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RESEARCH ARTICLE - ELECTRICAL ENGINEERING

MGF Approach to Compute the Packet Error Probability and Throughput for OFDM, CDMA and MC-CDMA Systems

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Abstract
In this paper, we present a new approach to evaluate the packet error probability (PEP) of code-division multiple access (CDMA), multi-carrier CDMA (MC-CDMA) and orthogonal frequency-division multiple access systems. The PEP is computed using the moment-generating function of signal-to-noise ratio. Our approach uses an approximation of the Marcum Q-function and allows to evaluate the PEP in closed form without any integration. The obtained theoretical results were compared to simulations, and their validity was confirmed.

Keywords Packet error probability · CDMA · MC-CDMA · OFDM

1 Introduction

The performance analysis of wireless communication systems can be evaluated by different approaches and presented using several metrics. Given that, the moment-generating function (MGF) of signal-to-noise ratio (SNR) has been identified as a powerful tool for simplifying some of the analysis [1–10]. Based on the MGF of SNR, different methods have been discussed in the literature to derive the symbol and bit error probabilities, (SEP) and (BEP) respectively, [1–10]. The MGF of SNR has been used to evaluate symbol and bit error probabilities for different channels such as Alpha–Mu, Nakagami and Beckmann fading [11–13]. The same approach has been used to study the bit error probability of free-space optical (FSO) communications [14]. Also, the MGF of SNR has been used to compute the channel capacity in [15–20]. Another possible application is to determine the area under the curve of the receiver operating characteristic (ROC) of spectrum sensing algorithms using the energy detector [21]. Nevertheless, the MGF approach has not been yet used to derive the packet error probability (PEP), which is the main objective of this paper. BEP computation using the MGF as initially suggested in [1] requires that some integrals be computed numerically. In this paper, the PEP is expressed in closed form as a function of MGF of SNR, and there is no need to compute any integral. Our results are also valid for CDMA, MC-CDMA and OFDM systems. Indeed, the MGF is the Laplace transform (LT) of the PDF of instantaneous SNR. Thereby, in order to compute the PEP, most of the researchers use an integral, which is calculated numerically, and there is no closed-form expression. This paper proposes a closed-form mathematical expression based on the MGF for computation of the PEP for wireless communications systems.

The proposed MGF approach is a simple tool to estimate the PEP for different systems, such as:

– Direct-sequence code-division multiple access systems (DS-CDMA).
– Multi-carrier CDMA (MC-CDMA) systems.
– Orthogonal frequency-division multiple access (OFDMA) systems.

The contributions of our paper are:

– Deriving the packet error probability using the MGF of SNR. Only the SEP and BEP were studied in [1–21].
– Studying DS-CDMA, MC-CDMA and OFDMA systems. The results presented in [1–21] are valid for single-carrier systems.

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- Evaluating BEP using the MGF as suggested in [1] requires that some integrals be computed numerically. In this paper, the PEP is derived in closed form without any integral.

The paper is organized as follows: the PEP for single-carrier systems is presented in Sect. 2. Sections 3, 4 and 5 derive the packet error probability for direct-sequence code-division multiple access, multi-carrier code-division multiple access (MC-CDMA) and orthogonal frequency-division multiple access (OFDM) systems, respectively. Section 6 provides some simulation and theoretical results. Section 7 concludes the paper.

2 PEP for Single-Carrier and Frequency-Nonselective Channels

2.1 System Model

For frequency-nonselective channels, the received signal can be written as

$$r = hs + n \tag{1}$$

where h is the channel coefficient, s is the transmitted symbol and n is the additive white Gaussian noise (AWGN) with PSD N_0.

Therefore, the SNR is written as

$$\Gamma = \frac{E_s |h|^2}{N_0}. \tag{2}$$

For Rayleigh fading channels, the SNR is exponentially distributed.

