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Coefficient bounds and Fekete–Szegö Inequality for a new family of bi-univalent functions defined by Horadam polynomials

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Abstract
In the current article, we introduce and investigate a new family $\mathcal{K}_{\Sigma}(\delta, \lambda, x)$ of analytic and bi-univalent functions by using the Horadam polynomials defined in the open unit disk $\mathbb{U}$. We determine upper bounds for the initial Taylor–Maclaurin coefficients. Further we obtain the Fekete–Szegö inequality of functions belonging to this family. We also point out several certain special cases for our results.

Keywords Bi-univalent function · Horadam polynomials · Upper Bounds · Fekete–Szegö problem · Subordination

Mathematics Subject Classification Primary 30C45; Secondary 30C50

1 Introduction

Denote by $\mathcal{A}$ the collection of analytic functions in the open unit disk $\mathbb{U} = \{z \in \mathbb{C} : |z| < 1\}$ that have the following normalized form:

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n. \tag{1.1}$$

Further, let S indicate the subclass of $\mathcal{A}$ consisting of functions which are univalent in $\mathbb{U}$. According to the Koebe one-quarter theorem [6] every function $f \in S$ has an inverse f^{-1} defined by $f^{-1}(f(z)) = z$, $(z \in \mathbb{U})$ and $f(f^{-1}(w)) = w$, $(|w| < r_0(f), r_0(f) \geqq \frac{1}{4})$,

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where

$$g(w) = f^{-1}(w) = w - a_2w^2 + \left(2a_2^2 - a_3\right) w^3 - \left(5a_2^3 - 5a_2a_3 + a_4\right) w^4 + \cdots . \quad (1.2)$$

A function $f \in \mathcal{A}$ is said to be bi-univalent in $\mathbb{U}$ if both f and f^{-1} are univalent in $\mathbb{U}$. Let Σ stands for the class of bi-univalent functions in $\mathbb{U}$ given by (1.1). In fact, Srivastava et al. [25] have actually revived the study of analytic and bi-univalent functions in recent years, it was followed by such works as those by Caglar et al. [5], Bulut [4], Adegani et al. [2] and others (see, for example [18, 20–22, 29, 30, 32]). From the work of Srivastava et al. [25], we choose to recall the following examples of functions in the class Σ :

$$\frac{z}{1-z}, \quad -\log(1-z) \quad \text{and} \quad \frac{1}{2}\log\left(\frac{1+z}{1-z}\right).$$

We notice that the class Σ is not empty. However, the Koebe function is not a member of Σ.

The problem to find the general coefficient bounds on the Taylor–Maclaurin coefficients $|a_n|$ $(n \in \mathbb{N};\ n \geqq 3)$ for functions $f \in \Sigma$ is still not completely addressed for many of the subclasses of the bi-univalent function class Σ (see, for example, [20, 26, 28]).

The Fekete–Szegö functional $\left|a_3 - \mu a_2^2\right|$ for $f \in S$ is well known for its rich history in the field of Geometric Function Theory. Its origin was in the disproof by Fekete and Szegö [7] of the Littlewood–Paley conjecture that the coefficients of odd univalent functions are bounded by unity. The functional has since received great attention, particularly in the study of many subclasses of the family of univalent functions. This topic has become of considerable interest among researchers in Geometric Function Theory (see, for example, [14, 17, 23, 24, 27]).

With a view to recalling the principle of subordination between analytic functions, let the functions f and g be analytic in $\mathbb{U}$. We say that the function f is subordinate to g, if there exists a Schwarz function ω, which is analytic in $\mathbb{U}$ with $\omega(0) = 0$ and $|\omega(z)| < 1$ $(z \in \mathbb{U})$, such that $f(z) = g\big(\omega(z)\big)$. This subordination is denoted by $f \prec g$ or $f(z) \prec g(z)$ $(z \in \mathbb{U})$. It is well known that (see [16]), if the function g is univalent in $\mathbb{U}$, then

$$f \prec g \quad (z \in \mathbb{U}) \iff f(0) = g(0) \quad \text{and} \quad f(\mathbb{U}) \subseteq g(\mathbb{U}).$$

