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ORIGINAL ARTICLE

Developing measurement concepts within context: Children's representations of length

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Abstract This article presents data gathered from an investigation which focused on the experiences children have with measurement in the early years of schooling. The focus of this article is children's understandings of length at this early stage. 32 children aged 4–6 years at an Australian primary school were asked to draw a ruler and describe their drawing, once in February at the beginning of school, and again in November towards the end of their first year of school. The drawings and their accompanying descriptions are classified within a matrix which, informed by Bronfenbrenner's ecological theory and literature regarding the development of length concepts, considers conceptual understanding and contextual richness. The responses revealed that children have a good understanding of length at the start of school, but that as their ability to contextualise develops so too does their conceptual understanding. This article suggests that participation in tasks such as these allows children to create their own understandings of length in meaningful ways. Additionally, the task and its matrix of analysis provide an assessment strategy for identifying children's understandings about length and the contexts in which these understandings develop.

Keywords Young children · Measurement · Representations · Context

The intuitive mathematics that students learn through personal experiences is the most influential connection for early mathematics development and supports the conceptual development of mathematics introduced in school contexts (Clements and Sarama 2000). As mathematics becomes more abstract the relationship between school and

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out-of-school mathematics tends to become less tangible (Lowrie 2004a). Schoenfeld (1989) argued that mathematics is an act of sense making and consequently involves cultural, social, and cognitive phenomena that are essentially interwoven. It is often difficult for teachers to include the social and cultural contexts of learning in classroom situations since these experiences are personal (Civil 2002; Lowrie 2004a), although these social and cultural experiences are so critical to early concept development. If designed in a manner that considers these social and cultural contexts, mathematical activities can become personalised and provide opportunities for deep learning and consequently enhance mathematical meaning; and if the activities are open-ended in nature, there is certainly scope for this to occur (Sullivan et al. 2005). Unfortunately, as concepts become more abstract there is a tendency for teaching-learning experiences to be context free—and thus free from cultural and social dimensions. Nevertheless, for young children, and particularly within the measurement strand of the curriculum, these cultural and social dimensions are critical. The present investigation considers the importance of promoting such phenomena as young children develop understandings of measurement and, specifically, length.

Children engage in a range of everyday activities that involve length measurement, and consequently most children have some understanding of the concept of length before they are taught about it formally. These understandings are often fragmentary and limited, and they can lead children into error in some situations; but nevertheless they are genuine in the sense that in every case the child understands something about the relations involved in the concept and can apply this understanding logically (Nunes and Bryant 1996). These early understandings should be viewed as a basis for children to learn more about length concepts.

In this article we present responses provided by children aged 4–6 years at an Australian primary school to an open-ended drawing task which focused on the concept of length. Data are classified within a matrix which considers conceptual understanding and, as we encouraged students to represent ideas and understandings within personal contexts, also considers contextual richness. The drawings revealed that children's understandings of length were influenced by contextual information, and that the richness of the contextual information had implications for the sophistication of length knowledge. This article suggests that participation in tasks such as this allows children to create their own understandings of length and link classroom learning to their own experiences in meaningful ways.

Literature context

Length is the most easily understood of the measurement concepts and refers to how long something is, whether along a single plane or through two dimensions (Reys et al. 2007; Zevenbergen et al. 2004). Linear measurement involves six important concepts which include: 1) partitioning; 2) unit iteration; 3) transitivity; 4) conservation; 5) accumulation of distance; and 6) relation to number (Stephan and Clements 2003). *Partitioning* is the mental activity of dividing an object into equal-sized units. This idea is not obvious to children, as it involves mentally seeing the object as something that can be partitioned before even physically measuring (Clements and Stephan 2004). *Unit iteration* is the ability to think of the length of a unit as part of the length of the object being measured and to place the unit repeatedly along the length of the object (Kamii and Clark 1997). *Transitivity* is the understanding that if the length of object A is equal to (or greater/less than) the length of object B and object B is the same length as (or greater/less than) object C, then object A is the same length as (or greater/less than) object C (Clements and Stephan 2004). A child who can reason transitively can take a third or middle item as a referent by which to compare the heights or lengths of other objects, and most researchers argue that students must reason transitively before they can understand measurement (Boulton-Lewis 1987; Hiebert 1981; Kamii and Clark 1997. *Conservation* of length is the understanding that as an object is moved, its length does not change (Piaget 1999). Although researchers agree that conservation is essential for a complete understanding of measurement, it has been cautioned that students do not necessarily need to develop transitivity and conservation before they can learn some measurement ideas (Boulton-Lewis 1987; Clements 1999; Hiebert 1981). *Accumulation of distance* is the understanding that as one iterates a unit along the length of an object and counts the iteration, the number words signify the space covered by all units counted up to that point (Clements and Stephan 2004). Finally, *relation to number* requires students to reorganise their understanding from the counting of discrete units to the measure of continuous units.

