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Effective neutrosophic soft set theory and its application to decision-making

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Abstract

The neutrosophic soft set is the most powerful and effective extension of soft sets, which deals with parameterized values of options since it also takes into account uncertain membership and negative membership degrees. Many decision-making models have been created about this set structure, but these models have been processed over the criteria and options and external effects have not been taken into account. But even in daily life, although a option often seems to depend on a parameter, it is obvious that some external influences supporting these parameters are also taken into account. For example, if a disease is to be diagnosed, the symptoms are examined first, but also the patient's medical history, severity of symptoms, genetic and environmental factors, countries recently visited, etc. taking into account a decision. In this study, an effective neutrosophic soft cluster structure will be created that takes into account external effects and assigns truth-membership, indeterminacy-membership and falsity-membership degrees to them. In addition, the Topsis method, which has a very important place in decision-making problems on this structure, will be applied on a hypothetical example.

Keywords Decision making · TOPSIS · Neutrosophic sets · Effective neutrosophic soft sets

Mathematics Subject Classification 03E72 · 03E75 · 03E99 · 54A99 · 20F10 · 90B50

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1 Introduction

Traditional mathematical methods may be insufficient to resolve uncertainties in robotics, engineering, economics, medicine and environment. In order to cope with these inadequacies, researchers searched for appropriate solutions and methods in mathematics. Many more theories have been put forward, the main ones being fuzzy set theory [1], intuitionistic fuzzy set theory [2], soft set theory [3], neutrosophic soft set theory [4]. Fuzzy set theory [1], one of the solutions to uncertainty, was introduced by Zadeh in 1965 and since then this theory has been a powerful tool for solving decision-making problems. But for many researchers, this approach was still insufficient for modeling data and eliminating uncertainty so new set theories continued to be developed. only the positive membership degree of an element is when thought to be insufficient, the intuitionistic fuzzy set theory [2] which takes the negative membership degree in the range [0,1] and defines the sum of positive membership and negative membership degrees as less than and equal to 1, was created by Atanassov. When it was concluded that this theory was insufficient in terms of parameterization, soft set theory [3] was defined by Molodtsov in 1999 and it quickly found its place in many fields. Then, fuzzy soft set [5] and intuitionistic fuzzy soft set [6] structures, which are the binary combinations of this set theory, are presented and intensive studies have been made on these set structures in all sub-branches of mathematics and continue to be done. However, these set theories were still insufficient as they did not evaluate the degree of uncertainty membership of an element, so the neutrosophic set theory [7], which is a generalization of these set theories, was created by Samarandache. Later, the neurososophic soft set theory [4], which is a combination of soft set and neutrosophic set, was first made by Maji, revised and reinterpreted by Deli and Broumi in 2015 [9]. In 2017, Bera introduced neutrosophic soft topological spaces [8], but realizing that there are some structural shortcomings regarding operations on this set, Öztürk redefined some basic concepts such as intersection, union, 'and', 'or' on neutrosophic soft sets [10]. They also reconstructed neutrosophic soft topological spaces using concepts defined in the [10] study and studied separation axioms [11] on this structure. These still-very-popular structures and their combinations are the subjects of new investigations [12–19]. One of them that the concept of efficient fuzzy soft set (EFSS) [20], which was recently defined by Alkhazaleh. In addition, a new algorithm is given to solve decision-making problems based on this concept in the same study.

However, it is obvious that considering only the degree of truth-membership is insufficient to remove uncertainty, and that a more precise decision will be reached thanks to the neutrosophic soft set structure, taking into account the degrees of uncertainty-membership and false-membership. In addition, the concept of the effective neutrosophic soft set will be introduced by considering the effect of external effects on neutrosophic soft sets instead of ignoring the external effects that may affect the decision and dealing with the parameters and the universal set. Finally, The value of each alternative and the weight of each criterion will be characterized by neutrosophic numbers. The importance of criteria and alternatives will be defined by collecting individual opinions of decision-makers through the weighted average operator. It will be shown that the Multi-Criteria Group Decision-making problem with Neutrosophic soft data is easy to use and precise. Finally, to demonstrate the applicability of this method, a numerical experiment on this set structure for supplier selection will be given as an application of the TOPSIS method, which has an important place in decision-making, at the end of this article.

The motivation of this article is that there are various natural disasters in the world and unfortunately many citizens of the world lost their lives because of this. One of these disasters is the earthquake that recently caused great destruction and pain in our country. Since seismic movements have begun to occur frequently and increase in the world, the need for development in earthquake detection and warning technologies has gradually increased. The earthquake warning system does not give the news that an earthquake will occur by making an earthquake prediction as expected. It is a computer system designed to notify nearby areas of a significant earthquake at the time of an earthquake. Thanks to the rapid development of the internet and technology, if we want we can even turn even our phone into a seismometer, so that it can detect earthquakes and send warnings. The event that proved the importance of this system was experienced in Japan, which has the most advanced early warning system. In the earthquake with a magnitude of 9.0 that occurred in the country, the system activated and warned the residents of Tokyo about 80 s before the earthquake. And the country has allocated a large budget to ensure this success. However, recent studies on the internet and cloud systems have reduced these costs considerably. Thanks to the Internet of Things, it is possible to get an earlier earthquake warning while reducing the cost. In this study, an urban transformation project has been rehearsed in a region that is likely to experience destruction during an earthquake. Let's assume that the companies that made a request to realize this project are evaluated. After all companies have been evaluated, the four selected companies will be re-evaluated by two decision makers and the final decision will be made. The suitability of the building materials to be used by these companies for the region will be examined. In addition, some external factors (environmentally friendly, sound insulation, thermal insulation, compressive strength etc.) that will make this material more attractive will be taken into consideration and the most suitable company will be decided.

The article is organized as follows: In the first section, commonly used general definitions of neutrosophic soft sets are given. In Sect. 2 introduces the effective neutrosophic soft set and some general definitions on the set. In Sect. 3, first the algorithm of the TOPSIS method that can be applied on the effective neutrosophic soft set, and then a numerical study in which this algorithm is applied is given.

2 Preliminaries

Definition 1 [7] A neutrosophic set A on the universe of discourse Σ is defined as:

$$A = \{\langle \varepsilon, T_A(\varepsilon), I_A(\varepsilon), F_A(\varepsilon)\rangle : \varepsilon \in \Sigma\},$$

where $T, I, F : \Sigma \to]^{-}0, 1^{+}[$ and $^{-}0 \leq T_A(\varepsilon) + I_A(\varepsilon) + F_A(\varepsilon) \leq 3^{+}$.

Definition 2 [3] Let Σ be an initial universe, Υ be a set of all parameters and $P(\Sigma)$ denotes the power set of Σ. A pair (H, Υ) is called a soft set over Σ, where F is a mapping given by $F : \Upsilon \to P(\Sigma)$.