2.2 PEP Evaluation: MGF-Based Approach

For BPSK modulation and a block-fading channel, the packet error probability is written as

$$\text{Pbloc}(L, \overline{\Gamma}) = 1 - \int_0^{+\infty} \left[1 - Q(\sqrt{2x})\right]^L f_\Gamma(x)\mathrm{d}x \tag{3}$$

The channel is assumed to be block fading, i.e., constant during L symbols and independent between two packets, and $f_\Gamma(x)$ is the PDF of the SNR. For Rayleigh fading channels, it is given by

$$f_\Gamma(x) = \frac{e^{-\frac{x}{\overline{\Gamma}}}}{\overline{\Gamma}} \tag{4}$$

$\overline{\Gamma}$ is the average SNR.

The previous equation can be written as

$$\text{Pbloc} = \int_0^{+\infty} \sum_{k=1}^{L} \binom{L}{k} (-1)^k Q\left(\sqrt{2x}\right)^k f_\Gamma(x)\mathrm{d}x \tag{5}$$

where $Q(x)$ is the Marcum Q-function defined by

$$Q(x) = \int_x^{+\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{u^2}{2}} \mathrm{d}u. \tag{6}$$

The next step is to use an approximation of the complementary error function, which is related to Marcum Q-function [22,23]

$$\text{erfc}(x) \simeq \frac{e^{-x^2}}{6} + \frac{e^{-\frac{4x^2}{3}}}{2}. \tag{7}$$

We have

$$Q(x) = \frac{1}{2}\text{erfc}\left(\frac{x}{\sqrt{2}}\right) \tag{8}$$

and

$$Q\left(\sqrt{2x}\right)^k \simeq \frac{1}{4^k} \sum_{p=0}^{k} \binom{k}{p} \frac{1}{3^{k-p}} e^{-x(k-p)} e^{-\frac{4xp}{3}}. \tag{9}$$

Finally, using previous equations, we have

$$\text{Pbloc}(L, \overline{\Gamma}) \simeq \sum_{k=1}^{L} \binom{L}{k} (-1)^k \frac{1}{4^k} \sum_{p=0}^{k} \binom{k}{p} \times \frac{1}{3^{k-p}} \int_0^{+\infty} e^{-x(k+\frac{p}{3})} f_\Gamma(x)\mathrm{d}x \tag{10}$$

and

$$\text{Pbloc}(L, \overline{\Gamma}) \simeq \sum_{k=1}^{L} \binom{L}{k} (-1)^k \frac{1}{4^k} \sum_{p=0}^{k} \binom{k}{p} \times \frac{1}{3^{k-p}} M_\Gamma\left(k + \frac{p}{3}\right) \tag{11}$$

where $M_\Gamma(s)$ is the moment-generating function (MGF) of SNR (Laplace transform LT of the probability density function of SNR) defined by

$$M_\Gamma(s) = LT(f_\Gamma(x)) = \int_0^{+\infty} e^{-sx} f_\Gamma(x)\mathrm{d}x. \tag{12}$$

The throughput can be easily deduced from the packet error probability

$$\text{Thr} = R(1 - \text{Pbloc}) \tag{13}$$

where R is the data rate.

3 Application to CDMA Systems

3.1 System Model

We study a direct-sequence CDMA (DS-CDMA) system composed of two nodes: a transmitter T and a receiver R. The transmitted signal by T is written as

$$e(t) = \sum_k s_k c^k(t - kT_s), \tag{14}$$

where s_k is the k-th transmitted symbol, T_s is the symbol period, $c^k(t)$ is the spreading waveform given by

$$c^k(t) = \sum_{q=0}^{N-1} c(kN+q) g(t - qT_c), \tag{15}$$

N is the spreading factor, $g(t)$ is the shaping filter, T_c is the chip period and $c(k)$ is the spreading sequence.

For DS-CDMA systems, the received signal at node R can be expressed as

$$r(t) = \sum_{l=1}^{P} f^l(t) \sum_k s_k c^k(t - kT_s - \tau^l) + n(t), \tag{16}$$

where P is the number of paths, τ^l is the delay of l-th path, $f^l(t)$ is the complex gain of l-th path at time t and $n(t)$ is an AWGN with one-sided PSD equal to N_0,

The m-th correlation of the rake receiver for k-th symbol period can be expressed as [24]

$$z^m(k) = s_k \sum_{l=1}^{P} f^l(kT_s) q(\tau_m - \tau_l) + n^m(k), \tag{17}$$

where $n^m(k)$ is a noise term, $q(\tau) = (g \otimes g)(\tau)$ and $\otimes$ is the convolution between two signals.