Although researchers debate the order of the development of these concepts and the ages at which they are developed, they tend to agree that these ideas form the foundation for linear measurement (Stephan and Clements 2003). There are many developmental sequences for length learning presented in the literature (see for example Bobis et al. 2009; Zevenbergen et al. 2004), but most are similar in their progression from identification of the attribute and use of informal measurement through to the use of formal units, with application to contexts being the final stage of development. Although developmental sequences such as these are of vital significance, another important consideration when discussing development of understanding of length is the role of context. It is important to emphasise that "application" is typically regarded as the final, most sophisticated stage of development. However, when the role of context is emphasised, it often becomes the case that "applications" become highlighted in the earlier stages of length understanding. This will be discussed in greater detail later in the article.

The basis for these types of sequences, whether explicitly or implicitly, is Piaget et al. (1960) developmental theory of measurement (Clements and Stephan 2004). As Clements and Stephan outline, this approach motivates students to see the need for a standard measuring unit. However, a somewhat different approach is suggested by other research that questions the wisdom of concentrating first on non-standard units. Boulton-Lewis et al. (1996) found that children used non-standard units unsuccessfully, and were in fact successful at an earlier age with standard units and measuring instruments. Nunes et al. (1995) research suggested that children can meaningfully use rulers before they "reinvent" such ideas as units and iteration. They concluded that rather than making measurement more difficult, the children benefited from the numerical representation provided by the conventional units already built into the ruler. As opposed to the Piagetian argument that children must conserve length before they can make sense of standard systems such as rulers, findings of these studies support a Vygotskian perspective in which rulers are viewed as cultural instruments that children can appropriate (Clements and Stephan 2004).

In light of these differing perspectives, Clements and Stephan (2004) offer several recommendations for the teaching of length measurement. Firstly, length measurement should not be taught as a simple skill; rather, it is a complex combination of concepts and skills that develops over time. Secondly, initial informal activities should establish the attribute of length and develop concepts such as "longer" and "shorter" and strategies such as direct comparison. Thirdly, an emphasis on children solving real measurement problems helps children develop strong concepts and skills. Finally, teachers should help children closely connect the use of manipulative units and rulers.

An effective way of engaging students in length measurement is through the use of contexts. Guided by Bronfenbrenner's ecological theory (cf. 1974, 1979, 1988), we use the term *contexts* to refer to the environments in which children exist, and the connections between these. Bronfenbrenner (1974, 1979) describes three key elements of contexts: 1) the physical space and the materials within this; 2) people, in differing roles and relationships with the child; and 3) activities in which the child is engaging. There are many contexts out of which mathematics can be learned, and school mathematics should be learned through contexts that are meaningful to the learner (Masingila and de Silva 2001). As highlighted in the Introduction, we have suggested that some of the most meaningful contexts for learning mathematics are students' out-of-school experiences. When children's mathematics learning in- and out-of-school are connected, they can support one another, helping children to become more mathematically powerful (Masingila and de Silva 2001). An important consideration, however, is the way in which said contexts are applied. Teachers need to be wary of imposing contrived contexts upon children; rather, mathematical activities should allow children to apply their *own* meaningful contexts. Children can more appropriately bring contexts to bear on situations when those contexts are not directly given by or contained within the problem (Carraher and Schliemann 2002).