Recently, Hörçum and Koçer [11] considered the Horadam polynomials $h_n(x)$, which are given by the following recurrence relation (see also Horadam and Mahon [10]):

$$h_n(x) = pxh_{n-1}(x) + qh_{n-2}(x) \quad (x \in \mathbb{R};\ n \in \mathbb{N} - \{1, 2\}), \quad (1.3)$$

with $h_1(x) = a$ and $h_2(x) = bx$, for some real constant a, b, p and q. The characteristic equation of repetition relation (1.3) is $t^2 - pxt - q = 0$. This equation has two real roots $x_1 = \frac{px+\sqrt{p^2x^2+4q}}{2}$ and $x_2 = \frac{px-\sqrt{p^2x^2+4q}}{2}$.

Remark 1.1 By selecting the particular values of a, b, p and q, the Horadam polynomial $h_n(x)$ reduces to several polynomials. Some of these special cases are recorded below.

1. Taking $a = b = p = q = 1$, we obtain the Fibonacci polynomials $F_n(x)$.
2. Taking $a = 2$ and $b = p = q = 1$, we attain the Lucas polynomials $L_n(x)$.
3. Taking $a = q = 1$ and $b = p = 2$, we have the Pell polynomials $P_n(x)$.
4. Taking $a = b = p = 2$ and $q = 1$, we get the Pell–Lucas polynomials $Q_n(x)$.
5. Taking $a = b = 1$, $p = 2$ and $q = -1$, we obtain the Chebyshev polynomials $T_n(x)$ of the first kind.
6. Taking $a = 1$, $b = p = 2$ and $q = -1$, we have the Chebyshev polynomials $U_n(x)$ of the second kind.

These polynomials, the families of orthogonal polynomials and other special polynomials as well as their generalizations are potentially important in a variety of disciplines in many of sciences, specially in the mathematics, statistics and physics. For more information associated with these polynomials see [8–10, 12, 13].

The generating function of the Horadam polynomials $h_n(x)$ (see [11]) is given by

$$\Pi(x,z)=\sum_{n=1}^{\infty}h_n(x)z^{n-1}=\frac{a+(b-ap)xz}{1-pxz-qz^2}. \tag{1.4}$$

In fact, Srivastava et al. [19] have already these the Horadam polynomials in a similar context involving analytic and bi-univalent functions in recent years, it was followed by such works as those by Magesh et al. [15], Al-Amoush [3], Wanas and Alina [31] and Abirami et al. [1].

2 Main results

We begin this section by defining the family $\mathcal{K}_\Sigma(\delta,\lambda,x)$ as follows:

Definition 2.1 For $\delta \geqq 0$, $0 \leqq \lambda \leqq 1$ and $x \in \mathbb{R}$, a function $f \in \Sigma$ is said to be in the family $\mathcal{K}_\Sigma(\delta,\lambda,x)$ if it satisfies the subordinations:

$$Z(1-\delta)\left[(1-\lambda)\frac{zf'(z)}{f(z)}+\lambda\left(1+\frac{zf''(z)}{f'(z)}\right)\right]+\delta\frac{\lambda z^2f''(z)+zf'(z)}{\lambda zf'(z)+(1-\lambda)f(z)}\prec\Pi(x,z)+1-a$$

and

$$(1-\delta)\left[(1-\lambda)\frac{wg'(w)}{g(w)}+\lambda\left(1+\frac{wg''(w)}{g'(w)}\right)\right]+\delta\frac{\lambda w^2g''(w)+wg'(w)}{\lambda wg'(w)+(1-\lambda)g(w)}\prec\Pi(x,w)+1-a,$$

where a is real constant and the function $g=f^{-1}$ is given by (1.2).

Remark 2.1 For $\lambda=1$, a function $f\in\Sigma$ is in the family $\mathcal{K}_\Sigma(\delta,1,x)=:\mathcal{K}_\Sigma(x)$ which was considered recently by Magesh et al. [15], if the following conditions are satisfied:

$$1+\frac{zf''(z)}{f'(z)}\prec\Pi(x,z)+1-a$$

and

$$1+\frac{wg''(w)}{g'(w)}\prec\Pi(x,w)+1-a,$$

where $z,w\in\mathbb{U}$ and the function g is described in (1.2).

