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RESEARCH ARTICLE - MECHANICAL ENGINEERING

Frequency and Deflection Responses of Shear Deformable Skew Sandwich Curved Shell Panel: A Finite Element Approach

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Abstract

The eigenfrequency and transverse deflection values of the sandwich shell panel structure including the skew angle effect are examined numerically in this article. The sandwich shell panel is modelled via the higher-order displacement polynomial functions in the framework of the equivalent single-layer theory including the thickness stretching term effect. The numerical solutions are obtained via an own finite element code (MATLAB platform) in association with the derived mathematical model. The variational technique has been adopted to solve the sandwich structural equilibrium equation and the eigenvalue parameter under the influence of mechanical loading. The solution stability including the validity of the current numerical solutions has been verified via solving the adequate number of examples as same as the available published data. Finally, the current model is extended further to explore the probable effect of one or more parameters (geometrical, material and end constraint) on the final structural performances (frequency, deflection and stresses) including the fibre skew angle.

Keywords Skew sandwich composite · Bending · Vibration · Skew angle · Finite element analysis · HSDT

1 Introduction

In today's world most of the engineering high-performance structural parts/members are made of laminated and sandwich composite materials due of their unmatched properties (low density, light in weight, high stiffness and strength) in comparison to their metal counterpart. The skew curved/flat panels are the trivial choice for the design engineers utilized for the swept wings of an aircraft through an introduction of the substructures in the form of the oblique plates. It is also important to mention that the structures made of layered material always exposed to the combined loading during their operational life. These structural components are exposed to vibration and bending type of loading, which may affect the full structural strength due to the cyclic loading effect. Therefore, the vibration and bending analysis of skew sandwich structure becomes more important to study, and it is necessary to predict the actual deflection as well as the frequency parameters. However, the mathematical modelling and subsequent analysis of such structural components generally associated with different kinds of mathematical difficulties due to the presence of skew angle.

Further, to overcome the shortcomings of the former research work and to bridge the necessary gap, various studies have been performed in the past. Moreover, different kinds of mathematical modelling and subsequent solution steps are provided for the transient and free vibration analysis of the sandwich structures. In general, the classical theory, shear deformation theory and the refined higher-order theories are already implemented earlier to analyse the numerical responses of the laminated and sandwich structures. These studies are even improved now and then for an accurate and realistic prediction of the desired responses. In this regard, a few relevant research corresponding to the bending and the frequency responses of the sandwich structures have been presented here to highlight the insufficiencies and the necessity of the current research.

Researcher around the globe already implemented different kinematic models, namely the classical laminate theory (CLT), shear deformation theory including the refined higher-order theories in association with the finite element (FE) technique previously to analyse the various structural responses of the laminated and sandwich components [1–5].

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The frequency and the stability responses are obtained using the refined non-conforming quadrilateral thin plate bending element named as RPQ4 [6]. The free vibration responses of the skew sandwich plate with orthotropic core are computed [7] using the p-Ritz method. On the other hand, the static responses of the laminated composite plate are evaluated by developing the layer-wise, mixed, 18-noded FE model [8]. Similarly, the flexural and vibration responses of the laminated and sandwich plates are investigated [9,10] with the help of the refined higher-order shear deformation theory (RHSDT). In addition, the free vibration frequencies of the doubly curved sandwich composite shell panels are obtained [11] using the higher-order shear deformation theory (HSDT). Additionally, the nonlinear frequencies of the functionally graded (FG) plates are reported using the first-order shear deformation theory (FSDT) kinematics and von Karman type of strain–displacement relations [12]. The frequency values of the rectangular plate structure are investigated [13] using an auxiliary nodal surface (ANS) technique including the higher-order kinematics and finite element method (FEM). The flexural responses of the laminated and composite sandwich plates examined [14,15] using a quadrilateral element based on the third-order zigzag theory (TOZT). The flexural responses of the composite and sandwich laminates under uniformly distributed loading (UDL) are studied [16] based on the solution of a two-point boundary value problem (BVP) governed by a set of linear first-order ordinary differential equations. Further, to improve the solution accuracy a new curved beam model has been derived [17] for the frequency and the dynamic characteristics. Similarly, a multi-body system algorithm is proposed to obtain the eigenvalues and the eigenvectors [18] by linearizing the nonlinear system equations of motion computationally. Further, the static bending behaviour of the functionally graded material (FGM) sandwich plate composed of isotropic material is studied [19] with the help of a new displacement numerical model via the refined shear deformation theory (RSDT). Subsequently, a new model is derived using the high-order theory for the analysis of the sandwich panel structure including the effect of the external loading on the free vibration frequency and buckling load parameter of the circular cylindrical composite sandwich shell with transversely compliant core [20]. The free vibration responses of the multi-layered cross-ply laminated plate structure [21] using a Levy-type solution in the framework of Carrera's Unified Formulation (CUF) and the layer-wise kinematics, and additionally Navier's solution technique in the framework of the new HSDT [22,23] type of displacement model for the computation of the bending deflection of the FG rectangular plate. Similarly, a C^0 FE model was established to compute the bending deflection of the laminated composite and sandwich shell panel using Sander's approximations [24] in the framework of the higher-order zigzag theory (HOZT). The shear responses of the laminated composite sandwich panel structure including the polyvinyl chloride (PVC) foam core are investigated numerically [25] and verified with experimental data. The optimal frequency responses of the skew laminated sandwich plate structure are analysed [26] using the MFD method in conjunction with the FEM and the FSDT kinematic model. The thermoelastic bending responses of the FG sandwich plates are examined [27] including the temperature variation using a refined trigonometric shear deformation theory (RTSDT). Subsequently, the nonlinear static and dynamic responses of the skew sandwich and FG plate structure are investigated considering the moderate rotation via von Karman strain and the HSDT [28] kinematics including the stretching effects [29] and C^0 FE model based on the HOZT [30] polynomial. The impact responses of the aluminium foam sandwich panel with fibre metal laminate skin structure are modelled numerically [31] using the commercial FE tool (LS-Dyna) and the solution accuracy verified with subsequent experimental values. The vibration frequencies of beam structure (Timoshenko and Euler) are reported [32] considering the variable cross section and the non-prismatic configuration under the influence of the inconstant axial loading. The vibration characteristics of the bimodular conical/cylindrical laminated panel structure are investigated [33] numerically considering the combination of Jone's weighted compliance and Bert's technique. Similarly, the nonlinear vibration frequencies of the doubly curved composite shell panels are investigated [34] using Green–Lagrange nonlinear strain kinematics and the HSDT mid-plane kinematics. The free vibration responses of the isotropic and laminated composite skew plates are obtained [35] experimentally and numerically via commercial FE package MSC/NASTRAN. Thermomechanical nonlinear bending of thick FG circular plates resting on Winkler elastic foundation is investigated [36] based on the sinusoidal shear deformation theory. In the recent past, the static, the free vibration and the transient behaviour of the laminated composite shell panels have been studied [37] using the HSDT type of mid-plane theory and the commercial FE package (ANSYS). Similarly, the bending and the free vibration responses of the carbon nanotube-reinforced composite (CNTRC) plate were investigated [38] using the FEM in conjunction with the FSDT and the HSDT mid-plane kinematics. In continuation to that, few research is reported [39–49] on the bending, vibration and buckling behaviour of the layered and FGM structures using discrete singular convolution technique, a unified approach and layer-wise differential quadrature (LW-DQ) method in the framework of the FSDT. Similarly, the structural responses (deflection, frequency and stability) of the isotropic, layered composite and FG sandwich structures are studied [50–61] using the different higher-order and hyperbolic shear deformation polynomial kinematic theories including the mid-plane stretching effect.

Based on the available knowledge gap from the comprehensive review, the present article aims to develop a generic mathematical model to investigate the static bending, the frequency and the stresses (normal and shear) values of the skew sandwich shell panels with laminate facings and different types of core (isotropic and orthotropic) layer. The comprehensive review of the current and the recent past also confirms that no study has been reported yet on the bending and the subsequent free vibration frequencies of the skew sandwich shell panels using the HSDT kinematic model via the equivalent single-layer theory including the thickness stretching effect. Hence, a general mathematical model has been proposed and derived using the HSDT kinematics for the skew sandwich composite shell panels with laminate facings. The generalized structural equations (eigenvalue and equilibrium) are obtained, and the necessary order is reduced further with the help of an isoparametric FFM (nine-noded quadrilateral Lagrangian element with ten degrees of freedom per node). Now, the desired deflection and stress including the frequencies are computed with the help of an in-house suitable computer code developed in MATLAB platform in conjunction with the current higher-order FE model. The convergences and the corresponding validity of the obtained numerical solution model have also been checked to show the robustness. Finally, the bending and the vibration responses are computed for different geometrical parameters (the thickness ratios, the curvature ratios, the lamination schemes, the skew angles and the support conditions) to show the importance of the presently developed higher-order mathematical model.