One way of incorporating contexts in mathematical activities is through the implementation of open-ended tasks. Open-ended tasks can contribute to teachers' appreciation of mathematical and social learning of students (Stephens and Sullivan 1997). Guided by the work of Sullivan and Lilburn (1997), we refer to open-ended tasks as the type of classroom activity that: 1) requires students to do more than recall known facts or reproduce a skill; 2) has an educative component in that the task allows students to learn from the process of doing the task and informs teachers about students' capabilities; and 3) has more than one possible solution. In general terms, a "good" open-ended task is one in which a teacher receives enlightening responses—work samples that provide insights into students' understandings that could not have been predicted (Smith and MacDonald 2009). Specifically, this investigation involved the implementation of a task which was deliberately vague, allowing children to apply their own meaningful and personalised contexts.

Method

Methodological framework

Kendrick and McKay (2004) have suggested that as teachers seek to acknowledge children's diverse experiences, they must also embrace children's multifaceted ways of knowing, stating that teachers' major pedagogical challenge is to help children transform what they know into modes of representation that allow for a full range of human experience. Goldin and Kaput (1996) define a *representation* as a configuration of some kind that symbolises, interacts with, or otherwise represents something else; and they suggest that representations do not occur in isolation, and "usually belong to highly structured systems, either personal and idiosyncratic or cultural and conventional" (p. 398). Given this definition of representation, Bronfenbrenner's (cf. 1974, 1979, 1988) ecological perspective becomes all the more pertinent—that is, representations should be viewed as constructions of knowledge resulting from children's participation in a variety of contexts.

Vygotsky (1978) viewed representation as a way of knowing, and emphasised the critical role of representation in young children's concept development. For the young child, activities such as drawing bring ideas to the surface (Woleck 2001) and allow for the translation between internal and external representation. Informed by the work of Goldin and Kaput (1996), we refer to *internal* representations as "the mental configurations of individuals [which] are not directly observable" (p. 399). In contrast, *external* representations are "physically embodied, observable configurations… accessible to observation by anyone with suitable knowledge" (Goldin and Kaput 1996, p. 400). With this in mind, the notion of "representation" serves two purposes: 1) the means by which the internal representation is constructed and communicated; and 2) the final external artefact.

Often, representation requires multimodal engagement, integrating several forms of understanding. While young children frequently engage in multimodal expression through a range of activities such as music, dance, drama, and other disciplines (Wright 2006), the focus of this article is on children's meaning making and representation through drawing and storytelling. This process, which Wright terms "drawing-telling," gives children the opportunity to create and share meaning using both verbal and nonverbal modes. This crossover of modes increases children's capacity to use many forms of representational thinking and to mentally manipulate and organise images, ideas (Wright 2007), and experiences.

Data gathering

Thirty-two children in their first year of formal schooling, known as Kindergarten in NSW, participated in this study. Children in NSW commence Kindergarten in late January, and they

> must start school by the time they are 6 years old but they may start in the year they turn 5, provided their fifth birthday is before July 31 of that year. Hence, it is possible for a new Kindergarten class to contain children aged between 4 years 6 months and 6 years. (Perry and Dockett 2005, p. 65)

The children were first visited in February, at the beginning of the first year of formal schooling and before any formal teaching about length had taken place. At this point in time, the children were aged 4 and 5 years. During this visit they were asked to produce a drawing of a ruler, and provide a description of their drawing. The task was deliberately vague to allow for individual interpretation by the children. Rather than the teacher applying a contrived context, these open-ended tasks allowed students to contextualise the concept of length in their own way. In this way, the tasks were inclusive, allowing for individual understandings and experiences.

The design of the task was based on the "drawing-telling" process outlined by Wright (2007), and was inspired by the work of Helen Pengelly (1985, cited in Clarke 1998a) with her "draw a clock" task. Pengelly asked children aged 3–7 years to create a clock face using a variety of resources. The aim of Pengelly's work was to identify stages of development in regards to children's understanding of the clock (Smith and MacDonald 2009). We felt that by gaining insight into children's perceptions of length by asking them to create meaningful representations, we could capture the processes through which children make sense of the concept of length and identify the kinds of background knowledge that children bring to the concept (Woleck 2001). Asking the children to "tell us about your drawing" provided an additional opportunity for those children to reveal their understandings in different but complementary ways. The information revealed during this "drawing-telling" process proved to be far more informative than the drawing was on its own.

The task was then repeated in November of the same year, towards the end of the children's first year of formal schooling, at which point they were aged 5 and 6 years. Once again, the children were encouraged to talk with the researcher throughout the production of their drawing, and/or provide a description of the drawing upon its completion. These descriptions were annotated on the drawing, so that the drawing and annotations could be considered as a single entity.

