Afrika Matematika (2022) 33:3
https://doi.org/10.1007/s13370-021-00954-x

Hypersurfaces immersed in a golden Riemannian manifold

Mohammad Aamir Qayyoom[1] · **Mobin Ahmad**[1]

Received: 3 October 2019 / Accepted: 6 December 2021 / Published online: 19 January 2022

Abstract
The purpose of this paper is to study properties of hypersurfaces of a Riemannian manifold equipped with a golden structure. We discuss the necessary and sufficient conditions for a hypersurface to be a golden Riemannian manifold in terms of invariance. The totally geodesics and totally umbilical hypersurfaces in golden Riemannian manifolds are analyzed. Also, we construct an example on hypersurface of the Euclidean space endowed with a golden Riemannian structure.

Keywords Golden structure · Riemannian manifold · Invariant hypersurfaces · Non-invariant hypersurfaces · Totally geodesics · Umbilical hypersurfaces · Normal induced structure · Killing vector fields

Mathematics Subject Classification 53C25 · 53B25

1 Introduction

The theory of submanifolds was initiated by Fermat in his study of geometry of plane curves. Nowadays, the theory of submanifolds is an active and vast research field playing an important role in the development of modern differential geometry. Moreover, the theory of hypersurfaces of an almost contact manifold have been studied by geometers in [1,3,6,19,20].

The notion of golden structure was introduced by Crasmareanu and Hretcanu in [7] and they studied its properties in [8,17]. Then several geometers studied golden Riemannian manifolds (see [9–12,17,21]). Crasmareanu and Hretcanu [18] also defined and studied metallic structure as generalization of golden stucture. Then, the metallic structures were studied by many geometers (see [5,15,16]).

Gezer et al. [13] investigated the integrability conditions of the golden Riemannian manifold. Hretcanu studied submanifolds of Riemannian manifold with golden structure [14].

✉ Mobin Ahmad
mobinahmad68@gmail.com

Mohammad Aamir Qayyoom
aamir.qayyoom@gmail.com

1 Department of Mathematics and Statistics, Faculty of Science, Integral University, Lucknow 226026, India

Bahadir and Uddin [4] studied slant submanifolds of golden Riemannian manifolds. Ahmad and Qayyoom [2] studied submanifolds in a Riemannian manifold with golden structure.

Motivated by above studies, in this paper we study some properties of hypersurfaces of a golden Riemannian manifold. The paper is organized as follows.

In Sect. 2, we define golden Riemannian manifolds.

In Sect. 3, we establish several properties of induced structure $\sum = (J, g, \xi, u, a)$ on the hypersurface immersed in golden Riemannian manifold. In last section, we construct an example of golden Riemannian structure on Euclidean space and obtain its hypersurfaces.

2 Definitions and preliminaries

Definition 2.1 [7] Let $(\overline{M}, g)$ be a Riemannian manifold. A golden structure on $(\overline{M}, g)$ is a non-null tensor $\overline{J}$ of type $(1, 1)$ which satisfies the equation

$$\overline{J}^2 = \overline{J} + I, \tag{2.1}$$

where I is the identity transformation. We say that the metric g is $\overline{J}$-compatible if

$$g(\overline{J}X, Y) = g(X, \overline{J}Y) \tag{2.2}$$

for all X,Y vector fields on $\overline{M}$. If we substitute $\overline{J}X$ into X in (2.2), then we have

$$g(\overline{J}X, \overline{J}Y) = g(\overline{J}X, Y) + g(X, Y). \tag{2.3}$$

The Riemannian metric (2.2) is called $\overline{J}$-compatible and $(\overline{M}, \overline{J}, g)$ is called a Golden Riemannian structure.

Definition 2.2 If $(\overline{M}, g, \overline{J})$ is a golden Riemannian manifold and $\overline{J}$ is parallel with respect to Levi-Civita connection $\overline{\nabla}$ on M $(i.e. \overline{\nabla}\overline{J}) = 0$, then we call $(\overline{M}, g, \overline{J})$ is a locally golden Riemannian manifold.

Proposition 2.3 [7] *The eigenvalues of a golden structure $\overline{J}$ are the golden ratio ϕ and $1-\phi$.*

Let M be an n-dimensional hypersurface isometrically immersed in an $(n+1)$-dimensional golden Riemannian manifold $(\overline{M}, g, \overline{J})$ with $n \in \mathbb{N}$. We denote by $T_x M$ the tangent space of M at a point $x \in M$ and $T_x^{\perp} M$ the normal space of x in M. Let i_* be the differential of immersion $i_* : M \to \overline{M}$. The induced Riemannian metric g on M is given by $g(X, Y) = g(i_* X, i_* Y)$ for every $X, Y \in \chi(M)$.