2 Mathematical Modelling

In this present work, the geometry of the skew sandwich shell panel with laminate facings and the core layer is provided in Fig. 1 for the analysis of the desired bending and vibration responses. The dimension of the sandwich shell panels [38] is length 'a', width 'b' and the total thickness considered to be 'h', whereas the total thickness is the sum of the facesheet 'h_f' and thickness of the core 'h_c'. The R_x and R_y are the radii of the curvature of the sandwich shell panel in the respective directions. The curved shell panels are generally classified on the basis of the curvature parameter, i.e. spherical (both the curvatures are same), cylindrical (one curvature is infinite), elliptical (one curvature is twice of the other curvature), hyperboloidal (both the curvatures are same in magnitude but opposite in direction) and flat (both the curvature are infinite).

The mathematical model for the proposed skew sandwich shell panel is developed using the higher-order mid-plane kinematics including the thickness stretching [38,64]:

$$\begin{aligned}\bar{U}(X,Y,Z,t) &= U_0(X,Y,t) + z\theta_X(X,Y,t) \\ &\quad + z^2\xi_X(X,Y,t) + z^3\zeta_X(X,Y,t) \\ \bar{V}(X,Y,Z,t) &= V_0(X,Y,t) + z\theta_Y(X,Y,t) \\ &\quad + z^2\xi_Y(X,Y,t) + z^3\zeta_Y(X,Y,t) \\ \bar{W}(X,Y,Z,t) &= W_0(X,Y,t) + z\theta_Z(X,Y,t),\end{aligned} \tag{1}$$

where $\bar{U}$, $\bar{V}$ and $\bar{W}$ are the displacement of any point within the panel along X, Y and Z directions, respectively. U_0, V_0 and W_0 are the mid-plane displacement of any point within the panel along X, Y and Z directions, respectively. θ_X, θ_Y and θ_Z are the rotation of normal to the mid-plane and extension terms, respectively. The functions ξ_X, ξ_Y, ζ_X and ζ_Y are higher-order terms of Taylor series expansion in the mid-plane of the panel. The 't' represents time.

Further, Eq. (1) in the matrix form can be represented as:

$$\{\delta\} = [f]\{\delta_0\} \tag{2}$$

where $\{\delta\} = \left\{ U \;\; V \;\; W \right\}^{\mathrm{T}}$, $[f]$ and $\{\delta_0\} = \left[U_0 \;\; V_0 \;\; W_0 \;\; \theta_X \;\; \theta_Y \;\; \theta_Z \;\; \xi_X \;\; \xi_Y \;\; \zeta_X \;\; \zeta_Y\right]^{\mathrm{T}}$ are the displacement field vector at any point, thickness coordinate matrix and displacement field vector within the mid-plane.

FEM is a widely appreciated numerical tool for any complex geometrical structural analysis due to their adaptive nature. For the discretization purpose, a nine-noded isoparametric element is adopted with ten degrees of freedom (DOF) per node. The displacement field of the present model can be expressed in terms of desired field variables. The displacement vector $\{\delta_0\}$ at any point on the mid-surface is given by [38]:

$$\{\delta_0\} = \sum_{i=1}^{9} N_i\{\delta_{0i}\} \tag{3}$$

where $\{\delta_{0i}\} = \left[U_{0_i} \;\; V_{0_i} \;\; W_{0_i} \;\; \theta_{X_i} \;\; \theta_{Y_i} \;\; \theta_{Z_i} \;\; \xi_{X_i} \;\; \xi_{Y_i} \;\; \zeta_{X_i} \;\; \zeta_{Y_i}\right]^{\mathrm{T}}$ is the nodal displacement vector of the model and N_i is the interpolating function associated with node 'i'.

In the present analysis of skew sandwich panel, one of the edges is not parallel to the global axis system (X and Y). In addition to that, the transformation of the nodal unknowns of the skew edges from global axes system (X and Y) to local axes system (X' and Y') is necessary to carry out at the element level through appropriate transformation. At the end, the transformed matrices formed at an elemental level and assembled further to obtain the global matrices using the general assembly procedure as in general FEM. In addition, the transformation is not necessary for the elements that are not lying on the skew edges. Figure 2 represents the skew sandwich flat panel configuration where the skew angle is denoted as 'φ'. Now, the transformed nodal coordinates in a

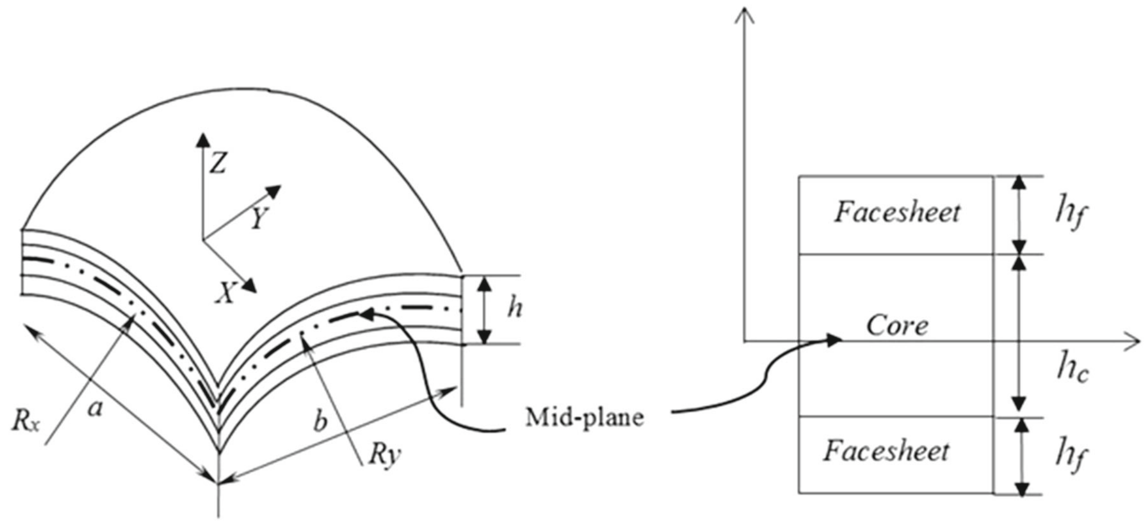

Fig. 1 Configuration of sandwich composite shell panel

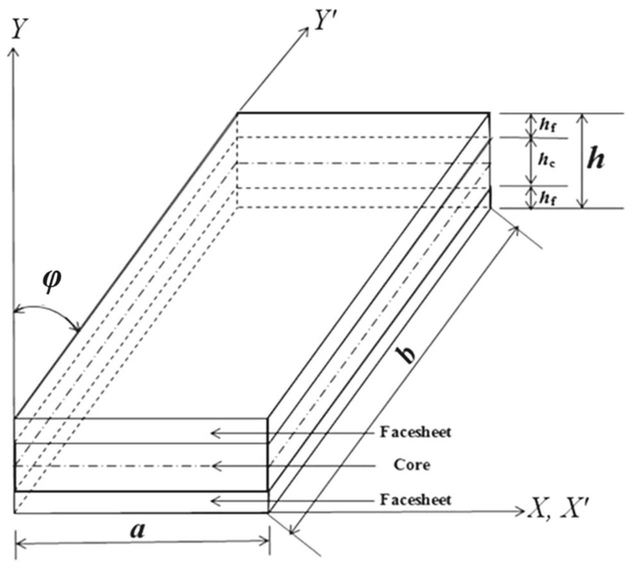

Fig. 2 Configuration of skew sandwich composite flat panel

Cartesian coordinate system are defined using the transformation matrix $[T_S]$ and conceded as:

$$\left\{\delta_0^*\right\} = [T_S]\left\{\delta_0\right\} \tag{4}$$

where

$$[T_S] = \begin{bmatrix} \cos\varphi & -\sin\varphi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \sin\varphi & \cos\varphi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos\varphi & -\sin\varphi & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sin\varphi & \cos\varphi & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \cos\varphi & -\sin\varphi & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \sin\varphi & \cos\varphi & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cos\varphi & -\sin\varphi \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sin\varphi & \cos\varphi \end{bmatrix}$$

3 Strain–Displacement Relations

The generalized strain–displacement relation is utilized to introduce the displacement behaviour within the panel as [34]:

$$\{\varepsilon\} = \left\{\varepsilon_{xx}\ \varepsilon_{yy}\ \varepsilon_{zz}\ \gamma_{yz}\ \gamma_{xz}\ \gamma_{xy}\right\}^{\mathrm{T}}$$

$$\{\varepsilon\} = \left\{\frac{\partial u}{\partial x}\ \frac{\partial v}{\partial y}\ \frac{\partial w}{\partial z}\ \frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\ \frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\ \frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right\}^{\mathrm{T}} \tag{5}$$

$$\{\varepsilon\} = [T]\{\bar{\varepsilon}\} \tag{6}$$

Now, the mid-plane strain vector can be written in terms of nodal displacement vector and defined as [64]:

$$\{\bar{\varepsilon}\} = [B]\left\{\delta_0^*\right\} \tag{7}$$

where $[B]$ is product form of the differential operators and the shape functions in the strain terms.

