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Reliability Evaluation by a Dependent Competing Failure Model Including a Time-Varying Rate for Sudden Degradation Increments

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Abstract

In this study, an effective dependent competing failure model is proposed for systems suffering from shocks. Under worse system degradation, shocks with the same magnitudes can bring larger sudden degradation increments. However, this relationship was ignored in most existing research. To address this problem, in the proposed failure model, a time-dependent rate is included for the sudden degradation increments by shocks. This time-varying rate is applied for the consideration that system degradation is closely related to operation time. Two dependent competing failure processes, i.e., soft failure and hard failure, are involved in the dependent competing failure model. The distribution of the total sudden degradation increments is then deduced, and its accuracy is verified by Monte Carlo simulation. The developed reliability model is illustrated by the reliability analysis of a microelectromechanical system. The sensitivity analyses of important parameters are also performed. The analysis results show that the proposed time-varying model effectively considers the impact of system degradation on sudden degradation increments, and by using this model, the change of sudden degradation increments can be well reflected under different system performances. These advantages make the reliability model more practical and help achieve more effective maintenance policies.

Keywords Dependent competing failure · Time-varying rate · Shock threshold · Monte Carlo simulation

1 Introduction

Systems suffer from failures usually due to internal degradation (e.g., corrosion, fatigue or wear) and external shocks [1]. The failure process is then a comprehensive result of various influencing factors which dependently and competitively contribute to failures. Also, when different failure modes exist, the system will fail with the type that occurs first, i.e., competing relationship [2]. Hence, if this dependency between shocks and degradation is ignored, the system reliability may be overestimated, which can cause unexpected failures and unpunctual maintenance activities, and followed by an economic loss [3]. Therefore, the dependent competing failure model with shocks and degradation is essential especially for systems requiring high safety and accuracy. In this paper, a new and effective dependent competing failure model is proposed for systems suffering from shocks, considering the impact of system degradation on sudden degradation increments by shocks.

1.1 Background

There have been significant research studies on shock and degradation models. Huang et al. [4] evaluated the reliability of elective devices subject to catastrophic failure and degradation failure, and they derived a probability function to predict the significant failures of product. Some scholars focused their studies on competing failure modes. For example, Bunea and Mazzuchi [5] analyzed the failure rate of items in accelerated life testing, based on Bayesian estimation under competing failure modes. Cui et al. [6] gave some reliability indexes for Markov repairable systems, under competing risks modes composed of N-Phase-type distribution and τ-Phase-type distribution. Various methods were used to analyze the reliability of systems experiencing dependent competing failure. Che et al. [2] developed a facilitation model-based Markov point process, to study the reliability of systems undergoing dependent competing failures. Liu et al. [7] discussed the reliability of a system

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subject to dependent competing failure processes under four shock-failure modes, considering Weibull inter-arrival shock process. Liu et al. [8] calculated the reliability of degraded systems subject to multiple dependent competing failures, by using Monte Carlo simulation and considering the degradation self-healing phenomenon. Kong et al. [9] made an improvement of system reliability under dependent competing failure processes by the calibrations of degradation process. Wang and Pham [10] investigated a system subject to multiple degradation processes by time-varying copulas.

Some research considered the mutual relationship of failure processes and discussed different kinds of random shocks. Hao et al. [11] developed a reliability model for systems suffering from continuous degradation and random shocks, with considering the mutual relationship between dependent competing failures. Fan et al. [12] analyzed the reliability of systems experiencing dependent competing failures, in which random shocks were classified into three types according to the magnitude. An et al. [13] concluded that for the components with high reliability, random shocks have an influence on the degradation process of components, if and only if the magnitudes of shocks exceed a critical level under multiple dependent competing failure modes. Hao et al. [14] discussed the reliability of systems subject to dependent competing failures, where the system degradation rate and hard failure threshold are non-constant. There is some research evaluated system reliability by using soft computing-based technique [35] and analyzed reliability, availability, and maintainability together by using PSO and Fuzzy Methodology [36].

It is worth mentioned that stochastic process has been extensively used to model the whole degradation process. For example, Zhang et al. [15] predicted the remaining useful life of systems with nonlinear multi-degradation states based on Wiener process. Li et al. [16] developed a nonlinear Wiener process degradation model for system degradation analysis considering autoregressive measurement errors. Ni et al. [17] modeled the shock damage by a Gamma distribution for a two-stage degradation system with a nonlinear degradation process and a time-varying degradation rate. Mercier and Castro [18] developed a degradation model according to a non-homogeneous gamma process and analyzed two imperfect repair models under this degradation model. For some other important research on system reliability, Niwas and Garg [19] presented a reliability model for a single-unit system, by using a Markov decision process and under a cost-free warranty policy. Garg [20] solved the reliability redundancy allocation problems for systems subject to nonlinear constraints by a penalty guided Biogeography-based optimization method. Garg [21] investigated the fuzzy multi-objective reliability optimization problem for systems with uncertain data by cuckoo search algorithm. Garg [22] developed fuzzy Kolmogorov's differential equations to evaluate system reliability by using vague data and based on a Markov decision process. Garg et al. [23] considered the effect of preventive maintenance actions on system reliability in the maintenance scheduling of a pulping unit.

1.2 Motivation of this Study

Although plenty of contributions have been devoted to the reliability analysis for systems suffering from shocks, most of them considered a constant rate for sudden degradation increments. However, the system performance or state gets worse with increasing running time, and the ability to withstand shocks also decrease. In this situation, considering a constant rate for sudden degradation increments can cause a certain error in reliability assessment. In [10], a Gamma distribution with a time-varying parameter was used to model the shock damage. This model described the shock damage from the perspective of variable Gamma distribution, but the time-varying parameter was not focused on the relationship between shock damage and the magnitude of shocks. Besides, Wei et al. [24] considered binary-state deteriorating systems, where the shock damage follows a normal distribution with different means and variances in different degradation states. Although the effect of system states on shock damage was considered in [24], it seems that no further study is focused on the growth rate between shock damage and the magnitude of shocks.

The study is performed under a reasonable assumption that the sudden degradation increments caused by shocks are dependent with the current system degradation. An effective dependent competing failure model is proposed. This model includes an innovative stochastic model for the sudden degradation increments by shocks, to reflect the relationship between sudden degradation and system degradation. However, the current system degradation is really complicated, consisting of normal degradation and sudden degradation accumulations. During the previous operation time, a system may undergo various random shocks with different magnitudes, subject to the change of temperature and humidity, overloading, vibration and so on. Also, the normal degradation is significantly influenced by changing environment. Hence, when modeling the relationship between sudden degradation increments and system degradation, it is very difficult to utilize the current degradation amount directly.

Therefore, in this study, we transform the impact of the current degradation into a simplified form when modeling the sudden degradation increments by shocks. Then, the current operation time when shocks arrive is used to reflect this impact approximately. It is reasonable, because in practical systems the current degradation amount is closely related to the operation time. First, the accumulated normal degradation gets larger with running time. Also, during a larger operation time, more shocks occur and followed with more sudden degradation. This means the accumulated sudden degradation is closely related to the current operation time. Second, both the effects of normal degradation and sudden degradation on total system degradation are regularly dependent with time, under the common assumption that the occurrences of normal degradation and shocks both follow some known distributions. All the above reasons imply the close relationship between operation time and total system degradation. Actually, the total degradation amount over time has been provided, for example see [13]. However, very few of existing research considered this transformation for the impact of system degradation on sudden degradation. In this study, the effect of the current degradation amount is indirectly reflected by operation time, and then a time-varying model is constructed for the sudden degradation increments caused by shocks. In the proposed time-varying model, the rate of sudden degradation increment varies over time, and it is described by a nonlinear regression model. In this study, the power-law model is applied here, because power-law model is a commonly and effectively used nonlinear regression model, and its character is very suitable for the assumption that the rate of sudden degradation increment increases with time [25].

Additionally, in the proposed dependent competing failure model, the normal degradation is assumed to be a general degradation path or linear degradation path, similar to most research, for example see [12]. Random shocks in this study are classified into three types according to their magnitudes: safe shock, damage shock and destructive shock, similar to [24]. Two dependent competing failure processes, soft failure and hard failure [26], are both involved in the dependent competing failure model. Specifically, the total accumulative degradation amount of a system is assumed to be the sum of normal degradation and sudden degradation increments caused by damage shocks.

The remainder of this study is structured as follows. Section 2 provides a description for the system, including normal degradation process and random shock process. Section 3 develops a reliability model for the system subject to dependent competing failure, where the rate of sudden degradation increments is modeled by a time-varying model. In Sect. 4, the developed reliability model is illustrated by an example of a microelectromechanical system (MEMS), and sensitivity analyses are also performed. Section 5 provides a conclusion and gives future work.