In other words, the soft set is a parameterized family of subsets of the set Σ. For $\tau \in \Upsilon$, $H(\tau)$ may be considered as the set of $\tau-$elements of the soft set (H, Υ), or as the set of $\tau-$approximate elements of the soft set, i.e.,

$$(H, \Upsilon) = \{(\tau, H(\tau)) : \tau \in \Upsilon, H : \Upsilon \to P(\Sigma)\}.$$

Firstly, neutrosophic soft set defined by Maji [4] and later this concept has been modified by Deli and Bromi [9] as given below:

Definition 3 Let Σ be an initial universe set and Υ be a set of parameters. Let $P(\Sigma)$ denote the set of all neutrosophic sets of Σ. Then, a neutrosophic soft set $\left(\widetilde{H}, \Upsilon\right)$ over Σ is a set defined by a set valued function $\widetilde{H}$ representing a mapping $\widetilde{H}:\Upsilon\rightarrow P(\Sigma)$ where $\widetilde{H}$ is called approximate function of the neutrosophic soft set $\left(\widetilde{H},\Upsilon\right)$. In other words, the neutrosophic soft set is a parameterized family of some elements of the set $P(\Sigma)$ and therefore it can be written as a set of ordered pairs,

$$\left(\widetilde{H},\Upsilon\right)=\left\{\left(\tau,\left\langle\varepsilon,T_{\widetilde{H}(\tau)}(\varepsilon),I_{\widetilde{H}(\tau)}(\varepsilon),F_{\widetilde{H}(\tau)}(\varepsilon)\right\rangle:\varepsilon\in\Sigma\right):\tau\in\Upsilon\right\}$$

where $T_{\widetilde{H}(\tau)}(\varepsilon)$, $I_{\widetilde{H}(\tau)}(\varepsilon)$, $F_{\widetilde{H}(\tau)}(\varepsilon)\in[0,1]$, respectively called the truth-membership, indeterminacy-membership, falsity-membership function of $\widetilde{H}(\tau)$. Since supremum of each T,I,F is 1 so the inequality $0\leq T_{\widetilde{H}(\tau)}(\varepsilon)+I_{\widetilde{H}(\tau)}(\varepsilon)+F_{\widetilde{H}(\tau)}(\varepsilon)\leq 3$ is obvious.

Definition 4 [8] Let $\left(\widetilde{H},\Upsilon\right)$ be neutrosophic soft set over the universe set Σ. The complement of $\left(\widetilde{H},\Upsilon\right)$ is denoted by $\left(\widetilde{H},\Upsilon\right)^c$ and is defined by:

$$\left(\widetilde{H},\Upsilon\right)^c=\left\{\left(\tau,\left\langle\varepsilon,F_{\widetilde{H}(\tau)}(\varepsilon),1-I_{\widetilde{H}(\tau)}(\varepsilon),T_{\widetilde{H}(\tau)}(\varepsilon)\right\rangle:\varepsilon\in\Sigma\right):\tau\in\Upsilon\right\}.$$

Obvious that, $\left(\left(\widetilde{H},\Upsilon\right)^c\right)^c=\left(\widetilde{H},\Upsilon\right)$.

Definition 5 [4] Let $\left(\widetilde{H},\Upsilon\right)$ and $\left(\widetilde{G},\Upsilon\right)$ be two neutrosophic soft sets over the universe set Σ. $\left(\widetilde{H},\Upsilon\right)$ is said to be neutrosophic soft subset of $\left(\widetilde{G},\Upsilon\right)$ if $T_{\widetilde{H}(\tau)}(\varepsilon)\leq T_{\widetilde{G}(\tau)}(\varepsilon)$, $I_{\widetilde{H}(\tau)}(\varepsilon)\leq I_{\widetilde{G}(\tau)}(\varepsilon)$, $F_{\widetilde{H}(\tau)}(\varepsilon)\geq F_{\widetilde{G}(\tau)}(\varepsilon)$, $\forall\tau\in\Upsilon$, $\forall\varepsilon\in\Sigma$. It is denoted by $\left(\widetilde{H},\Upsilon\right)\subseteq\left(\widetilde{G},\Upsilon\right)$.

$\left(\widetilde{H},\Upsilon\right)$ is said to be neutrosophic soft equal to $\left(\widetilde{G},\Upsilon\right)$ if $\left(\widetilde{H},\Upsilon\right)$ is neutrosophic soft subset of $\left(\widetilde{G},\Upsilon\right)$ and $\left(\widetilde{G},\Upsilon\right)$ is neutrosophic soft subset of $\left(\widetilde{H},\Upsilon\right)$. It is denoted by $\left(\widetilde{H},\Upsilon\right)=\left(\widetilde{G},\Upsilon\right)$.

Definition 6 [10] Let $\left(\widetilde{H}_1,\Upsilon\right)$ and $\left(\widetilde{H}_2,\Upsilon\right)$ be two neutrosophic soft sets over the universe set Σ. Then their union is denoted by $\left(\widetilde{H}_1,\Upsilon\right)\Cup\left(\widetilde{H}_2,\Upsilon\right)=\left(\widetilde{H}_3,\Upsilon\right)$ and is defined by:

$$\left(\widetilde{H}_3,\Upsilon\right)=\left\{\left(\tau,\left\langle\varepsilon,T_{\widetilde{H}_3(\tau)}(\varepsilon),I_{\widetilde{H}_3(\tau)}(\varepsilon),F_{\widetilde{H}_3(\tau)}(\varepsilon)\right\rangle:\varepsilon\in\Sigma\right):\tau\in\Upsilon\right\}$$

where

$$T_{\widetilde{H}_3(\tau)}(\varepsilon)=\max\left\{T_{\widetilde{H}_1(\tau)}(\varepsilon),T_{\widetilde{H}_2(\tau)}(\varepsilon)\right\},$$
$$I_{\widetilde{H}_3(\tau)}(\varepsilon)=\max\left\{I_{\widetilde{H}_1(\tau)}(\varepsilon),I_{\widetilde{H}_2(\tau)}(\varepsilon)\right\},$$
$$F_{\widetilde{H}_3(\tau)}(\varepsilon)=\min\left\{F_{\widetilde{H}_1(\tau)}(\varepsilon),F_{\widetilde{H}_2(\tau)}(\varepsilon)\right\}.$$

Definition 7 [10] Let $\left(\widetilde{H}_1,\Upsilon\right)$ and $\left(\widetilde{H}_2,\Upsilon\right)$ be two neutrosophic soft sets over the universe set Σ. Then their intersection is denoted by $\left(\widetilde{H}_1,\Upsilon\right)\Cap\left(\widetilde{H}_2,\Upsilon\right)=\left(\widetilde{H}_3,\Upsilon\right)$ and is defined by:

$$\left(\widetilde{H}_3,\Upsilon\right)=\left\{\left(\tau,\left\langle\varepsilon,T_{\widetilde{H}_3(\tau)}(\varepsilon),I_{\widetilde{H}_3(\tau)}(\varepsilon),F_{\widetilde{H}_3(\tau)}(\varepsilon)\right\rangle:\varepsilon\in\Sigma\right):\tau\in\Upsilon\right\}$$

where

$$T_{\widetilde{H}_3(\tau)}(\varepsilon)=\min\left\{T_{\widetilde{H}_1(\tau)}(\varepsilon),T_{\widetilde{H}_2(\tau)}(\varepsilon)\right\},$$
$$I_{\widetilde{H}_3(\tau)}(\varepsilon)=\min\left\{I_{\widetilde{H}_1(\tau)}(\varepsilon),I_{\widetilde{H}_2(\tau)}(\varepsilon)\right\},$$
$$F_{\widetilde{H}_3(\tau)}(\varepsilon)=\max\left\{F_{\widetilde{H}_1(\tau)}(\varepsilon),F_{\widetilde{H}_2(\tau)}(\varepsilon)\right\}.$$

3 Effective neutrosophic soft sets

Definition 8 A neutrosophic set ∇ in a universe of discourse Υ^*, where $\nabla : \Upsilon^* \to [0, 1]$ is a function, is an effective set. Υ^* is a set of effective parameters that can change membership and it's written in the following way;

$$\nabla = \left\{< \tau^*, (T_\nabla(\tau^*), I_\nabla(\tau^*), F_\nabla(\tau^*)) >: \tau^* \in \Upsilon^*\right\}$$