The desired constitutive relation of any arbitrary kth layer of the sandwich shell panel is given by [63]:

$$\{\sigma\}^k = [Q]^k\{\varepsilon\}^k \tag{8}$$

where $[Q] = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} & Q_{14} & 0 & 0 \\ Q_{12} & Q_{22} & Q_{23} & Q_{24} & 0 & 0 \\ Q_{13} & Q_{23} & Q_{33} & Q_{34} & 0 & 0 \\ Q_{14} & Q_{24} & Q_{34} & Q_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & Q_{55} & Q_{56} \\ 0 & 0 & 0 & 0 & Q_{56} & Q_{66} \end{bmatrix}$, $\{\sigma\}^k$ is the stress vector and $\{\varepsilon\}^k$ is the strain vector.

The kinetic energy (T) of the sandwich shell panel can be expressed as:

$$T = \frac{1}{2}\int_V \rho\left\{\delta_0^*\right\}^{\mathrm{T}}\left\{\dot{\delta}_0^*\right\}\mathrm{d}V \tag{9}$$

where ρ represents the density. $\{\delta_0^*\}$ and $\{\dot{\delta}_0^*\}$ are the displacement vector and the first-order differential of the displacement vector with respect to time, respectively.

Now, Eq. (2) is used in Eq. (9) and kinetic energy (T) for 'N' number of the layered shell panel can be conceded as [64]:

$$T = \frac{1}{2}\int_A \left(\sum_{k=1}^{N}\int_{z_{k-1}}^{z_k} \{\dot{\delta}_0^*\}^{\mathrm{T}} [f]^{\mathrm{T}} \rho^k [f] \{\dot{\delta}_0^*\} dz\right) \mathrm{d}A$$
$$= \frac{1}{2}\int_A \{\dot{\delta}_0^*\}^{\mathrm{T}} [m] \{\dot{\delta}_0^*\} \mathrm{d}A \tag{10}$$

where $[m] = \int_{z_{k-1}}^{z_k} \left([f]^{\mathrm{T}} \rho^k [f]\right) dz$ represents the inertia matrix.

The strain energy $(U_{S.E.})$ of the skew sandwich shell panel can be expressed as:

$$U_{S.E.} = \frac{1}{2}\int_v \{\varepsilon\}^{\mathrm{T}} \{\sigma\} \mathrm{d}V \tag{11}$$

Now, the total strain energy expression can be obtained by putting the strains and the stresses as given in Eqs. (6) and (8) into Eq. (11) and expressed as [64]:

$$U_{S.E} = \frac{1}{2}\int_V \left\{\varepsilon^{\mathrm{T}}\right\}^k \left[\bar{Q}\right]^k \{\varepsilon\}^k \mathrm{d}V \tag{12}$$

Now, the total work done expression for any sandwich shell panel can be linearized and expressed as [62]:

$$W.D. = \frac{1}{2}\int_A \{\varepsilon_G\}^{\mathrm{T}} [D_G] \{\varepsilon_G\} \mathrm{d}A \tag{13}$$

where $\{\varepsilon_G\}$ represents the geometric strain and $[D_G]$ represents the material property matrix.

The elemental equations for kinetic energy (T^e), strain energy $(U_{S.E.}^e)$ and the work done $(W.D.^e)$ are obtained by putting Eq. (11) into Eqs. (6), (9) and (10) and further represented as [64]:

$$T^e = \frac{1}{2}\int_A \left\{[N_i]^{\mathrm{T}} [m] [N_i] \mathrm{d}A\right\} \{\dot{\delta}_0^*\}_i \tag{14}$$

$$U_{S.E.}^e = \frac{1}{2}\int_A \left(\{\delta_0^*\}_i^T [B]_i^T [D] [B]_i \{\delta_0^*\}_i\right) \mathrm{d}A \tag{15}$$

$$W.D.^e = \frac{1}{2}\int_A \left(\{\delta_0^*\}_i^T [B_G]_i^T [D_G] [B_G]_i \{\delta_0^*\}_i\right) \mathrm{d}A \tag{16}$$

4 System Governing Equation and Solution Approach

The generalized governing equation can be obtained using the variational principle, and this can be expressed as:

$$\delta \int_{t_1}^{t_2} (T^e - (U_{S.E.}^e + W.D.^e)) \mathrm{d}t = 0 \tag{17}$$

The equilibrium equation for any element within the panel can be obtained by putting Eqs. (14), (15), and (16) in Eq. (17) as:

$$\left([K]^e\right) \{\delta_0^*\} + [M]^e \{\ddot{\delta}_0^*\} = \{F\}^e \tag{18}$$

The above elemental equation can be rewritten in the global form as:

$$([K]) \{\delta^*\} + [M]\{\ddot{\delta}^*\} = \{F\} \tag{19}$$

where $\{\delta^*\}$and$\{\ddot{\delta}^*\}$are the global displacement and acceleration vector. The global force vector is represented by $\{F\}$. $[K]$ and $[M]$ are the corresponding global matrices of $[K]^e$ and $[M]^e$, respectively, and can be additionally represented as [37]:

$$[K] = \int_A [B]^{\mathrm{T}}[D][B]\mathrm{d}A$$
$$[M] = \int_A [N]^{\mathrm{T}}[N]\rho \mathrm{d}A \tag{20}$$

Now, the free vibration responses are obtained by using Eq. (19) by dropping appropriate terms. The equilibrium equation for the free vibration behaviour of skew sandwich shell panel can be obtained by dropping the load vector term from Eq. (19) and presented as:

$$\left[[K] - \omega^2[M]\right] \{\Delta\} = 0 \tag{21}$$

where ω is the eigenvalue (frequency) of the free vibrated skew sandwich shell panel and Δ is the corresponding eigenvector (mode shapes).

The final formula of the motion equation for bending analysis of skew sandwich shell panel is obtained by minimizing the energy expression by variation principle and represented as [37]:

$$\delta \Pi = \delta U - \delta W.D. \tag{22}$$

where δ is the variational symbol and Π is the total potential energy.

The motion equation for the static analysis of sandwich composite shell panel is obtained by dropping the inertia matrix from Eq. (19). The final form of the equilibrium equation may be expressed as [37,64]:

$$([K]) \{\delta^*\} = \{F\} \tag{23}$$

5 Results and Discussion

Now, the effectiveness and the corresponding validity of the numerical solution have been verified by matching the results with that of the available published literature. The following material properties are utilized for the current numerical experimentation of the bending and the vibration problems.

Case I: M1 (graphite–epoxy)—Kumar et al. [24]

Face: $E_1/E_2 = 25$, $G_{12} = G_{13} = 0.5E_2$, $G_{23} = 0.2E_2$, $\upsilon_{12} = 0.25$

Core: $E_1/E_2 = 0.04$, $G_{12} = 0.5E_2$, $G_{13} = G_{23} = 0.06E_2$, $\upsilon_{12} = 0.25$

M2 (graphite–epoxy)—Chalak et al. [30]:

Face: $E_1 = 72.4$GPa, $E_2 = 6.895$GPa, $E_3 = 6.895$GPa, $G_{12} = 3.450$GPa, $G_{23} = 1.400$GPa, $G_{13} = 3.450$GPa, $\upsilon_{12} = \upsilon_{23} = \upsilon_{13} = 0.25$

Core: $E_1 = 0.2758$GPa, $E_2 = 0.2758$GPa, $E_3 = 0.2758$GPa, $G_{12} = 0.1103$GPa, $G_{23} = 0.4137$GPa, $G_{13} = 0.4137$GPa, $\upsilon_{12} = \upsilon_{23} = \upsilon_{13} = 0.25$

Case II: M1—Garg et al. [10]:

Face: $E_1 = 19 \times 10^6$ lb/in^2, $E_2 = E_3$, $G_{12} = G_{23} = 1 \times 10^6$ lb/in^2, $G_{13} = 0.90 \times 10^6$ lb/in^2, $\upsilon_{12} = \upsilon_{13} = 0.22$, $\upsilon_{23} = 0.49$, $\rho = 0.057$ lb/in^2

Core: $E_1 = E_2 = E_3 = 1000$ lb/in^2, $G_{12} = G_{23} = G_{13} = 500$ lb/in^2, $\upsilon_{12} = \upsilon_{23} = \upsilon_{13} = 0$, $\rho = 0.003403$ lb/in^2

M2 (graphite–epoxy)—Wang et al. [7]:

$$\frac{E_1}{E_2} = 40.0, \frac{G_{12}}{E_2} = 1.0, \upsilon_{12} = 0.25, \upsilon_{12} = 0.3,$$
$$\frac{G_x^c}{E_c} = \frac{1.173}{6.279}, \frac{G_y^c}{E_c} = \frac{2.415}{6.279}, \frac{G_x^c}{E_c} = \frac{1.173}{6.279}, \frac{\rho_f}{\rho_c} = 0.6818$$

The desired responses (deflection and frequency) and stresses are non-dimensionalized using the following formulae:

Non-dimensional central displacement: $W = \frac{100E_{Tf}h^3W_c}{a^4q_0}$

Non-dimensional fundamental frequency: $\varpi = 100\omega a\sqrt{\rho_c/E_1}$

$$Q = q_0a^4/E_2h^4; \bar{\sigma}_{XX}(a/2, b/2, Z) = \sigma_{XX}h^2/(qa^2); \bar{\sigma}_{YY}(a/2, b/2, Z) = \sigma_{YY}h^2/(qa^2)$$
$$\bar{\tau}_{XZ}(0, b/2, Z) = \tau_{XZ}h/(qa); \bar{\tau}_{YZ}(a/2, 0, Z) = \tau_{YZ}h/(qa); \bar{\tau}_{XY}(a, b, Z) = \tau_{XY}h^2/(qa^2)$$

Now, the following support conditions are employed to reduce the number of unknowns and to derive the mathematical form of the physical model.

Simply supported (S)
$V_0 = W_0 = \theta_Y = \theta_Z = \xi_Y = \zeta_Y = 0$ at $X = 0, a$ and $U_0 = W_0 = \theta_X = \theta_Z = \xi_X = \zeta_X = 0$ at $Y = 0, b$.

Clamped (C)
$U_0 = V_0 = W_0 = \theta_X = \theta_Y = \theta_Z = \xi_X = \xi_Y = \zeta_X = \zeta_Y = 0$ at $X = 0, a$ and $Y = 0, b$.

5.1 Convergence and Validation Study

The convergence and the validity of the derived model have been established for the transverse bending deflection including the frequency parameters. In general, the lamination scheme for the computation of final output is considered as the symmetric and the anti-symmetric cross-ply configuration including the core, i.e. $(0°/90°/C/0°/90°)$ and $(0°/90°/C/90°/0°)$, respectively, where 'C' represents the core layer. In addition, the thickness of the core material is considered to be 0.8 times of the total thickness (h), whereas the face sheets took as $0.1h$ on each side of the panel. The responses are obtained for all edges simply supported (SSSS), clamped (CCCC) and two opposite edges clamped and simply supported (CSCS).

The bending analysis of the square sandwich spherical panel with laminated facing has been obtained using the M1 properties of Case I. The responses are computed for $(0°/90°/C/0°/90°)$ lamination scheme by considering two curvature ratios ($R/a = 5$ and 20) and two thickness ratios ($a/h = 10$ and 100) under uniformly distributed load (UDL). The non-dimensional deflection parameters are obtained for different mesh divisions numerically using the currently established model and presented in Fig. 3.

In addition to that, another problem of the skew sandwich cylindrical shell panel has been solved to represent the convergence behaviour of the currently developed mathematical model. The responses are obtained for the square symmetric sandwich cylindrical shell panel $(0°/90°/C/90°/0°)$ for the different support conditions under the mechanical UDL. For the computational purpose, the material property is considered as M2 of Case II and other parameters are three skew angles ($\varphi = 15°$, $30°$ and $45°$), $a/h = 10$ and $R/a = 5$. The non-dimensional deflections are computed using the presently developed numerical model for different mesh sizes and presented in Fig. 4. The convergence studies indicate that the numerical results obtained are converging well with the refinement of mesh divisions under various boundary conditions. It is believed that a (6×6) mesh size is enough to obtain the final responses and utilized for the computational purpose throughout the analysis.

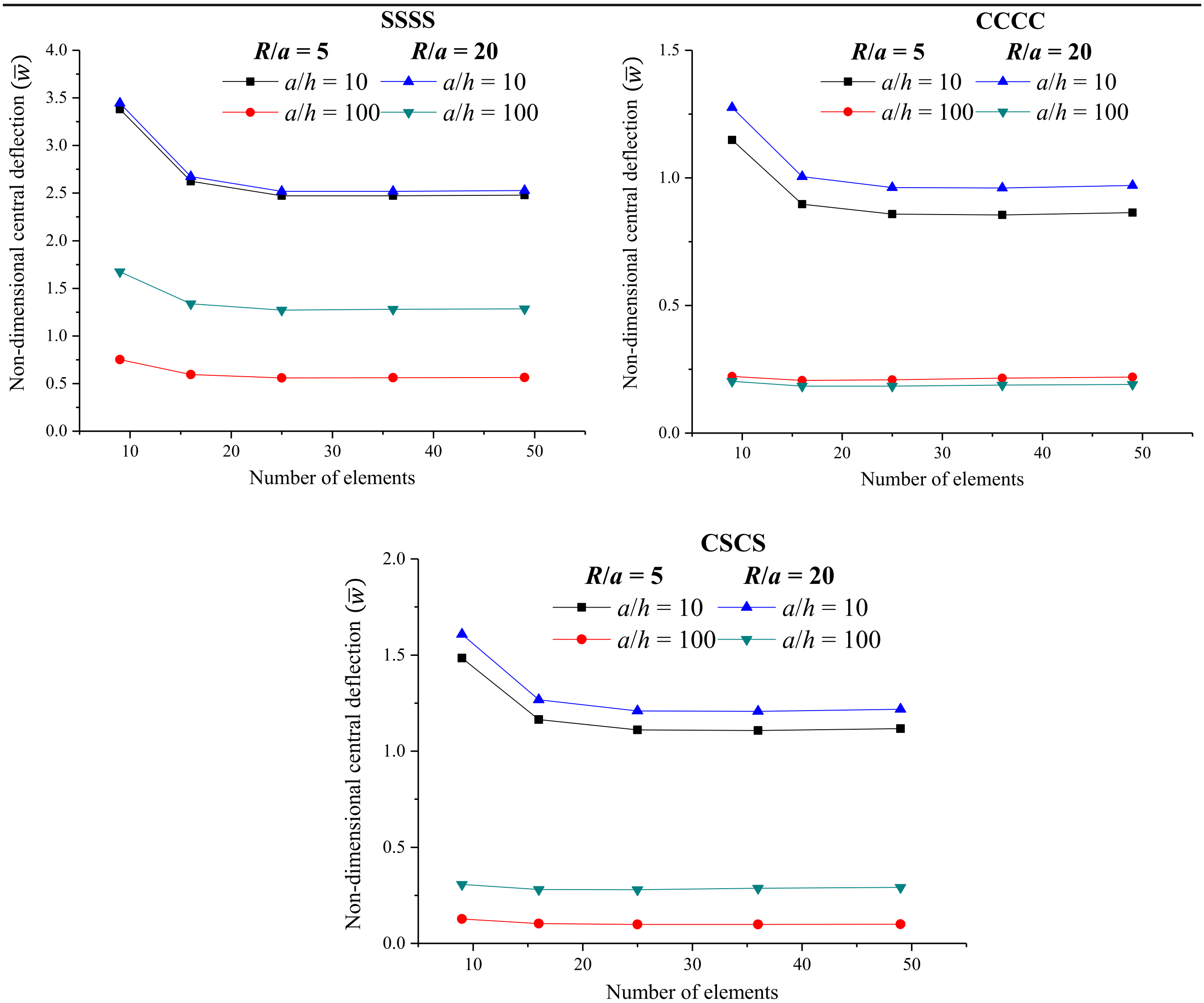

Fig. 3 Convergence study of bending responses of square $(0°/90°/C/0°/90°)$ sandwich spherical shell panel for different curvature ratios (R/a) and support conditions