The rake output can be written as

$$\begin{aligned} \widetilde{s}_k &= \sum_{m=1}^{P} f^{m*}(kT_s) z^m(k) \\ &= s_k \mathbf{f}^\dagger \mathbf{Q} \mathbf{f} + \sum_{m=1}^{P} f^{m*} n^m(k) \end{aligned} \tag{18}$$

where $\mathbf{Q}$ is an $P \times P$ matrix, $\mathrm{Q}(k,l) = q(\tau^k - \tau^l)$,

$$\mathbf{f} = \left(f^1(kT_s), \ldots, f^P(kT_s)\right)^T. \tag{19}$$

The SNR is given by [24]

$$\Gamma = \frac{E_s}{N_0} \mathbf{f}^\dagger \mathbf{Q} \mathbf{f}, \tag{20}$$

where E_s is the transmitted energy per symbol.

3.2 PEP Derivation for CDMA Systems

Using (17), the SNR can be expressed as

$$\Gamma = \frac{E_s}{N_0} \mathbf{g}^\dagger \mathbf{g},$$

where $\mathbf{g} = \sqrt{\mathbf{Q}}\mathbf{f}$.

Using the Karhunen–Loève (KL) orthogonal expansion, we have

$$\Gamma = \frac{E_S}{N_0} \mathbf{h}^\dagger \mathbf{h}, \tag{21}$$

where $\mathbf{h} = \left(h^1, \ldots, h^P\right)^T = \mathbf{U}^\dagger \mathbf{g}$, $\mathbf{U} = \left[\mathbf{u}^1 \ldots \mathbf{u}^P\right]$ is a unitary matrix, $\mathbf{u}^i$ is the i-th normalized eigenvector of the covariance matrix of $\mathbf{g}$:

$$E\left(\mathbf{g}\mathbf{g}^\dagger\right) = \sqrt{\mathbf{Q}} \mathbf{R_f} \sqrt{\mathbf{Q}} = \mathbf{U} \boldsymbol{\Delta} \mathbf{U}^\dagger, \tag{22}$$

$\mathbf{R_f} = E(\mathbf{f}\mathbf{f}^\dagger)$, $\boldsymbol{\Delta} = \mathrm{diag}(\lambda^{(1)}, \ldots, \lambda^{(P)})$ and $\lambda^{(i)}$ is the i-th eigenvalue.

Since the components of $\mathbf{h}$ are independent, the SNR is the sum of P-independent exponential random variables. The moment-generating function (MGF) of the SNR is therefore given by

$$M_\Gamma(s) = E(e^{-s\Gamma}) = \prod_{j=1}^{P} \frac{1}{1 + s\beta^{(j)}}, \tag{23}$$

where

$$\beta^{(j)} = \lambda^{(j)} \frac{E_s}{N_0}. \tag{24}$$

Using a fraction decomposition, we obtain

$$M_\Gamma(s) = \sum_{j=1}^{P} \frac{\pi^{(j)}}{1 + s\beta^{(j)}} \tag{25}$$

where the residue is given by

$$\pi^{(j)} = \prod_{1 \le k \le P, k \ne j} \frac{\lambda^{(j)}}{\lambda^{(j)} - \lambda^{(k)}}. \tag{26}$$

By using the inverse Laplace transform (LT), we deduce the PDF of the SNR [25]

$$p_{\Gamma}(\gamma) = \sum_{j=1}^{P} \frac{\pi^{(j)}}{\beta^{(j)}} \exp\left(-\frac{\gamma}{\beta^{(j)}}\right), \text{ if } \gamma \geq 0 \tag{27}$$

Using the results of Sect. 2, we deduce the PEP

$$\text{Pbloc}_{\text{DS-CDMA}} = \sum_{j=1}^{P} \pi^{(j)} \text{Pbloc}(L, \beta^{(j)}), \tag{28}$$

where $\text{Pbloc}(L, a)$ has been defined in (11).

