in mathematics, Jurdak and Zein (1998) examined how journaling affected attitudes toward mathematics and achievement (including conceptual understanding, procedural knowledge, problem solving, mathematics school achievement, and mathematical communication). This study of 11–13-year old Lebanese students required treatment group students to write for up to 10 min at the end of three class periods a week for 12 weeks. The authors found that journal writing had a positive impact on some measures of mathematics achievement, but not on problem-solving ability.

These two studies illustrate the ambiguity surrounding the effects of journal writing on mathematics achievement and metacognitive skills. In fact, Herrick (2005) conducted a literature review on this topic as her thesis project. She argues that while a great deal has been written about writing in mathematics, much of it is subjective rather than evidence based, and very little research actually exists in relation to the effects of writing on the metacognitive and problem-solving skills of students. Herrick (2005) concludes that, though there are some exceptions, there is an overall positive effect of writing on problem solving and metacognitive skills. She warns, however, that these results must be taken cautiously due to the relatively low number of studies, the differences in duration and type of the interventions, different sample sizes, and the lack of a statistical meta-analysis. Finally, Herrick (2005) argues that more studies examining the intersection of writing, problem solving, and metacognition are needed.

This study contributes to the furtherance of knowledge by detailing one way of incorporating mathematics and diary writing, which is one type of journaling (Hiemstra 2001), in an attempt to support both students' metacognitive skills and their abilities to problem solve. It is necessary to identify ways to support children's problem solving, and understanding how diary writing may assist in this endeavour is important. Additionally, this study is carried out with diverse students (ethnically, culturally, and socioeconomically) and thus holds potential for educators and researchers alike to gain deeper understanding into their sense making and mathematical learning. Furthermore, Polya's (1985) heuristic for problem solving is of seminal importance in mathematics education, but the four phases were what he observed expert problem solvers (e.g., mathematicians) engaging in. These phases have been taken for granted by some as the only approach to solving problems and appropriate for students of all levels and ages. Examining how (and if) this heuristic is enacted with young learners, rather than just expert problem solvers, and the effect of doing so, is an important issue. Specifically, the present study investigated whether engaging students in structured diary writing focusing on Polya's (1985) heuristic and discussion would serve to improve their problem-solving abilities. Structured diaries differ from the traditional free-write diaries in that they include prompts for students to respond to rather than allowing them to write without a prompt. The diary format was chosen for several reasons. First, writing "can provide unique insight into the way students are thinking" (Roberts and Tayeh 2007, p. 236). Secondly, it encourages students to reflect on the processes in which they have engaged, which is essential for the improvement of their problem-solving skills (Roberts and Tayeh 2007; Schoenfeld 1987). A third benefit of the use of a diary is that it provides the teacher with a written record of the students' thinking that may be used for the purposes of evaluation and assessment. Finally, writing in their diaries provides students a chance to rehearse their explanations before engaging in a class discussion (Van de Walle 2003). The prompts used in the present study were intended to focus students on the first, second, and fourth phases of the heuristic (i.e., understanding the problem, devising a plan, and looking back) because, unlike the third phase (i.e., carrying out the plan), they are often neglected by teachers and students (Jacobbe 2008; Schoenfeld 1987), and inattention to these phases can lead to inefficient and unsuccessful problem solving (Schoenfeld 1987). Additionally, this study sought to understand students' thinking as they solve problems using Polya's heuristic. Specifically, the following research questions were examined:

- How do students respond to prompts designed to guide them through Polya's heuristic for problem solving?
- Through writing about and discussing their problem-solving experiences, and being prompted to analyse what they did, will students be able to solve more problems?

- Through writing about and discussing their problem-solving experiences, and being prompted to analyse what they did, will students' problem-solving skills improve?

Method

Participants

Participants in this study were seven (two male and five female) African-American students enrolled in a voluntary summer school enrichment program at an elementary school (ages 5–11) in the south-eastern United States. The goal of the enrichment program was to provide an opportunity for struggling learners to build their basic skills as well as prepare for the next grade level. The school serves a predominantly African-American population. Of the students who attend the school, 96% receive free or reduced lunch. The school was one of the lowest performing in the area as measured by standardised state achievement tests. Participants were entering grade 4 (ages 9–11) in August. Based on their prior achievement test scores, class grades the year prior, and feedback from their summer school teacher, the participants were classified as struggling in mathematics. Details about students' reading level were not available. All students in the class received the intervention, though some did not return parental informed consent waivers and thus were not included in the study.

Instruments

Pretest and posttest A pretest was administered on the first day of the study, before the intervention began, and a posttest was administered after the completion of the intervention at the end of the fourth day. Each test consisted of three word problems that covered similar mathematical and computational skills (see Appendix A). To align with the literature on problem solving described earlier, these word problems were designed to be challenging yet attainable, have multiple entry points and potential solution strategies, and be "contextually relevant" (Bostic and Jacobbe 2010, p. 36), thus making them appropriate for assessing problem solving. A mathematics education professor examined the questions for these criteria and for content validity and found them to be acceptable. The mathematical skills required to solve these problems (e.g., whole-number computation) were skills students entering fourth grade were assumed to have mastered. Thus, the questions were designed to assess problem-solving ability rather than mathematical skill.

Structured diaries Students were prompted when they began solving problems to keep diaries, in order to engage them in self-monitoring and reflecting on their work, as well as to guide them through Polya's heuristic for problem solving. The diaries, which were handouts with prompts and space for students to write on them, were filled out before and after each problem-solving instance. In the "before" phase, students wrote their answer to the following questions: (1) What is the problem asking? (Restate the problem using your own words.) (2) What information is provided in the problem that can help you? (3) What do you still need to find out in order to answer the problem? (4) What are some ideas you have to solve this problem?

After solving the entire problem (during the "after" phase), the students once again wrote in diaries to answer: (1) What did you do to solve the problem? (2) Does your answer make sense? How do you know? (3) Did you try anything that did not work? What happened?

Diary questions from the "before" phase aimed at helping students to analyse and understand the problem and instructed them to design and explain a plan. Questions for the "after" phase prompted students to reflect upon their thinking and solution strategies.

Questions for the diary were designed to be general enough that they would apply to any problem-solving situation, but specific enough to be helpful for a student who was struggling with a problem (Jacobbe 2008; Polya 1985). The ultimate goal of the diary was for students to internalise or assimilate the questions, allowing them to develop a mental habit of self-questioning based on Polya's heuristic while engaged in the problem-solving process (Polya 1985). Therefore, the diaries were used not during the pretest or posttest, but rather as part of the intervention to aid students in the problem-solving process.