2 System Degradation Process and Assumptions

The investigated system works in a relatively stable environment and deteriorates gradually under prescribed situations, such as rated voltage, normal humidity, permitted load and so on. The normal continuous working environment makes the system suffer from a normal degradation process. Besides, the investigated system subjects to some external damage factors, such as high voltage, shock load, and so on, which can accelerate system degradation or even destroy the system directly. For example, a working gear may suddenly break down when encountering a huge shock load. Additionally, in the investigated system, the shocks with the same magnitudes can bring larger sudden degradation increments under a worse system performance. Therefore, the investigated system simultaneously undergoes normal degradation and external shocks and will suffer from different failure processes.

The related notations in this study are as the following.

Notations			
$X(t)$	Normal degradation at time t	$N(t)$	Total number of random shock at time t
$S(t)$	Cumulative degradation amount caused by damage shocks at time t	P_1	Occurrence probability of safe shock
$X_S(t)$	Total degradation amount of system at time t	P_2	Occurrence probability of damage shock
D	Threshold of hard failure	P_3	Occurrence probability of destructive shock
H	Threshold of soft failure	λ	Arrival intensity of random shock
S	Threshold of damage shock	$T_{(i)}$	The time interval between the i th and $(i-1)$ th damage shocks
Y_i	Sudden degradation increment caused by the i th random shock	T_i	Arrival time of the i th damage shock
W_i	the i th random shock	φ	Initial degradation of normal degradation
$N_1(t)$	Number of safe shock at time t	β	Degradation rate of normal degradation
$N_2(t)$	Number of damage shock at time t	μ_β, σ_β	Mean and standard deviation of β
$N_3(t)$	Number of destructive shock at time t	μ_w, σ_w	Mean and standard deviation of W_I
		a, b	Variable parameters

For the investigated system, its total degradation $X_S(t)$ is the accumulated effect by normal degradation and sudden degradation. In the following, we use Fig. 1a, Fig. 1a, and c to illustrate the system degradation processes more clearly.

Figure 1a shows the normal degradation process of this system. In this figure, $X(t)$ represents the degradation amount over time, while H is the allowable maximum amount of degradation. The normal degradation process is closely related to system operation time, and then a longer operation time can cause a larger degradation. In fact, the degradation of a system is very complicated, and most existing degradation processes were fitted based on a large amount of degradation data. Different degradation models can be referred to [27]. This study considers the normal degradation process as a continuous linear process for simplicity, similar to [13].

Figure 1b presents the random shock process of this system. The classification method of the three types of shocks is similar to [24]. In this figure, D indicates the allowable maximum magnitude of random shock, while S represents the damage shock threshold. The safe shock (W_2), with a magnitude lower than S, has no effect on degradation. The damage shocks (W_1, W_3, W_4 and W_5), with magnitudes greater than S but less than D, can accelerate the degradation process. The destructive shock (W_6) with a magnitude reaching or exceeding D will cause a system failure immediately.

Figure 1c shows the total degradation process of the investigated system. The total degradation process is composed of normal degradation and shock degradation. $X_S(t)$ represents the total degradation amount over time. The symbols Y_1, Y_3, Y_4, and Y_5, respectively, denote the sudden degradation increment caused by W_1, W_3, W_4, and W_5.

From the above, two dependent competing failure processes are involved in the system: (1) the system encounters a failure when a destructive shock arrives with a magnitude not lower than D, i.e., hard failure [26]; (2) The system suffers from a failure when the total accumulative degradation amount reaches the threshold H, i.e., soft failure [26]. The system fails when any type of failure happens first [13]. In contrast, the system normally operates when all the random shocks are not destructive shocks, and the total accumulative degradation amount is below the soft failure threshold H.

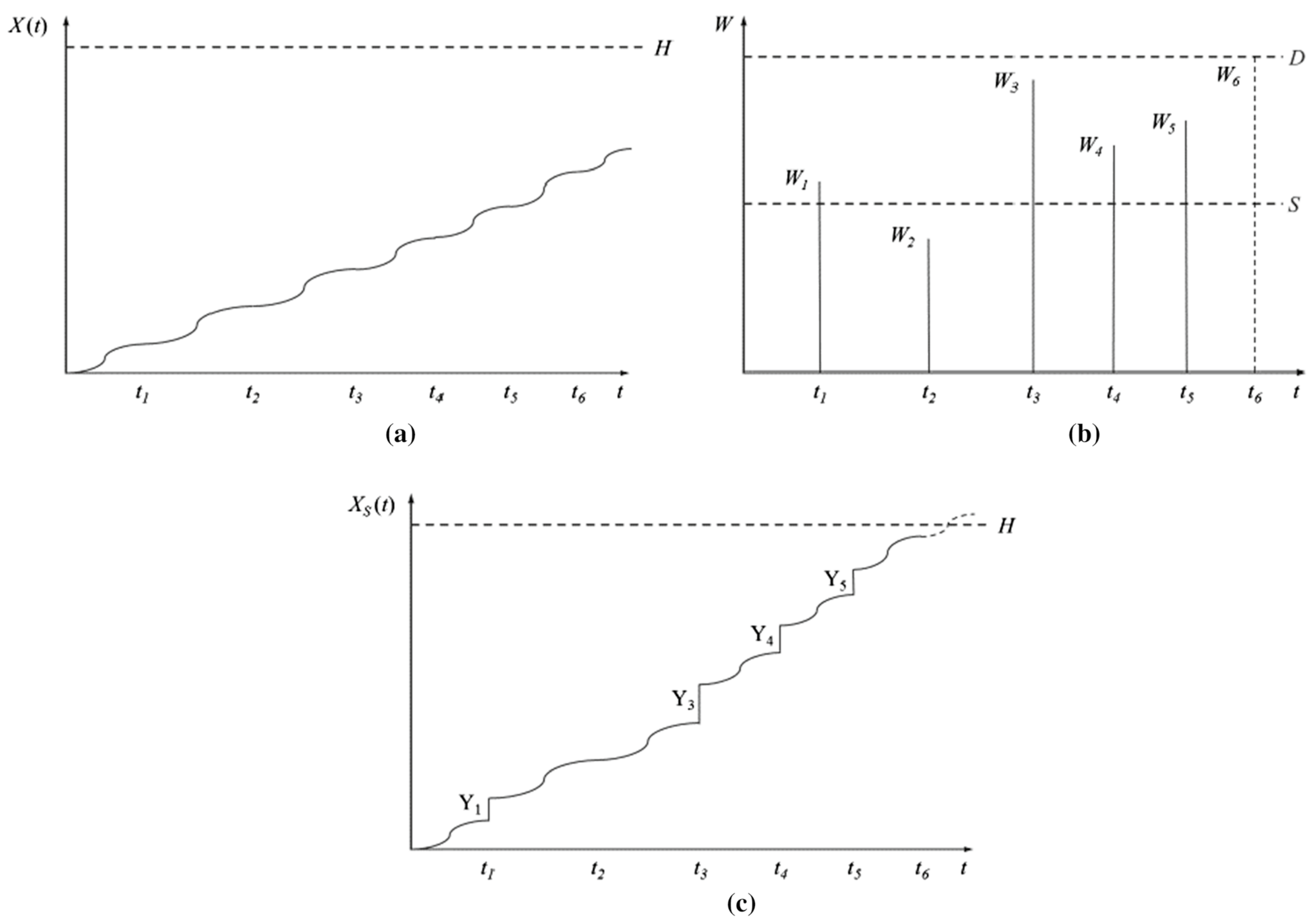

Fig. 1 Dependent competing failure processes: **a** normal degradation process; **b** random shock process; **c** total degradation process under random shocks

3 Reliability Modeling

In this section, the reliability analyses for a hard failure process and a soft failure process are performed for the investigated system, respectively. Further, the system reliability model is derived. A time-varying model is included to describe the change rate for sudden degradation increments. This makes the reliability model can well reflect the changes of sudden degradation increments.

3.1 Reliability Analysis for a Hard Failure Process

Safe shocks and damage shocks cannot lead to failures directly, while the system encounters a hard failure if a destructive shock arrives. $F(\cdot)$ is the cumulative distribution function of the magnitude of a random shock W_i. Hence, when experiencing the i th shock, the probability that a system does not break down due to a hard failure is as the following:

$$P\left(W_i < D\right) = F(D), \quad i = 1, 2, \ldots, \infty \tag{1}$$

Most research considered W_i as an independent and identically distributed random variable following a normal distribution, for example see [13]. A normal distribution is also adopted for the assumption of W_i to develop our model. Then, Eq. (1) can be rewritten as Eq. (2), when the magnitude of a random shock follows a normal distribution, i.e., $W_i \sim N(\mu_w, \sigma_w^2)$:

$$P\left(W_i < D\right) = F(D) = \Phi\left(\frac{D-\mu_w}{\sigma_w}\right), \quad i = 1, 2, \ldots, \infty \tag{2}$$

where $\Phi(\cdot)$ denotes the cumulative distribution function of random variables following a standard normal distribution.