Remark 2.2 For $\lambda=0$, a function $f\in\Sigma$ is in the family $\mathcal{K}_\Sigma(\delta,0,x)=:\mathcal{W}_\Sigma(x)$ which was considered recently by Srivastava et al. [19], the following conditions are satisfied:

$$\frac{zf'(z)}{f(z)}\prec\Pi(x,z)+1-a \quad\text{and}\quad \frac{wg'(w)}{g(w)}\prec\Pi(x,w)+1-a, \tag{2.1}$$

where $z,w\in\mathbb{U}$ and the function g is described in (1.2).

Remark 2.3 For $\delta = 0$, a function $f \in \Sigma$ is in the family $\mathcal{K}_{\Sigma}(0, \lambda, x) =: M_{\Sigma}(\lambda, x)$ which was considered recently by Magesh et al. [15] if the following conditions are satisfied:

$$(1-\lambda)\frac{zf'(z)}{f(z)} + \lambda\left(1 + \frac{zf''(z)}{f'(z)}\right) \prec \Pi(x, z) + 1 - a$$

and

$$(1-\lambda)\frac{wg'(w)}{g(w)} + \lambda\left(1 + \frac{wg''(w)}{g'(w)}\right) \prec \Pi(x, w) + 1 - a,$$

where $z, w \in \mathbb{U}$ and the function g is described in (1.2).

Theorem 2.1 *For $\delta \geqq 0$, $0 \leqq \lambda \leqq 1$ and $x \in \mathbb{R}$, let $f \in \mathcal{A}$ be in the family $\mathcal{K}_{\Sigma}(\delta, \lambda, x)$. Then*

$$|a_2| \leqq \frac{|bx|\sqrt{|bx|}}{\sqrt{\left|\left[(\lambda + 1 - \lambda\delta(\lambda - 1))\,b - p\,(\lambda + 1)^2\right] bx^2 - qa\,(\lambda + 1)^2\right|}}$$

and

$$|a_3| \leqq \frac{|bx|}{2(2\lambda + 1)} + \frac{b^2x^2}{(\lambda + 1)^2}.$$

Proof Let $f \in \mathcal{K}_{\Sigma}(\delta, \lambda, x)$. Then there are two analytic functions $u, v : \mathbb{U} \longrightarrow \mathbb{U}$ given by

$$u(z) = u_1 z + u_2 z^2 + u_3 z^3 + \cdots \quad (z \in \mathbb{U}) \tag{2.2}$$

and

$$v(w) = v_1 w + v_2 w^2 + v_3 w^3 + \cdots \quad (w \in \mathbb{U}), \tag{2.3}$$

with $u(0) = v(0) = 0$ and $\max\{|u(z)|, |v(w)|\} < 1$, $z, w \in \mathbb{U}$ such that

$$(1-\delta)\left[(1-\lambda)\frac{zf'(z)}{f(z)} + \lambda\left(1 + \frac{zf''(z)}{f'(z)}\right)\right] + \delta\frac{\lambda z^2 f''(z) + zf'(z)}{\lambda zf'(z) + (1-\lambda)f(z)}$$
$$= \Pi(x, u(z)) + 1 - a$$

and

$$(1-\delta)\left[(1-\lambda)\frac{wg'(w)}{g(w)} + \lambda\left(1 + \frac{wg''(w)}{g'(w)}\right)\right] + \delta\frac{\lambda w^2 g''(w) + wg'(w)}{\lambda wg'(w) + (1-\lambda)g(w)}$$
$$= \Pi(x, v(w)) + 1 - a.$$

Or, equivalently

$$(1-\delta)\left[(1-\lambda)\frac{zf'(z)}{f(z)} + \lambda\left(1 + \frac{zf''(z)}{f'(z)}\right)\right] + \delta\frac{\lambda z^2 f''(z) + zf'(z)}{\lambda zf'(z) + (1-\lambda)f(z)}$$
$$= 1 + h_1(x) + h_2(x)u(z) + h_3(x)u^2(z) + \cdots \tag{2.4}$$

and

$$(1-\delta)\left[(1-\lambda)\frac{wg'(w)}{g(w)} + \lambda\left(1 + \frac{wg''(w)}{g'(w)}\right)\right] + \delta\frac{\lambda w^2 g''(w) + wg'(w)}{\lambda wg'(w) + (1-\lambda)g(w)}$$
$$= 1 + h_1(x) + h_2(x)v(w) + h_3(x)v^2(w) + \cdots. \tag{2.5}$$