Intervention

The lead author acted as the teacher during the four-day intervention. Constraints placed upon the researchers by both the time limit of the summer program and other programs occurring at the school (e.g., preservice teachers were attending an on-site course on problem solving at the same time as the intervention) unfortunately did not allow for a longer study. The summer school program required students to meet as a class all day with two teachers, similar to the structure of a regular school day. It was in one of these classes that the intervention was carried out. The study was carried out several weeks into the summer school program, so the group had established themselves as a class. The teacher-researcher attended the class's "morning meetings" to build rapport, and the regular classroom teacher observed from her desk as the intervention was carried out.

The intervention for this study was twofold. First, students were provided the opportunity to solve problems during their mathematics class. Second, students kept a diary and engaged in class discussions to develop their thinking. The diary required them to engage in self-questioning and reflect upon the problem-solving process.

In order to improve students' problem-solving skills, they must be provided with the opportunity to practice problem-solving behaviours. Thus, students were given as many problems as time would allow over the four-day period. The teacher worked with the students each day for approximately one and a half hours. Students solved five problems during the intervention (one on the first day, and two on each of the second and third days). The lead author crafted all the problems, with the exception of The Ups and Downs of Shopping (Johnson and Herr 2001, p. 24). In accordance with previous research (Hiebert 2003; Roberts and Tayeh 2007), problems were designed to be relevant, challenge students, encourage discussion, allow for multiple solution strategies, and be accessible to students of differing ability levels (see Appendix B).

These characteristics of the problems were evaluated by a mathematics education professor and found to be acceptable. Time constraints and the fact that the summer enrichment program occurs for only a few weeks each year did not allow for the problems or intervention to be piloted prior to the actual study.

On the first day of the intervention, students were given as much time as they needed to complete the pretest. When everyone finished, they solved the Ups and Downs of Shopping problem together as a class. Specifically, after discussing the problem with students to ensure they understood it, the teacher modelled how to fill out the “before” portion of the diary based on that discussion. Next, she modelled her thinking as she carried out the plan the class devised. Finally, following another discussion about the strategy used, the teacher modelled filling out the “after” portion of the diary as students filled out their own diaries at their tables. The second day consisted of two problem-solving sessions. The first session was similar to the session of the previous day: The teacher modelled filling out the diary and solved the problem with the class. For the second session, the students read the problem individually, after which the teacher read the problem aloud to the class. They then discussed it in order to understand the question, with the teacher asking questions similar to those on the before portion of the diary and students answering. They then filled out their diaries and solved the problem individually. Students sitting in the same group were allowed to communicate with each other about their progress, although their writing and actual solving of the problem were done individually. The teacher circulated the classroom and monitored student progress. She also answered questions they had about the problem and their diary while being careful not to give any answers to the problem. When students were finished, the teacher facilitated a discussion, encouraging students to share their solution strategies with one another. The third day was similar to the second session of day two: Students solved problems individually after a brief class discussion. They worked on two separate problems and wrote a separate entry in their diary each time.

On the final day (day 4) of the intervention, students did not engage in problem solving. Instead, they participated in a “think, pair, share” discussion, the purpose of which was for students to share their solution strategies with others, as well as reflect on what they had learned during the week of problem solving. First, students worked individually to answer written questions about what they had experienced. Some of the questions were as follows: “What was the most difficult problem this week?” “What was hard about it?” “Were you able to solve this problem?” “If so, how did you solve it?” Students then shared their responses in groups of two or three. Following the small-group discussion, groups shared what they had discussed with the class; all students were encouraged to participate in this whole-group discussion. They were then asked to give advice about problem solving under the hypothetical scenario that a new student was coming into the classroom. Finally, students completed the posttest.

Data collection

The pretests, posttests, and diaries all served as sources of data. The students’ written reflections completed on day four of the intervention were also collected. Additionally, the teacher-researcher wrote a reflection at the end of each day during the intervention. These reflections included details regarding students’ successes and difficulties, common mistakes, students' reactions to problems, their writing processes, and any noteworthy event that occurred that day.

Data analysis

Pre/post achievement comparison To determine whether students were able to solve more problems correctly after the intervention, students' overall scores on the posttest were compared to their score on the pretest. Items on the pretest and posttest were given a score of zero or one depending on whether the students' solution was incorrect or correct, respectively. Students could thus earn up to three points on each test. Statistical analyses were not conducted due to the small sample size. However, each student's score on an individual item on the posttest was compared to his or her corresponding score on the pretest (e.g., item 1 score on posttest versus item 1 on pretest) to examine growth on that particular problem type.

Pre/post solution strategy comparison While the achievement results are important and of value to teachers, into problem solving, the final solution is not the most important part of the process. What should be focused on are the strategies and thinking that students use to arrive at their solution (Hiebert 2003; Schoenfeld 1987; Van de Walle 2003). Thus, to determine whether students' problem-solving skills improved and to gain insight into students' thinking as they solved problems, their solution strategies on the pretest and posttest were compared. These comparisons were made independently of the "correctness" of their answers. Items were classified according to (a) whether there was no solution (i.e., the problem was left blank), (b) whether there was there was a solution but no strategy evident (indicating either a guess or mental computation), or (c) by type of strategy used (e.g., draw a picture, make a chart, look for a pattern, etc.). A comparison was then made between the pretest and posttest solution strategies used on corresponding problems (e.g., item 1 strategy on the pretest versus item 1 strategy on the posttest). A student was said to improve on a problem and given a score of one if their posttest classification on that item was ranked higher than their classification on the corresponding pretest item. For example, a student who provided a solution on the pretest with no strategy evident but provided both a solution and made a chart on the posttest would be considered to have improved in their strategy use, regardless of whether either solution was correct. If the classification did not change or was ranked lower from pretest to posttest, then the student was given a score of zero for that item.

Diary analysis To examine students' problem-solving skills and determine how they responded to prompts designed to guide them through Polya's heuristic for problem solving, the structured diaries the students wrote were collected. The researcher attempted to adopt the viewpoint of a classroom teacher when examining the diaries, looking to see that prompts were answered completely, what types of strategies students used, and whether they were able to answer the prompts on their own without guidance from the teacher or a peer. Evidence of understanding of the question, such as whether the student answered in their own words or copied text from the problem, and whether what was written adequately reflected the work shown on the problem and in later class discussions (as captured in daily reflections), was noted.