3.2 Reliability Analysis for a Soft Failure Process

A homogeneous Poisson process with intensity λ is employed to describe the arrival number of random shocks in the time interval $[0, t]$, similar to [13]. Then, the probability that a system has undergone n random shocks from the initial time to time t is as the following:

$$\mathrm{P}(\mathrm{N}(\mathrm{t}) = n) = \frac{e^{-\lambda t}(\lambda t)^n}{n!}, n = 1, 2 \ldots \tag{3}$$

The number of safe shocks, damage shocks and destructive shocks are expressed as $N_1(t)$, $N_2(t)$ and $N_3(t)$, respectively. Obviously, the sum of the three numbers is equal to $N(t)$:

$$N_1(t) + N_2(t) + N_3(t) = N(t) \tag{4}$$

When a shock arrives, the occurrence probability of a safe shock, a damage shock or a destructive shock are, respectively, P_1, P_2 or P_3, and the sum of them is 1:

$$P_1 + P_2 + P_3 = 1 \tag{5}$$

where $P_1 = P(W_i < \mathrm{S}) = F(S)$, $P_2 = P(\mathrm{S} < W_i < \mathrm{D}) = F(\mathrm{D}) - F(S)$ and $P_3 = P(W_i \geq D) = 1 - F(D)$.

Based on the decomposition approach of Poisson process [28], the arrival times of safe shocks, damage shocks and destructive shocks follow Poisson processes with intensity λP_1, λP_2 and λP_3, respectively. Additionally, the arrival of one shock, no matter of which type, does not affect the occurrence probabilities of other two types of shocks, i.e., $N_1(t)$, $N_2(t)$ and $N_3(t)$ are independent of each other.

3.2.1 Sudden Degradation Increments Based on a Time-Varying Model

The sudden degradation increment caused by one damage shock is represented by Y_i, $i = 1, 2, \ldots, \infty$. Most studies assumed that Y_i is a linear function of the magnitude of the random shock, with a constant change rate during the whole shock-degradation process; or assumed that Y_i is a random variable following a normal distribution, for example see [12]. However, the degradation process in shock environments is quite complicated, and the related errors may rise during reliability evaluation if ignoring the impacts of operating states or system degradation. In the following section, we provide a time-varying model (developed based on a power-law model), for the sudden degradation increments, by using operation time to reflect the effect of system degradation on Y_i.

Symbol T_i represents the operation time when the *ith* damage shock arrives. Then, according to the general form of a power-law model, the proposed time-varying model for sudden degradation increments is shown as:

$$Y_i = \left(aT_i^b\right) \cdot \left(W_i - S\right), \quad W_i > S \tag{6}$$

in which, a and b are two parameters reflecting the influence of operation time on Y_i, and they can be adjusted based on practical systems.aT_i^b denotes the rate of sudden degradation increments caused by the ith damage shock. This model implies a linear relationship between the sudden degradation increment and the shock magnitude, similar to [13]. Differently, this model considers the effect of the current degradation, and it includes a time-varying rate for sudden degradation increments rather than a constant rate.

The random shock arrivals are assumed to follow a known homogeneous Poisson process. Then, the time intervals between two successive random shocks follow an exponential distribution [13]. Let $T_{(i)}$ represent the time interval between the ith and $(i-1)th$ damage shocks, and then the operation time when the ith damage shock arrives can be written as:

$$T_i = T_{(1)} + T_{(2)} + T_{(3)} + \cdots + T_{(i)} \tag{7}$$

As mentioned, the arrival intensity of damage shocks is λP_2, so we can have:

$$T_{(i)} \sim E\left(\lambda P_2\right) \tag{8}$$

where E(·) represents an exponential distribution.

Referring to [29], the moment generating function of $T_{(i)}$ is as the following:

$$\varphi_{T_{(i)}}(t) = \frac{\lambda P_2}{\lambda P_2 - t} \tag{9}$$

where t denotes the operation time, and its unit can be determined depending on actual systems. Due to the independence between different $T_{(i)}$, and the relationship between $T_{(i)}$ and T_i, the moment generating function of T_i can be derived as the following:

$$\varphi_{T_i}(t) = \left(\varphi_{T_{(i)}}(t)\right)^i = \left(\frac{\lambda P_2}{\lambda P_2 - t}\right)^i \tag{10}$$

which is exactly the moment generating function of a Gamma distribution. Hence, T_i follows a Gamma distribution, since the distribution of T_i can be determined by its moment generating function [29]. Therefore, the distribution of T_i is written as Eq. (11):

$$T_i \sim Ga\left(i, \lambda P_2\right) \tag{11}$$

The expected value of T_i is as the following:

$$E\left(T_i\right) = \frac{i}{\lambda P_2} \tag{12}$$

For simplicity, in the time-varying model for sudden degradation increments (see Eq. (6)), T_i is approximately replaced by the expectation of T_i, as the following:

$$T_i \approx E\left(T_i\right) = \frac{i}{\lambda P_2} \tag{13}$$

By using the expression of T_i in Eqs. (13), (6) can be transformed as:

$$Y_i = \left(a \cdot \left(\frac{i}{\lambda P_2}\right)^b\right) \cdot \left(W_i - S\right), \quad W_i > S \tag{14}$$

This transformation is reasonable, because the occurrence time of a random shock is approximately equal to its expectation in a long run. After the approximate replacement of T_i, the right side of Eq. (14) only has a random variable W_i. Hence, Y_i is a random variable following a normal distribution, and its distribution is derived as the following:

$$Y_i \sim N\left(a\left(\frac{i}{\lambda P_2}\right)^b \left(\mu_w - S\right), a^2\left(\frac{i}{\lambda P_2}\right)^{2b} \sigma_w^2\right) \quad i = 1, 2, 3, \ldots, \infty \tag{15}$$

Furthermore, referring to [30], the cumulative degradation amount caused by damage shocks from operation time 0 to t, can be written as Eq. (16):

$$S(t) = \begin{cases} \sum_{i=1}^{N_2(t)} Y_i \text{ if } N_2(t) > 0 \\ 0 \text{ if } N_2(t) = 0 \end{cases} \tag{16}$$

In the situation of $N_2(t) > 0$, $S(t)$ also follows a normal distribution due to the additivity of normal distributions and is obtained as:

$$S(t) \sim N\left(\frac{a\left(\mu_w - S\right)}{\left(\lambda P_2\right)^b} \sum_{i=1}^{N_2(t)} i^b, \frac{a^2\sigma_w^2}{\left(\lambda P_2\right)^{2b}} \sum_{i=1}^{N_2(t)} i^{2b}\right) \tag{17}$$

where the expectation and variance of $S(t)$ are expressed as follows:

$$E(S(t)) = \frac{a\left(\mu_w - S\right)}{\left(\lambda P_2\right)^b} \sum_{i=1}^{N_2(t)} i^b \tag{18}$$

$$Var(S(t)) = \frac{a^2\sigma_w^2}{\left(\lambda P_2\right)^{2b}} \sum_{i=1}^{N_2(t)} i^{2b} \tag{19}$$

With the replacement of T_i by $E\left(T_i\right)$ in Eq. (6), Eq. (17) is obtained as an approximate distribution of $S(t)$. We verify the accuracy of this approximation by comparing the expectation and variance of $S(t)$ got by Monte Carlo simulation and by Eq. (17). In the Monte Carlo simulation, the expectation and variance of $S(t)$ are obtained when not replacing T_i in Eq. (6). The result shows that this replacement only generates very few errors in both the expectation and variance of $S(t)$. Please see the Appendix for the detailed verification process and result.

3.2.2 The Total Degradation and Survival Probability

A continuous linear degradation is used as the normal degradation process, and then the normal degradation amount at operation time t is as follows:

$$X(t) = \varphi + \beta t \tag{20}$$

Similar to [18], the initial degradation φ is considered to be a constant value and the degradation rate β is assumed as a random variable following a normal distribution, i.e.,

$\beta \sim N\left(\mu_\beta, \sigma_\beta^2\right)$. Then, the normal degradation X(t) also follows a normal distribution:

$$X(t) \sim N\left(\mu_\beta t + \varphi, t^2\sigma_\beta^2\right) \tag{21}$$

in which $\mu_{X(t)} = \mu_\beta t + \varphi$ and $\sigma_{X(t)} = t\sigma_\beta$.