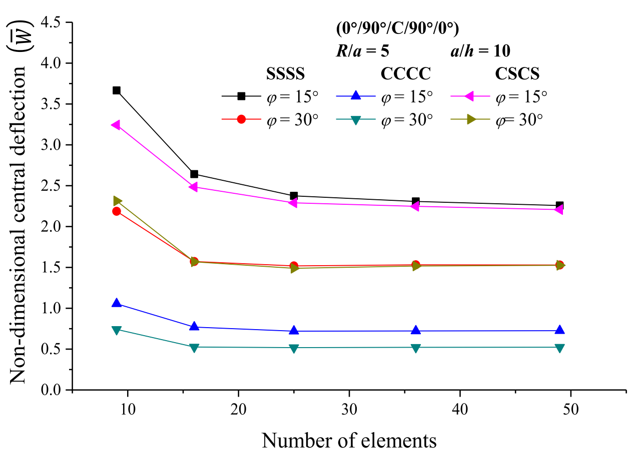

Fig. 4 Convergence study of bending responses of square symmetric skew sandwich composite cylindrical shell panel for different support conditions

Similarly, the frequency responses are computed for the square sandwich composite $(0°/90°/C/0°/90°)$ cylindrical shell panel using the M1 properties of the Case II. The non-dimensional frequencies are computed for two thickness ratios ($a/h = 4$ and 10), four curvature ratios ($R/a = 1,\ 3,\ 5$ and 10) and the same support conditions as in the case of the bending. The responses are obtained for the different mesh sizes and presented in Fig. 5. In addition to that, another problem of symmetric skew sandwich $(0°/90°/C/90°/0°)$ cylindrical shell panel is also solved and presented in Fig. 6. The responses are obtained using the material property M2 of Case II for three skew angles ($\varphi = 15°$, $30°$ and $45°$), $a/h = 20$, $R/a = 5$ and $a/b = 2$. Figures 5 and 6 indicate that the results computed using the present mathematical model are converging well with the mesh refinement and a (6×6) mesh size is enough to obtain the frequency responses, so it is utilized throughout the frequency analysis.

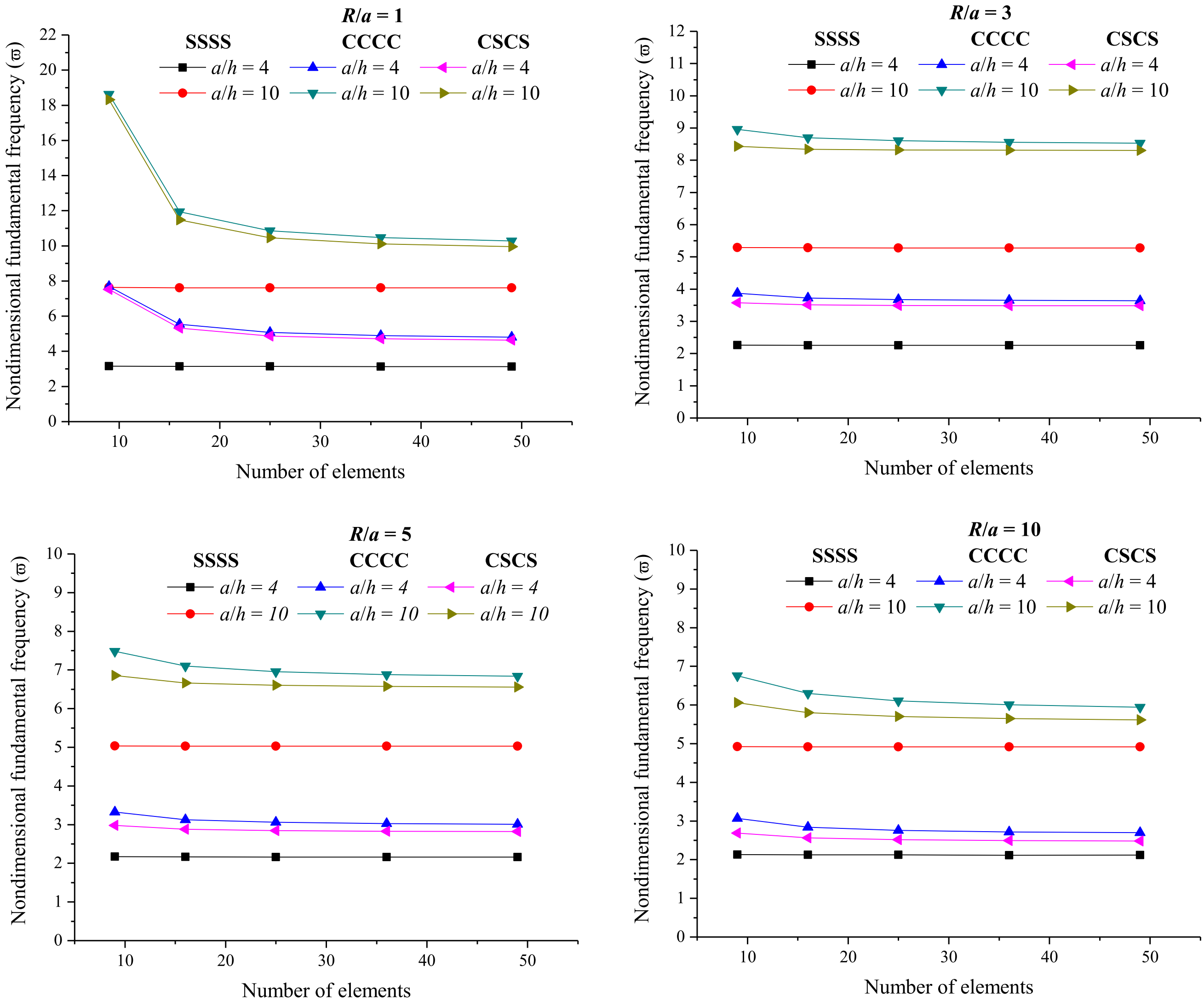

Fig. 5 Convergence study of frequency responses of square $(0°/90°/C/0°/90°)$ sandwich cylindrical shell panel for different curvature ratios (R/a) and support conditions

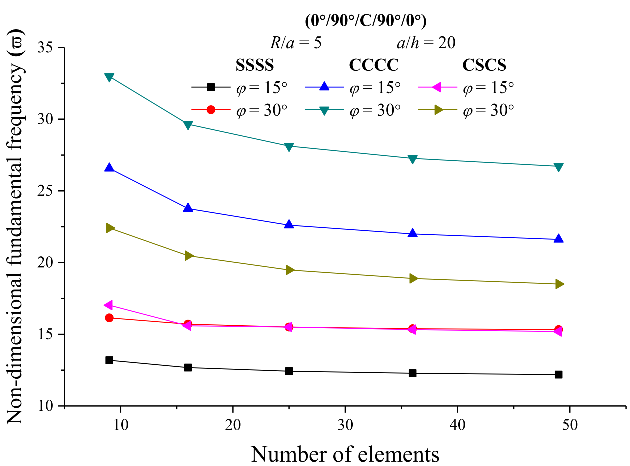

Fig. 6 Convergence study of frequency responses of symmetric skew sandwich composite cylindrical shell panel for different support conditions $(a/b = 2)$

Now the proposed higher-order mathematical model is further extended to compute the desired responses and compared with that of the available published results. The bending responses are computed for the simply supported sandwich composite spherical shell panel under UDL including M1 (Case I) properties and presented in Table 1. It is easily seen from the comparison study that the results obtained using the HSDT are in good agreement with the available published FEM results obtained using the HOZT kinematics.

In addition to that, another bending comparison study has been conducted and presented in Table 2 for the skew sandwich flat panel. In this example, the deflections are obtained for the simply supported square skew sandwich flat panel with laminate facing and isotropic core. The responses were computed for the square symmetric $(0°/90°/C/90°/0°)$ and anti-symmetric $(0°/90°/C/0°/90°)$ skew sandwich flat

Table 1 Comparison study of the bending responses of SSSS square sandwich (0°/90°/C/0°/90°) spherical shell panel under UDL

R/a	Source	a/h	
		10	100
5	Present	2.4713	0.5621
	Kumar et al. [24]	2.572	0.5615
20	Present	2.5184	1.2796
	Kumar et al. [24]	2.625	1.296

Table 4 Comparison study of the frequency responses of SSSS symmetric skew sandwich $(0°/90°/C/90°/0°)$ composite flat panel for different lamination schemes

Skew angle (φ)	Present	Wang et al. [7]
0°	12.8713	12.063
15°	12.2782	12.767
30°	15.3874	15.196
45°	22.9841	20.572

panel for two skew angles ($\varphi = 15°$ and 30°) and $a/h = 10$ and M2 (Case I) material properties. The responses were compared with those of the available published results, and the comparison indicates that the present results obtained using the higher-order model are in good agreement. It is important to note that the results obtained in the references are using various shear deformation kinematic theories, i.e. the HOZT, the RHSDT and the TOZT.