Definition 9 Let Σ be an initial universe, Υ be a set of all parameters, Υ^* be a set of effective parameters, ∇ be a effective set over $\Upsilon^*$and $P(\Sigma)$ represent the power set of Σ. In this case $(\widetilde{H}, \Upsilon)_\nabla$ is called on effective neutrosophic soft set over Σ, where $\widetilde{H}$ is a mapping represented by $H : \Upsilon \to P(\Sigma)$ and it may be expressed as a collection of ordered pairs;

$$(\widetilde{H}_\nabla, \Upsilon) = \left\{\left(\tau_j, \left\langle \varepsilon_j,\ T_{\widetilde{H}(\tau_i)}(\varepsilon_j)_\nabla, I_{\widetilde{H}(\tau_i)}(\varepsilon_j)_\nabla, F_{\widetilde{H}(\tau_i)}(\varepsilon_j)_\nabla \right\rangle\right) : \varepsilon_i \in \Sigma,\ \tau_j \in \Upsilon\right\}$$

and $T_{\widetilde{H}(\tau_i)}(\varepsilon_j)_\nabla, I_{\widetilde{H}(\tau_i)}(\varepsilon_j)_\nabla, F_{\widetilde{H}(\tau_i)}(\varepsilon_j)_\nabla$ membership values for $\forall \tau_j^* \in \Upsilon^*$ is calculated as

$$T_{\widetilde{H}(\tau)}(\varepsilon_i)_\nabla = \begin{cases} T_{\widetilde{H}(\tau)}(\varepsilon_i) + \left(\frac{(1-T_{\widetilde{H}(\tau)}(\varepsilon_i)).\sum T_{\nabla_{\Sigma_i}}(\tau_j^*)}{|A|}\right) & if\ T_{\widetilde{H}(\tau)}(\varepsilon_i) \in (0, 1) \\ T_{\widetilde{H}(\tau)}(\varepsilon_i) & otherwise \end{cases}$$

$$I_{\widetilde{H}(\tau)}(\varepsilon_i)_\nabla = \begin{cases} I_{\widetilde{H}(\tau)}(\varepsilon_i) + \left(\frac{(1-I_{\widetilde{H}(\tau)}(\varepsilon_i)).\sum I_{\nabla_{\Sigma_i}}(\tau_j^*)}{|A|}\right) & if\ I_{\widetilde{H}(\tau)}(\varepsilon_i) \in (0, 1) \\ I_{\widetilde{H}(\tau)}(\varepsilon_i) & otherwise \end{cases}$$

$$F_{\widetilde{H}(\tau)}(\varepsilon_i)_\nabla = \begin{cases} (1 - F_{\widetilde{H}(\tau)}(\varepsilon_i)) + \left(\frac{(F_{\widetilde{H}(\tau)}(\varepsilon_i)).\sum F_{\nabla_{\Sigma_i}}(\tau_j^*)}{|A|}\right) & if\ F_{\widetilde{H}(\tau)}(\varepsilon_i) \in (0, 1) \\ F_{\widetilde{H}(\tau)}(\varepsilon_i) & otherwise \end{cases}$$

Example 1 Suppose that $\Sigma = \{\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4\}$ be a set of universe and $\Upsilon = \{\tau_1, \tau_2, \tau_3, \tau_4\}$ be a set of all parameters and $\Upsilon^* = \{\tau_1^*, \tau_2^*, \tau_3^*\}$ be a set of effective parameters. Assume the as following is the effective set over Υ^* for each element of set Σ;

$$\nabla(\varepsilon_1) = \left\{\frac{\tau_1^*}{(0.5, 0.6, 0.7)}, \frac{\tau_2^*}{(0.1, 0.7, 0.3)}, \frac{\tau_3^*}{(0.7, 0.4, 0.1)}\right\}$$
$$\nabla(\varepsilon_2) = \left\{\frac{\tau_1^*}{(0.9, 0.1, 0.3)}, \frac{\tau_2^*}{(0.6, 0.4, 0.7)}, \frac{\tau_3^*}{(0.3, 0.8, 0.1)}\right\}$$
$$\nabla(\varepsilon_3) = \left\{\frac{\tau_1^*}{(0.6, 0.2, 0.4)}, \frac{\tau_2^*}{(0.9, 0.3, 0.7)}, \frac{\tau_3^*}{(0.8, 0.2, 0.4)}\right\}$$
$$\nabla(\varepsilon_4) = \left\{\frac{\tau_1^*}{(0.5, 0.5, 0.5)}, \frac{\tau_2^*}{(0.5, 0.1, 0.3)}, \frac{\tau_3^*}{(0.4, 0.1, 0.7)}\right\}$$

and let the neutrosophic soft set $(\widetilde{H}, \Upsilon)$ be given as;

$$(\widetilde{H}, \Upsilon) = \left\{\begin{array}{l} \left(\tau_1, \left\{\frac{\varepsilon_1}{(0.56,0.72,0.10)}, \frac{\varepsilon_2}{(0.59,0.26,0.61)}, \frac{\varepsilon_3}{(0.79,0.61,0.32)}, \frac{\varepsilon_4}{(0.80,0.70,0.67)}\right\}\right) \\ \left(\tau_2, \left\{\frac{\varepsilon_1}{(0.73,0.15,0.60)}, \frac{\varepsilon_2}{(0.76,0.50,0.33)}, \frac{\varepsilon_3}{(0.17,0.80,0.65)}, \frac{\varepsilon_4}{(0.93,0.62,0.77)}\right\}\right) \\ \left(\tau_3, \left\{\frac{\varepsilon_1}{(0.80,0.71,0.17)}, \frac{\varepsilon_2}{(0.80,0.60,0.50)}, \frac{\varepsilon_3}{(0.78,0.60,0.26)}, \frac{\varepsilon_4}{(0.81,0.36,0.13)}\right\}\right) \\ \left(\tau_4, \left\{\frac{\varepsilon_1}{(0.90,0.13,0.31)}, \frac{\varepsilon_2}{(0.35,0.47,0.15)}, \frac{\varepsilon_3}{(0.83,0.30,0.18)}, \frac{\varepsilon_4}{(0.76,0.87,0.93)}\right\}\right) \end{array}\right\}$$

Using the definition 9, the effective set of the set x is calculated as;

$$T_{\widetilde{H}(\tau_1)}(\varepsilon_1)_\nabla = \left\{0.56 + \left(\frac{(1-0.56)(0.50+0.10+0.70)}{3}\right)\right\} = 0.751$$

$$I_{\widetilde{H}(\tau_1)}(\varepsilon_1)_\nabla = \left\{0.72 + \left(\frac{(1-0.72)(0.60+0.70+0.40)}{3}\right)\right\} = 0.876$$

$$F_{\widetilde{H}(\tau_1)}(\varepsilon_1)_\nabla = \left\{(1-0.10) + \left(\frac{(0.10)(0.70+0.30+0.10)}{3}\right)\right\} = 0.937$$

in the same way,

$$T_{\widetilde{H}(\tau_1)}(\varepsilon_2)_\nabla = \left\{0.73 + \left(\frac{(1-0.73)(0.90+0.60+0.30)}{3}\right)\right\} = 0.836$$

$$I_{\widetilde{H}(\tau_1)}(\varepsilon_2)_\nabla = \left\{0.15 + \left(\frac{(1-0.15)(0.10+0.40+0.80)}{3}\right)\right\} = 0.581$$