Combining (2.2), (2.3), (2.4) and (2.5) yields

$$(1-\delta)\left[(1-\lambda)\frac{zf'(z)}{f(z)}+\lambda\left(1+\frac{zf''(z)}{f'(z)}\right)\right]+\delta\frac{\lambda z^2 f''(z)+zf'(z)}{\lambda zf'(z)+(1-\lambda)f(z)}$$
$$=1+h_2(x)u_1z+\left[h_2(x)u_2+h_3(x)u_1^2\right]z^2+\cdots \tag{2.6}$$

and

$$(1-\delta)\left[(1-\lambda)\frac{wg'(w)}{g(w)}+\lambda\left(1+\frac{wg''(w)}{g'(w)}\right)\right]+\delta\frac{\lambda w^2 g''(w)+wg'(w)}{\lambda wg'(w)+(1-\lambda)g(w)}$$
$$=1+h_2(x)v_1w+\left[h_2(x)v_2+h_3(x)v_1^2\right]w^2+\cdots. \tag{2.7}$$

It is quite well-known that if $\max\{|u(z)|,|v(w)|\}<1$, $z,w\in\mathbb{U}$, then

$$|u_i|\leqq 1 \quad \text{and} \quad |v_i|\leqq 1 \quad (\forall\, i\in\mathbb{N}). \tag{2.8}$$

Comparing the corresponding coefficients in (2.6) and (2.7), after simplifying, we have

$$(\lambda+1)a_2=h_2(x)u_1, \tag{2.9}$$
$$2(2\lambda+1)a_3-(\lambda\delta(\lambda-1)+3\lambda+1)\,a_2^2=h_2(x)u_2+h_3(x)u_1^2, \tag{2.10}$$
$$-(\lambda+1)a_2=h_2(x)v_1 \tag{2.11}$$

and

$$2(2\lambda+1)(2a_2^2-a_3)-(\lambda\delta(\lambda-1)+3\lambda+1)\,a_2^2=h_2(x)v_2+h_3(x)v_1^2. \tag{2.12}$$

It follows from (2.9) and (2.11) that

$$u_1=-v_1 \tag{2.13}$$

and

$$2(\lambda+1)^2a_2^2=h_2^2(x)(u_1^2+v_1^2). \tag{2.14}$$

If we add (2.10) to (2.12), we find that

$$2\,(\lambda+1-\lambda\delta(\lambda-1))\,a_2^2=h_2(x)(u_2+v_2)+h_3(x)(u_1^2+v_1^2). \tag{2.15}$$

Substituting the value of $u_1^2+v_1^2$ from (2.14) in the right hand side of (2.15), we deduce that

$$a_2^2=\frac{h_2^3(x)(u_2+v_2)}{2\left[h_2^2(x)\,(\lambda+1-\lambda\delta(\lambda-1))-h_3(x)\,(\lambda+1)^2\right]}. \tag{2.16}$$

Further computations using (1.3), (2.8) and (2.16), we obtain

$$|a_2|\leqq\frac{|bx|\sqrt{|bx|}}{\sqrt{\left|\left[(\lambda+1-\lambda\delta(\lambda-1))\,b-p\,(\lambda+1)^2\right]bx^2-qa\,(\lambda+1)^2\right|}}.$$

Next, if we subtract (2.12) from (2.10), we can easily see that

$$4(2\lambda+1)(a_3-a_2^2)=h_2(x)(u_2-v_2)+h_3(x)(u_1^2-v_1^2). \tag{2.17}$$

In view of (2.13) and (2.14), we get from (2.17)

$$a_3=\frac{h_2(x)(u_2-v_2)}{4(2\lambda+1)}+\frac{h_2^2(x)(u_1^2+v_1^2)}{2\,(\lambda+1)^2}.$$