Data analysis

We set about to establish students' sense-making and representational understandings of linear measurement within a learning matrix. Both authors worked together to devise a matrix which would allow the data (both the drawings and their descriptions) to be coded according to both content knowledge development and degree of contextual richness and, as such, uncover any emerging relationships between the development of content knowledge and the richness of contextual information. The resultant matrix had one axis which described the particular understandings and concepts students possessed or displayed while the other axis described the richness of the context in which the representation or scenario was embedded.

From a measurement concept perspective, we evaluated the work in relation to a body of research literature and Australian curriculum material which has described concept development in relation to attributes of length. The development levels for content reflect the progression from acknowledging the attributes of length, through to recognition of units, and finally recognition of the need for units of equal size (Bobis et al. 2009; Zevenbergen et al. 2004). A similar analysis has been conducted by Mulligan and Mitchelmore (2009) as part of their *Pattern and Structure Assessment* (PASA) which required students to complete a number of tasks, including drawing the features on an empty ruler. In the PASA, representations were categorised according to four stages of structural development, ranging from the representations lacking any evidence of numerical or spatial structure through to the representations correctly integrating numerical and spatial structural features. In this analysis, our content descriptors evolve using a similar progression, but in a manner specific to length concepts.

With respect to context, we made decisions based on Bronfenbrenner's (1974, 1979) descriptions of contexts, and the extent to which the students were able to use applications of measurement understandings in relation to personal contexts (it is worth noting that some of these personalised contexts were not real events). We looked closely at the extent to which specific people, places, and events, and the self, were represented. While ecological analysis in the strictest sense is in and of itself complex and multifaceted, the element of analysis most crucial to this investigation is what Bronfenbrenner describes as *chronosystem analysis*, or the analysis of developmental changes over time. This focuses on: 1) identifying the particular experiences that influenced the person's development; and 2) the specific effects that these experiences produced (Bronfenbrenner 2005).

As a consequence of this two-phase analysis, we were able to assess the tasks in relation to both content knowledge and contextual richness. Table 1 outlines the attributes and identifiable characteristics which allowed the worksamples to be placed in a particular position on the matrix.

Results

As detailed in the previous section, a matrix was used to classify the responses according to both sophistication of content knowledge and richness of contextual information. We were not surprised that students who demonstrated the most limited form of understanding—recognition of the attributes of a ruler—did not represent their ideas in a highly rich vein. These students—for example, Nick, who was able to represent the shape of a ruler but was unable to give any description or explanation, showed a basic understanding of a ruler and its attributes, with some giving background information, usually pertaining to where they had seen a ruler. Responses such as these included that given by Phoebe (Fig. 1), who described her drawing as "This is a measure and it is measuring a house. The dots are the numbers," and Lachlan, who also showed a house being measured.

At the next level of understanding—sequencing of numbers—no students were able to represent a ruler showing sequencing of numbers with only marginal contextual richness. All of the children demonstrating the sequencing of numbers on their ruler embedded either moderate or highly contextual information in their representation, such as Tahlia, who explained "Mum uses the ruler to measure me," and Harry (Fig. 2) in his elaborate drawing and explanation of the many different rulers he has encountered both in and out of school:

> This is the classroom ruler. It's straight and it has numbers on it. This is my Grandma's ruler. It's swirly so you can make round things. This is also my Grandma's ruler. Sometimes she measures stuff and sometimes I measure stuff. Sometimes she doesn't want me to touch it.

Real-life circumstances were increasingly evident at this level of understanding, as evidenced by Chloe, who explained her drawing as "I'm standing on a chair and measuring Mum," and Emily-Rose, who gave the description "My Dad's in the