According to [30], the total degradation of a system from 0 to t can be calculated as:

$$X_s(t) = X(t) + S(t) \tag{22}$$

Even though we consider the influence of system degradation on sudden degradation, this influence and relationship have been transformed into operation times at the time-vary model. Therefore, it is assumed that $S(t)$ is independent with X(t), and the total degradation can still be calculated as Eq. (22). Consequently, the total degradation amount of a system from 0 to t can be derived as a random variable following a normal distribution, shown as the following:

$$X_s(t) \sim N\left(\mu_\beta t + \varphi + \frac{a\left(\mu_w - S\right)}{\left(\lambda P_2\right)^b}\sum_{i=1}^{N_2(t)} i^b, t^2\sigma_\beta^2 + \frac{a^2\sigma_w^2}{\left(\lambda P_2\right)^{2b}}\sum_{i=1}^{N_2(t)} i^{2b}\right) \tag{23}$$

From the above, if the system does not encounter a soft failure, the total degradation will not reach the soft failure threshold H. Then, the survival probability of the system is as the following:

$$P\left(X_s(t) < H\right) = \sum_{i=1}^{\infty} P\left(X(t) + S(t) < H|N_2(t) = i\right) \cdot P\left(N_2(t) = i\right) \tag{24}$$

Combining Eqs. (24) and (20), the survival probability of the system can be further derived as:

$$\begin{aligned} P\left(X_s(t) < H\right) &= \sum_{i=1}^{\infty} P\left(X(t) + S(t) < H|N_2(t) = i\right) \cdot P\left(N_2(t) = i\right) \\ &= \sum_{i=1}^{\infty} \Phi\left(\frac{H - \left(\mu_\beta t + \varphi + \frac{a(\mu_w - S)}{(\lambda P_2)^b}\sum_{i=1}^{i} i^b\right)}{\sqrt{t^2\sigma_\beta^2 + \frac{a^2\sigma_w^2}{(\lambda P_2)^{2b}}\sum_{i=1}^{i} i^{2b}}}\right) \cdot \frac{e^{-\lambda P_2 t}\left(\lambda P_2 t\right)^i}{i!} \end{aligned} \tag{25}$$

3.3 Reliability Modeling for Systems Undergoing a Hard Failure Process and a Soft Failure Process

To model the reliability for systems undergoing a hard failure process and a soft failure process, two situations of random shocks are investigated:

Situation 1: No shocks occur during the operation period [0, t].

In this situation, a system does not experience any random shock. This is an idea situation, and the system only fails due to normal degradation. If the total degradation amount is greater than the threshold H, then the system will break down. Therefore, the reliability of the system in this situation is as the following:

$$\begin{aligned} R_1(t) &= P\left(X_s(t) < H\right) \\ &= P\left(X(t) + S(t) < H|N_1(t) = 0, N_2(t) = 0, N_3(t) = 0\right) \\ &\quad P\left(N_1(t) = 0\right) \cdot P\left(N_2(t) = 0\right) \cdot P\left(N_3(t) = 0\right) \\ &= P\left(X(t) + S(t) < H|N_2(t) = 0, N_3(t) = 0\right) \cdot P\left(N_2(t) = 0\right) \cdot P\left(N_3(t) = 0\right) \\ &= P\left(X(t) < H|N_2(t) = 0, N_3(t) = 0\right) \cdot P\left(N_2(t) = 0\right) \cdot P\left(N_3(t) = 0\right) \\ &= P(X(t) < H) \cdot P\left(N_2(t) = 0\right) \cdot P\left(N_3(t) = 0\right) \\ &= \Phi\left(\frac{H - \left(\mu_\beta t + \varphi\right)}{t\sigma_\beta}\right) \cdot \frac{e^{-\lambda P_2 t}\left(\lambda P_2 t\right)^0}{0!} \cdot \frac{e^{-\lambda P_3 t}\left(\lambda P_3 t\right)^0}{0!} \\ &= \Phi\left(\frac{H - \left(\mu_\beta t + \varphi\right)}{t\sigma_\beta}\right) \cdot e^{-\lambda(P_2 + P_3)t} \end{aligned} \tag{26}$$

Situation 2: Shocks occur during the operation period [0, t], but not contain destructive shocks.

In this situation, the system experiences random shocks, when the number of damage shock and safe shock are, respectively, i and k. The safe shocks have no influence on the total degradation. If a system operates normally, the total degradation amount should not exceed the critical threshold H. So, in this situation, the system reliability is as the following:

$$\begin{aligned} R_2(t) &= P\left(X_s(t) < H\right) \\ &= P\left(X(t) + S(t) < H|N_1(t) = k, N_2(t) = i, N_3(t) = 0\right) \\ &\quad P\left(N_1(t) = k\right) \cdot P\left(N_2(t) = i\right) \cdot P\left(N_3(t) = 0\right) \\ &= P\left(X(t) + S(t) < H|N_2(t) = i, N_3(t) = 0\right) \cdot P\left(N_2(t) = i\right) \cdot P\left(N_3(t) = 0\right) \\ &= P\left(X(t) + S(t) < H|N_2(t) = i\right) \cdot P\left(N_2(t) = i\right) \cdot P\left(N_3(t) = 0\right) \\ &= \sum_{i=1}^{\infty} \Phi\left(\frac{H - \left(\mu_\beta t + \varphi + \frac{a(\mu_w - S)}{(\lambda P_2)^b}\sum_{i=1}^{i} i^b\right)}{\sqrt{t^2\sigma_\beta^2 + \frac{a^2\sigma_w^2}{(\lambda P_2)^{2b}}\sum_{i=1}^{i} i^{2b}}}\right) \cdot \frac{e^{-\lambda P_2 t}\left(\lambda P_2 t\right)^i}{i!} \cdot \frac{e^{-\lambda P_3 t}\left(\lambda P_3 t\right)^0}{0!} \\ &= \sum_{i=1}^{\infty} \Phi\left(\frac{H - \left(\mu_\beta t + \varphi + \frac{a(\mu_w - S)}{(\lambda P_2)^b}\sum_{i=1}^{i} i^b\right)}{\sqrt{t^2\sigma_\beta^2 + \frac{a^2\sigma_w^2}{(\lambda P_2)^{2b}}\sum_{i=1}^{i} i^{2b}}}\right) \cdot \frac{e^{-\lambda(P_2 + P_3)t}\left(\lambda P_2 t\right)^i}{i!} \end{aligned} \tag{27}$$

Finally, integrating the above two situations, the system reliability under dependent competing failure processes can be obtained as:

$$
\begin{aligned}
R(t) &= R_1(t) + R_2(t) \\
&= \Phi\left(\frac{H - (\mu_\beta t + \varphi)}{t\sigma_\beta}\right) \cdot e^{-\lambda(P_2+P_3)t} \\
&+ \sum_{i=1}^{\infty} \Phi\left(\frac{H - \left(\mu_\beta t + \varphi + \frac{a(\mu_w - S)}{(\lambda P_2)^b} \sum_{i=1}^{i} i^b\right)}{\sqrt{t^2\sigma_\beta^2 + \frac{a^2\sigma_w^2}{(\lambda P_2)^{2b}} \sum_{i=1}^{i} i^{2b}}}\right) \cdot \frac{e^{-\lambda(P_2+P_3)t}(\lambda P_2 t)^i}{i!}
\end{aligned}
\tag{28}
$$

where $P_2 = P(S < W_i < D) = F(D) - F(S) = \Phi\left(\frac{D-\mu_w}{\sigma_w}\right) - \Phi\left(\frac{S-\mu_w}{\sigma_w}\right)$ and $P_3 = P(W_i \geq D) = 1 - F(D) = 1 - \Phi\left(\frac{D-\mu_w}{\sigma_w}\right)$.

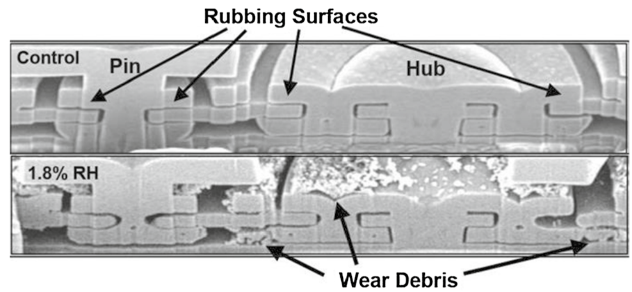

Fig. 3 Scanned electron microscopy image of a worn pin taken from [31]

4 Numerical Example: A MEMS Application

In this section, the proposed model is applied to a microelectromechanical system (MEMS), which was exploited in Sandia National Laboratories, and has been widely used in the research on reliability evaluation [30]. As shown in Fig. 2 taken from [31], A MEMS is composed of orthogonal linear comb drive actuators mechanically connected to a rotating gear. During operation, the linear displacement of comb drive actuators is converted into the circular motion of gear via pin joints.