Daily reflections The teacher-researcher's daily written reflections were not used formally in the analysis. However, they were used to provide details regarding students' experiences, attitudes, and diary-writing processes. These reflections were often referred to during the analyses of other data to ensure results reflected what the researcher observed and experienced in the classroom.

Results

Pretest and posttest achievement comparison

Results from the pretest and posttest are shown in Table 1. All names have been changed to pseudonyms. As exhibited, three of the seven students solved more questions correctly on the posttest than the pretest.

Strategy use and student thinking

Through a comparison of students' pretest and posttest strategies on each problem, six students were found to have improved in their solution methods (see Tables 2 and 3).

To illustrate the improvement in solution strategies, we consider the cases of Joejoe, Anthony, and Keisha, and their responses to the same questions from the pretest and posttest. These students were chosen because their responses offer typical examples of the improvement seen in solution strategies from the students in the study.

Joejoe is an unusual case because he scored lower on the posttest (see Table 1) yet was considered to have improved in his ability to solve problems (see Table 2). Consider both versions of question two:

Pretest: Tasha is at the store buying 15 bottles of soda. Cola is on sale: "3 bottles for $2!" Diet cola is on sale also: "5 bottles for $4!" Should Tasha buy cola or diet cola if she wants to spend the least amount of money? Explain why.

Posttest: Tanita's mom asked her to buy 12 hamburgers for dinner. McDonald's is advertising: "4 burgers for $3!" Wendy's is offering, "6 burgers for $4!"

Table 1 Pretest and posttest results

Participant	Pretest			Posttest		
	Q 1	Q 2	Q 3	Q 1	Q 2	Q 3
Keisha[a]	1[b]	0	0	1	1	1
Joejoe	1	1	0	0	0	0
Nicole	0	0	0	0	0	0
Jarquavius[a]	0	0	0	1	0	0
Simone	1	0	0	0	0	0
Tyreisha[a]	1	0	0	1	1	0
Anthony	0	0	0	0	0	0

[a]Demonstrated an increase in number of correct answers from pretest to posttest[b]A "1" indicates a correct answer

Table 2 Solution strategy improvement from pretest to posttest

Participant	Demonstrated improvement		
	Q1	Q2	Q3
Keisha	0	1	1
Joejoe	0	1	1
Nicole	0	0	0
Jarquavius	1	1	0
Simone	1	0	0
Tyreisha	0	1	1
Anthony	1	1	0

Where should Tanita buy her burgers if she wants to spend the least amount of money? Explain why.

Both of these problems require students to determine two different total costs and compare them. On the pretest, Joejoe technically got the correct answer ("Cola it is less") but it is not clear how he arrived at his solution. While he did not provide any explanation, it is possible he compared the two prices, noticed that $2 is less than $4, and thus assumed the cola would be cheaper. While it is equally possible Joejoe thought through the problem correctly but simply did not share his strategy, it became clear in the class discussions that surfaced after the test that the incorrect strategy described above was a very common mistake made by students in the class. Regardless of the strategy Joejoe used, it was not evident at all on the pretest. On the posttest, Joejoe's solution was only partially correct, yet it provides more evidence of his thinking. He worked out the total that Tanita would spend at Wendy's: "buy 12 of Wendy's burgers for eight dollars." This reasoning is correct, but does not fully answer the question asked of him regarding which restaurant requires one to spend the least amount of money. First, it is unclear whether Joejoe compared this cost ($8) to what it would cost for the same number of burgers at McDonald's and second, it is unclear whether he is indicating that Wendy's is the better buy or just worked out the total to be spent at Wendy's. Such an answer demonstrates that while Joejoe may not have fully understood the question, he did realise he needed to determine a total, not just take the exact dollar values stated in the problem and compare those, as he appeared to have done on the pretest. His strategy was correct; he simply needed to continue working on the problem. Another interpretation may be that Joejoe had simply gotten better at recording his strategy use from pretest to posttest. With the data collected for this study, it is impossible to determine which is the correct interpretation. However, as one of the goals

Table 3 Total number of correct solutions and improved strategies by problem

Question	Number correct on pretest	Number correct on posttest	Number of students who improved[a]
1	4	3	3
2	1	2	5
3	0	1	3

[a]From pretest to posttest

of this study was to understand students' thinking during problem solving, Joejoe's solution to the pretest question provides better evidence of this and thus serves as an example of improved strategy use.

Similar to Joejoe, Anthony did not respond to question 2 on the pretest, but through drawing a picture on the posttest, was able to determine how much 12 burgers would cost at Wendy's through the use of a picture and (presumably) calculating the cost mentally (see Fig. 1).

Anthony's solution strategy was valid; he simply did not finish answering the question, even though the lack of time constraints on the tests afforded him the opportunity to do so had he chosen to. Anthony did not explicitly state Wendy's as the restaurant with the better buy, nor did he explain why. Both students demonstrated progress toward understanding the mathematics needed to solve the problem, but neither fully carried out the process. Their procedural skills were not lacking, and their strategies were a step towards the final solution, but it appears they did not reflect on their solution and ask themselves whether they answered the question completely. With more time to develop a habit of self-questioning and reflection on their solution, Anthony and Joejoe may have realised the incompleteness of their answers and thus may have answered these questions correctly. One can conclude Anthony and Joejoe did not internalise the problem-solving process.

Finally, consider Keisha's solutions to these same two questions (see Fig. 2).

On the pretest, Keisha seems to have not understood the problem or devised a plan to solve it, since she merely restated part of the problem ("two dollars for 3") without calculating a total price or comparing the price of cola to diet cola. However, on the posttest, it is evident that not only did Keisha understand that she needed to compare how much Tanita would have to pay at each restaurant, but she also correctly grouped the hamburgers and labelled the individual price of each group before writing out the total. Her final answer, "I think she should get Wendy because you can pay less for more and McDonald you pay more for less [sic]," and her pictures demonstrate that Keisha took time to understand the problem before working on it, devised an appropriate plan (draw a picture), and employed her plan correctly. While it is possible that Keisha is simply writing more on the posttest after an intervention that encouraged her to represent complete answers, the statement on the pretest focusing on the sale value ($2 for 3 bottles) rather than the total value ($10 for 15 bottles) indicates that she likely did not calculate the total value and thus may not have fully understood the problem or carried out an appropriate plan. Following the