We consider a local orthonormal basis N of the normal space $T_x^{\perp} M$. For every $X \in T_x M$, the vector fields $\overline{J}(i_* X)$ and $\overline{J}(N)$ can be decomposed in tangential and normal components as follows:

$$\overline{J}(i_* X) = i_*(JX) + u(X)N, \tag{2.4}$$

$$\overline{J}(N) = i_*(\xi) + aN, \tag{2.5}$$

where J is a $(1, 1)$ tensor field on M, ξ is tangent vector field on M, u is $1 - form$ on M and a is 1×1 matrix of smooth real function on M.

We denote the covariant derivative in $\overline{M}$ by $\overline{\nabla}$ and covariant derivative in M by ∇. We denote by A the Weingarten operator on TM with respect to the local unit normal vector field N of M in $\overline{M}$. The Gauss and Weingarten formulae are given by

$$\overline{\nabla}_X Y = \nabla_X Y + h(X, Y)N, \tag{2.6}$$
$$\overline{\nabla}_X N = -AX + \nabla_X^{\perp} N, \tag{2.7}$$

where $h(X, Y) = g(AX, Y)$ is the second fundamental form in $T^{\perp}M$ and $X, Y \in \chi(M)$.

Proposition 2.4 [17] *If M is a hypersurface isometrically immersed in a golden Riemannian manifold $(\overline{M}, g, \overline{J})$, from (2.4) and (2.5), we obtain the structure $\sum = (J, g, u, \xi, a)$ induced on M by the golden structure $\overline{J}$, satisfies the following inequalities:*

$$J^2(X) = J(X) + X - u(X)\xi, \tag{2.8}$$
$$u(JX) = (1-a)u(X), \tag{2.9}$$
$$u(\xi) = 1 + a - a^2, \tag{2.10}$$
$$J(\xi) = (1-a)\xi, \tag{2.11}$$
$$u(X) = g(X, \xi), \tag{2.12}$$
$$g(JX, Y) = g(X, JY), \tag{2.13}$$
$$g(JX, JY) = g(JX, Y) + g(X, Y) - u(X)u(Y). \tag{2.14}$$

for every $X, Y \in \chi(M)$.

Proposition 2.5 [17] *If M is a hypersurface in a golden Riemannian manifold $(\overline{M}, g, \overline{J})$ and $\overline{J}$ is parallel with respect to the Levi Civita connection $\overline{\nabla}$ on $\overline{M}$ $(\overline{\nabla}\ \overline{J} = 0)$, then the structure $\sum = (J, g, u, \xi, a)$ induced on M by the structure $\overline{J}$ has the following properties:*

$$(\nabla_X J)(Y) = g(AX, Y)\xi + u(Y)AX, \tag{2.15}$$
$$(\nabla_X u)(Y) = -g(AX, JY) + ag(AX, Y), \tag{2.16}$$
$$\nabla_X \xi = -J(AX) + aAX, \tag{2.17}$$
$$X(a) = -2u(AX) = -2g(AX, \xi) = -2g(X, A\xi) \tag{2.18}$$

for every $X, Y \in \chi(M)$.

Theorem 2.6 [17] *If the structure $\sum = (J, g, \xi, u, a)$ induced by structure $\overline{J}$ on an umbilical hypersurface M in a golden Riemannian manifold $(\overline{M}, g, \overline{J})$ with $\overline{\nabla}\,\overline{J} = 0$ we have*

$$(\nabla_X J)(Y) = \lambda[g(X, Y)\xi + g(Y, \xi)X], \tag{2.19}$$
$$(\nabla_X u)(Y) = \lambda[ag(X, Y) - g(X, JY)], \tag{2.20}$$
$$\nabla_X \xi = \lambda(aX - J(X)), \tag{2.21}$$
$$\nabla_\xi \xi = \lambda(2a-1)\xi, \tag{2.22}$$
$$X(a) = -2\lambda g(X, \xi) \tag{2.23}$$

for any $X, Y \in \chi(M)$.