$$F_{\widetilde{H}(\tau_1)}(\varepsilon_2)_\nabla = \left\{(1-0.60) + \left(\frac{(0.60)(0.30+0.70+0.10)}{3}\right)\right\} = 0.614$$

$$T_{\widetilde{H}(\tau_1)}(\varepsilon_3)_\nabla = \left\{0.80 + \left(\frac{(1-0.80)(0.60+0.90+0.80)}{3}\right)\right\} = 0.951$$

$$I_{\widetilde{H}(\tau_1)}(\varepsilon_3)_\nabla = \left\{0.71 + \left(\frac{(1-0.71)(0.20+0.30+0.20)}{3}\right)\right\} = 0.701$$

$$F_{\widetilde{H}(\tau_1)}(\varepsilon_4)_\nabla = \left\{(1-0.17) + \left(\frac{(0.17)(0.40+0.70+0.40)}{3}\right)\right\} = 0.840$$

$$T_{\widetilde{H}(\tau_1)}(\varepsilon_4)_\nabla = \left\{0.90 + \left(\frac{(1-0.90)(0.50+0.50+0.40)}{3}\right)\right\} = 0.893$$

$$I_{\widetilde{H}(\tau_1)}(\varepsilon_4)_\nabla = \left\{0.13 + \left(\frac{(1-0.13)(0.50+0.10+0.10)}{3}\right)\right\} = 0.770$$

$$F_{\widetilde{H}(\tau_1)}(\varepsilon_4)_\nabla = \left\{(1-0.31) + \left(\frac{(0.31)(0.50+0.30+0.70)}{3}\right)\right\} = 0.665$$

Similarly, when the calculations are continued, the effective neutrosophic soft set is found as follows;

$$(\widetilde{H}_\nabla, \Upsilon) = \left\{\begin{array}{l} \left(\tau_1, \left\{\frac{\varepsilon_1}{(0.75,0.88,0.94)}, \frac{\varepsilon_2}{(0.84,0.58,0.61)}, \frac{\varepsilon_3}{(0.95,0.70,0.84)}, \frac{\varepsilon_4}{(0.89,0.77,0.67)}\right\}\right) \\ \left(\tau_2, \left\{\frac{\varepsilon_1}{(0.85,0.63,0.62)}, \frac{\varepsilon_2}{(0.90,0.72,0.79)}, \frac{\varepsilon_3}{(0.81,0.85,0.68)}, \frac{\varepsilon_4}{(0.96,0.71,0.62)}\right\}\right) \\ \left(\tau_3, \left\{\frac{\varepsilon_1}{(0.89,0.87,0.89)}, \frac{\varepsilon_2}{(0.92,0.77,0.68)}, \frac{\varepsilon_3}{(0.95,0.69,0.87)}, \frac{\varepsilon_4}{(0.90,0.51,0.94)}\right\}\right) \\ \left(\tau_4, \left\{\frac{\varepsilon_1}{(0.94,0.62,0.80)}, \frac{\varepsilon_2}{(0.74,0.70,0.91)}, \frac{\varepsilon_3}{(0.96,0.46,0.91)}, \frac{\varepsilon_4}{(0.87,0.90,0.54)}\right\}\right) \end{array}\right\}$$

Definition 10 Let $\nabla = \{< \tau^*, (T_\nabla(\tau^*), I_\nabla(\tau^*), F_\nabla(\tau^*)) >: \tau^* \in \Upsilon^*\}$ be an effective set over Υ^* and its complement denoted by ∇^c. Then ∇^c is defined as $\left\{ < \tau^*, (F_\nabla(\tau^*), 1 - I_\nabla(\tau^*), T_\nabla(\tau^*)) >: \tau^* \in \Upsilon^* \right\}$

In this case $(\widetilde{H}_{\nabla^c}, \Upsilon)$ is obtained by using the complement of the effective set and without changing the given neutrosophic soft set and $(\widetilde{H}_{\nabla^c}, \Upsilon)$ effective neutrosophic soft set is called $\nabla_{complement}$ of the $(\widetilde{H}_\nabla, \Upsilon)$ effective neutrosophic soft set.

Similarly, $(\widetilde{H}^c_\nabla, \Upsilon)$ is obtained by using the complement of the given neutrosophic soft set and without changing the effective set and $(\widetilde{H}^c_\nabla, \Upsilon)$ effective neutrosophic soft set is called $NS_{complement}$ of the $(\widetilde{H}_\nabla, \Upsilon)$ effective neutrosophic soft set.

And $(\widetilde{H}^c_{\nabla^c}, \Upsilon)$ is obtained by using both the complement of the given neutrosophic soft set and the complement of the effective set and $(\widetilde{H}^c_{\nabla^c}, \Upsilon)$ effective neutrosophic soft set is called $Total_{complement}$ of the $(\widetilde{H}_{\nabla}, \Upsilon)$ effective neutrosophic soft set.

Example 2 If we take into account example1 $(\widetilde{H}_{\nabla^c}, \Upsilon)$, $(\widetilde{H}^c_{\nabla}, \Upsilon)$ and $(\widetilde{H}^c_{\nabla^c}, \Upsilon)$ are calculated as follows, respectively;

$$(\widetilde{H}_{\nabla^c}, \Upsilon) = \left\{ \begin{array}{l} \left(\tau_1, \left\{\frac{\varepsilon_1}{(0.72,0.84,0.94)}, \frac{\varepsilon_2}{(0.74,0.68,0.76)}, \frac{\varepsilon_3}{(0.90,0.91,0.93)}, \frac{\varepsilon_4}{(0.90,0.93,0.64)}\right\}\right) \\ \left(\tau_2, \left\{\frac{\varepsilon_1}{(0.83,0.52,0.66)}, \frac{\varepsilon_2}{(0.85,0.78,0.87)}, \frac{\varepsilon_3}{(0.59,0.95,0.85)}, \frac{\varepsilon_4}{(0.97,0.91,0.59)}\right\}\right) \\ \left(\tau_3, \left\{\frac{\varepsilon_1}{(0.87,0.84,0.90)}, \frac{\varepsilon_2}{(0.87,0.83,0.80)}, \frac{\varepsilon_3}{(0.89,0.91,0.94)}, \frac{\varepsilon_4}{(0.91,0.85,0.93)}\right\}\right) \\ \left(\tau_4, \left\{\frac{\varepsilon_1}{(0.94,0.51,0.82)}, \frac{\varepsilon_2}{(0.59,0.77,0.94)}, \frac{\varepsilon_3}{(0.92,0.84,0.96)}, \frac{\varepsilon_4}{(0.88,0.97,0.50)}\right\}\right) \end{array} \right\}$$

$$(\widetilde{H}^c_{\nabla}, \Upsilon) = \left\{ \begin{array}{l} \left(\tau_1, \left\{\frac{\varepsilon_1}{(0.49,0.69,0.65)}, \frac{\varepsilon_2}{(0.84,0.85,0.63)}, \frac{\varepsilon_3}{(0.84,0.53,0.61)}, \frac{\varepsilon_4}{(0.82,0.46,0.60)}\right\}\right) \\ \left(\tau_2, \left\{\frac{\varepsilon_1}{(0.77,0.94,0.54)}, \frac{\varepsilon_2}{(0.73,0.72,0.52)}, \frac{\varepsilon_3}{(0.92,0.39,0.92)}, \frac{\varepsilon_4}{(0.88,0.53,0.54)}\right\}\right) \\ \left(\tau_3, \left\{\frac{\varepsilon_1}{(0.53,0.69,0.49)}, \frac{\varepsilon_2}{(0.80,0.66,0.49)}, \frac{\varepsilon_3}{(0.83,0.54,0.61)}, \frac{\varepsilon_4}{(0.54,0.72,0.60)}\right\}\right) \\ \left(\tau_4, \left\{\frac{\varepsilon_1}{(0.61,0.94,0.43)}, \frac{\varepsilon_2}{(0.66,0.73,0.78)}, \frac{\varepsilon_3}{(0.81,0.77,0.59)}, \frac{\varepsilon_4}{(0.96,0.33,0.62)}\right\}\right) \end{array} \right\}$$