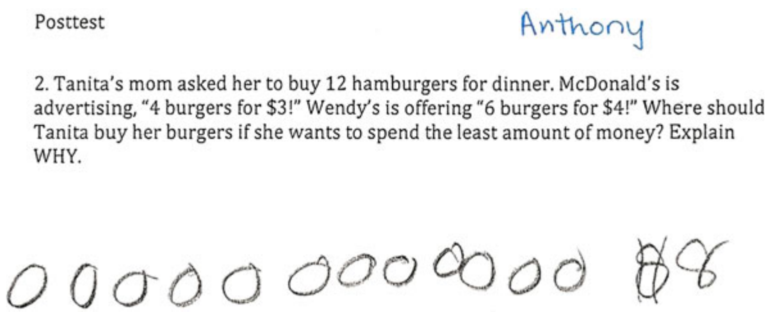
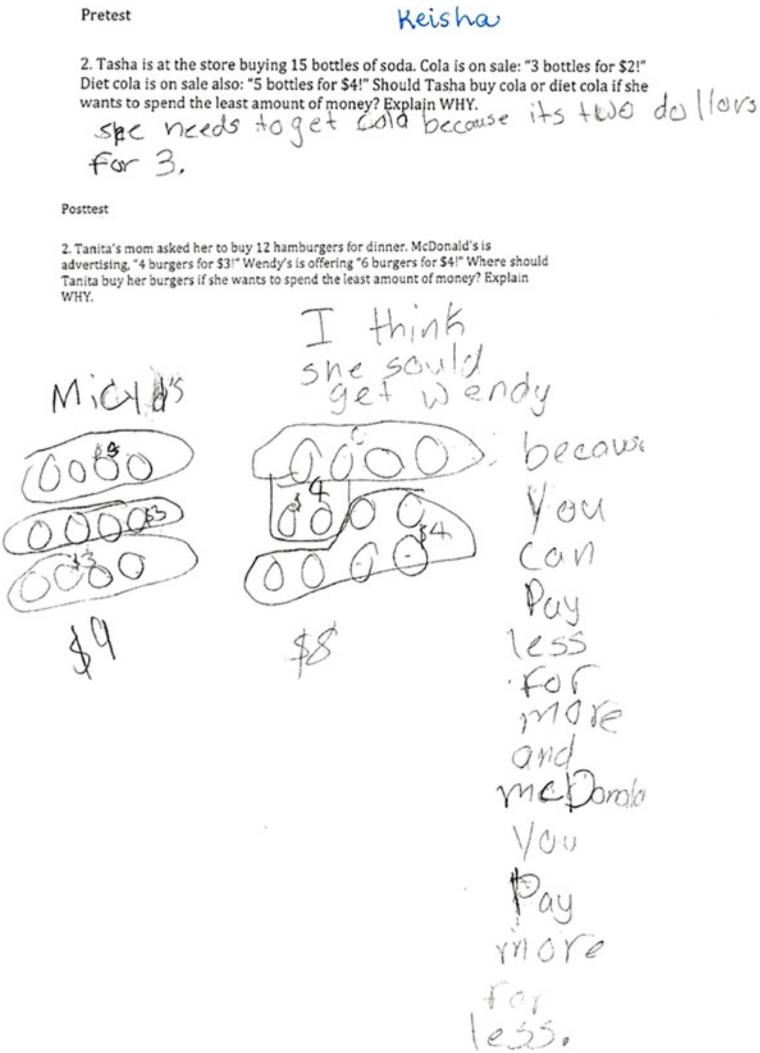


Fig. 1 Anthony's posttest solution


Pretest Keisha

2. Tasha is at the store buying 15 bottles of soda. Cola is on sale: "3 bottles for $2!" Diet cola is on sale also: "5 bottles for $4!" Should Tasha buy cola or diet cola if she wants to spend the least amount of money? Explain WHY.

she needs to get cola because its two dollors for 3.

Posttest

2. Tanita's mom asked her to buy 12 hamburgers for dinner. McDonald's is advertising, "4 burgers for $3!" Wendy's is offering "6 burgers for $4!" Where should Tanita buy her burgers if she wants to spend the least amount of money? Explain WHY.

I think she sould get wendy because you can pay less for more and mcDonalds you pay more for less.

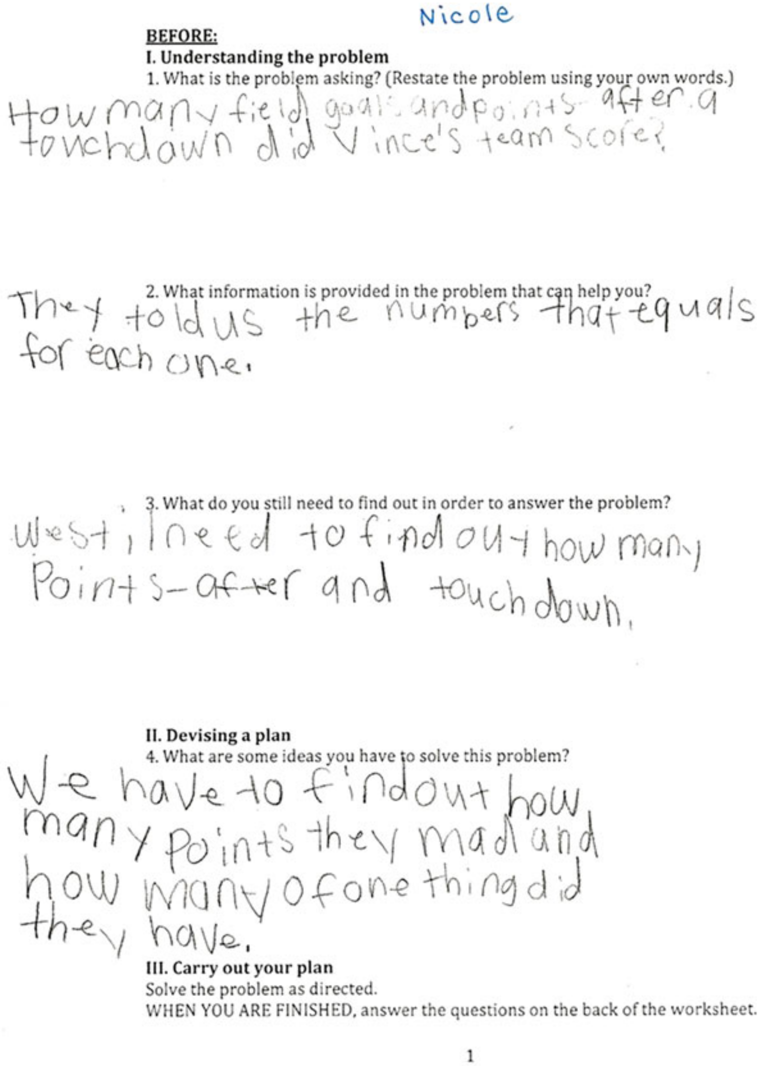
Fig. 2 Keisha's pretest and posttest solutions

intervention, Keisha was able to solve the problem on the posttest more successfully and better explain her strategy.