Corollary 2.7 *Let M be a totally umbilical submanifold in a golden Riemannian manifold $(\overline{M}, g, \overline{J})$ with structure $\sum = (J, g, \xi, u, a)$ induced by $\overline{J}$ on M and $\overline{\nabla}\,\overline{J} = 0$, follows that*

$$(\nabla_X J)(\xi) = \lambda(1 + a - a^2)X, \tag{2.24}$$
$$(\nabla_\xi J)(Y) = 2\lambda g(Y, \xi)\xi), \tag{2.25}$$
$$(\nabla_X u)\xi = 2a\lambda g(X, \xi) - \lambda g(X, \xi), \tag{2.26}$$

for any $X, Y \in \chi(M)$.

Proof For totally umbilical manifold, $A = \lambda I$, then equation (2.15) reduces to

$$(\nabla_X J)(Y) = \lambda[g(X, Y) + u(Y)X].$$

Taking $Y = \xi$, we get

$$(\nabla_X J)\xi = \lambda[g(X, \xi)\xi + u(\xi)X].$$

Using equality (2.10), we obtain

$$(\nabla_X J)\xi = \lambda(1 + a - a^2)X,$$

which is (2.24). Using equality (2.19) and if $X = \xi$, we obtain

$$(\nabla_\xi J)(Y) = \lambda[g(Y, \xi)\xi + g(\xi, Y)\xi]$$

$$(\nabla_\xi J)Y = 2\lambda g(Y, \xi)\xi,$$

which gives (2.25). Using (2.20) and taking $Y = \xi$, we have

$$(\nabla_X u)\xi = \lambda[ag(X, \xi) - g(X, J(\xi))].$$

Using equality (2.11), we obtain

$$(\nabla_X u)\xi = 2\lambda ag(X, \xi) - \lambda g(X, \xi).$$

which proves (2.26). □

3 Properties of induced structures on hypersurfaces in golden Riemannian manifolds

Proposition 3.1 *Let M be a hypersurface on a golden Riemannian manifold* $(\overline{M}, g, \overline{J})$ *with structure* $\sum = (J, g, \xi, u, a)$ *induced on M by the structure* $\overline{J}$ *with* $\xi \neq 0$. *A necessary and sufficient condition for M to be totally geodesic in M is that* $\nabla_X J = 0$ *for any* $X \in \chi(M)$.

Proof If M is totally geodesic, then $A = 0$. Using equality (2.15), we obtain

$$\nabla_X J = 0.$$

Conversely, we suppose that $\nabla_X J = 0$ and from (2.15), we have

$$g(AX, Y)\xi + g(Y, \xi)AX = 0.$$

We may have one of the following conditions:

(i) If AX and ξ are linearly dependent vector fields, then there exist a real number α such that $AX = \alpha\xi$ and from this we obtain

$$g(\alpha\xi, Y)\xi + g(Y, \xi)\alpha\xi = 0.$$

That is,

$$g(Y, \xi) = 0$$

for any $Y \in \chi(M)$. Then for $Y = \xi$, we obtain $g(\xi, \xi) = 0$, which is equivalent with $\xi = 0$. But this is impossible.

(ii) If AX and ξ are linearly independent vector fields, then

$$g(AX, Y) = 0$$

for any $X, Y \in \chi(M)$. Thus, $A = 0$ and from this we have that M is a totally geodesic hypersurface in $\overline{M}$. □

Proposition 3.2 *If M is a hypersurface in a golden Riemannian manifold $(\overline{M}, g, \overline{J})$ with structure $\sum = (J, g, \xi, u, a)$ induced on M by $\overline{J}$, then following equalities are equivalent:*

$$\nabla_X u = 0 \Leftrightarrow \nabla_X \xi = 0$$

for each $X \in \chi(M)$.

Proof $\nabla_X u = 0$, then by using equality (2.16) we obtain

$$g(AX, JY) = ag((AX), Y).$$

Using (2.13) in above equation, we get

$$g(J(AX) - aAX, Y) = 0$$

for any $X, Y \in \chi(M)$. By use of (2.17), we have

$$g(\nabla_X \xi, Y) = 0.$$

As $y \neq 0$, we get

$$\nabla_X \xi = 0.$$

Conversely, we suppose that$\nabla \xi = 0$ and we have

$$g(\nabla_X \xi, Y) = 0.$$

By using equality (2.17), we get

$$g(J(AX) - aAX, Y) = 0,$$

$$g(J(AX), Y) - g(aAX, Y) = 0,$$
$$g(AX, JY) - ag(AX, Y) = 0.$$

In view of (2.16), above equation reduces to

$$\nabla_X u = 0.$$

□

Proposition 3.3 *If M is a totally umbilical hypersurface in a golden Riemannian manifold $(\overline{M}, g, \overline{J})$ with the structure $\sum = (J, g, \xi, u, a)$ induced on M by $\overline{J}$. Then the $1 - form$ u is closed.*