4 Application to MC-CDMA Systems

4.1 System Model

We study a MC-CDMA system composed of two nodes: a transmitter T and a receiver R. The transmitted signal by node T can be expressed as [26,27]

$$e(t) = \sqrt{\frac{E_s}{S}} \sum_{k} s_k g(t - kT_s) \sum_{m=0}^{S-1} c_{kS+m} e^{j2\pi f_m t}, \tag{29}$$

where S is the number of subcarriers, s_k is the k-th transmitted symbol, T_s is the symbol period, $\{c_{kS+m}\}_{m=0}^{S-1}$ is the spreading sequence, $g(t)$ is a rectangular pulse response with temporal support $[0T_s]$, $T_s = T_s^u + \Delta$ where T_s^u is the useful symbol duration and Δ is the guard interval, $f_m = f_0 + m\Delta f$ is the m-th carrier frequency and $\Delta f = 1/T_s^u$ is the carrier separation.

The received signal by node R on k-th symbol period is expressed as

$$r(t) = \sqrt{\frac{E_s}{S}} \frac{s_k}{\sqrt{T_s - \Delta}} \sum_{m=0}^{S-1} c_{kS+m} e^{j2\pi f_m t} F(f_m; t) + n(t), \tag{30}$$

where

$$F(f_m; t) = \int f(\tau; t) e^{-j2\pi f_m \tau} \mathrm{d}\tau, \tag{31}$$

$f(\tau; t)$ is the Rayleigh multipath fading channel and $n(t)$ is an AWGN.

The discrete Fourier transformation (DFT) output is written as

$$\mathbf{z}^k = \left(z_0^k, \ldots z_{S-1}^k\right)^T = \sqrt{\frac{E_s}{S}} s_k \mathbf{F} + \mathbf{n}, \tag{32}$$

$\mathbf{F} = (F(f_0; kT_s), \ldots, F(f_{S-1}; kT_s))^T$, $\mathbf{n} = (n_0^k, \ldots, n_{S-1}^k)^T$ is a zero-mean noise vector with covariance matrix $N_0 \mathbf{I}_S$.
The receiver output is written as

$$\Lambda^k = \sqrt{\frac{E_s}{S}} \frac{\mathbf{F}^{\dagger} \mathbf{z}^k}{N_0} \tag{33}$$

where $(.)^{\dagger}$ denotes the Hermitian transpose operator.

4.2 PEP Derivation for MC-CDMA Systems

Using (33), the instantaneous SNR at the output of the receiver is given by

$$\Gamma = \frac{E_s}{S} \frac{\mathbf{F}^{\dagger} \mathbf{F}}{N_0}. \tag{34}$$

Assuming an uncorrelated scattering [28], the (q,l)-th entry of $\mathbf{F}$ correlation matrix $\mathbf{Q} = E\left(\mathbf{F}\mathbf{F}^{\dagger}\right)$ is given by

$$\mathbf{Q}(q, l) = \phi_F(f_q - f_l) = \int \phi_f(\tau) e^{-j2\pi(f_q - f_l)\tau} \mathrm{d}\tau, \tag{35}$$

where $\phi_f(\tau)$ is the multipath intensity profile of the channel [28] and $\phi_F(\Delta f)$ is the spaced frequency correlation function of the multipath channel.