The major failure mechanisms of a MEMS are as follows: (1) The considerable wear of the contact surface between the gear and the pin joint can cause a damaged pin joint (Fig. 3), and the system encounters a soft failure; (2) Destructive shocks such as loads with great magnitudes can lead to spring fracture (Fig. 4), and the system fails directly, i.e., hard failure. In addition, smaller shocks also can produce certain amounts of wear debris between the pin joint and the gear, when these wear debris are large enough. The two failure modes both can cause system failure [13]. Therefore, a MEMS experiences dependent competing failure processes. According to [33], a MEMS can withstand shock loads due to system structure and material strength. Hence, a safe shock should be considered in this example.

In the following numerical examples, the reliability analysis by the proposed model is given and compared to existing research, and sensitivity analyses on related parameters are also conducted.

4.1 Reliability Model Application and Comparison

In this section, the reliability of MEMS is analyzed by the proposed model in Sect. 3. The related parameters are presented in Table 1.

By using Eq. (28) and the parameters in Tables 2, the reliability curve of the system can be obtained, as shown in Fig. 5. It shows that the system reliability $R(t)$ decreases monotonically with increasing operation time t. When the operation time is close to 1.7e5, the reliability decreases to 0. It is seen that the trend of reliability presents two styles on different sides of time 1.3e5. From 0 to 1.3e5, the reliability decreases with increasing operation time in an approximately constant rate. When arriving at the time 1.3e5, the reliability starts to decline dramatically, and the rate is much greater than the former rate.

This phenomenon can be explained as the following. Based on the proposed time-varying rate model, the rate of sudden degradation increments aT_i^b get larger with system operation time. Hence, when the system operation time is not large (from 0 to 1.3e5 in the example), this rate is

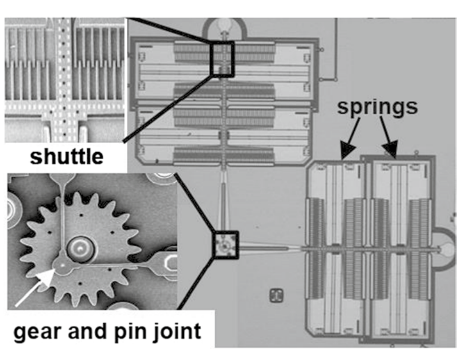

Fig. 2 Scanned electron microscopy image of the micro-engine taken from [31]

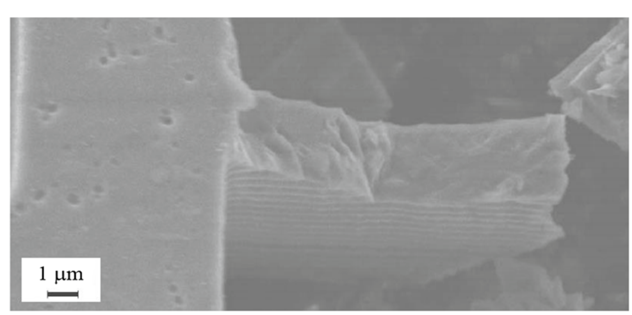

Fig. 4 Scanned electron microscopy image of a shock-induced failure of a suspension spring taken from [32]

Table 1 Related parameters for the numerical example of a MEMS

Parameters	Description	Value	Sources
H	threshold of soft failure	$0.00125\mu m^3$	Tanner et al. [31]
D	threshold of hard failure	$1.5Gpa$	Tanner et al. [31]
S	Threshold of damage shock	$0.2Gpa$	An et al. [13]
$X(t)$	Normal degradation at time t	$X(t) = \varphi + \beta t$	An et al. [13]
φ	Initial degradation of normal degradation	0	Tanner et al. [31]
μ_w	Mean of W_i	$1.2Gpa$	An et al. [13]
σ_w	Standard deviation of W_i	$0.4Gpa$	An et al. [13]
λ	Arrival intensity of random shock	5×10^{-5}	Jiang et al. [34]
μ_β	Mean of β	$8.4823 \times 10^{-9}\mu m^3$	Peng et al. [30]
σ_β	Standard deviation of β	$6.0016 \times 10^{-10}\mu m^3$	Peng et al. [30]
a	Variable parameter	$1.0e-12$	Assumption
b	Variable parameter	1.2	Assumption

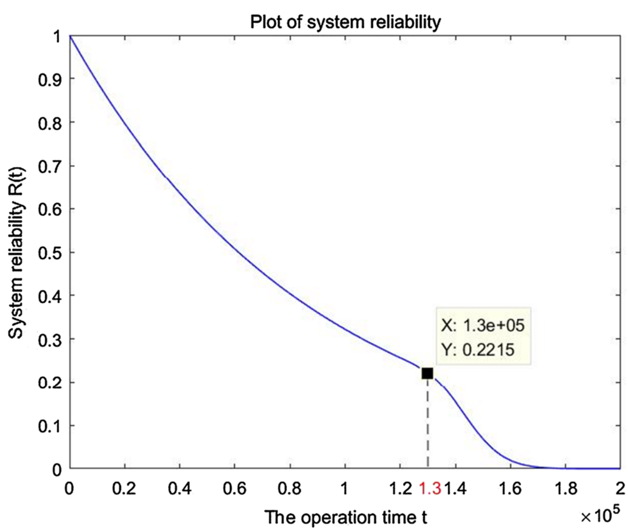

Fig. 5 Change of reliability with increasing operation time

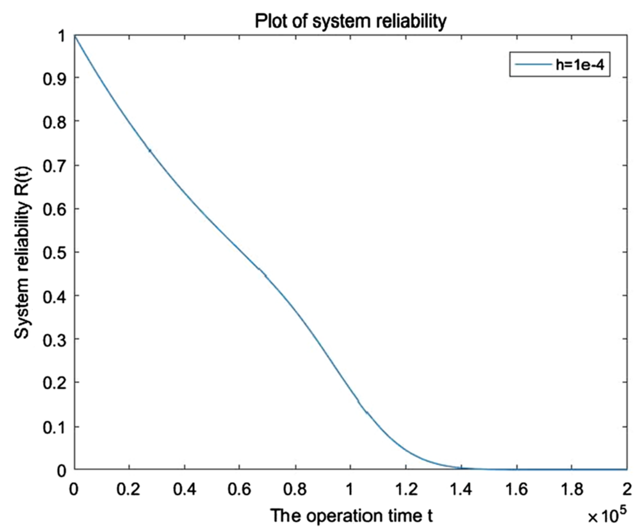

Fig. 6 Change of reliability with a constant rate of sudden degradation increment h

relatively small, and also results in a small sudden degradation increment, under the assumption that the shock magnitudes follow a normal distribution with a small mean. In this situation, the system total degradation process is mainly influenced by the normal degradation process, and it decreases at an approximately constant rate during the operation time from 0 to 1.3e5 (see Fig. 5). In contrast, when the system performance gets worse (during the operation time after 1.3e5), sudden degradation increments can increase rapidly with a greater T_i^b. Then, the system total degradation process is accelerated obviously by the large sudden degradation increments. This finally makes the system undergo an increased risk of failure, and also makes the reliability decrease much faster during the operation time after 1.3e5 (see Fig. 5).

In order to compare with the situation containing a constant rate for sudden degradation increments by shocks, Fig. 6 is presented based on the reliability model in [13]. It provides a reliability curve generated with a constant rate h, which is set to be 1e−4, for sudden degradation increments. When comparing Figs. 5 and 6, the system reliabilities decrease to 0 at different operation time points, because the rates for sudden degradation increments in them are set very differently although other parameters are the same.

Additionally, under the assumption of normal distribution, most random shocks occur with close magnitudes. Then, in the proposed model (See Fig. 5), the sudden degradation increments caused by shocks are influenced much by the rate aT_i^b. Differently, in the constant rate model (see Fig. 6), the sudden degradation increments increase with a fixed rate during the whole system lifetime, and then the system reliability decreases almost with a constant rate.