Proof As M is totally umbilical hypersurface, that is $A = \lambda I$. Then

$$(\nabla_X u)(Y) = -\lambda g(X, JY) + a\lambda g(X, Y). \tag{3.1}$$

Since,

$$du(X, Y) = (\nabla_X u)(Y) - (\nabla_Y u)(X).$$

Using (2.13) and (3.1) in above equation, we get

$$\begin{aligned} du(X,Y) &= [-\lambda g(X,JY) + a\lambda g(X,Y)] - [-\lambda g(Y,JX) + a\lambda g(Y,X)] \\ du(X,Y) &= -\lambda g(X,JY) + \lambda g(Y,JX) \\ du(X,Y) &= \lambda[-g(JX,Y) + g(JX,Y)] \\ du(X,Y) &= 0. \end{aligned} \tag{3.2}$$

Thus, $1 - form\ u$ is closed. □

Proposition 3.4 *Let M be a hypersurface in a golden Riemannian manifold* $(\overline{M}, g, \overline{J})$ *with* $\overline{\nabla}\,\overline{J} = 0$ *and the structure* $\sum = (J, g, \xi, u, a)$ *induced on M by* $\overline{J}$*. Then* ξ *is a Killing vector field with respect to g on M if and only if we have*

$$2aA = JA + AJ, \tag{3.3}$$

where A is the Weingarten operator on M.

Proof We have that ξ is a Killing vector field on M if and only if

$$(L_\xi g)(Y, Z) = 0$$

for all $Y, Z \in \chi(M)$. Then

$$\begin{aligned} &L_\xi g(Y,Z) - g(L_\xi Y, Z) - g(Y, L_\xi Z) = 0 \\ &\xi g(Y,Z) - g([\xi,Y],Z) - g(Y,[\xi,Z]) = 0 \\ &\xi g(Y,Z) - g(\nabla_\xi Y - \nabla_Y \xi, Z) - g(Y, \nabla_\xi Z - \nabla_Z \xi) = 0 \\ &\xi g(Y,Z) - g(\nabla_\xi Y, Z) - g(Y, \nabla_\xi Z) + g(\nabla_Y \xi, Z) + g(Y, \nabla_Z \xi) = 0 \\ &g(\nabla_Y \xi, Z) + g(Y, \nabla_Z \xi) = 0 \end{aligned}$$

for all $Y, Z \in \chi(M)$.

Using equality (2.17) in above equation we obtain

$$g(-JAY + aAY, Z) + g(-JAZ + aAZ, Y) = 0$$

or,

$$g(2aAY - JAY - A(JY), Z) = 0,$$

which is equivalent to

$$2aA = JA + AJ.$$

□

Theorem 3.5 [16] *Let M be a hypersurface in a locally golden Riemannian manifold* $(\overline{M}, g, \overline{J})$*. If the normal connection* $\nabla^\perp$ *vanishes identically on the normal bundle* $T^\perp(M)$ *and M is a non-invariant hypersurface with respect to the golden structure J then the induced structure* $\sum = (J, g, \xi, u, a)$ *on M is normal if and only if the tensor field J commutes with every Weingarten operator A* $(i.e.\ JA = AJ)$.

Proposition 3.6 *Let M be a hypersurface of a golden Riemannian manifold* $(\overline{M}, g, \overline{J})$ *with* $\overline{\nabla}\ \overline{J} = 0$ *and the normal induced structure* $\sum = (J, g, \xi, u, a)$ *on M by* $\overline{J}$*. Then* ξ *is a Killing vector field with respect to g on M if only if*

$$aA = JA = AJ,$$

where A is the Weingarten operator on M.

Proof A necessary and sufficient condition for the normality of structure $\sum = (J, g, \xi, u, a)$ is that the tensor field J commutes by the Weingarten operator A. That is, $JA = AJ$. Then the proof of the proposition is a consequence of (3.3). □

Proposition 3.7 *Let M be a hypersurface of a golden Riemannian manifold $(\overline{M}, g, \overline{J})$ with $\overline{\nabla}\,\overline{J} = 0$ and structure $\sum = (J, g, \xi, u, a)$ induced on M by $\overline{J}$ and ξ is a Killing vector field. If $a \neq \phi$, then rank $A = 1$ and ξ is an eigenvector of the Weingarten operator A with the eigenvalue $\frac{\xi(a)}{2(a^2-a-1)}$.*