Diary writing

One of the reasons for keeping a diary was to compel students to ask themselves the same questions each time they solved a problem. The goal was that, with enough repeated use of the diary, students would begin to internalise the questions and develop a mental habit of self-reflection during problem solving that they would then use to solve problems during the posttest. It was evident from their posttest solutions that some of the students did not internalise the questions as intended. Indeed, developing a mental habit after just 3 days of problem solving and a fourth day of discussion is a difficult thing to do. Even so, being required to explain their understanding of a problem, devise a plan, and reflect upon the problem may have helped students in their ability to problem solve. Consider, for example, the following problem about American football that students completed individually on the second day of the intervention and Nicole's diary and solution to the problem (see Figs. 3, 4, and 5):

> Vince plays football for the Falcons. In football, a touchdown is worth 6 points, points-after a touchdown are worth 1 point, conversions are worth 2 points, and a field goal is worth 3 points. In the last game they played, the falcons scored a total of 27 points. They did not score any field goals or 2-point conversions. How many field goals and points-after a touchdown did Vince's team score?


Nicole

BEFORE:

I. Understanding the problem

1. What is the problem asking? (Restate the problem using your own words.)

How many field goals and points after a touchdown did Vince's team score?

2. What information is provided in the problem that can help you?

They told us the numbers that equals for each one.

3. What do you still need to find out in order to answer the problem?

We still need to find out how many Points-after and touchdown.

II. Devising a plan

4. What are some ideas you have to solve this problem?

We have to find out how many points they mad and how many of one thing did they have.

III. Carry out your plan

Solve the problem as directed.

WHEN YOU ARE FINISHED, answer the questions on the back of the worksheet.

1

Fig. 3 Nicole's diary from the "before" phase

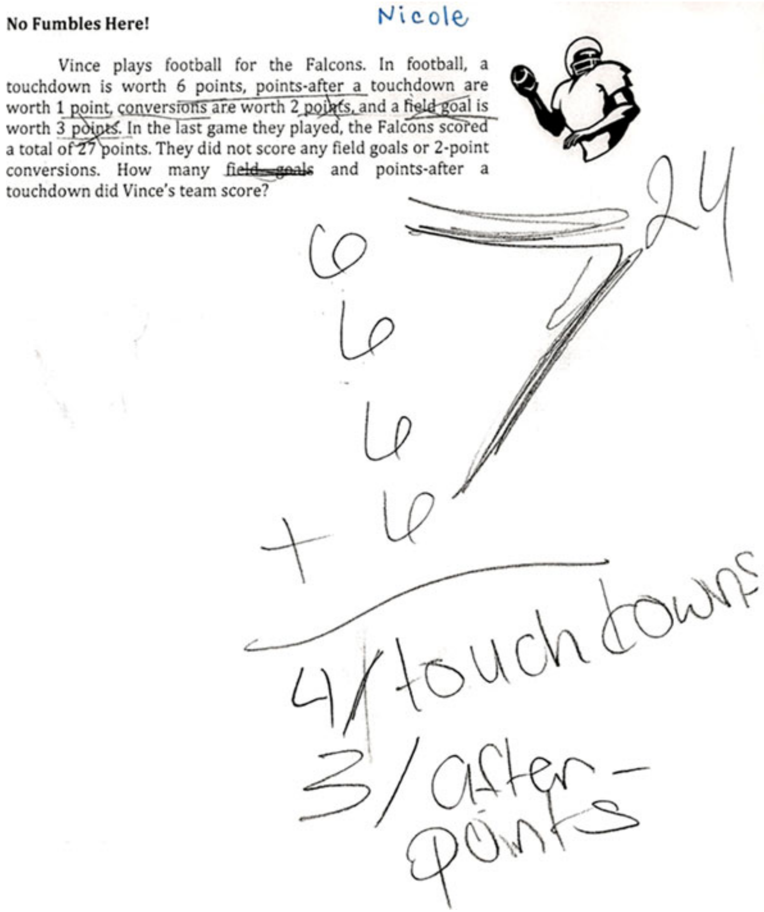
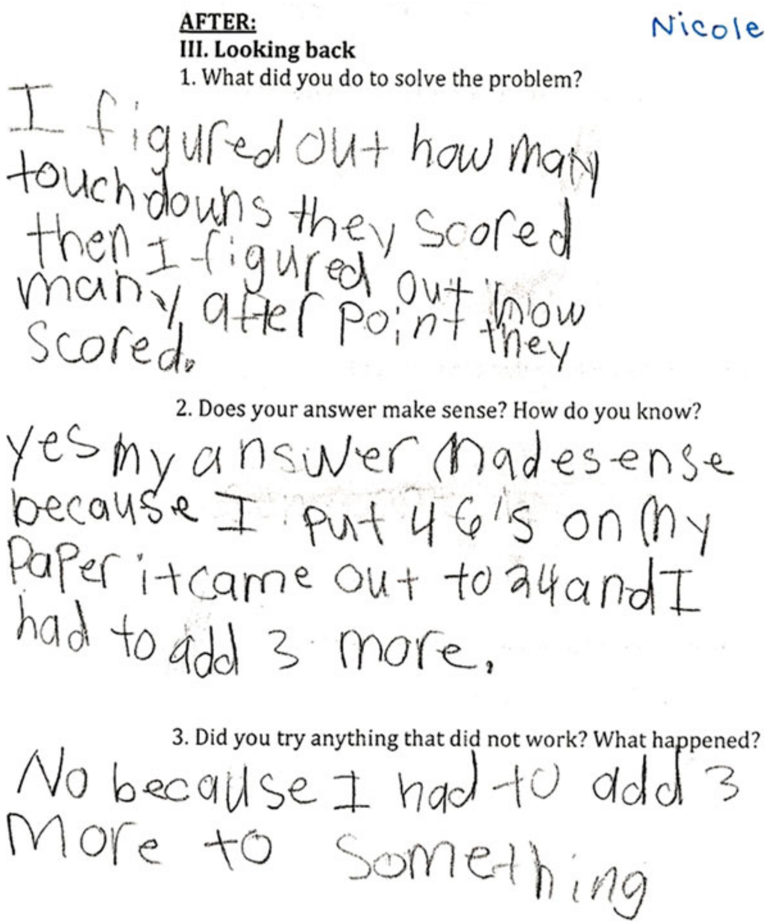


Fig. 4 Nicole's solution

This diary was typical of what Nicole produced during the intervention, and demonstrates that she understands the given information in the problem when she writes, "they told us the numbers that equals for each one;" in other words, how many points each different way to score is worth. She also had a clear plan in mind. As evidenced by her work, the strategy Nicole chose was appropriate. After determining the number of touchdowns the Falcons scored to total 24 points, Nicole worked out mentally that three more points were needed to reach the desired 27 points. However, since no field goals were scored, Nicole determines three "after-points" were scored. In her reflection, she not only explains vividly how she arrived at her solution, but she also reworks the computations: "4 6's…came out to 24 and I had to add 3 more" to get 27, as the final phase of Polya's (1985) heuristic requires.