Similarly, the comparison study has also been carried out for the vibration case using the current higher-order FE model. The frequency responses obtained for all-side simply support panel using M1 properties (Case II) including other geometrical parameters as same as the initial one. The results computed via utilizing the currently derived FE model including the reference data are provided in Table 3 for the comparison purpose. It is clear from the table that the present values are showing good agreement with those of the available published literature. Subsequently, another problem has been solved to establish the adequate effectiveness of the presently derived numerical model for the computation of the output values. The frequency responses are obtained for the square simply supported skew sandwich $(0°/90°/C/90°/0°)$ flat panel and presented in Table 4. The responses are obtained using M2 properties (Case II), $a/h = 20$ and $a/b = 2$.

Once again to show the model efficacy, another validity study has been carried out for a square skew laminated composite plate $(0°/90°)_{10}$ utilizing the property and an associated input parameter as same as the [35]. The comparison table (Table 5) shows the present numerical and published experimental including the FEM results. It is observed from the responses that the present results are within the expected line and the deviation between the results may arise due to the type of numerical modelling approaches adopted between the present and the reference.

Table 2 Comparison study of the bending responses of SSSS square symmetric skew sandwich composite flat panel for different lamination schemes

	(0°/90°/C/90°/0°)		(0°/90°/C/0°/90°)	
	$\varphi = 15°$	$\varphi = 30°$	$\varphi = 15°$	$\varphi = 30°$
Present	2.6566	1.7530	2.4094	1.6012
Chalak et al. [30]	2.3744	1.7790	2.3645	1.7471
Chakrabarti and Sheikh [9]	2.3969	1.8100	2.3870	1.7706
Kapuria and Kulkarni [14]	2.3968	1.8053	–	–
Kulkarni and Kapuria [15]	2.2968	1.7119	–	–

Table 3 Comparison study of the frequency responses of simply supported square sandwich $(0°/90°/C/0°/90°)$ cylindrical shell panel

R/a	a/h	Present	Garg et al. [10][a]	Garg et al. [10][b]	Garg et al. [10][c]	Garg et al. [10][d]
1	4	3.1364	3.15525	3.16713	3.29165	8.67183
	10	7.6111	7.71269	7.72971	7.78323	14.16395
3	4	2.2544	2.28894	2.29066	2.30664	9.12889
	10	5.2783	5.36149	5.36505	5.37074	14.00424
5	4	2.1625	2.19846	2.19911	2.19893	9.17231
	10	5.0274	5.10821	5.10959	5.10823	13.99486
10	4	2.1125	2.15938	2.15954	2.15145	9.192
	10	4.9172	4.99708	4.99744	4.99274	13.99235

[a] Higher-order shear deformation theory with 12 DOF (HOST12)
[b] Higher-order shear deformation theory with 11 DOF (HOST11)
[c] Higher-order shear deformation theory with 9 DOF (HOST9)
[d] FSDT

Table 5 Comparison study of the frequency responses of a square $(0°/90°)_{10}$ skew laminated flat panel

Model	$\varphi = 15°$	$\varphi = 30°$	$\varphi = 45°$
Present	13.3275	13.7274	15.9089
Srinivasa et al. [35][a]	12.0320	13.0200	16.0620
Srinivasa et al. [35][b]	12.4040	13.4920	16.7310

[a]Experimental data, [b]FEM

Further, an in-plane and out-of-plane stresses are obtained for square simply supported three-layered (0°/C/0°) sandwich flat panel under the sinusoidal loading. The responses are compared with those available benchmark solutions (elasticity solution [1]; FEM HSDT and FSDT [2]; mixed FEM [5,8] for 11 and 6 DOF; semi-analytical model [16]) and presented in Table 6. For the computational purpose, the material properties are considered the same as M2 of Case I and the geometrical parameters are considered the same as reference [16]. Also, the sinusoidal loading is considered as: $Q = q_0 \sin\left(\frac{\pi X}{a}\right) \sin\left(\frac{\pi Y}{b}\right)$. It is understood from this comparison that the present responses are close to the reference results and the small deviation may be due to the different displacement kinematic models.

From the comparison studies, it is understood that the current FE results and the reference (obtained via p-Ritz method) values are in good agreement. However, the differences are higher when compared with FSDT results and it is due to the fact that the model becomes stiffer or overestimate the frequencies for the FSDT mid-plane kinematics. This, in turn, indicates the necessity of the higher-order kinematic model for the analysis of the sandwich structure for the accurate prediction of the structural responses.

5.2 Numerical Examples

Based on the convergence and comparison study, the presently developed higher-order FE model is further employed to investigate the influence of individual or the combined effect of the geometrical parameters on the bending and frequency responses of the skew sandwich panel structure. The bending and free vibration responses are computed for the symmetric and unsymmetric skew sandwich shell panels with different lamination schemes of face sheets. In general, the bending and vibration responses of two symmetric $(0°/C/0°$ and $0°/90°/C/90°/0°)$ and an unsymmetric $(0°/90°/C/0°/90°)$ sandwich shell panels are obtained using the given material properties. In addition, the thickness of the core and the face sheets is considered to be $0.8h$ and $0.1h$, respectively, for each case of the lamination schemes. Further, three skew angles ($\varphi = 0°$, 15°, and 45°), two thickness ratios ($a/h = 10$ and 100), two curvature ratios ($R/a = 10$ and 100) and three support conditions (SSSS, CCCC and CSCS) are employed throughout the analysis.

The bending responses are computed for the symmetric and anti-symmetric skew sandwich curved (cylindrical, spherical, elliptical and hyperboloidal) shell panels for two curvature ratios ($R/a = 10$ and 100) and presented in Tables 7, 8, 9 and 10, 11, 12, respectively. From the computed results, it is noticed that the deflection parameters are decreasing when the thickness ratio increases. Further, the panel deflections increase with the increase in curvature ratio (R/a) values. This is because of the fact that the stiffness of the panel structure decreases and/or increases due to the increase in the corresponding thickness and the curvature ratios. It is also observed that the deflection parameters of the skew sandwich shell panels follow a declining trend when the skew angle values increase, i.e. $\varphi = 15°$ to 45°. The

Table 6 Comparison study of the maximum stresses of simply supported square symmetric (0°/C/0°) sandwich flat panel under sinusoidal transverse loading

a/h	Model	W	$\bar{\sigma}_{XX}\left(\frac{a}{2},\frac{b}{2},\frac{h}{2}\right)$	$\bar{\sigma}_{YY}\left(\frac{a}{2},\frac{b}{2},\frac{h}{6}\right)$	$\bar{\tau}_{XY}\left(0,0,\frac{h}{2}\right)$	$\bar{\tau}_{XZ}\left(0,\frac{b}{2},0\right)$	$\bar{\tau}_{XY}\left(\frac{a}{2},0,0\right)$
4	Present	7.0526	1.509	0.1838	−0.1378	0.2256	0.0961
	Kant et al. [16]		1.556	0.259	−0.144	0.239	0.107
	Pagano [1]		1.556	0.259	−0.144	0.239	0.107
	Pandya and Kant [2]		1.523	0.241	−0.142	0.275	–
	Wu and Kuo [5]		1.548	0.249	–	–	−0.134
	Ramtekkar et al. [8]		1.57	0.26	–	0.237	0.104
10	Present	2.0666	1.1563	0.0912	−0.0679	0.2812	0.0506
	Kant et al. [16]		1.153	0.11	−0.0707	0.3	0.0527
	Pagano [1]		1.153	0.11	−0.071	0.3	0.053
	Pandya and Kant [2]		1.166	0.105	−0.069	0.34	–
	Wu and Kuo [5]		1.21	0.111	−0.071	0.324	–
	Ramtekkar et al. [8]		1.159	0.111	−0.071	0.303	0.055

Table 7 Non-dimensional deflection of the different curved sandwich ($0°/C/0°$) shell panels for different thickness ratios, skew angles and support conditions ($R/a = 10$)