Since the components of $\mathbf{F}$ are correlated, we use the Karhunen–Loève orthogonal expansion to obtain

$$\Gamma = \frac{E_s}{S} \frac{\mathbf{H}^{\dagger} \mathbf{H}}{N_0}, \tag{36}$$

where $\mathbf{H} = \mathbf{U}^{\dagger} \mathbf{F}$, $\mathbf{U} = \left[\mathbf{u}^1 \cdots \mathbf{u}^S\right]$ is a unitary matrix, $\mathbf{u}^i$ is the i-th normalized eigenvector of $\mathbf{Q}$

$$\mathbf{Q} = \mathbf{U} \boldsymbol{\Delta} \mathbf{U}^{\dagger}, \tag{37}$$

$\boldsymbol{\Delta} = \text{diag}(\lambda^{(1)}, \ldots, \lambda^{(S)})$ and $\lambda^{(i)}$ is the i-th eigenvalue. The eigenvalues are assumed to be ordered in a decreasing manner. $\mathbf{H}$ is a zero-mean complex Gaussian random vector with correlation matrix $\boldsymbol{\Delta}$.

Since the components of $\mathbf{H}$ are independent, the SNR is the sum of S-independent exponential random variables. The moment-generating function (MGF) of the SNR is therefore given by

$$M_{\Gamma}(s) = E(e^{-s\Gamma}) = \prod_{j=1}^{S} \frac{1}{1 + s\beta^{(j)}}, \tag{38}$$

where

$$\beta^{(j)} = \lambda^{(j)} \frac{E_s}{SN_0}, \tag{39}$$

Using a fraction decomposition, we obtain

$$M_\Gamma(s) = \sum_{j=1}^{S} \frac{\pi^{(j)}}{1+s\beta^{(j)}} \tag{40}$$

where the residue is given by

$$\pi^{(j)} = \prod_{1\le k\le S, k\ne j} \frac{\beta^{(j)}}{\beta^{(j)}-\beta^{(k)}}. \tag{41}$$

Using the inverse Laplace transform (LT), we deduce the PDF of the SNR

$$p_\Gamma(\gamma) = \sum_{j=1}^{S} \frac{\pi^{(j)}}{\beta^{(j)}} \exp\left(-\frac{\gamma}{\beta^{(j)}}\right), \text{ if } \gamma \ge 0. \tag{42}$$

Using the results of Sect. 2, [28] and (42), we obtain the PEP

$$\text{Pbloc}_{MCCDMA} \simeq \sum_{j=1}^{S} \pi^{(j)} \text{Pbloc}(L, \beta^{(j)}), \tag{43}$$

where $\text{Pbloc}(L, a)$ is defined in (11).

5 Application to OFDM Systems

5.1 System Model

We study an OFDM system composed of two nodes: a transmitter T and a receiver R.

It consists to decompose the allocated band B, in S subbands $\Delta f = \frac{B}{S}$. Δf represents the separation between consecutive subcarriers. Each subcarrier transmits data rate $\frac{R_s}{S}$ where R_s is the total data rate of S subcarriers.

The transmitted signal by node T can be expressed as

$$x(t) = \sum_{i=0}^{S-1} x_i(t) e^{2\pi j f_i t} \tag{44}$$

where

$$x_i(t) = \sum_k s_{kS+i}\, g(t-kT). \tag{45}$$

Subcarrier spacing is equal to $f_{i+1} - f_i = 1/T$. Data rate per carrier is equal to $\frac{1}{T} = \frac{R_s}{S} = \frac{1}{ST_s}$. Therefore, $T = ST_s$. We choose the number of subcarriers S so that $T >> T_m$ where T_m is the delay spread of the channel. In the frequency domain, this condition becomes $\frac{1}{ST_s} < B_c$, where B_c is coherence band.

$$\frac{R_s}{S} << B_c = \frac{1}{5\tau_{RMS}} \tag{46}$$

where B_c is the coherence bandwidth of the multipath channel and τ_{RMS} is the delay spread.

5.2 Transmitter Based on IFFT

$\forall\, 0 \le q \le S-1, \forall\, l \in Z$, we have

$$x\left(lT + \frac{T}{S}q\right) = \frac{1}{\sqrt{T}} \sum_{i=0}^{S-1} s_{lS+i} e^{2\pi j \frac{i}{T}\left(\frac{T}{S}q + lT\right)} = \frac{1}{\sqrt{T_s}\sqrt{S}} \sum_{i=0}^{S-1} s_{lS+i} e^{\frac{2\pi jiq}{S}}.$$

Therefore, we can make an inverse Fourier transform of transmitted symbols in order to obtain samples of transmitted signal. Then, a digital-to-analog converter is used to obtain signal $x(t)$.