Nicole was a very quiet student with little confidence in her mathematics abilities. Before writing in her diary, she would tell the teacher she did not know what to write. The teacher responded only by reading a question from the diary out loud to Nicole. She would tell the teacher her answer, who responded by suggesting that she write down what she just stated. This verbal exchange and encouragement to trust her own


AFTER: Nicole

III. Looking back

1. What did you do to solve the problem?

I figured out how many touchdowns they scored then I figured out how many after point they scored.

2. Does your answer make sense? How do you know?

yes my answer made sense because I put 4 6's on my Paper it came out to 24 and I had to add 3 more.

3. Did you try anything that did not work? What happened?

No because I had to add 3 More to something

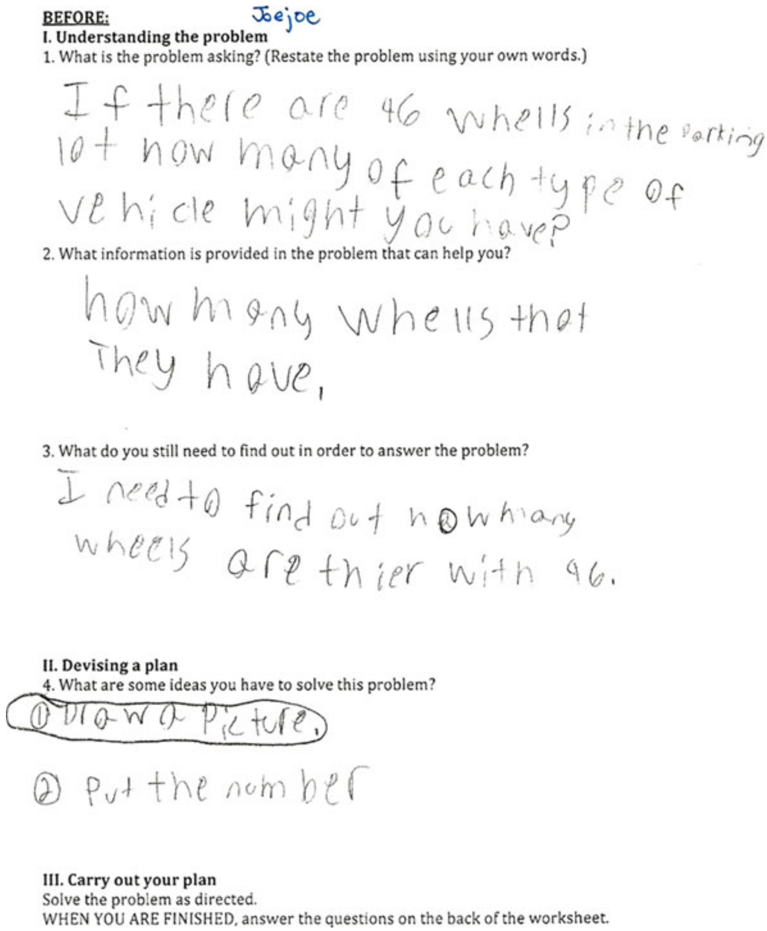
Fig. 5 Nicole's diary from the "after" phase

thinking seemed to be helpful to Nicole's writing and problem-solving process, even though the teacher provided no answers. The diary questions, first answered verbally and then written, may have helped Nicole develop her solution strategy.

Next, consider the Tires Problem, which was administered on the third day of the intervention for students to work on individually:

> There are many vehicles in the parking lot. Some, like motorcycles, have 2 wheels. Other vehicles, such as cars, have 4 wheels. There are even some vehicles, such as school busses, that have 6 wheels. If there are 46 wheels in the parking lot, how many of each type of vehicle might you have?

Similar to Nicole, Joejoe (see Figs. 6, 7, and 8) demonstrated on the Tires Problem that keeping a diary may have helped him to solve this problem. From the diary it is clear that he devised two plans and chose between them. Joejoe determined he wanted to draw a picture, yet he realised he did not know how to do so. Thus, the teacher showed him how to draw a representation of a vehicle and its wheels, and he proceeded to solve the problem on his own.



BEFORE: Joejoe

I. Understanding the problem

1. What is the problem asking? (Restate the problem using your own words.)

If there are 46 whells in the parking lot how many of each type of vehicle might you have?

2. What information is provided in the problem that can help you?

how many whells that they have,

3. What do you still need to find out in order to answer the problem?

I need to find out how many wheels are thier with 46.

II. Devising a plan

4. What are some ideas you have to solve this problem?

① Draw a picture,

② Put the number

III. Carry out your plan

Solve the problem as directed.

WHEN YOU ARE FINISHED, answer the questions on the back of the worksheet.

Fig. 6 Joejoe's diary from the "before" phase

Joejoe's work suggests an understanding of the problem. His answer to the prompt regarding what he needed to find out is somewhat unclear ("I need to find out how many wheels are their [sic] with 46"), but his plan, solution, and response to the question "Does it make sense?" indicate he understood he was finding out how many vehicles had a total of 46 wheels. Joejoe stumbled initially on carrying out his plan, but once he learned to draw a picture, he carried it out completely and accurately. What is interesting is that, while his drawing displayed the correct answer and his solution is rich with information, Joejoe failed to write the proper solution (indicating 4 buses were needed rather than the 5 he drew). It is unclear whether he made a careless error when counting the buses, or if he added the sixth bus after totalling his results. Yet Joejoe was convinced that his answer made sense, because he checked that the number of wheels he drew totalled 46. This information from his diary indicates Joejoe did indeed understand the final goal of the problem and carried his plan out correctly.