Table 1 Classification of responses to the “Draw a ruler” task

	Level 1: Recognition of attributes	Level 2: Sequencing of numbers	Level 3: Ordering of numbers/ Equal intervals
High context (c)	Level 1c:	Level 2c:	Level 3c:
	Highly unlikely to occur, because in order to provide a highly contextualised description the response must go beyond simply recognising the attributes.	Evidence of numbers.	Numbers placed in correct order, with equal partitioning.
		Evidence of sequencing, but numbers not necessarily in order.	Typically identifies the usefulness of rulers.
		Typically identifies the usefulness of rulers.	Real-life context consistently evident, with reference to specific people and/or places.
		Real-life context consistently evident, with reference to specific people and/or places.	Identification of self within the context.
		Identification of self within the context.	
Moderate context (b)	Level 1b:	Level 2b:	Level 3b:
	Recognition of a ruler, often with some description of its attributes.	Evidence of numbers.	Numbers placed in correct order, with evidence of equal partitioning.
	Often some reference to the usefulness of rulers.	Evidence of sequencing, but numbers not necessarily in order.	Often some reference to the usefulness of rulers.
	Real-life context more evident, with reference to people and/or places.	Often some reference to the usefulness of rulers.	Real-life context more evident, with reference to people and/or places.
		Real-life context more evident, with reference to people and/or places.	
Limited context (a)	Level 1a:	Level 2a:	Level 3a:
	Recognition of a ruler, often with some description of its attributes.	Evidence of numbers.	Numbers placed in correct order, often with evidence of equal partitioning.
	Little or no reference to context.	Evidence of sequencing, but numbers not necessarily in order.	Little or no reference to context.
		Little or no reference to context.	

camping house measuring wood. He wants to measure it every day. He’s going to fix the bird cage.”

We see the most profound difference between poorly contextualised responses and highly contextualised responses at the most sophisticated level of understanding—ordering of numbers/equal intervals. Very few responses at this level of sophistication were able to provide multiple comparisons with little or

Fig. 1 Phoebe's drawing

moderate contextual information. For example, Jade explained "It is a long ruler. It has numbers on it to see how much it is. No, how *long* it is." The majority of

Fig. 2 Harry's drawing

the responses at this level of understanding showed high contextual richness, and these drawings stood out as having increased evidence of real-life circumstances, as well as a demonstration of the usefulness of rulers. In Willis's drawing (Fig. 3), not only do we see a ruler with numbers accurately represented and equally partitioned, but we also see a real-life situation in which a ruler has been used:

> I put some numbers. They help us find how long and how short things are. I've seen my Daddy use a long ruler before, to measure the door. He took the front flyscreen door out to put new gauze in. He had to measure it to see how much gauze to use.

Zac produced a similarly rich representation, explaining: "It has numbers and lines. They help us measure things. I'm measuring a board, a big board, with a ruler, at my house."

The distribution of responses among the classifications varied between those collected in February and those collected in November. As can be drawn from Table 2, when the task was first implemented in February, 24 out of 32 students (75%) were able to demonstrate levels of understanding beyond simple recognition of the attributes of length, despite the fact they had been at school for only a short period of time and had received no formal instruction in this area. However, only 10 out of 32 students (31%) demonstrated the highest level of understanding at this early stage. Interestingly, 56% of students were able to represent their ideas in a highly rich vein at this early stage. The classification with the highest frequency at this early stage was Level 2c—that is, comparing lengths in addition to identifying the basic attributes of length, with high contextual richness.

Fig. 3 Willis's drawing

Table 2 Responses per category (N=32)

		Level 1: Recognition of attributes	Level 2: Sequencing of numbers	Level 3: Ordering of numbers/ Equal intervals	Total
Feb	(c) High context	0 (0%)	10 (31%)	8 (25%)	18 (56%)
	(b) Moderate context	4 (13%)	4 (13%)	1 (3%)	9 (28%)
	(a) Low context	4 (13%)	0 (0%)	1 (3%)	5 (16%)
	Total	8 (25%)	14 (44%)	10 (31%)	32 (100%)
Nov	(c) High context	0 (0%)	14 (44%)	16 (50%)	30 (94%)
	(b) Moderate context	1 (3%)	0 (0%)	0 (0%)	1 (3%)
	(a) Low context	1 (3%)	0 (0%)	0 (0%)	1 (3%)
	Total	2 (6%)	14 (44%)	16 (50%)	32 (100%)

The task was then repeated in November. Now nearing the end of their Kindergarten year, the children had experienced some formal learning about length, and also had an increased awareness of length due to their participation in the activity earlier in the year. As can be seen in Table 2, there was an increase in the number of children demonstrating Level 3 understanding in their representations. While 31% of students demonstrated Level 3 understanding in February, by November 50% of students demonstrated this level of sophistication in their representations of length. Another significant shift occurred with regard to Level 1. When the task was first completed in February, 25% of students were demonstrating Level 1 knowledge, and this decreased to only 6% when the task was repeated in November.