Besides, we take Fig. 7 from [24] for a better comparison and illustration. In [24], two linear models with different but

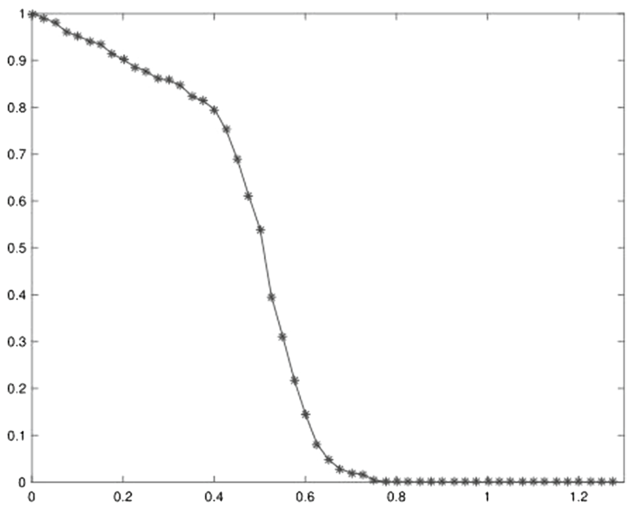

Fig. 7 The plot of the reliability function taken from [24]

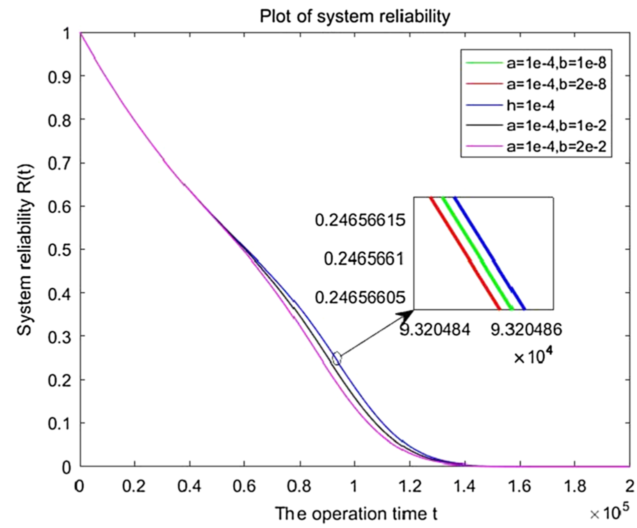

Fig. 8 Comparison of the time-varying rate and the constant-rate model

constant rates of sudden degradation increments are presented. These two models are corresponding to two different system states during the whole operation time, which makes the reliability curve present two styles (see Fig. 7). The phenomenon of two styles is similar to the proposed reliability model (see Fig. 5).

This means that the proposed time-varying model is effective to describe the sudden degradation increments caused by random shocks. Compared to [24], the reliability curve by the proposed reliability model decreases faster before the system enters a considerably poor state (see Fig. 5). This is because the rate in the time-varying model increases with operation time, while the rate in [24] is a fixed value in the first system state, then the system total degradation process is more accelerated in the proposed model. Additionally, the proposed time-varying model is considered to involve continuously changing rates of sudden degradation increments, while the models in [24] had two discrete rates. Hence, the proposed time-varying model is more general and can consider much more different system states.

4.2 Further Comparisons with Different Rate Parameter Settings

In this section, a further comparison for the proposed model and existing research is given in situations with different rate parameter settings. Compared to the sudden degradation increments model in [13], the proposed model differently includes a constant rate or a time-varying rate. For clarity, the proposed model is called as the time-varying model here, and the model of [13] is represented as the constant-rate model. Figure 8 presents several reliability curves according to the two models. Parameters a (in the time-varying model) and h (in the constant-rate model) are both equal to 1e−4, while parameter b (in the time-varying model) is changed from 1e−8 to 2e−2. We can observe that the features of these reliability curves are similar, but the results from the two models get closer when parameter b becomes smaller.

The explanation of Fig. 8 can be given as the following. In the proposed time-varying model, the rate for sudden degradation increment aT_i^b is closely related to two parameters, i.e., a and b. Differently, the corresponding rate in the constant-rate model is a fixed value h. Hence, in the example of Fig. 8 that a and h are equal, aT_i^b is highly close to the fixed value h when b is small enough. Then, the reliability curves are also close, such as the red, green, and blue reliability curves in Fig. 8. This implies that under some special situations, i.e., a quite small parameter b, the time-varying model's behavior is similar to models with a constant rate for sudden degradation increments. However, the reliability model based on the proposed time-varying model can present several different features with different parameter b, even when the parameter a is equal to the constant rate h.

Therefore, compared to constant rate models, the proposed time-varying model is more general and can better describe the change of sudden degradation increments. And then it is necessary to consider the time-varying rate model for the sudden degradation increment, to reflect systems' sensitivity due to shocks.

4.3 Sensitivity Analysis

In this section, sensitivity analyses are conducted to evaluate the influences of important parameters on system reliability. These parameters include the arrival intensity of random shock λ, the two parameters related to the proposed time-vary model, a and b, and the threshold of damage shock S.

The values of the parameters used in the sensitivity analysis are most set as those in Table 1. And in Table 1, parameters are taken from existing research (Please see the references in Table 1 On Page 16), except that the parameters *a* and *b* are especially assumed for our model. For the parameters taken from literature, their authors obtained them according to actual situations and requirements. Therefore, these values of parameters are reasonable, and they can well reflect practical situations, which can be used in this study. For a few parameters, they are changed in certain examples for special purposes and not set as Table 1. For example, in the sensitivity analysis for the threshold of damage shock *S*, the value of *S* is changed in a reasonable range according to that in Table 1 (Please see Fig. 11 on Page 23), while other parameters are still set as those in Table 1. These settings can make the analysis reasonable and related to actual situations.

(1) Sensitivity analysis for the arrival intensity of shocks
The effect of random shocks is an essential part for system total degradation. And the occurrence of random shocks is directly influenced by the arrival intensity of shocks. For this reason, the arrival intensity of shocks is very important, and its impact on system reliability is necessary to be analyzed.

Figure 9 exhibits three reliability curves with different arrival intensities of random shock λ. It is observed that the system reliability is susceptible to the value of λ. When the arrival intensity is increased from 2.0e-5 to 5.0e-5, the reliability curve moves to the left. It implies that the reliability will decrease rapidly when random shocks arrive with a higher intensity. This phenomenon is reasonable, because the higher arrival intensity makes the occurrence probability of damage shocks λP_2 larger. Then, the system is more likely to be impacted by shocks. Correspondingly, the system will deteriorate faster. Therefore, the reliability decreases quickly compared to the situation with a lower arrival intensity of shocks. Additionally, the reliability decreases more rapidly in the later stage when the system enters a considerably poor state with a longer running time.

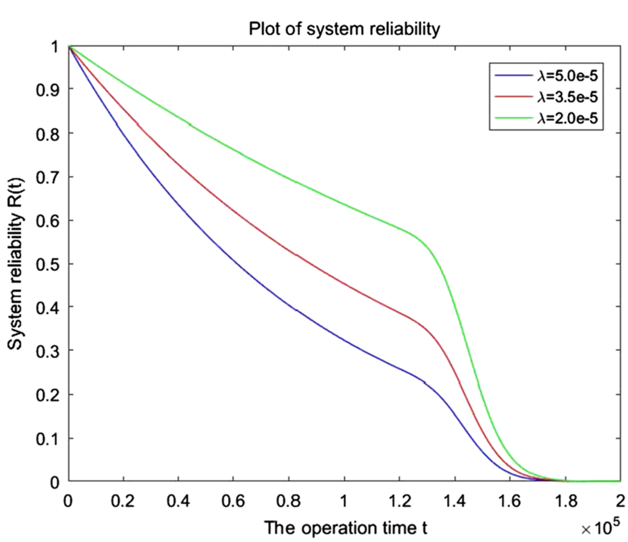

Fig. 9 The change of R(t) with different arrival intensities of random shock λ

(2) The sensitivity analysis for parameters a and b
The parameters, *a* and *b*, are two critical factors in the proposed time-varying rate model and indirectly influence the sudden degradation increments caused by damage shocks. They are especially set in our model and combined to determine the change rate of sudden degradation increments. Although they are not directly from true systems, they are closely related to actual situations and exiting research. First, the values of *a* and *b* can be adjusted according to actual systems when the degradation data are available. When the actual data of sudden degradation increments are known, the rate of sudden degradation increments aT_i^b can be fitted by adjusting parameters *a* and *b*. Second, in the numerical examples, we assumed parameters *a* and *b* according to existing literature of constant-rate models with a constant rate. Therefore, system reliability is closely related to the two parameters, and it is meaningful to perform the sensitivity analysis of parameters *a* and *b* for system reliability.

Figure 10 includes two subplots, respectively, related to the sensitivity analysis of *a* and *b*. Each subplot presents three or four reliability curves, corresponding to the situation with different *a* and the same *b*, or the situation with different *b* and the same *a*.

As shown in Fig. 10a, the value of parameter *b* is 1.2, while parameter *a* is changed from 1.0e-12 to 11.0e-12 in increments of 5.0e-12. It is seen that the reliability is not sensitive to *a* in the early stage (approximately before the time 1.2e5). After the time 1.2e5, some differences emerge among the three reliability curves. This shows that the corresponding reliability gradually decreases with increasing *a*. The phenomenon is possible, because in the proposed reliability model, the parameter *a* can impact the rate of shock degradation. If *a* is greater, then the rate of shock degradation becomes larger, and the system reliability will be reduced. However, this difference is not very big, and it only appears when the system has run for a long time. This implies that parameters *a* can be determined according to the operation time. Especially, when a system has operated for a long time, it is more necessary to estimate an appropriate *a*, because it can be meaningful for effective maintenance planning.