Proof We have $JA = aA$, then $J^2(AX) = a^2(AX)$ for all $X \in \chi(M)$. Using equality (2.6), we get

$$-u(AX)\xi = (a^2 - a - 1)AX.$$

Using equality (2.18) in above equation, we obtain

$$\frac{X(a)}{2}\xi = (a^2 - a - 1)AX$$

or,

$$AX = \frac{X(a)}{2(a^2 - a - 1)}\xi$$

for all $X \in \chi(M)$. If we put $X = \xi$ in above equation, we obtain

$$A(\xi) = \frac{\xi(a)}{2(a^2 - a - 1)}\xi$$

for all $X \in \chi(M)$.

Thus, ξ is an eigenvector of Weingarten operator A and its eigenvalue is $\frac{\xi(a)}{2(a^2-a-1)}$. □

Theorem 3.8 *Let M is a hypersurface of a golden Riemannian manifold $(\overline{M}, g, \overline{J})$. Then M is invariant if and only if the normal of M is an eigenvector of the matrix $\overline{J}$.*

Proof Suppose $a = \phi$. Using equality (2.10), we have

$$u(\xi) = 1 + \phi - \phi^2$$

or,

$$u(\xi) = g(X, \xi) = 0,$$

which is equivalent to $X \perp \xi$.

As X and ξ are both tangential, then there is only one possibility that $\xi = 0$ because $X \neq 0$. From (2.5), we have

$$\overline{J}N = i_*\xi + aN,$$

which gives

$$\overline{J}N = aN$$

for $\xi = 0$. Hence N is the eigenvector for $\overline{J}$. □

Remark 3.9 Let M be a hypersurface in a golden Riemannian manifold $(\overline{M}, g, \overline{J})$ with the induced structure $\sum = (J, g, \xi, u, a)$ on M by $\overline{J}$. We suppose that $(e_1, \ldots, e_n)$ is an orthonormal basis of the tangent space $T_x M$ for any $x \in M$. Then

$$div\xi = trace(e_i \to \nabla_{e_j}\xi)$$

using $\nabla_X \xi = -\lambda JX + a\lambda X$, we obtain

$$\nabla_{e_i}\xi = \lambda(aI - J)(e_i).$$

So,

$$div\xi = \lambda trace(aI - J),$$
$$div\xi = \lambda(na - traceJ).$$

4 Example of induced structure on a hypersurface in a golden Riemannian manifold

Example 4.1 We suppose that the ambient space is E^{2a+b} $(a, b \in N^*)$ and for any point of E^{2a+b}, we have its coordinates

$$(x^1, \ldots, x^a, y^1, \ldots, y^a, z^1, \ldots, z^b) = (x^i, y^i, z^j),$$

where $i \in 1, \ldots, a$ and $j \in 1, \ldots, b$. The tangent space $T_x(E^{2a+b})$ is isomorphic with E^{2a+b}.

Let $\overline{J} : E^{2a+b} \to E^{2a+b}$ be the golden structure on E^{2a+b} defined as.

$$\overline{J}(x^1, \ldots, x^a, y^1, \ldots, y^a, z^1, \ldots, z^b) = (\phi x^1, \ldots, \phi x^a, \phi y^1, \ldots, \phi y^a, (1-\phi)z^1, \ldots, (1-\phi)z^b).$$

Then

$$\overline{J}^2(x^1, \ldots, x^a, y^1, \ldots, y^a, z^1, \ldots, z^b) = (\phi^2 x^1, \ldots, \phi^2 x^a, \phi^2 y^1, \ldots, \phi^2 y^a, (1-\phi)^2 z^1, \ldots, (1-\phi)^2 z^b). \quad (4.1)$$

Since ϕ and $(1-\phi)$ are roots of $x^2 = x + 1$ then $\phi^2 = \phi + 1$ and $(1-\phi)^2 = (1-\phi) + 1$. Then, (4.1) gives

$$\overline{J}^2(x^1, \ldots, x^a, y^1, \ldots, y^a, z^1, \ldots, z^b) = ((\phi+1)x^1, \ldots, (\phi+1)x^a, (1+\phi)y^1, \ldots, (1+\phi)y^a, ((1-\phi)+1)z^1, \ldots, ((1-\phi)+1)z^b)$$
$$\overline{J}^2(x^1, \ldots, x^a, y^1, \ldots, y^a, z^1, \ldots, z^b) = (\phi x^1, \ldots, \phi x^a, \phi y^1, \ldots \phi y^a(1-\phi)z^1, \ldots, (1-\phi)z^b) + (x^1, \ldots, x^a, y^1, \ldots, y^a, z^1, \ldots, z^b)$$
$$\overline{J}^2(x^i, y^i, z^j) = J(x^i, y^i, z^j) + I(x^i, y^i, z^j)$$

or,

$$\overline{J}^2 = \overline{J} + I.$$

It follows that $(E^{2a+b}, g, \overline{J})$ is a golden Riemannian manifold.