5.3 Receiver Based on FFT

For fixed l and i, we compute

$$\frac{\sqrt{T}}{S} \sum_{q=0}^{S-1} x\left(lT + \frac{T}{S}q\right) e^{-\frac{2\pi jiq}{S}}.$$

Since

$$x\left(lT + \frac{T}{S}q\right) = \frac{1}{\sqrt{T}} \sum_{p=0}^{S-1} s_{lS+p} e^{\frac{2\pi jqp}{S}},$$

we deduce that

$$\frac{\sqrt{T}}{S} \sum_{q=0}^{S-1} x\left(lT + \frac{T}{S}q\right) e^{-\frac{2\pi jiq}{S}} = \frac{1}{S} \sum_{q=0}^{S-1} \sum_{p=0}^{S-1} s_{lS+p} e^{\frac{2\pi jq(p-i)}{S}} = s_{lS+i} + \frac{1}{S} \sum_{p\ne i} s_{lS+p} \sum_{q=0}^{S-1} e^{\frac{2\pi jq(p-i)}{S}} = s_{lS+i}.$$

5.4 Transmission Over Multipath Channel

The received signal at node R can be written as

$$u(t) = \int f(\tau) x(t-\tau) \mathrm{d}\tau$$

where

$$f(\tau) = \sum_{i=1}^{P} f_i \delta(\tau - i T_s).$$

Therefore,

$$u(t) = \sum_{i=1}^{P} f_i x(t - i T_s).$$

We deduce that

$$u\left(kT + \frac{T}{S}q\right) = u_k(q) = \sum_{i=1}^{P} f_i x\left(kT + (q-i)\frac{T}{S}\right) = \sum_{i=1}^{P} f_i x_{q-i}(k) \quad (47)$$

where $x_{q-i}(k) = x(kS + q - i)$.

In the presence of multipath propagation, a cyclic prefix is added at the transmitter to avoid intersymbol interference.

The transmitted OFDM symbol is written as

$$\begin{pmatrix} x_0(k) \\ x_1(k) \\ \vdots \\ x_{S-1}(k) \end{pmatrix}$$

where $x_l(k) = x(kS + l)$,

$$x_l(k) = \sum_{n=0}^{S-1} s_n(k) e^{2j\pi \frac{nl}{S}}.$$

We have

$$\begin{pmatrix} x_0(k) \\ x_1(k) \\ \vdots \\ x_{S-1}(k) \end{pmatrix} = \mathbf{A} \begin{pmatrix} s_0(k) \\ s_1(k) \\ \vdots \\ s_{S-1}(k) \end{pmatrix}$$

where

$$A_{i,k} = \frac{1}{\sqrt{S}} W^{(i-1)(k-1)}$$

et $W = e^{\frac{2\pi j}{S}}$.

We add the cyclic prefix to obtain canal:

$$\begin{pmatrix} x_{S-L+1}(k) \\ \vdots \\ x_{S-1}(k) \\ x_0(k) \\ x_1(k) \\ \vdots \\ x_{S-1}(k) \end{pmatrix}.$$

At the receiver, we remove the cyclic prefix and we make a FFT to obtain

$$y_{kS+l} = F(l) s_{kS+l} + n_l(k)$$

where $n_l(k)$ is a term due to noise.

$$F(l) = \sum_{i=1}^{P} f_i e^{-2j\pi \frac{li}{N}}$$

5.5 PEP Derivation for OFDM Systems

For OFDM systems, L data symbols are sent over S subcarriers and L/S symbols are transmitted over each carrier.