Thus applying (1.3), we obtain

$$|a_3| \leqq \frac{|bx|}{2(2\lambda+1)} + \frac{b^2x^2}{(\lambda+1)^2}.$$

This completes the proof of Theorem 2.1 □

By taking $\lambda = 1$, we state

Corollary 2.1 [15] *Let f given by 1.1 be in the family $\mathcal{K}_\Sigma(\delta, 1, x) =: \mathcal{K}_\Sigma(x)$. Then*

$$|a_2| \leqq \frac{|bx|\sqrt{|bx|}}{\sqrt{|(2b-4p)bx^2 - 4qa|}},$$

$$|a_3| \leqq \frac{|bx|}{6} + \frac{b^2x^2}{4}.$$

By taking $\lambda = 0$, we state

Corollary 2.2 [19] *Let f given by 1.1 be in the family $\mathcal{K}_\Sigma(\delta, 0, x) =: \mathcal{W}_\Sigma(x)$. Then*

$$|a_2| \leqq \frac{|bx|\sqrt{|bx|}}{\sqrt{|(b-p)bx^2 - qa|}},$$

$$|a_3| \leqq \frac{|bx|}{2} + b^2x^2.$$

By taking $\delta = 0$, we state

Corollary 2.3 [15] *Let f given by 1.1 be in the family $\mathcal{K}_\Sigma(0, \lambda, x) =: M_\Sigma(\lambda, x)$. Then*

$$|a_2| \leqq \frac{|bx|\sqrt{|bx|}}{\sqrt{\left|\left[(\lambda+1)\,b - p\,(\lambda+1)^2\right]bx^2 - qa\,(\lambda+1)^2\right|}}$$

and

$$|a_3| \leqq \frac{|bx|}{2(2\lambda+1)} + \frac{b^2x^2}{(\lambda+1)^2}.$$

In the next theorem, we present the Fekete–Szegö inequality for the family $\mathcal{K}_\Sigma(\delta, \lambda, x)$.

Theorem 2.2 *For $\delta \geqq 0$, $0 \leqq \lambda \leqq 1$ and $x, \mu \in \mathbb{R}$, let $f \in \mathcal{A}$ be in the family $\mathcal{K}_\Sigma(\delta, \lambda, x)$. Then*

$$\left|a_3 - \mu a_2^2\right| \leqq \begin{cases} \frac{|bx|}{2(2\lambda+1)}; \\ \qquad for\ \ |\mu - 1| \leqq \frac{\left|\left[(\lambda+1-\lambda\delta(\lambda-1))b - p(\lambda+1)^2\right]bx^2 - qa(\lambda+1)^2\right|}{2b^2x^2(2\lambda+1)}, \\ \\ \frac{|bx|^3|\mu-1|}{\left|\left[(\lambda+1-\lambda\delta(\lambda-1))b - p(\lambda+1)^2\right]bx^2 - qa(\lambda+1)^2\right|}; \\ \\ \qquad for\ \ |\mu - 1| \geqq \frac{\left|\left[(\lambda+1-\lambda\delta(\lambda-1))b - p(\lambda+1)^2\right]bx^2 - qa(\lambda+1)^2\right|}{2b^2x^2(2\lambda+1)}. \end{cases}$$

Proof It follows from (2.16) and (2.17) that

$$\begin{aligned} a_3 - \mu a_2^2 &= \frac{h_2(x)(u_2 - v_2)}{4(2\lambda+1)} + (1-\mu)\, a_2^2 \\ &= \frac{h_2(x)(u_2 - v_2)}{4(2\lambda+1)} + \frac{h_2^3(x)(u_2+v_2)\,(1-\mu)}{2\left[h_2^2(x)\,(\lambda+1-\lambda\delta(\lambda-1)) - h_3(x)\,(\lambda+1)^2\right]} \\ &= \frac{h_2(x)}{2}\left[\left(\psi(\mu,x) + \frac{1}{2(2\lambda+1)}\right)u_2 + \left(\psi(\mu,x) - \frac{1}{2(2\lambda+1)}\right)v_2\right], \end{aligned}$$

where

$$\psi(\mu,x) = \frac{h_2^2(x)\,(1-\mu)}{h_2^2(x)\,(\lambda+1-\lambda\delta(\lambda-1)) - h_3(x)\,(\lambda+1)^2}.$$