$$(\widetilde{H}^c_{\nabla^c}, \Upsilon) = \left\{ \begin{array}{l} \left(\tau_1, \left\{\frac{\varepsilon_1}{(0.43,0.59,0.68)}, \frac{\varepsilon_2}{(0.75,0.89,0.76)}, \frac{\varepsilon_3}{(0.66,0.86,0.82)}, \frac{\varepsilon_4}{(0.84,0.84,0.57)}\right\}\right) \\ \left(\tau_2, \left\{\frac{\varepsilon_1}{(0.75,0.92,0.59)}, \frac{\varepsilon_2}{(0.58,0.79,0.70)}, \frac{\varepsilon_3}{(0.83,0.81,0.96)}, \frac{\varepsilon_4}{(0.89,0.86,0.50)}\right\}\right) \\ \left(\tau_3, \left\{\frac{\varepsilon_1}{(0.47,0.60,0.55)}, \frac{\varepsilon_2}{(0.68,0.74,0.68)}, \frac{\varepsilon_3}{(0.63,0.86,0.82)}, \frac{\varepsilon_4}{(0.57,0.92,0.57)}\right\}\right) \\ \left(\tau_4, \left\{\frac{\varepsilon_1}{(0.56,0.93,0.49)}, \frac{\varepsilon_2}{(0.46,0.80,0.86)}, \frac{\varepsilon_3}{(0.59,0.93,0.81)}, \frac{\varepsilon_4}{(0.97,0.80,0.60)}\right\}\right) \end{array} \right\}$$

Definition 11 Let $(F_{\nabla_1}, \Upsilon_1)$ and $(G_{\nabla_2}, \Upsilon_2)$ be two effective neutrosophic soft set over Σ and the union of this two sets be denoted (H_{∇_u}, Υ).

$$H_{\nabla_u}(\tau) = \begin{cases} F_{\nabla_u}(\tau) & if\ \tau \in \Upsilon_1 - \Upsilon_2 \\ G_{\nabla_u}(\tau) & if\ \tau \in \Upsilon_2 - \Upsilon_1 \\ (F \cup G)_{\nabla_u}(\tau) & if\ \tau \in \Upsilon_1 \cap \Upsilon_2 \end{cases}$$

where $\Upsilon = \Upsilon_1 \cup \Upsilon_2$, $\forall \tau \in \Upsilon$ and u is the union of the effective sets ∇_1 and ∇_2.

Definition 12 Let $(F_{\nabla_1}, \Upsilon_1)$ and $(G_{\nabla_2}, \Upsilon_2)$ be two effective neutrosophic soft set over Σ and the intersection of this two sets be denoted (K_{∇_u}, Υ).

$$K_{\nabla_\iota}(\tau) = \begin{cases} F_{\nabla_\iota}(\tau) & if\ \tau \in \Upsilon_1 - \Upsilon_2 \\ G_{\nabla_\iota}(\tau) & if\ \tau \in \Upsilon_2 - \Upsilon_1 \\ (F \cap G)_{\nabla_\iota}(\tau) & if\ \tau \in \Upsilon_1 \cap \Upsilon_2 \end{cases}$$

where $\Upsilon = \Upsilon_1 \cup \Upsilon_2$, $\forall \tau \in \Upsilon$ and ι is the intersection of the effective sets ∇_1 and ∇_2.

Example 3 We take into account example1 and let

$$\begin{aligned} \nabla_1(x_1) &= \left\{\frac{e_1^*}{(0.5,0.6,0.7)}, \frac{e_2^*}{(0.1,0.7,0.3)}, \frac{e_3^*}{(0.7,0.4,0.1)}\right\} \\ \nabla_1(x_2) &= \left\{\frac{e_1^*}{(0.9,0.1,0.3)}, \frac{e_2^*}{(0.6,0.4,0.7)}, \frac{e_3^*}{(0.3,0.8,0.1)}\right\} \\ \nabla_1(x_3) &= \left\{\frac{e_1^*}{(0.6,0.2,0.4)}, \frac{e_2^*}{(0.9,0.3,0.7)}, \frac{e_3^*}{(0.8,0.2,0.4)}\right\} \\ \nabla_1(x_4) &= \left\{\frac{e_1^*}{(0.5,0.5,0.5)}, \frac{e_2^*}{(0.5,0.1,0.3)}, \frac{e_3^*}{(0.4,0.1,0.7)}\right\} \end{aligned}$$

and

$$
\begin{aligned}
\nabla_2(x_1) &= \left\{\frac{e_1^*}{(0.1,0.6,0.3)}, \frac{e_2^*}{(0.7,0.1,0.4)}, \frac{e_3^*}{(0.5,0.1,0.2)}\right\}\\
\nabla_2(x_2) &= \left\{\frac{e_1^*}{(0.3,0.8,0.7)}, \frac{e_2^*}{(0.9,0.7,0.6)}, \frac{e_3^*}{(0.5,0.7,0.3)}\right\}\\
\nabla_2(x_3) &= \left\{\frac{e_1^*}{(0.6,0.5,0.9)}, \frac{e_2^*}{(0.9,0.6,0.7)}, \frac{e_3^*}{(0.5,0.6,0.4)}\right\}\\
\nabla_2(x_4) &= \left\{\frac{e_1^*}{(0.7,0.1,0.5)}, \frac{e_2^*}{(0.6,0.2,0.4)}, \frac{e_3^*}{(0.8,0.4,0.3)}\right\}
\end{aligned}
$$

$$
(\widetilde{H}, \Upsilon_1) = \left\{\begin{array}{l}
\left(\tau_1, \left\{\frac{\varepsilon_1}{(0.56,0.72,0.10)}, \frac{\varepsilon_2}{(0.59,0.26,0.61)}, \frac{\varepsilon_3}{(0.79,0.61,0.32)}, \frac{\varepsilon_4}{(0.80,0.70,0.67)}\right\}\right)\\
\left(\tau_2, \left\{\frac{\varepsilon_1}{(0.73,0.15,0.60)}, \frac{\varepsilon_2}{(0.76,0.50,0.33)}, \frac{\varepsilon_3}{(0.17,0.80,0.65)}, \frac{\varepsilon_4}{(0.93,0.62,0.77)}\right\}\right)\\
\left(\tau_3, \left\{\frac{\varepsilon_1}{(0.80,0.71,0.17)}, \frac{\varepsilon_2}{(0.80,0.60,0.50)}, \frac{\varepsilon_3}{(0.78,0.60,0.26)}, \frac{\varepsilon_4}{(0.81,0.36,0.13)}\right\}\right)\\
\left(\tau_4, \left\{\frac{\varepsilon_1}{(0.90,0.13,0.31)}, \frac{\varepsilon_2}{(0.35,0.47,0.15)}, \frac{\varepsilon_3}{(0.83,0.30,0.18)}, \frac{\varepsilon_4}{(0.76,0.87,0.93)}\right\}\right)
\end{array}\right\},
$$