In Fig. 10b, the value of parameter *a* is 1.0e-12, while parameter *b* is changed from 0.7 to 2.2 in increments

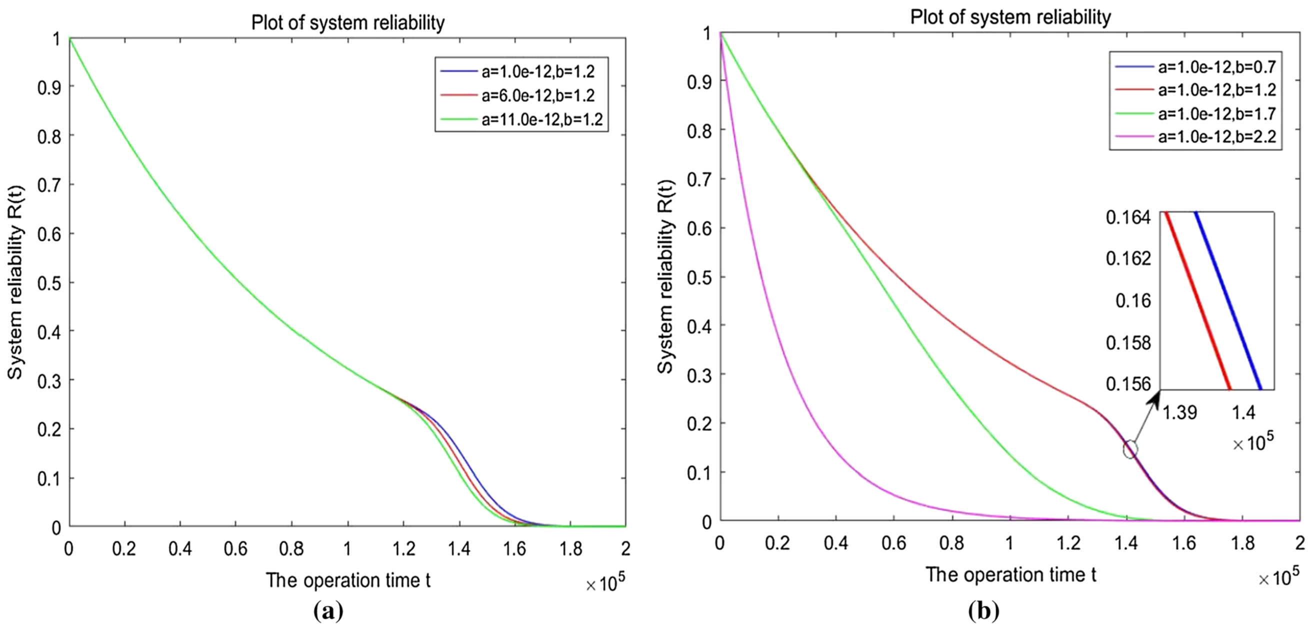

Fig. 10 Sensitivity analysis of *a* and *b*: **a** Sensitivity analysis of *a*; **b** Sensitivity analysis of *b*

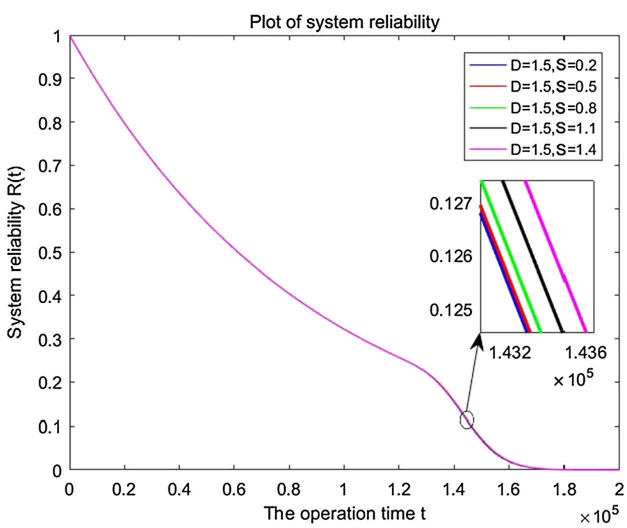

Fig. 11 Sensitivity analysis for the threshold of damage shock (S)

of 0.5. Obviously, the reliability curve shifts to the left when parameter *b* increases. This phenomenon is reasonable, because the parameter *b* is positively related to the rate of shock degradation. When *b* becomes greater, the rate of shock degradation also gets greater, and the total degradation of system is accelerated. It is noted that the reliability is not very sensitive to *b* when *b* is relatively small. This situation can be explained as follows. First, when *b* is relatively small, the impact of the sudden degradation rate on degradation process is also small. In contrast, when the system enters a considerably poor state, the reliability can show obvious difference for situations with different b. Second, when the parameter *b* is large, the impact of the sudden degradation rate on degradation process is also large. Then, the system reliability will be significantly different even after a short running time. Figure 10b shows that, there is some relationship between parameter *b* and system lifetime. If the lifetime is larger, it corresponds to a smaller *b*; otherwise, it corresponds to a larger b. Therefore, in engineering applications, the parameter *b* can be estimated by considering the lifetime.

(3) <u>Sensitivity analysis for the thresholds of damage shock and hard failure</u>

The damage shock threshold determines whether one shock can influence the system reliability, while the hard failure threshold determines whether one shock results in a sudden failure. Hence, the sensitivity analyses of the two parameters for system reliability are necessary. The analysis result will also be beneficial to improve system performance when designing system reliability.

In Fig. 11, 5 reliability curves are given for situations with a constant threshold of hard failure (*D*) and different thresholds of damage shock (*S*). It shows that the system reliability shifts to the right while the threshold of damage shock is changed from 0.2 to 1.4 in increments of 0.3. It means that the system reliability becomes better with increasing damage shock threshold. However, the reliability is not sensitive to *S* in the early stage, approximately before the time 1.35e5 (see Fig. 11).

The phenomenon in Fig. 11 can be explanted as follows: First, if the system has not run a long time and its state is not so poor, the effect of the damage shock threshold on system reliability is very small. Second, when the system enters a considerably poor state, the system resistance can be obviously different between situations with high and different damage shock thresholds (e.g., when S is larger than 0.8 in Fig. 11). Systems with a higher damage shock threshold mean a stronger ability to resist shocks and followed with a better reliability. Consequently, in engineering applications, improving the damage shock threshold can be an effective way to extend the lifetime of deteriorating systems.

Figure 12 presents 3 reliability curves considering different thresholds of hard failure D and a constant threshold of damage shock S. It is seen that the system reliability shifts to the right when the threshold of hard failure D is changed from 1.5 to 1.9 in increments of 0.2. Evidently, the reliability is sensitivity to the threshold of hard failure. The reason is that larger values of D make the occurrence probability of hard failures lower. However, the difference becomes much smaller when the operation time exceeds a certain time point. This means that a system with a high threshold of hard failure can have a relatively high reliability. Besides, improving the threshold D can be very effective when the operation time is close to a certain value (around 1.3e5 in this example).

From the above analyses, there are several advantages and findings of our model: First, the proposed time-varying model can reflect different change rates for sudden degradation increments in different operation times. Second, the two parameters a and b in the time-varying model are closely related to the operation time. Third, the system reliability can be improved effectively when increasing the threshold of hard failure around a certain operation time.

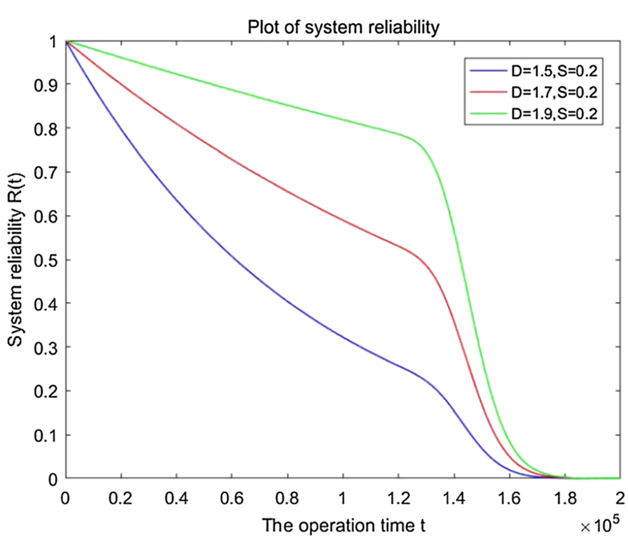

Fig. 12 Sensitivity analysis for the threshold of hard failure (D)

5 Conclusions

In this study, an innovative dependent competing failure model is developed for deteriorating systems. Two failure processes, i.e., hard failure and soft failure, are involved. A time-varying rate of sudden degradation increments is included in the failure model, to describe the sudden degradation increments caused by damage shocks. This study considers the effect of system degradation on sudden degradation increments, and the operation time when shocks arrive is used to reflect this effect indirectly. The distribution of the accumulated sudden degradation increments is deduced, and its accuracy is verified by Monte Carlo simulation. Based on the above, the total degradation containing normal degradation and sudden degradation is obtained, and the reliability model is constructed.