The qualitative results of Nicole and Joejoe's posttests indicate that they did not benefit from the intervention, but their diaries suggest otherwise. From their writing,

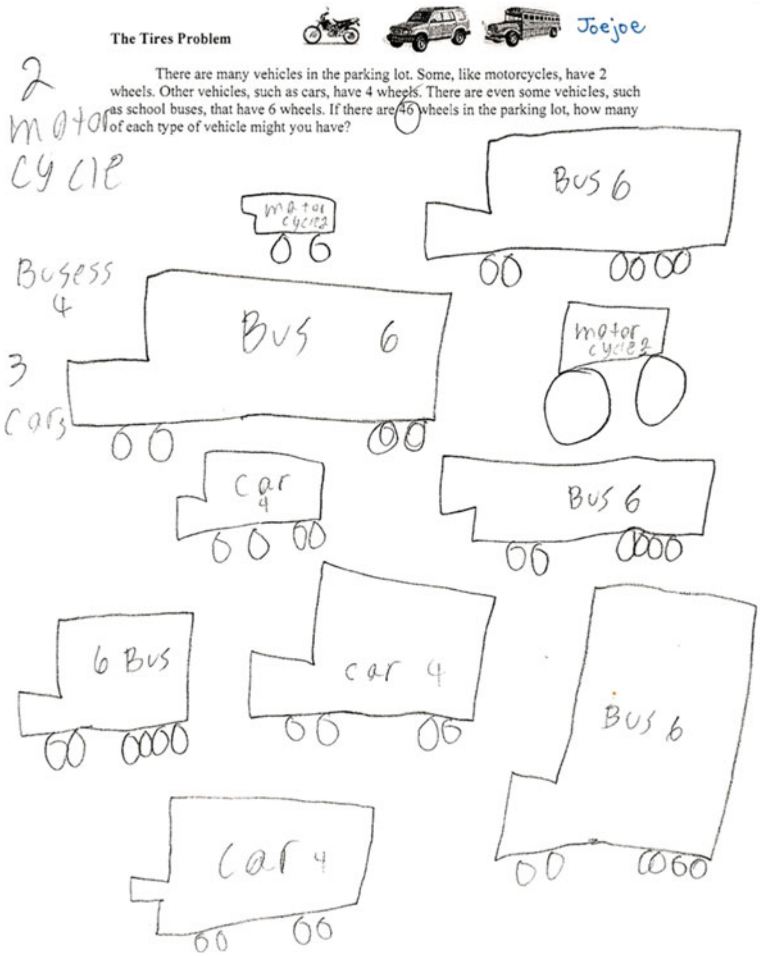


Fig. 7 Joejoe’s solution

it appears that they understood the problem, knew how to solve it, were able to carry out their plans to find the correct solution, and were confident that it was solved correctly. These students were able to solve problems successfully when prompted with the diaries, suggesting that were they to ask themselves the diary questions or keep a diary while they worked on the posttest, they may have scored higher.

Class discussion

The final results come from the group questions and class discussion held on the fourth day of the intervention. One of the questions students were asked required them to think about the types of things that might help someone engage in problem solving. Validating Polya’s first phase, several students commented on the importance of helping other students understand the problem. For example, Keisha stated, “I would help them understand it how I do,” and Jarquavius wrote, “I will help them understand it better. We will talk about it.” These suggestions make it clear that they

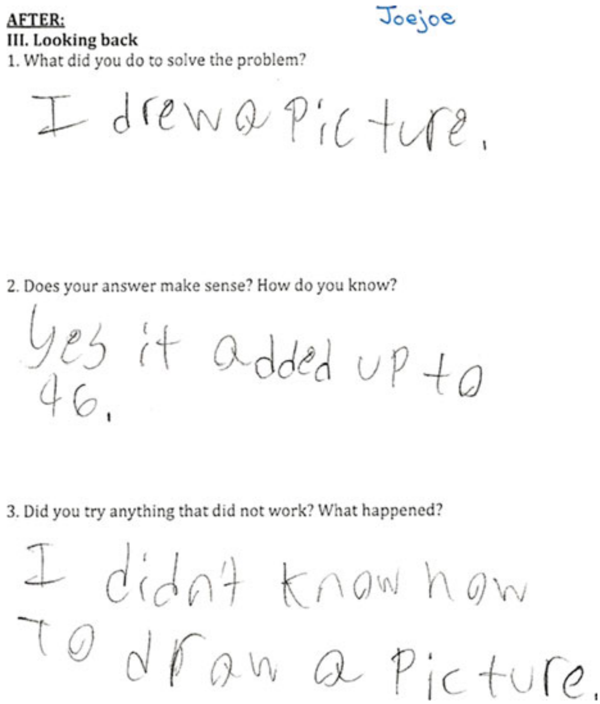

AFTER: Joejoe
III. Looking back
1. What did you do to solve the problem?
I drew a picture.
2. Does your answer make sense? How do you know?
Yes it added up to 46.
3. Did you try anything that did not work? What happened?
I didn't know how to draw a picture.

Fig. 8 Joejoe's diary from the "after" phase

value understanding a problem before it can be solved. Jarquavius also implied in his statement that vocalisation aids in thinking, understanding, and problem solving, which supports findings of previous research (e.g., NRC 2001; Schoenfeld 1987). Additionally, Tyreisha's suggestion, "I would tell them to draw picture or write the problems out," may support the use of the diary, which requires students to write about their problems. Explaining the problem (including the given information and what they needed to find) was the focus of the "before" portion of the diary. Tyreisha's comment suggests that writing about the problem before solving it was helpful to her, and would therefore be helpful to others.

Conclusions

Taken together, these results indicate students' problem-solving strategies did improve. While only three of the seven students received higher scores on the posttest than on the pretest, a closer analysis of their work indicated that six students showed improvement in their solution strategies following the intervention. Comparing the quality of their solutions when prompted to work through Polya's heuristic during the intervention to the quality of their solutions to the posttest, where they were not prompted, made clear that most students did not internalise the questions and processes from their diaries. However, these examples (i.e., Figs. 3, 4, 5, 6, 7 and 8) support the notion that keeping a diary during problem solving help to make students' understanding of problems and their development and implementation of a solution plan clearer to the teacher. Additionally, the scaffolding provided by being prompted to work through Polya's heuristic while problem solving, rather than simply solving the problem without planning or reflection, may have benefited these students by allowing them to work in their zone of proximal development. It can be inferred from these results that encouraging students to follow Polya's heuristics and engage in writing throughout the process may help to increase students' problem-solving skills.