$$
(\widetilde{G}, \Upsilon_2) = \left\{\begin{array}{l}
\left(\tau_1, \left\{\frac{\varepsilon_1}{(0.25,0.42,0.50)}, \frac{\varepsilon_2}{(0.65,0.54,0.21)}, \frac{\varepsilon_3}{(0.60,0.14,0.67)}, \frac{\varepsilon_4}{(0.14,0.52,0.80)}\right\}\right)\\
\left(\tau_2, \left\{\frac{\varepsilon_1}{(0.44,0.45,0.52)}, \frac{\varepsilon_2}{(0.25,0.35,0.50)}, \frac{\varepsilon_3}{(0.47,0.23,0.15)}, \frac{\varepsilon_4}{(0.58,0.44,0.40)}\right\}\right)\\
\left(\tau_3, \left\{\frac{\varepsilon_1}{(0.15,0.54,0.12)}, \frac{\varepsilon_2}{(0.84,0.54,0.20)}, \frac{\varepsilon_3}{(0.90,0.21,0.30)}, \frac{\varepsilon_4}{(0.81,0.12,0.23)}\right\}\right)\\
\left(\tau_4, \left\{\frac{\varepsilon_1}{(0.14,0.52,0.80)}, \frac{\varepsilon_2}{(0.58,0.44,0.40)}, \frac{\varepsilon_3}{(0.74,0.21,0.41)}, \frac{\varepsilon_4}{(0.50,0.41,0.51)}\right\}\right)
\end{array}\right\}
$$

where $\Upsilon_1, \Upsilon_2 \subset \Upsilon$. With the given information, the effective neutrosophic soft combination is calculated as follows;

$$
\nabla_1 \cup \nabla_2 = \nabla = \begin{aligned}
\nabla_u(\varepsilon_1) &= \left\{\frac{\tau_1^*}{(0.5,0.6,0.3)}, \frac{\tau_2^*}{(0.7,0.7,0.3)}, \frac{\tau_3^*}{(0.7,0.4,0.1)}\right\}\\
\nabla_u(\varepsilon_2) &= \left\{\frac{\tau_1^*}{(0.9,0.8,0.3)}, \frac{\tau_2^*}{(0.9,0.7,0.6)}, \frac{\tau_3^*}{(0.5,0.8,0.1)}\right\}\\
\nabla_u(\varepsilon_3) &= \left\{\frac{\tau_1^*}{(0.6,0.2,0.4)}, \frac{\tau_2^*}{(0.9,0.6,0.7)}, \frac{\tau_3^*}{(0.8,0.6,0.4)}\right\}\\
\nabla_u(\varepsilon_4) &= \left\{\frac{\tau_1^*}{(0.7,0.5,0.5)}, \frac{\tau_2^*}{(0.6,0.2,0.3)}, \frac{\tau_3^*}{(0.8,0.4,0.3)}\right\}
\end{aligned}
$$

$$
(\widetilde{H}, \Upsilon_1) \cup (\widetilde{G}, \Upsilon_2) = (\widetilde{S}, \Upsilon) =
$$

$$
\left\{\begin{array}{l}
\left(\tau_1, \left\{\frac{\varepsilon_1}{(0.56,0.72,0.10)}, \frac{\varepsilon_2}{(0.65,0.54,0.21)}, \frac{\varepsilon_3}{(0.79,0.61,0.32)}, \frac{\varepsilon_4}{(0.80,0.70,0.67)}\right\}\right)\\
\left(\tau_2, \left\{\frac{\varepsilon_1}{(0.73,0.45,0.52)}, \frac{\varepsilon_2}{(0.76,0.50,0.33)}, \frac{\varepsilon_3}{(0.47,0.80,0.15)}, \frac{\varepsilon_4}{(0.93,0.62,0.40)}\right\}\right)\\
\left(\tau_3, \left\{\frac{\varepsilon_1}{(0.80,0.71,0.12)}, \frac{\varepsilon_2}{(0.84,0.60,0.20)}, \frac{\varepsilon_3}{(0.90,0.60,0.26)}, \frac{\varepsilon_4}{(0.81,0.36,0.13)}\right\}\right)\\
\left(\tau_4, \left\{\frac{\varepsilon_1}{(0.90,0.13,0.14)}, \frac{\varepsilon_2}{(0.74,0.60,0.15)}, \frac{\varepsilon_3}{(0.83,0.30,0.18)}, \frac{\varepsilon_4}{(0.76,0.87,0.51)}\right\}\right)
\end{array}\right\}.
$$

Then, $(\widetilde{S}_{\nabla_u}, \Upsilon)$ effective neutrosophic soft set x is found as follows;

$$
(\widetilde{S}_{\nabla_u}, \Upsilon) = \left\{\begin{array}{l}
\left(\tau_1, \left\{\frac{\varepsilon_1}{(0.84,0.88,0.92)}, \frac{\varepsilon_2}{(0.92,0.89,0.86)}, \frac{\varepsilon_3}{(0.95,0.83,0.84)}, \frac{\varepsilon_4}{(0.94,0.81,0.58)}\right\}\right)\\
\left(\tau_2, \left\{\frac{\varepsilon_1}{(0.90,0.76,0.60)}, \frac{\varepsilon_2}{(0.94,0.88,0.78)}, \frac{\varepsilon_3}{(0.88,0.91,0.93)}, \frac{\varepsilon_4}{(0.98,0.76,0.75)}\right\}\right)\\
\left(\tau_3, \left\{\frac{\varepsilon_1}{(0.93,0.87,0.91)}, \frac{\varepsilon_2}{(0.96,0.91,0.87)}, \frac{\varepsilon_3}{(0.98,0.83,0.87)}, \frac{\varepsilon_4}{(0.94,0.60,0.92)}\right\}\right)\\
\left(\tau_4, \left\{\frac{\varepsilon_1}{(0.96,0.62,0.89)}, \frac{\varepsilon_2}{(0.94,0.91,0.90)}, \frac{\varepsilon_3}{(0.96,0.70,0.91)}, \frac{\varepsilon_4}{(0.93,0.92,0.68)}\right\}\right)
\end{array}\right\}.
$$

Similarly, when the calculations are made, $(\widetilde{K}_{\nabla_\iota}, \Upsilon)$ is obtained as follows;

$$(\widetilde{K}_{\nabla_\iota}, \Upsilon) = \left\{ \begin{array}{l} \left(\tau_1, \left\{\frac{\varepsilon_1}{(0.43,0.58,0.72)}, \frac{\varepsilon_2}{(0.75,0.56,0.74)}, \frac{\varepsilon_3}{(0.87,0.34,0.78)}, \frac{\varepsilon_4}{(0.54,0.57,0.63)}\right\}\right) \\ \left(\tau_2, \left\{\frac{\varepsilon_1}{(0.57,0.38,0.66)}, \frac{\varepsilon_2}{(0.55,0.61,0.78)}, \frac{\varepsilon_3}{(0.72,0.41,0.78)}, \frac{\varepsilon_4}{(0.78,0.50,0.64)}\right\}\right) \\ \left(\tau_3, \left\{\frac{\varepsilon_1}{(0.35,0.66,0.90)}, \frac{\varepsilon_2}{(0.88,0.72,0.78)}, \frac{\varepsilon_3}{(0.93,0.39,0.90)}, \frac{\varepsilon_4}{(0.86,0.29,0.81)}\right\}\right) \\ \left(\tau_4, \left\{\frac{\varepsilon_1}{(0.68,0.35,0.82)}, \frac{\varepsilon_2}{(0.61,0.68,0.90)}, \frac{\varepsilon_3}{(0.94,0.33,0.92)}, \frac{\varepsilon_4}{(0.73,0.47,0.57)}\right\}\right) \end{array} \right\}.$$