Furthermore, the proposed reliability model is illustrated by a MEMS application, and the sensitivity analyses of some parameters are performed. According to the results of numerical examples, some important conclusions are given as follows:

(1) The proposed time-varying model is more general, compared with situations including a constant rate for sudden degradation increments. This implies the significance of our model to consider the effect of system degradation on sudden degradation increments by shocks.

(2) The system reliability can be effectively improved by reducing the occurrence intensity of shocks, and by increasing the threshold of hard failure. These improvements are especially obvious when near a certain operation time. The greater threshold of damage shock can also provide a higher reliability, only when the system enters a considerably poor state.

(3) The parameter b has a greater impact on system reliability than the parameter a, while parameter a only makes some differences under a longer operation time. It is reasonable due to the form of the power-law model used in the time-varying model. Based on the reliability data obtained in different operation stages, the values of a and b can be estimated and adjusted.

Then the importance of the proposed reliability model can be summarized as the following. The proposed time-varying model considers a more comprehensive system degradation process, and the effect of system degradation level on sudden degradation increments is included. Compared with constant-rate models, the proposed time-varying model

Table 2 The relative errors of expectations and variances for 20 sets of time

Time (t)	Expectations obtained by Monte Carlo simulation (E_M)	Expectations obtained by Eq. (17) (E_R)	Variances obtained by Monte Carlo simulation (V_M)	Variances obtained by Eq. (17) (V_R)	The relative errors of expectations (R_E)	The relative errors of variances (R_V)
691.6400	8.030325e+05	8.047403e+05	2.126179e+08	2.138333e+08	2.126664e−03	5.716202e−03
701.6400	8.289680e+05	8.306903e+05	2.233242e+08	2.245746e+08	2.077610e−03	5.598698e−03
711.6400	8.552758e+05	8.570084e+05	2.343718e+08	2.356538e+08	2.025817e−03	5.469922e−03
721.6400	8.818412e+05	8.835888e+05	2.457167e+08	2.470350e+08	1.981797e−03	5.364796e−03
731.6400	9.089335e+05	9.107000e+05	2.574804e+08	2.588389e+08	1.943490e−03	5.275879e−03
741.6400	9.363566e+05	9.381294e+05	2.695850e+08	2.709735e+08	1.893278e−03	5.150353e−03
751.6400	9.645131e+05	9.662868e+05	2.822163e+08	2.836308e+08	1.839032e−03	5.012270e−03
761.6400	9.930823e+05	9.948676e+05	2.952388e+08	2.966892e+08	1.797796e−03	4.912838e−03
771.6400	1.022078e+06	1.023870+e06	3.086655e+08	3.101481e+08	1.753326e−03	4.803299e−03
781.6400	1.051484e+06	1.053286e+06	3.224954e+08	3.240135e+08	1.713388e−03	4.707407e−03
791.6400	1.081179e+06	1.082986e+06	3.366773e+08	3.382278e+08	1.672011e−03	4.605291e−03
801.6400	1.111516e+06	1.113339e+06	3.513877e+08	3.529796e+08	1.639872e−03	4.530352e−03
811.6400	1.141967e+06	1.143794e+06	3.663758e+08	3.679986e+8	1.599736e−03	4.429157e−03
821.6400	1.173025e+06	1.174858+06	3.818907e+08	3.835472e+08	1.562455e−03	4.337667e−03
831.6400	1.204706e+06	1.206540e+06	3.979493e+08	3.996348e+08	1.521855e−03	4.235283e−03
841.6400	1.237183e+06	1.239022e+06	4.146504e+08	4.163703e+08	1.486221e−03	4.147786e−03
851.6400	1.269862e+06	1.271715+06	4.316970e+08	4.334607e+08	1.459046e−03	4.085314e−03
861.6400	1.302888e+06	1.304757e+06	4.491691e+08	4.509787e+08	1.433866e−03	4.028782e−03
871.6400	1.336136e+06	1.338017+06	4.670051e+08	4.688583e+08	1.407701e−03	3.968297e−03
881.6400	1.370004e+06	1.371887e+06	4.854262e+08	4.873120e+08	1.374208e−03	3.884864e−03

is more general and can dynamically describe the change rate of sudden degradation increments with different operation times. Then, by using the proposed time-varying model, system reliability can be evaluated more comprehensively and can help achieve more effective maintenance strategies for systems.

For the further research, this study can be extended as follows: (1) Analyze the reliability and maintenance of industrial systems by using PSO and fuzzy methodology based on the proposed model; (2) Evaluate multi-item system reliability by using soft computing-based technique.

Appendix: Accuracy verification of the derived distribution of $S(t)$

Because of the replacement of T_i by $E(T_i)$, Eq. (17) is an approximate distribution of $S(t)$, i.e., the accumulated sudden degradation increments by damage shocks at operation time t. We verify the accuracy here, by comparing the expectation and variance obtained from Monte Carlo and from Eq. (17). In the Monte Carlo simulation, we obtain the expectation and variance of $S(t)$ when not replacing T_i in Eq. (6).

The Monte Carlo simulation and comparison method are described as follows:

Step 1. Set parameters and generate random shocks.

Five thousand sets of damage shocks are generated, while each set includes 200 arrival time points, corresponding to the operation times when damage shocks arrive. Let λP_2 ,S, μ_w and σ_w be 1, 0.2, 1.2 and 0.4, representing the arrival rate of damage shock, the threshold of safe shock, the mean of random shock and the standard deviation of random shock, respectively. The two variable parameters a and b are assumed to be 1 and 1.2, respectively. Clearly, the arrival time of the *ith* damage shock in each set is the sum of the previous i time intervals.

Step 2. Determine the comparison time span.

Without loss of generality, the comparisons are performed at 20 time points, i.e., 20 different values of t, because the expectation and variance of $S(t)$ is different with time t. The investigated range of t is from 691.6400 to 881.6400 here, with 881.6400 not exceeding the arrival time of the last shock in each set.

Step 3. Calculate the expectation and variance of $S(t)$ when not replacing T_i.

For each time point t, if t is extremely great, all the damage shocks in each set are considered. Otherwise, only the damage shocks have arrived until t in each set are considered. For each set, we can calculate $S(t)$, the accumulated sudden degradation by shocks. Then, by dealing with the values of $S(t)$ in all set, we obtain the expectation and variance of $S(t)$ by Monte Carlo method.

Step 4. Calculate the expectation and variance of $S(t)$ when replacing T_i by $E\left[T_i\right]$.

For each t, number the arrival times of damage shocks from 0 to t in each set obtained by Step 1, starting from 1 in sequence. Then, calculate the expectation of each numbered arrival time of damage shocks in each set with Eq. (12). By using these expectations of the arrival times of shocks, for each set, calculate the expectation and variance of each shock degradation by Eq. (17). Finally, estimate the expectation and variance of $S(t)$ by average the expectation and variance obtained in all sets.

Step 5. Compare the expectation and variance of $S(t)$ obtained by Step 3 and Step 4.

For different t, calculate the relative errors of expectation and variance of $S(t)$ obtained by Step 3 and Step 4.

Table 2 presents the comparison results, including two types of relative errors. One is the relative errors of the expectation of $S(t)$, can be calculated as the following:

$$R_E = \frac{\left|E_M - E_R\right|}{E_M}$$

where E_M and E_R are the expectations obtained by Monte Carlo simulation and Eq. (17) for each t, respectively. The other is the relative errors of the variance of $S(t)$, which can be expressed as the following:

$$R_V = \frac{\left|V_M - V_R\right|}{V_M}$$

where V_M and V_R are the variances obtained by Monte Carlo simulation and Eq. (17) for each t, respectively. It is seen that the replacement of T_i with $E\left(T_i\right)$ only leads to a quite small error for both the expectation and variance of $S(t)$.

Based on the simulation data in Table 2, we can easily observe that the relative errors of expectation and variance are greatly small and decrease gradually with the incensement of operation time. So, combining the above analysis, the expectation of T_i can be used to replace T_i, approximately.

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