Shell panel	Skew angle (φ)	SSSS		CCCC		CSCS	
		a/h					
		10	100	10	100	10	100
Cylindrical	0°	3.1034	1.2555	1.7344	0.1635	1.978	0.1568
	15°	21.0637	0.3326	5.7417	0.3139	14.6185	0.3416
	45°	6.3267	0.2389	2.2003	0.1869	6.2558	0.1824
Spherical	0°	3.089	1.0488	1.7291	0.1587	1.9744	0.1539
	15°	20.5877	0.4104	5.6822	0.2873	14.5337	0.3209
	45°	6.3303	0.2406	2.1912	0.1821	6.2611	0.1894
Elliptical	0°	3.0973	1.1607	1.7327	0.1619	1.9767	0.1558
	15°	20.8182	0.3731	5.722	0.3045	14.5744	0.3304
	45°	6.3272	0.2399	2.1973	0.1852	6.2572	0.186
Hyperboloidal	0°	3.1091	1.3418	1.7323	0.1616	1.9776	0.1563
	15°	21.6026	0.2777	5.7195	0.303	14.7168	0.368
	45°	6.3333	0.2349	2.1972	0.1856	6.2607	0.175

Table 8 Non-dimensional deflection of the different curved sandwich ($0°/90°/C/90°/0°$) shell panels for different thickness ratios, skew angles and support conditions ($R/a = 10$)

Shell panel	Skew angle (φ)	SSSS		CCCC		CSCS	
		a/h					
		10	100	10	100	10	100
Cylindrical	0°	2.5102	1.2698	1.3482	0.222	1.7209	0.2949
	15°	4.201	0.5195	2.2159	0.2444	4.0883	0.4754
	45°	1.6664	0.2973	1.0856	0.1097	1.6647	0.2686
Spherical	0°	2.4994	1.0135	1.3214	0.175	1.719	0.2841
	15°	4.1947	0.5442	2.1351	0.1757	4.0925	0.4644
	45°	1.6686	0.2969	1.066	0.0946	1.667	0.2705
Elliptical	0°	2.5052	1.1489	1.3412	0.2076	1.7202	0.2912
	15°	4.1956	0.532	2.1944	0.2216	4.0884	0.4687
	45°	1.667	0.2971	1.0805	0.1054	1.6654	0.2695
Hyperboloidal	0°	2.5172	1.3875	1.3232	0.1781	1.7207	0.2913
	15°	4.2251	0.5089	2.1409	0.1796	4.1002	0.4991
	45°	1.6678	0.2975	1.0675	0.0957	1.6661	0.2668

Table 9 Non-dimensional deflection of the different curved sandwich ($0°/90°/C/0°/90°$) shell panels for different thickness ratios, skew angles and support conditions ($R/a = 10$)

Shell panel	Skew angle (φ)	SSSS		CCCC		CSCS	
		a/h					
		10	100	10	100	10	100
Cylindrical	0°	2.5132	1.2726	1.3491	0.2226	1.7466	0.3088
	15°	3.8527	0.5114	2.0726	0.2297	3.7618	0.4586
	45°	1.5446	0.2845	1.0207	0.1016	1.5429	0.2589
Spherical	0°	2.5027	1.0154	1.3221	0.1753	1.7446	0.2971
	15°	3.8485	0.5331	2.0018	0.1683	3.7665	0.4496
	45°	1.5467	0.2842	1.0034	0.0885	1.5451	0.2606
Elliptical	0°	2.5084	1.1512	1.342	0.2081	1.7458	0.3047
	15°	3.8485	0.5224	2.0538	0.2096	3.7623	0.4529
	45°	1.5453	0.2843	1.0162	0.0979	1.5436	0.2597
Hyperboloidal	0°	2.5203	1.3909	1.3243	0.1785	1.7465	0.3049
	15°	3.8734	0.5026	2.0065	0.1718	3.7721	0.4801
	45°	1.5459	0.2847	1.0045	0.0895	1.5441	0.2574

Table 10 Non-dimensional deflection of the different curved sandwich ($0°/C/0°$) shell panels for different thickness ratios, skew angles and support conditions ($R/a = 100$)

Shell panel	Skew angle (φ)	SSSS		CCCC		CSCS	
		a/h					
		10	100	10	100	10	100
Cylindrical	0°	3.1016	1.331	1.8209	0.2799	2.0934	0.272
	15°	26.7492	20.0314	6.907	4.3586	23.4646	13.8248
	45°	8.2043	5.6067	2.4043	1.1682	8.2022	5.35
Spherical	0°	3.1014	1.3282	1.8208	0.2798	2.0934	0.272
	15°	26.7415	19.5989	6.9062	4.328	23.4627	13.7358
	45°	8.2043	5.605	2.4042	1.1655	8.2023	5.3458
Elliptical	0°	3.1015	1.3298	1.8209	0.2798	2.0934	0.272
	15°	26.7453	19.8155	6.9067	4.3485	23.4636	13.7823
	45°	8.2043	5.6058	2.4043	1.1673	8.2022	5.3479
Hyperboloidal	0°	3.1017	1.3319	1.8209	0.2798	2.0934	0.272
	15°	26.7574	20.46	6.9067	4.3473	23.4667	13.8977
	45°	8.2044	5.6087	2.4043	1.1672	8.2023	5.3549

Table 11 Non-dimensional deflection of the different curved sandwich ($0°/90°/C/90°/0°$) shell panels for different thickness ratios, skew angles and support conditions ($R/a = 100$)

Shell panel	Skew angle (φ)	SSSS		CCCC		CSCS	
		a/h					
		10	100	10	100	10	100
Cylindrical	0°	2.5116	1.3837	1.3748	0.2963	1.7637	0.4488
	15°	4.3926	2.3353	2.311	0.4434	4.3983	2.3986
	45°	1.7252	0.6968	1.1112	0.1406	1.7247	0.6457
Spherical	0°	2.5115	1.38	1.3745	0.2953	1.7637	0.4485
	15°	4.3925	2.3306	2.3101	0.4403	4.3984	2.3958
	45°	1.7252	0.6968	1.111	0.1403	1.7247	0.6457
Elliptical	0°	2.5116	1.3822	1.3747	0.296	1.7637	0.4487
	15°	4.3925	2.3329	2.3108	0.4426	4.3983	2.3972
	45°	1.7252	0.6968	1.1111	0.1405	1.7247	0.6457
Hyperboloidal	0°	2.5117	1.385	1.3745	0.2953	1.7637	0.4487
	15°	4.3929	2.3406	2.3102	0.4405	4.3985	2.4013
	45°	1.7252	0.6968	1.111	0.1403	1.7247	0.6457

Table 12 Non-dimensional deflection of the different curved sandwich ($0°/90°/C/0°/90°$) shell panels for different thickness ratios, skew angles and support conditions ($R/a = 100$)

Shell panel	Skew angle (φ)	SSSS		CCCC		CSCS	
		a/h					
		10	100	10	100	10	100
Cylindrical	0°	2.515	1.3872	1.3755	0.297	1.7902	0.4795
	15°	4.0134	2.096	2.1553	0.3987	4.0238	2.1441
	45°	1.5941	0.6294	1.0429	0.1275	1.5933	0.5855
Spherical	0°	2.5149	1.3834	1.3752	0.296	1.7901	0.4792
	15°	4.0134	2.0923	2.1546	0.3963	4.0239	2.1419
	45°	1.5941	0.6294	1.0427	0.1272	1.5933	0.5855
Elliptical	0°	2.5149	1.3856	1.3754	0.2968	1.7902	0.4794
	15°	4.0134	2.0941	2.1552	0.3981	4.0239	2.143
	45°	1.5941	0.6294	1.0429	0.1274	1.5933	0.5855
Hyperboloidal	0°	2.515	1.3885	1.3753	0.2961	1.7902	0.4794
	15°	4.0136	2.1002	2.1546	0.3965	4.024	2.1463
	45°	1.5941	0.6294	1.0427	0.1272	1.5933	0.5855

Table 13 Non-dimensional fundamental frequency of the different curved sandwich ($0°/C/0°$) shell panels for different thickness ratios, skew angles and support conditions ($R/a = 10$)

Shell panel	Skew angle (φ)	SSSS		CCCC		CSCS	
		a/h					
		10	100	10	100	10	100
Cylindrical	0°	24.0952	3.0307	42.8355	7.5995	38.9462	7.3089
	15°	16.8171	3.3473	26.9649	5.6706	19.9648	5.0954
	45°	27.0004	4.9090	43.0321	7.8282	30.7883	6.7655
Spherical	0°	24.2316	4.0196	42.9548	8.2578	38.978	7.5799
	15°	16.9493	4.0984	27.155	6.5238	20.0144	5.5135
	45°	27.0326	5.2039	43.1476	8.4415	30.7919	6.8635
Elliptical	0°	24.1549	3.4774	42.8673	7.7816	38.9583	7.401
	15°	16.8793	3.6932	27.0164	5.9177	19.9873	5.2736
	45°	27.0185	5.0409	43.0628	7.9956	30.7937	6.8088
Hyperboloidal	0°	24.0271	2.6172	42.9398	8.1682	38.9442	7.3955
	15°	16.7168	2.9446	27.125	6.3629	19.9343	5.0039
	45°	26.9522	4.7633	43.1338	8.3698	30.7562	6.7177