In E^{2a+b}, we can get the hypersphere

$$S^{2a+b-1}(r) = (x^1, \ldots, x^a, y^1, \ldots, y^a, \ldots, z^1, \ldots, z^b), \sum_{i=1}^{a} x^{i^2} + \sum_{i=1}^{a} y^{i^2} + \sum_{j=1}^{b} z^{j^2} = R^2,$$

where R is its radius and $(x^1, \ldots, x^a, y^1, \ldots, y^a, z^1, \ldots, z^b)$ are the coordinates of any point of $S^{2a+b-1}(R)$. We use following notations

$\sum_{i=1}^{a}(x^i)^2 = r_1^2$, $\sum_{i=1}^{a}(y^i)^2 = r_2^2$, $\sum_{j=1}^{b}(z^j)^2 = r_3^2$ and $r_1^2 + r_2^2 = r^2$. Thus, we have $r^2 + r_3^2 = R^3$. We remark that $N_1 = \frac{1}{R}(x^i, y^i, z^j)$, $i \epsilon (1, \ldots, a)$, $j \epsilon (1, \ldots, b)$ is a unit normal vector field on sphere $S^{2a+b-1}(R)$ and $J(N_1) = \frac{1}{R}(\phi y^i, \phi x^i, (1-\phi)z^j)$.

For a tangent vector field X on $S^{2a+b-1}(R)$, we use the following notation

$$X = (X^1, \ldots, X^a, Y^1, \ldots, Y^a, Z^1, \ldots, Z^b) = (X^i, Y^i, Z^j).$$

Hence, we have

$$\sum_{i=1}^{a} x^i X^i + \sum_{i=1}^{a} y^i Y^i + \sum_{j=1}^{b} z^j Z^j = 0.$$

If we decompose $\overline{J}(N)$ and $\overline{J}(X^i, Y^i, Z^j)$ respectively in tangential and normal components on $T_(x, y, z)S^{2a+b-1}(R)$, we obtain

$$\overline{J}(N) = \xi + AN, \overline{J}(X^i, Y^i, Z^j) = J(X^i, Y^i, Z^j) + u(X^i, Y^i, Z^j),$$

where (X^i, Y^i, Z^j) is a tangent vector field on $S^{2a+b-1}(R)$, u is $1 - form$ on $S^{2a+b-1}(R)$ and A is smooth real function on $S^{2a+b-1}(R)$.

Using $A = \langle\overline{J}(N), N\rangle$, $\xi = \overline{J}(N) - AN$, $u(X^i, Y^i, Z^j) = \langle(X^i, Y^i, Z^j), \xi\rangle$ and $J(X^i, Y^i, Z^j) = \overline{J}(X^i, Y^i, Z^J) - u(X^i, Y^i, Z^j)N$, the elements of the induced structure $\sum = (J, g, \xi, u, A)$ on $S^{2a+b-1}(R)$ by the golden Riemannian structure $(\overline{J}, g)$ on E^{2a+b} are given as follows:

(i) $A = \frac{\phi r^2 + (1-\phi)r_3^2}{R^2}$,
(ii) $\xi = \frac{1}{R}[\phi y^i - \frac{\phi r^2+(1-\phi)r_3^2}{R^2}x^i, \phi x^i - \frac{\phi r^2+(1-\phi)r_3^2}{R^2}y^i, \frac{(1-2\phi)r^2}{R^2}z^j]$,
(iii) $u(X) = \frac{1}{R}[\phi \sum_{i=1}^{a}(y^i X^i + x^i Y^i) + (1-\phi)\sum_{j=1}^{b} z^j Z^j]$,
(iv) $J(X) = (\phi Y^i - \frac{1}{R}u(X)x^i, \phi X^i - \frac{1}{R}u(X)y^i, (1-\phi)Z^j - \frac{1}{R}u(X)z^j)$.