An important consideration comes from the examination of Nicole's diary and the events that transpired around her writing. As noted earlier, Nicole needed encouragement and prompting, as well as to verbally articulate her thoughts, before she was confident enough to write in the diary. One wonders whether the writing was a necessary aspect of the intervention for Nicole, or if the act of articulating her thoughts out loud was enough. Teachers must be careful to attend to their students' diverse needs. Had Nicole been asked to "do her best" and quietly write down what she could without articulating it out loud first (something many teachers ask of their students) she may not have solved the problems as successfully as she did. It is also noteworthy that Nicole is the only student who did not improve at all in her strategy use from pretest to posttest. This was not for lack of ability, as demonstrated by her diaries and problem solving throughout the intervention. Perhaps it is a case of Nicole working in her zone of proximal development during problem solving that was scaffolded by the teacher; but her work on the posttest was independent and thus indicated a lack of progress. More research is needed to determine what further types of supports can be provided to a student such as Nicole to improve her achievement and strategy use on assessments and other individual problem-solving tasks.

There are some limitations to this study that need to be acknowledged. The first limitation to this study is the small sample size. As the treatment was conducted during a voluntary summer school program, there were very few students available to participate in the study. For this reason, these findings cannot be generalised to the broader community based on this study alone. However, since the students in this study were students of colour and struggled with mathematics, the results indicate there is potential for this type of intervention in other diverse settings.

Another limitation is that the time of the intervention was very short (4 days). Problem-solving skills take time to develop. However, even with such a short intervention, there were some indications of improvement in these skills, as evidenced by the strategy comparison analysis and Joejoe's, Anthony's and Keisha's solutions. A longer study is needed to determine whether a mental habit of self-questioning during the before and after phases of problem solving and the internalisation of a heuristic can be developed and sustained through the use of diaries. Additionally, it is unclear whether the improvement in students' solution strategies was due to the diary writing, class discussions, practice they received solving problems during the intervention, or some combination of these events and others. A study with multiple treatment groups would be better able to tease out what elements of the intervention caused the improvement. Furthermore, the diaries may not have had any impact on students' thinking at all, but rather simply encouraged them to make their thinking and answers explicit. For example, a student may have used a "mental" strategy on the pretest and only provided the solution, but during the intervention picked up on the culture of "strategy share" and thus decided to make the strategy explicit on the posttest. This scenario would result in the researcher classifying the student as having improved in strategy use when, in fact, strategy use may not have improved at all but was made explicit to the teacher. Such a scenario, however, is not necessarily an undesirable outcome of the intervention; when students' thinking is made clear, teachers are better able to determine what that student knows and support them to learn what they do not know.

A final and related limitation is due to the fact that it is difficult to determine the extent of the development of a mental habit through examining students' written work only. A future study in which students are videotaped and required to verbalise their thinking while solving problems would serve to more adequately gauge whether and to what extent they internalise Polya's problem-solving process and develop a mental habit of self-monitoring while engaging in problem solving.

The ability to problem solve is a vital skill that all mathematics teachers ought to help students develop. Teaching mathematics through problem solving enables students to think critically and connect school mathematics to the real world, enhances their understanding of concepts, and makes mathematics accessible to learners of all types. Results of this study indicate that diary writing that prompts students to follow Polya's heuristic may lead to richer solution strategies and thus increase their ability to solve problems.

Appendix A pretest and posttest questions

Pretest

1. Keisha and her class are taking a field trip to the Butterfly Rainforest. There are 41 students, 2 teachers, and 3 parents going on the trip. If they are taking vans to the museum, and each van holds 6 people, how many vans will they need?
2. Tasha is at the store buying 15 bottles of soda. Cola is on sale: "3 bottles for $2!" Diet cola is on sale also: "5 bottles for $4!" Should Tasha buy cola or diet cola if she wants to spend the least amount of money? Explain WHY.
3. Joey is at the pet store. They sell birds, puppies, and snakes. Joey counted up the legs of all the animals and noticed there are 66 legs in the store. If there are 31 pets in the store, 7 of which are snakes, how many birds and puppies combined are there? How many birds are there? How many puppies are there?

Posttest

1. Anthony's older sister is getting married, and he is helping set up tables for the party. There will be 13 kids and 45 adults at the party. If each table holds 5 people, how many tables does Anthony need to help set up?
2. Tanita's mom asked her to buy 12 hamburgers for dinner. McDonald's is advertising, "4 burgers for $3!" Wendy's is offering "6 burgers for $4!" Where should Tanita buy her burgers if she wants to spend the least amount of money? Explain WHY.
3. Katie is shipping Girl Scout cookies to her customers. She has a total of 72 boxes of cookies. There are several types of containers for shipping. Small containers hold 2 boxes, medium containers hold 4 boxes, and large containers hold 10 boxes. If Katie has a total of 26 containers, none of which are large, how many small containers does she use? How many medium containers?

Appendix B questions for problem solving

The ups and downs of shopping

Roberto is shopping in a large department store with many floors. He enters the store on the middle floor from a skyway and immediately goes to the banking department. After making sure he has enough money, Roberto goes up 3 floors to the housewares department. Then he goes down five floors to the children's department. Next, he goes up six floors to the TV department. Finally, Roberto goes down 10 floors to the main entrance of the store, which is on the first floor, and leaves to go to another store down the street. How many floors does the department store have?

Pay day

Your neighbor needs a babysitter for a week. She offers you two options for getting paid: you can work 7 days at $20 per day OR be paid $2 on the first day and have your salary doubled every day after that for a week. Which would you prefer?

No fumbles here!

Vince plays football for the Falcons. In football, a touchdown is worth 6 points, points-after a touchdown are worth 1 point, conversions are worth 2 points, and a field goal is worth 3 points. In the last game they played, the Falcons scored a total of 27 points. They did not score any field goals or 2-point conversions. How many touchdowns and points-after a touchdown did Vince's team score?

Counting cookies

Joel bought 2 boxes of cookies with 61 cookies in each box. Nikki bought 5 boxes of cookies with 24 in each box. Ke'shawn bought 3 boxes of cookies with 48 in each box. Who got the most cookies? Who got the fewest cookies?

The tires problem

There are many vehicles in the parking lot. Some, like motorcycles, have 2 wheels. Other vehicles, such as cars, have 4 wheels. There are even some vehicles, such as school buses, that have 6 wheels. If there are 48 wheels in the parking lot, how many of each type of vehicle might you have?

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