The PEP is written as

$$\mathrm{Pbloc}_{\mathrm{OFDM}} = 1 - \prod_{i=1}^{S} \left[1 - \mathrm{Pbloc}\left(\frac{L}{S}, \overline{\Gamma_i}\right)\right] \quad (48)$$

where $\mathrm{Pbloc}(\frac{L}{S}, \Gamma_i)$ is the PEP over i-th subcarrier and $\overline{\Gamma_i}$ is the average SNR over i-th subcarrier and $\mathrm{Pbloc}(L, a)$ has been defined in (11). The packet is correctly received if there are no errors over all S subcarriers.

6 Theoretical and Simulation Results

Figures 1, 2, 3 show the packet error probability (PEP) for CDMA, MC-CDMA and OFDM systems. Each packet contains 56 symbols.

Figure 1 shows the results for CDMA systems in the presence of 1, 2 and 3 paths. The delays of paths are separated T_c. The average of power gains of paths is 0.6 and 0.4 for $P = 2$ paths and 0.4, 0.35, 0.25 for $P = 3$. We notice a good accordance between simulation results and theoretical curves. We have made Monte Carlo simulations until 10^4 packet was erroneously received. We observe that the PEP decreases as the number of paths increase due to diversity. The rake receiver combines signals from all paths using maximum ratio combining, and the SNR is the sum of SNRs from different paths.

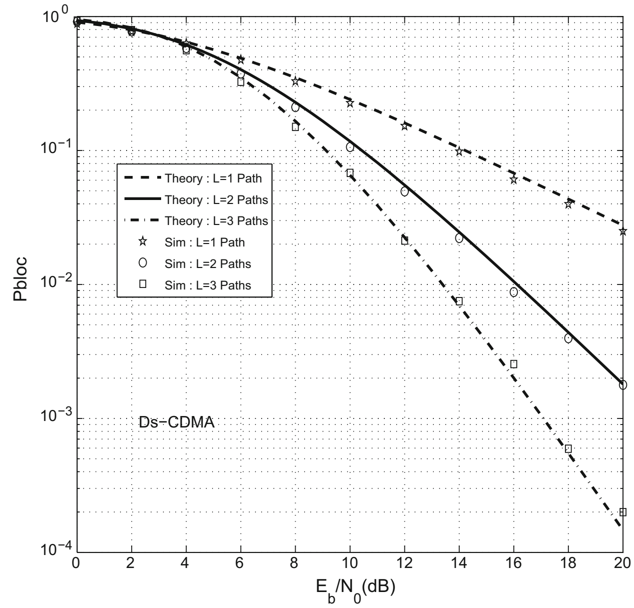

Fig. 1 PEP for DS-CDMA systems in the presence of $L = 1, 2, 3$ paths

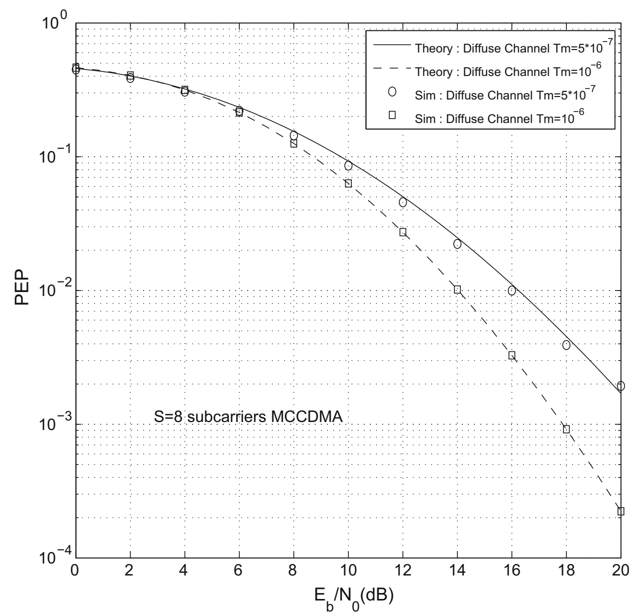

Fig. 2 PEP for MC-CDMA systems in the presence of a diffuse channel

Figure 2 shows the results for MC-CDMA channel for $S = 8$ subcarriers and in the presence of a diffuse channel with multipath intensity profile given by [28]

$$\phi_f(\tau) = \frac{1}{T_m} e^{\frac{-\tau}{T_m}} \tag{49}$$

with $T_m = 5 * 10^{-7}$ and $T_m = 10^{-6}$.