Now, we have

$$\begin{aligned}
A &= \langle\overline{J}(N), N\rangle \\
&\langle\overline{J}(N), N\rangle = \langle\xi + AN, N\rangle. \\
&\frac{1}{R^2}\langle\overline{J}(x^i, y^i, z^j), (x^i, y^i, z^j)\rangle = A\langle N, N\rangle \\
&\frac{1}{R^2}\langle(\phi x^1, \ldots, \phi x^a, \phi y^1, \ldots, \phi \\
&y^a, (1-\phi)z^1, \ldots, (1-\phi)z^b), (x^1, \ldots, x^a, y^1, \ldots, y^a, z^1, \ldots, z^b)\rangle \\
&= \frac{A}{R^2}\langle(x^i, y^i, z^j), (x^i, y^i, z^j)\rangle \\
&\frac{1}{R^2}(\phi\sum_{i=1}^{a}(x^i)^2 + \sum_{i=1}^{a}(y^i)^2 + (1-\phi)\sum_{j=1}^{b}(z^j)^2)
\end{aligned}$$

$$= \frac{A}{R^2}(\sum_{i=1}^{a}(x^i)^2 + \sum_{i=1}^{a}(y^i)^2 + \sum_{j=1}^{a}(z^j)^2)$$
$$\phi r_1^2 + \phi r_2^2 + (1-\phi) r_3^2 = A(r_1^2 + r_2^2 + r_3^2)$$
$$\phi r^2 + (1-\phi) r_3^2 = AR^2$$

or,

$$A = \frac{\phi r^2 + (1-\phi) r_3^2}{R^2}.$$

Since,

$$\xi = \overline{J}(N) - AN$$
$$\xi = \overline{J}(\frac{1}{R}(x^i, y^i, z^j) - A(\frac{1}{R}(x^i, y^i, z^j))$$
$$\xi = \frac{1}{R}[(\phi x^1, \ldots, \phi x^a, \phi y^1, \ldots, \phi y^a, (1-\phi) z^1, \ldots, (1-\phi) z^b) - \frac{\phi r^2 + (1-\phi) r_3^2}{R^2}(x^1, \ldots, x^a, y^1, \ldots, y^a, z^1, \ldots, z^b)]$$
$$\xi = \frac{1}{R}((\phi x^i, \phi y^i, (1-\phi) z^j) - \frac{\phi r^2 + (1-\phi) r_3^2}{R^2}(x^i, y^i, z^j))$$
$$\xi = \frac{1}{R}(\frac{(2\phi - 1) r_3^2}{R^2} x^i, \frac{(2\phi - 1) r_3^2}{R^2} y^i, \frac{(1-2\phi) r^2}{R^2} z^j)$$
$$\xi = \frac{(2\phi - 1)}{R^3}(r_3^2 x^i, r_3^2 y^i, -r^2 z^j).$$

Also,

$$u(X) = u(X^i, Y^i, Z^j) = \langle (X^i, Y^i, Z^j), \xi \rangle$$
$$u(X) = \langle (X^i, Y^i, Z^j), \overline{J}(N) - AN \rangle$$
$$u(X) = \langle (X^1, \ldots, X^a, Y^1, \ldots, Y^a, Z^1, \ldots, Z^b), \frac{1}{R} (\phi x^1, \ldots, \phi x^a, \phi y^1, \ldots, \phi y^a, (1-\phi) z^1, \ldots, (1-\phi) z^b) \rangle$$
$$u(X) = \frac{1}{R}(\sum_{i=1}^{a} \phi x^i X^i + \sum_{i=1}^{a} \phi y^i Y^i + \sum_{j=1}^{b}(1-\phi) z^j Z^j)$$
$$u(X) = \frac{1}{R}[\phi \sum_{i=1}^{a}(x^i X^i + y^i Y^i) + (1-\phi) \sum_{j=1}^{b} z^j Z^j)].$$

Since,

$$J(X) = J(X^i, Y^i, Z^j)$$
$$J(X) = \overline{J}(X^i, Y^i, Z^j) - u(X^i, Y^i, Z^j).N$$
$$J(X) = (\phi X^i, \phi Y^i, (1-\phi) Z^j) - \frac{1}{R} u(X)(x^i, y^i, z^j)$$
$$J(X) = (\phi X^i - \frac{1}{R} u(X) x^i, \phi Y^i - \frac{1}{R} u(X) y^i, (1-\phi) Z^j - \frac{1}{R} u(X) z^j),$$

where $X = (X^i, Y^i, Z^j)$ is a tangent vector on sphere at any point (x^i, y^i, z^j). Therefore, from the above relations we have $(J, <>, \xi, u, a)$ induced structure by $\overline{J}$ from E^{2a+b} on the sphere $S^{2a+b-1}(R)$ of codimension 1 in Euclidean space $E^{2a+b}(R)$.