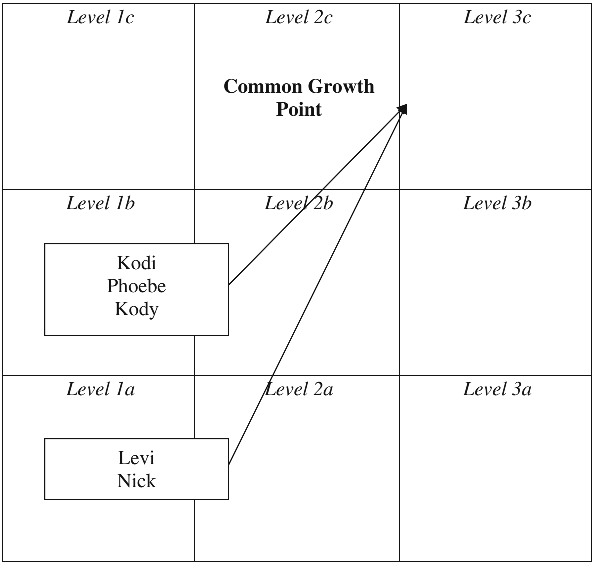
Of course, it could be argued that children are expected to develop their understanding of length during their first year of formal schooling. However, implicit in this expectation is the notion of children developing content knowledge alone. What may be overlooked is the children's increased ability to *contextualise* their mathematical understandings. It can be seen in Table 2 that in February, very few children could demonstrate a high level of understanding with only minimal contextual information, and by November, *all* of the children who demonstrated the highest level of understanding *also* displayed a high degree of contextual richness. It is important to note that when the individual students' classifications from February and November were compared, 50% of children whose content knowledge improved *also* demonstrated increased contextual richness in their representations. As an example of what is meant by this, Fig. 4 maps a selection of children's growth towards a common point—Level 2c.

This result may indicate that the more children can position a particular concept within a personalised context, the more sophisticated their understanding becomes. In this way, the application of personalised contexts guides children towards a personalised meaning of the mathematical concept, and, from this, understanding (Boaler 1993).

Discussion and implications

The results from this study have demonstrated that children possess a good understanding of length upon entering school, and that this understanding continues

Fig. 4 Example growth of selected students from February to November



to develop throughout the first year of schooling. Importantly, however, the results also demonstrate that there is a strong relationship between the richness of the contextual information and the development of measurement understanding. Responses were frequently categorised as demonstrating a highly sophisticated understanding of the concept when they also displayed detailed narrations of the context. This is a notion supported by Chinnappan (2008), who argued "there is now an emerging consensus that learning and the quality of learning is a function of the context and the activity in which learning takes place" (p. 185).

It was clear that the students who demonstrated the most profound development in their content knowledge *also* increased their ability to contextualise this content. While it is true that during the school year the children encountered formal teaching about length which would have impacted upon their understanding, it is important to recognise that children are also continually learning outside the classroom (Aldridge and White 2002). By implementing tasks such as the one described in this article, we are provided a window into the learning in which children engage in contexts outside of school. By analysing the representations in relation to both content *and* context, we are able to see any emerging relationships between children's personal experiences and their developing mathematical understanding. This assertion is supported by Masingila and de Silva (2001) who observe that when children's mathematics learning in and out of school is connected, these learning experiences can support each other, helping children to become more mathematically powerful.

It has been argued that knowledge cannot be separated from the context in which we learn it (Hughes et al. 2000), and it is true that many instructional programs advocate the importance of connecting the mathematics learned in school to the mathematics encountered in "the real world." This usually involves presenting the mathematics content in a traditional "school" manner, and then adjusting it for use in the real world. However, through the implementation of tasks such as the one presented in this article, we are reversing this process, starting with the children's prior "real world" mathematics learning and then considering how this can be utilised in classroom mathematics learning. Connections between in-school and out-of-school mathematics are most commonly formed when children are encouraged to recognise the mathematics learning in which they participate outside of school (Lowrie 2004b).