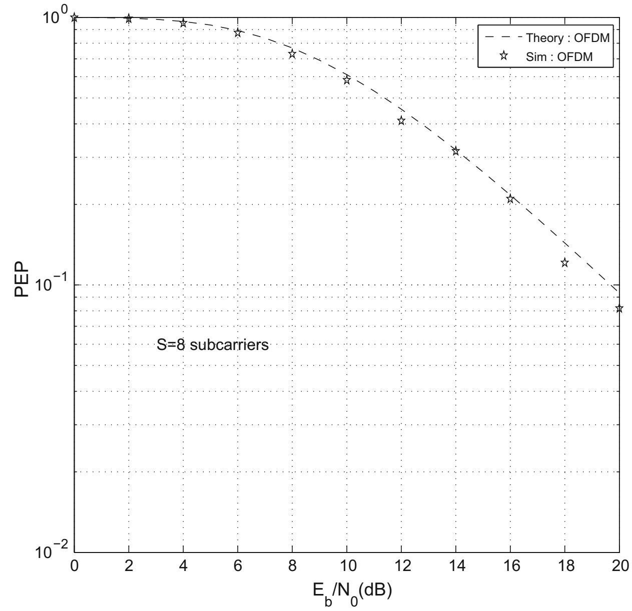

Fig. 3 PEP for OFDM systems

Matrix $\mathbf{Q}$ is equal to

$$\mathbf{Q}(q, l) = \frac{1}{2\pi j T_m (f_q - f_l) + 1}. \tag{50}$$

We notice that there is a good accordance between theoretical and simulation results for MC-CDMA systems. As the multipath spread of the channel T_m decreases as the coherence bandwidth increase and the channel coefficient of different carriers become more correlated so that the PEP increases.

Figure 3 shows the packet error probability for OFDM systems in the presence of two paths with average power gains 0.6 and 0.4. We observe a good accordance between theoretical and simulation results.

Figure 4 shows the packet error probability of OFDM for a longer packet containing $L = 400$ symbols and $S = 8$ subcarriers. A perfect match between theoretical and simulation results is observed. When packet length increases (Fig. 4), the packet error probability increases with respect to Fig. 3 ($L = 56$ symbols).

7 Conclusions

In this paper, we have presented a new approach that allows to derive the packet error probability in closed form using the MGF. The MGF of SNR is a well-known tool; it is equal to the Laplace transform of SNR PDF (Probability Density Function). Our approach has been applied for CDMA, MC-

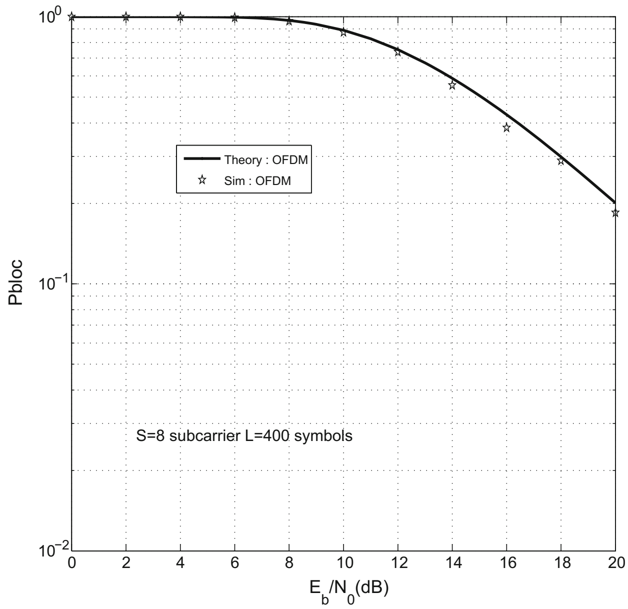

Fig. 4 PEP for OFDM systems: $L = 400$ symbols

CDMA and OFDM systems, and we have shown a perfect match with the simulation results.

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