In conclusion, $S^{2a+b-1}(r)$ is a totally umbilical hypersurface in E^{2a+b}.

Acknowledgements The authors are very much thankful to the referees for their valuable suggestions. Also authors would like to thank to Integral University, Lucknow, India, for providing the manuscript number IU/R&D/2018-MCN000449 to the present research work.

References

1. Ahmad, M., Jun, J.B., Haseeb, A.: Hypersurfaces of an almost r-paracontact manifold endowed with a quater symmetric metric connection. Bull. Korean Math. Soc. **46**(3), 477–487 (2009)
2. Ahmad, M., Qayyoom, M.A.: On submanifolds in Riemannian manifold with golden structure. Turk. J. Math Comput. Sci. **2**(11), 8–23 (2019)
3. Ahmad, M., Shaikh, A.A.: Transversal hypersurfaces of $(LCS)_n$-manifolds. Acta Math. Univ. Comnianae **I**(1), 107–116 (2018)
4. Bahadir, O., Uddin, S.: Slant submanifolds of golden Riemannian manifolds. J. Math. Extens. **13**(4), 23–39 (2019)
5. Blaga, A.M., Crasmareanu, M.: Invariant, anti-invariant and slant submanifolds of a metallic Riemannian manifold. Novi Sad J. Math. **48**(2), 55–80 (2018)
6. Blair, D.E., Ludden, G.D., Yano, K.: Induced strcutures on submanifolds. Kodai Math. Sem. Rep. **22**, 188–198 (1970)
7. Crasmareanu, M., Hretcanu, C.E.: Golden differential geometry. Chaos Solitons Frac. **38**(5), 1229–1238 (2008)
8. Crasmareanu, M., Hretcanu, C.E., Munteanu, M.I.: Golden and product shaped hypersurfaces in real space form. Int. J. Geom. Methods Mod. Phy. **10**(4), 13200069 (2013)
9. Erdoğan, F..E., Yıldırım, C.: On a study of the totally umbilical semi-invariant submanifolds of golden Riemannian manifolds. J. Polytech. **21**(4), 967–970 (2018)
10. Erdoǧan, F.E., Yıldırım, C., Semi-invariant submanifolds of golden Riemannian manifolds, AIP Conference Proceedings 1833(1), https://doi.org/10.1063/1.4981692,(2017)
11. Etayo, F., Santamaria, R., Upadhaya, A.: On the geometry of almost golden Riemannian manifolds. Mediterr. J. Math. **187**, 991–1007 (2017)
12. Gherici, B.: Induced structure on golden Riemannian manifolds. Beitr Algebra Geom. **18**, 366–392 (2018)
13. Gezer, A., Cengiz, N.: On integrability of golden Riemannian structures. Turk. J. Math. **37**, 693–703 (2013)
14. Hretcanu, C.E.: Submanifolds in Riemannian manifolds with golden structure. Workshop on Finsler geometry and its applications, Hungary (2007)
15. Hretcanu, C.E., Blaga, A.M.: Slant and semi-slant submanifolds in metallic Riemannian manifolds. Hindawi J. Funct. Sp. **2018**, 1–13 (2018)
16. Hretcanu, C.E., Blaga, A.M.: Submanifolds in metallic Riemannian manifolds. Differ. Geometry Dyn. Syst. **20**, 83–97 (2018)
17. Hretcanu, C.E., Crasmareanu, M., On some invariant submanifolds in a Riemannian manifolds with golden structure, An.S tiint. Univ. Al. I. cuza Iasi. Mat. (N.S) 53 (suppl.1) 199–211, (2007)
18. Hretcanu, C.E., Crasmareanu, M.: Metallic structure on Riemannian manifolds. Rev. Union Math. Argent. **54**(2), 15–27 (2013)
19. Khan, S.A., Ahmad, M., Zulekha, N.: Non-invariant hypersurfaces of a nearly hyperbolic Sasakian manifold with semi-symmetric non-metric connection. J. Sci. Arts **28**, 1–8 (2017)
20. Okumura, M.: Certain hypersurfaces of an odd-dimensional sphere. Tohoku Math. J. **19**, 381–395 (1967)
21. Ozkan, M.: Prolongations of golden structures to tangent bundles. Differ. Geometry Dyn. Syst. **16**, 227–238 (2014)

Publisher's Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.