In light of this, it is important to reconsider existing models of measurement development. When we take the stand point that "real world" measurement learning should be fore-grounded, we must reflect on the implications of this for the standard developmental sequence. "Real world" implies elements of usefulness and applicability, and this was certainly demonstrated in many of the responses to the task, for example Willis's description of using a ruler to find out how much gauze to use on the flyscreen door (see Fig. 3). In cases such as this, the "application" phase of the development sequence actually occurs *first* as opposed to last, informing the development of the other phases of measurement understanding. As such, we propose that measurement development "sequences" instead be considered "cycles," whereby "application" may come first rather than last, and the stages of development become blurred. This is befitting the calls from other researchers to reconsider the sequence in which children engage in measurement experiences, such as Clements (1999) who espoused the development of research activities designed to challenge traditional sequences of instruction. The notion of a developmental cycle also supports the notion of having various "entry points," thus validating the work of researchers whose results indicate children working at developmental levels which disrupt traditional sequences. Ginsburg et al. (1999) observed the sophistication of mathematical activity of children in play environments, and concluded that "young children engage in a variety of mathematical explorations and applications, some of which appear to involve surprisingly 'advanced' content and might even be considered developmentally inappropriate for a preschool or kindergarten curriculum, at least by conventional standards" (p. 89). Boulton-Lewis et al. (1996) found that while traditional sequences advise the use of non-standard units of measurement before advancing to standard units, the young children in their study preferred to use rulers, and could measure correctly with a ruler before they could devise a measurement strategy using non-standard units. Boulton-Lewis et al. concluded that early emphasis on various non-standard units may interfere with children's development of basic measurement concepts required to understand the need for standard units and, in contrast, using standard units and instruments such as rulers appears to be a more interesting and meaningful real-world activity for young children. In this study, we found that many children could represent a ruler with some accuracy, demonstrating a solid understanding of the concepts of equal partitioning and unit iteration. It is important to note, however, that our study also revealed that in order to achieve these higher levels of conceptual understanding, context is critical.

The task presented in this article has demonstrated that representation is a powerful tool for accessing children's knowledge of, and experiences with, measurement. Importantly, representation is not just a procedure by which children record their knowledge about a concept; it is also a process through which understandings can be constructed, re-considered, and applied in new ways. As Wright (2003, p. 24) describes, "children consider multiple interpretations, generate new meanings, and expand existing meanings while drawing and describing their drawings." This was demonstrated by Harry (see Fig. 2), who added to his drawing as his recollection of different rulers increased through the representation process. In this way, representations capture the process of constructing a mathematical concept or relationship, and allow the creator to record and reflect on their thinking (Woleck 2001).

While the learning matrix developed in this study has been used to analyse the responses, it could serve as an effective assessment tool for teachers wanting to implement open-ended tasks such as this in their own classroom. However, teachers wishing to initiate these sorts of assessment activities must be mindful of the need to make these tasks ongoing. As Clarke (1998b) cautions, a single completion of the task may not be a true representation of a child's understanding of the concept. Rather, it may represent a "snapshot" that shows which aspects of the concept the child was attending to at a given point in time, or their greater interest in aesthetic appeal rather than the mathematical ideas in which the teacher is interested (Clarke 1998b). By repeating the "Draw a ruler" task throughout the year, teachers are provided with information about the contexts and experiences that influence children's developing understandings about length, and are offered insight into the personalised ways in which children construct meaning about this complex and abstract mathematical concept.

It is important to note that it is the process of representation that is key, rather than the perceived "quality" of the drawing. However, we recognise that there may be strong relationship between a student's capacity to make sense of measurement understanding and their drawing ability. Given our advocacy for the implementation of drawing tasks such as the one presented in this article, we must consider the extent to which teachers can cope with representations that are not "accurate." Our response to this would be that all representations, regardless of quality, can be used as a catalyst for providing a connection between informal and formal learning, as part of concept development. Through the use of open-ended tasks that support the process of drawing-telling in mathematics, teachers can find out the ways in which students construct their understanding of a concept; identify the factors that influence student learning, such as prior knowledge about a topic and connections between home and schools contexts; provide opportunities for students to connect a mathematical concept with tangible, real-world applications; and encourage students to see themselves inside mathematics that is meaningful and personalised (Smith and MacDonald 2009).

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