4 Effective neutrosophic soft topsis method

4.1 Topsis algorithm in effective neutrosophic soft sets

Assume that $\Sigma = \{\varepsilon_1, \varepsilon_2, ..., \varepsilon_m\}$ is a set of options, $\Upsilon = \{\tau_1, \tau_2, ..., \tau_n\}$ ia a set of criteria and $\Upsilon^* = \{\tau_1^*, \tau_2^*, ..., \tau_k^*\}$ is a set of effective parameters. We create the following steps for the Effective Neutrosophic Soft set TOPSIS process:

Step 1. We determine the weight of decision makers by means of numerical values corresponding to linguistic variables. Assuming that the decision maker group is $D = \{D_1, D_2, ..., D_d\}$, we can calculate the each parameter weight of d decision makers as follows:

$$\epsilon_n = \left(\frac{\sum_{i=1}^{d} T_i}{d}, \frac{\sum_{i=1}^{d} I_i}{d}, \frac{\sum_{i=1}^{d} F_i}{d}\right)$$

Here, the weight of each parameter is calculated by considering the truth membership, indeterminacy membership and falsity membership values specified by the decision makers for that parameter.

Step 2. It is necessary to form a joint group decision by collecting the individual decisions of the decision makers. For this, the following calculation is made by taking into account the numerical equivalents of the linguistic importance rate stated by the decision makers;

$$(a_{ij}, b_{ij}, c_{ij}) = \left\{\min\{a_{ij}^d\}, \frac{\sum_{d=1}^{d} b_{ij}^d}{d}, \max\{c_{ij}^d\}\right\}$$

where (a_{ij}, b_{ij}, c_{ij}) is the numerical equivalent of the linguistic terms determined by the d decision makers for the i. option and j. criteria in the group decision result. Then, a normalized decision matrix is created by calculating $\left(\frac{a_{ij}}{c_j^*}, \frac{b_{ij}}{c_j^*}, \frac{c_{ij}}{c_j^*}\right)$, where $c^* = \max\{c_{ij}\}$. The matrix in the neutrosophic soft set format obtained here is processed with the effective parameter values, and a new matrix is obtained in the effective neutrosophic soft set format.

Step 3. Using the weights (W) of the criteria and the normalized decision matrix (N) in $ENSS(\Sigma, \Upsilon)$ format obtained, we obtain the normalized weighted decision matrix. Let suppose that $N^* = (n_{ij})$. Then its is defined by $N^* = W \otimes N$, here $n_{ij} = w_j \otimes n_{ij} = (a_{ij}, b_{ij}, c_{ij})$.Consequently, the aggregate effective neutrosophic soft matrix of the criteria can be stated as

$$N^* = \begin{pmatrix} \mathrm{uw}_{11} & \mathrm{uw}_{12} & \ldots & \mathrm{uw}_{1j} \\ \mathrm{uw}_{21} & \mathrm{uw}_{22} & \ldots & \mathrm{uw}_{2j} \\ \vdots & \vdots & \ddots & \vdots \\ \mathrm{uw}_{i1} & \mathrm{uw}_{i2} & \ldots & \mathrm{uw}_{ij} \end{pmatrix}$$

.

Step 4. In this step, the positive ideal solution and the negative ideal solution are calculated. According to the principle of the TOPSIS method, ENSS-PIS and ENSS-NIS can be defined respectively as follows;

$$A^{+}=(a_{u^{+}w}(\mu_j), b_{u^{+}w}(\mu_j), c_{u^{+}w}(\mu_j))$$
$$A^{-}=(a_{u^{-}w}(\mu_j), b_{u^{-}w}(\mu_j), c_{u^{-}w}(\mu_j))$$

where

$$A^{+}=\begin{cases}\left(\max(a_{u_i w}(\mu_j), \max(b_{u_i w}(\mu_j), \min(c_{u_i w}(\mu_j)\right) & \text{if j is benefit criteria}\\ (\min(a_{u_i w}(\mu_j), \min(b_{u_i w}(\mu_j), \max(c_{u_i w}(\mu_j)) & \text{if j is cost criteria}\end{cases}$$
$$A^{-}=\begin{cases}(\min(a_{u_i w}(\mu_j), \min(b_{u_i w}(\mu_j), \max(c_{u_i w}(\mu_j) & \text{if j is benefit criteria}\\ (\max(a_{u_i w}(\mu_j), \max(b_{u_i w}(\mu_j), \min(c_{u_i w}(\mu_j)) & \text{if j is cost criteria}\end{cases}$$

Step 5. In this step, the distance of each alternative to ENSS-PIS and ENSS-NIS is calculated. The equation used for this is as given below;

$$D^{+}=\sqrt{\frac{1}{3}\sum_{j=1}^{n}\left\{(\mid a_{ij}-a_j^{+}\mid)^2+(\mid b_{ij}-b_j^{+}\mid)^2+(\mid c_{ij}-c_j^{+}\mid)^2\right\}}$$
$$D^{-}=\sqrt{\frac{1}{3}\sum_{j=1}^{n}\left\{(\mid a_{ij}-a_j^{-}\mid)^2+(\mid b_{ij}-b_j^{-}\mid)^2+(\mid c_{ij}-c_j^{-}\mid)^2\right\}}$$

Step 6. Finally, using $CC_i=\frac{D^{-}}{D^{-}+D^{+}}$, we determine the relative proximity coefficient of each option to single-valued neutrosophic ideal solutions.

Step 7. The priority order of the options is determined by sorting all the options in descending order according to the relative closeness coefficient.

Theories	Disadvantage
Fuzzy sets [1]	Lacks information regarding the indeterminacy and non-membership
Intuitionistic fuzzy sets [2]	Lacks information regarding the indeterminacy
Pythagorean fuzzy sets [21]	Lacks information regarding the indeterminacy
Fermatean fuzzy sets [22]	Lacks information regarding the indeterminacy
Linear diophantine fuzzy sets [23]	Lacks information regarding the indeterminacy
Neutrosophic sets [7]	Lacks information regarding external effects
Effective fuzzy soft sets [20]	Lacks information regarding the indeterminacy and non-membership

4.2 Numerical example

Assume that the companies that make a request to realize the urban transformation project planned to be carried out in a city will be evaluated. the four companies selected as a result of the evaluation of all companies be re-evaluated by two decision makers and the final decision will be made (Tables 1, 2).

Let $\Sigma=\{\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4\}$ be set of four construction companies and $\Upsilon=\{\tau_1, \tau_2, \tau_3\}$ be set of building material. The characteristics of the specified Υ parameters are as follow; τ_1 =aerated, τ_2 =stone, τ_3 =brick. Moreover, let $\Upsilon^{*}=\{\tau_1^{*}, \tau_2^{*}, \tau_3^{*}\}$ be a set of effective parameters as follow; τ_1^{*} =lightness of the material, τ_2^{*} =heat and sound insulation feature, τ_3^{*} =cost. Since the aim of both experts is to make durable, comfortable, environmentally

Table 1 The decision criteria's importance weights

	D_1	D_2
e_1	M	VH
e_2	VH	L
e_3	VL	H

Table 2 Importance weight as linguistic veriable

Very high (VH)	(0.8, 0.9, 0.9)
High (H)	(0.6, 0.7, 0.8)
Medium (M)	(0.5, 0.6, 0.7)
Little (L)	(0.2, 0.3, 0.5)
Very little (VL)	(0, 0, 0.1)