According to (1.3), we find that

$$\left|a_3 - \mu a_2^2\right| \leqq \begin{cases} \frac{|bx|}{2(2\lambda+1)}, & 0 \leqq |\psi(\mu,x)| \leqq \frac{1}{2(2\lambda+1)}, \\ |bx|\,|\psi(\mu,x)|\,, & |\psi(\mu,x)| \geq \frac{1}{2(2\lambda+1)}. \end{cases}$$

After some computations, we obtain

$$\left|a_3 - \mu a_2^2\right| \leqq \begin{cases} \frac{|bx|}{2(2\lambda+1)}; \\ \qquad for\ \ |\mu-1| \leqq \frac{\left|\left[(\lambda+1-\lambda\delta(\lambda-1))b - p(\lambda+1)^2\right]bx^2 - qa(\lambda+1)^2\right|}{2b^2x^2(2\lambda+1)}, \\ \frac{|bx|^3|\mu-1|}{\left|\left[(\lambda+1-\lambda\delta(\lambda-1))b - p(\lambda+1)^2\right]bx^2 - qa(\lambda+1)^2\right|}; \\ \qquad for\ \ |\mu-1| \geqq \frac{\left|\left[(\lambda+1-\lambda\delta(\lambda-1))b - p(\lambda+1)^2\right]bx^2 - qa(\lambda+1)^2\right|}{2b^2x^2(2\lambda+1)}. \end{cases}$$

□

Putting $\mu = 1$ in Theorem 2.2, we obtain the following result:

Corollary 2.4 *For $\delta \geqq 0$, $0 \leqq \lambda \leqq 1$ and $x \in \mathbb{R}$, let $f \in \mathcal{A}$ be in the family $\mathcal{K}_\Sigma(\delta, \lambda, x)$. Then*

$$\left|a_3 - a_2^2\right| \leqq \frac{|bx|}{2(2\lambda+1)}.$$

Corollary 2.5 [15] *Let f given by 1.1 be in the family $\mathcal{K}_\Sigma(\delta, 1, x) =: \mathcal{K}_\Sigma(x)$. Then*

$$\left|a_3 - \mu a_2^2\right| \leqq \begin{cases} \frac{|bx|}{6}; & |\mu-1| \leqq \frac{|(2b-4p)bx^2 - 4qa|}{6b^2x^2}, \\ \frac{|bx|^3|\mu-1|}{|(2b-4p)bx^2-4qa|}; & |\mu-1| \geqq \frac{|(2b-4p)bx^2 - 4qa|}{6b^2x^2}. \end{cases}$$

Corollary 2.6 [19] *Let f given by 1.1 be in the family $\mathcal{K}_\Sigma(\delta, 0, x) =: \mathcal{W}_\Sigma(x)$. Then*

$$\left|a_3 - \mu a_2^2\right| \leqq \begin{cases} \frac{|bx|}{2}; & |\mu-1| \leqq \frac{|(b-p)bx^2 - qa|}{b^2x^2}, \\ \frac{|bx|^3|\mu-1|}{|(b-p)bx^2-qa|}; & |\mu-1| \geqq \frac{|(b-p)bx^2 - qa|}{b^2x^2}. \end{cases}$$

Corollary 2.7 [15] *Let f given by 1.1 be in the family $\mathcal{K}_\Sigma(0,\lambda,x) =: M_\Sigma(\lambda,x)$. Then*

$$\left|a_3-\mu a_2^2\right| \leqq \begin{cases} \frac{|bx|}{2(2\lambda+1)}; \\ \qquad for\ |\mu-1| \leqq \frac{\left|\left[(\lambda+1-)b-p(\lambda+1)^2\right]bx^2-qa(\lambda+1)^2\right|}{2b^2x^2(2\lambda+1)}, \\ \\ \frac{|bx|^3|\mu-1|}{\left|\left[(\lambda+1)b-p(\lambda+1)^2\right]bx^2-qa(\lambda+1)^2\right|}; \\ \\ \qquad for\ |\mu-1| \geqq \frac{\left|\left[(\lambda+1)b-p(\lambda+1)^2\right]bx^2-qa(\lambda+1)^2\right|}{2b^2x^2(2\lambda+1)}. \end{cases}$$

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