Table 3 Linguistic terms that indicate the importance of option

Criterias	Candidates	Decision D_1	Makers D_2
τ_1	ε_1	MG	VG
	ε_2	N	N
	ε_3	G	MG
	ε_4	W	VW
τ_2	ε_1	G	MG
	ε_2	VG	G
	ε_3	N	VG
	ε_4	G	VG
τ_3	ε_1	VG	G
	ε_2	G	G
	ε_3	VW	W
	ε_4	VG	G

friendly buildings and the best for their public, the effective set values for both will be the same and will be taken as follows;

$$\nabla(\varepsilon_1) = \left\{\frac{\tau_1^*}{(0.8, 0.5, 0.3)}, \frac{\tau_2^*}{(0.77, 0.34, 0.36)}, \frac{\tau_3^*}{(0.31, 0.02, 0.13)}\right\}$$
$$\nabla(\varepsilon_2) = \left\{\frac{\tau_1^*}{(0.23, 0.45, 0.94)}, \frac{\tau_2^*}{(0.71, 0, 0.23)}, \frac{\tau_3^*}{(0.6, 0.14, 1)}\right\}$$
$$\nabla(\varepsilon_3) = \left\{\frac{\tau_1^*}{(0.45, 0.54, 0.15)}, \frac{\tau_2^*}{(0.67, 0, 0.02, 0.8)}, \frac{\tau_3^*}{(0.5, 0.46, 0.36)}\right\}$$
$$\nabla(\varepsilon_4) = \left\{\frac{\tau_1^*}{(0.84, 0.5, 0.451)}, \frac{\tau_2^*}{(0.6, 0.14, 1)}, \frac{\tau_3^*}{(0.25, 0.8, 0.25)}\right\}$$

The decision-makers utilize a linguistic set of weights to determine the performance of each criterion. The information on weights provided to the three criteria by the two decision-makers are presented in Table 3.

Table 4 Importance rate of options as linguistic variable

Very good (VG)	(8, 9, 9)
Good (G)	(7, 7, 8)
Mid good (MG)	(6, 7, 8)
Neutral (N)	(5, 6, 7)
Mid weak (MW)	(3, 4, 5)
Weak (W)	(2, 3, 5)
Very weak (VW)	(1, 1, 2)

In addition, linguistic terms used to rate the importance of each option under each parameter for two decision makers are given in Table 3 and numerical values corresponding to these linguistic terms are given in Table 4.

Following that, we employ the efficient neutrosophic TOPSIS process, which is as follows:

Step 1: In order to calculate the weights of each parameter as specified by the decision makers, the values corresponding to the linguistic data are taken into account and the table below is created.

	D_1	D_2
τ_1	(0.50,0.60,0.70)	(0.80,0.90,0.90)
τ_2	(0.80,0.90,0.90)	(0.20,0.30,0.50)
τ_3	(0.00,0.00,0.10)	(0.60,0.70,0.80)

When the operation specified in step 1 is performed, the common weight of each parameter is found as follows;

τ_1	(0.6,0.75,0.80)
τ_2	(0.50,0.60,0.70)
τ_3	(0.30,0.35,0.45)

Step 2: Considering Tables 3 and 4 above, normalize decision matrix is calculated as follows;

	τ_1	τ_2	τ_3
ε_1	(0.67, 0.89, 1)	(0.67, 0.83, 1)	(0.78, 0.94, 1)
ε_2	(0.56, 0.67, 0.78)	(0.78, 0.94, 1)	(0.78, 0.89, 1)
ε_3	(0.67, 0.83, 1)	(0.56, 0.83, 1)	(0.11, 0.22, 0.56)
ε_4	(0.11, 0.22, 0.56)	(0.78, 0.94, 1)	(0.78, 0.94, 1)

Taking into account the effective set values given above and the table obtained in $NSS(\Sigma, \Upsilon)$ format in this step, the $ENSS(\Sigma, \Upsilon)$ table is created;

	τ_1	τ_2	τ_3
ε_1	(0.88, 0.92, 0.26)	(0.88, 0.88, 0.26)	(0.92, 0.84, 0.26)
ε_2	(0.81, 0.74, 0.65)	(0.91, 0.96, 0.55)	(0.91, 0.83, 0.55)
ε_3	(0.85, 0.89, 0.44)	(0.80, 0.89, 044)	(0.59, 0.41, 0.69)
ε_4	(0.61, 0.60, 0.76)	(0.90, 0.97, 0.57)	(0.90, 0.90, 0.57)

Step 3: The weighted normalized effective neutrosophic soft decision table is obtained by considering the weights found in step 1 and the last matrix obtained in the previous step.

	τ_1	τ_2	τ_3
ε_1	(0.57, 0.69, 0.21)	(0.44, 0.53, 0.18)	(0.28, 0.29, 0.12)
ε_2	(0.53, 0.56, 0.52)	(0.45, 0.57, 0.39)	(0.27, 0.29, 0.25)
ε_3	(0.55, 0.67, 0.35)	(0.40, 0.53, 0.31)	(0.18, 0.14, 0.31)
ε_4	(0.40, 0.45, 0.61)	(0.45, 0.58, 0.40)	(0.27, 0.32, 0.26)

Step 4: With the help of the table in step 3, the positive ideal solution (A^+) and the negative ideal solution (A^-) are calculated.

A^+	(0.57,0.69,0.61)	(0.45,0.58,0.40)	(0.28,0.32,0.31)
A^-	(0.40,0.45,0.21)	(0.40,0.53,0.18)	(0.18,0.14,0.12)

Step 5: The distance of each alternative to ENSS-PIS and ENSS-NIS, respectively, is calculated as follows;

	τ_1	τ_2	τ_3		τ_1	τ_2	τ_3
ε_1	0.23	0.13	0.11	ε_1	0.17	0.02	0.10
ε_2	0.09	0.01	0.04	ε_2	0.20	0.12	0.12
ε_3	0.15	0.07	0.11	ε_3	0.17	0.07	0.11
ε_4	0.17	0	0.03	ε_4	0.23	0.13	0.14

Step 6: Considering the table obtained in the previous step, the relative closeness coefficient of each option to the univalent neutrosophic ideal solutions is obtained as follows;

	D^+	D^-	CC_i
ε_1	0.47	0.30	0.39
ε_2	0.14	0.45	0.76
ε_3	0.33	0.35	0.52
ε_4	0.20	0.50	0.71

Step 7: When the relative closeness coefficient values are sorted by decreasing value, the line up of the four alternatives is $\varepsilon_2\rangle\varepsilon_4\rangle\varepsilon_3\rangle\varepsilon_1$. In this case, the most ideal one among these four alternatives is the ε_2 alternative.

5 Conclusion

The proposed effective neutrosophic soft decision-making method can be said to be suitable for science and engineering applications because we can process inconsistent information and ambiguous information that exist in the real world. In addition, all researchers are interested in parameters and the universal set, ignoring external influences that may affect their decisions. In this study, the effect of external effects on effective neutrosophic soft sets and the results of the decisions made by these sets were examined. Afterwards, Topsis, which is a very important technique for multi-criteria decision-making on this cluster, was used. This method measures distance directly, without selecting a priority order or reference point like the others. It is based on the principle that the solution with the shortest distance to the positive ideal value and the longest distance to the negative ideal value should be the best result. Therefore, it is thought that the most precise result will be obtained by considering many factors in the decision-making process. In this study, apart from indeterminacy-membership and falsity-membership degrees, external factors are also given a membership degree, and a more precise result is tried to be obtained thanks to a more competitive structure.

Data Availability A scenario has been adapted to show the real-life place of the theoretical knowledge presented in this article. This scenario is hypothetical so there is no data source to report.

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