Table 14 Non-dimensional fundamental frequency of the different curved sandwich ($0°/90°/C/90°/0°$) shell panels for different thickness ratios, skew angles and support conditions ($R/a = 10$)

Shell panel	Skew angle (φ)	SSSS		CCCC		CSCS	
		a/h					
		10	100	10	100	10	100
Cylindrical	0°	24.7798	3.0504	47.0195	7.1507	37.0204	6.1834
	15°	23.038	3.6712	39.2838	6.4138	26.233	5.1262
	45°	41.1127	6.2549	64.795	10.9207	45.0706	8.4629
Spherical	0°	24.9101	4.0708	47.2052	8.2853	37.0538	6.535
	15°	23.1327	4.3556	39.5071	7.6409	26.2715	5.4752
	45°	41.1234	6.457	64.9287	11.6757	45.0613	8.5313
Elliptical	0°	24.8383	3.5123	47.0677	7.4638	37.0335	6.3035
	15°	23.084	3.9808	39.3424	6.7585	26.2516	5.2655
	45°	41.1222	6.3407	64.8297	11.1204	45.072	8.4916
Hyperboloidal	0°	24.7028	2.6205	47.1914	8.1946	37.0155	6.2994
	15°	22.9548	3.3486	39.486	7.5247	26.2001	5.1109
	45°	41.0686	6.189	64.9186	11.6301	45.0313	8.4401

Table 15 Non-dimensional fundamental frequency of the different curved sandwich ($0°/90°/C/0°/90°$) shell panels for different thickness ratios, skew angles and support conditions ($R/a = 10$)

Shell panel	Skew angle (φ)	SSSS		CCCC		CSCS	
		a/h					
		10	100	10	100	10	100
Cylindrical	0°	24.8019	3.0497	47.1884	7.1494	36.4635	6.097
	15°	23.5347	3.7078	40.3428	6.5424	26.6959	5.1708
	45°	42.3674	6.4182	66.6205	11.2684	46.2897	8.6937
Spherical	0°	24.9299	4.0702	47.3748	8.2848	36.4969	6.4537
	15°	23.6265	4.389	40.562	7.7496	26.7333	5.5194
	45°	42.3755	6.6152	66.7532	12.0021	46.2782	8.76
Elliptical	0°	24.8595	3.5117	47.2372	7.4628	36.4766	6.2189
	15°	23.5795	4.0155	40.4008	6.8808	26.7141	5.3096
	45°	42.376	6.5017	66.6556	11.4622	46.2903	8.7216
Hyperboloidal	0°	24.7256	2.6196	47.3582	8.1937	36.4589	6.2147
	15°	23.4525	3.3895	40.5378	7.6348	26.6629	5.1562
	45°	42.3231	6.3539	66.7379	11.9577	46.2502	8.6713

Table 16 Non-dimensional fundamental frequency of the different curved sandwich $(0°/C/0°)$ shell panels for different thickness ratios, skew angles and support conditions $(R/a = 100)$

Shell panel	Skew angle (φ)	SSSS		CCCC		CSCS	
		a/h					
		10	100	10	100	10	100
Cylindrical	0°	24.0770	2.6258	42.5419	5.8374	38.6233	5.4620
	15°	16.5594	1.8665	26.5287	3.1419	19.3610	2.2846
	45°	26.7703	3.1443	42.7160	5.7356	30.3348	4.1348
Spherical	0°	24.0784	2.6391	42.5431	5.8464	38.6236	5.4657
	15°	16.5608	1.8791	26.5306	3.1583	19.3615	2.2903
	45°	26.7706	3.1479	42.7172	5.7443	30.3348	4.1362
Elliptical	0°	24.0776	2.6314	42.5422	5.8398	38.6234	5.4633
	15°	16.56	1.8721	26.5292	3.1463	19.3612	2.2868
	45°	26.7705	3.146	42.7163	5.7379	30.3349	4.1354
Hyperboloidal	0°	24.0763	2.6214	42.5429	5.8451	38.6233	5.4632
	15°	16.5584	1.8596	26.5303	3.1556	19.3607	2.2836
	45°	26.7698	3.1422	42.7171	5.7433	30.3345	4.1341

Table 17 Non-dimensional fundamental frequency of the different curved sandwich $(0°/90°/C/90°/0°)$ shell panels for different thickness ratios, skew angles and support conditions $(R/a = 100)$ (symmetric face sheets)

Shell panel	Skew angle (φ)	SSSS		CCCC		CSCS	
		a/h					
		10	100	10	100	10	100
Cylindrical	0°	24.7541	2.6292	46.836	5.8728	36.7871	4.6557
	15°	22.9229	2.6838	39.0872	4.9692	25.9342	3.2323
	45°	41.0309	5.2437	64.6559	9.9312	44.8645	7.1224
Spherical	0°	24.7554	2.643	46.8379	5.8878	36.7875	4.6605
	15°	22.9239	2.6932	39.0895	4.9866	25.9346	3.2372
	45°	41.031	5.2459	64.6572	9.9398	44.8644	7.1232
Elliptical	0°	24.7546	2.635	46.8365	5.8767	36.7873	4.6573
	15°	22.9234	2.6878	39.0878	4.9738	25.9344	3.2342
	45°	41.031	5.2446	64.6562	9.9334	44.8645	7.1227
Hyperbolical	0°	24.7533	2.6246	46.8378	5.8865	36.7871	4.6573
	15°	22.9221	2.6795	39.0893	4.985	25.9339	3.2321
	45°	41.0305	5.2429	64.6571	9.9392	44.8641	7.1221

Table 18 Non-dimensional fundamental frequency of the different curved sandwich of $(0°/90°/C/0°/90°)$ shell panels for different thickness ratios, skew angles and support conditions $(R/a = 100)$ (anti-symmetric face sheets)

Shell panel	Skew angle (φ)	SSSS		CCCC		CSCS	
		a/h					
		10	100	10	100	10	100
Cylindrical	0°	24.7738	2.6283	47.0063	5.8723	36.2274	4.5415
	15°	23.4242	2.7519	40.1535	5.1389	26.4056	3.3139
	45°	42.2899	5.4399	66.4874	10.3127	46.0912	7.3958
Spherical	0°	24.775	2.6421	47.0084	5.8873	36.2277	4.5465
	15°	23.4251	2.7611	40.1559	5.1557	26.4059	3.3188
	45°	42.2899	5.442	66.4889	10.321	46.091	7.3965
Elliptical	0°	24.7743	2.6341	47.0069	5.8762	36.2275	4.5432
	15°	23.4247	2.7559	40.1542	5.1433	26.4058	3.3158
	45°	42.2899	5.4408	66.4878	10.3149	46.0911	7.3961
Hyperboloidal	0°	24.7732	2.6237	47.0079	5.886	36.2274	4.5431
	15°	23.4235	2.7477	40.1553	5.1541	26.4053	3.3137
	45°	42.2895	5.4391	66.4883	10.3205	46.0909	7.3955

Table 19 Stresses of a simply supported (0°/C/0°) square sandwich spherical shell panel under the mechanical SSL

R/a	$\bar{\sigma}_{XX}(\frac{a}{2}, \frac{b}{2}, \pm\frac{h}{2})$		$\bar{\sigma}_{YY}(\frac{a}{2}, \frac{b}{2}, \pm\frac{h}{2})$		$\bar{\tau}_{XY}(\frac{a}{2}, \frac{b}{2}, \pm\frac{h}{2})$		$\bar{\tau}_{XZ}(0, \frac{b}{2}, \pm\frac{h}{2})$		$\bar{\tau}_{XZ}(\frac{a}{2}, 0, \pm\frac{h}{2})$	
5	− 0.6332	0.7045	0.0115	0.0857	0.0871	0.0113	− 0.1516	−0.1526	0.001	− 0.0067
10	−0.9202	0.9705	− 0.0181	0.0865	0.0801	− 0.0145	−0.2161	− 0.2168	0.0033	− 0.0021
20	−1.0398	1.0677	− 0.0393	0.0774	0.0655	− 0.0181	−0.2415	− 0.2419	0.0035	0.0005
50	− 1.0829	1.0944	− 0.0524	0.0681	0.053	− 0.0334	−0.2497	− 0.2499	0.0031	0.0018

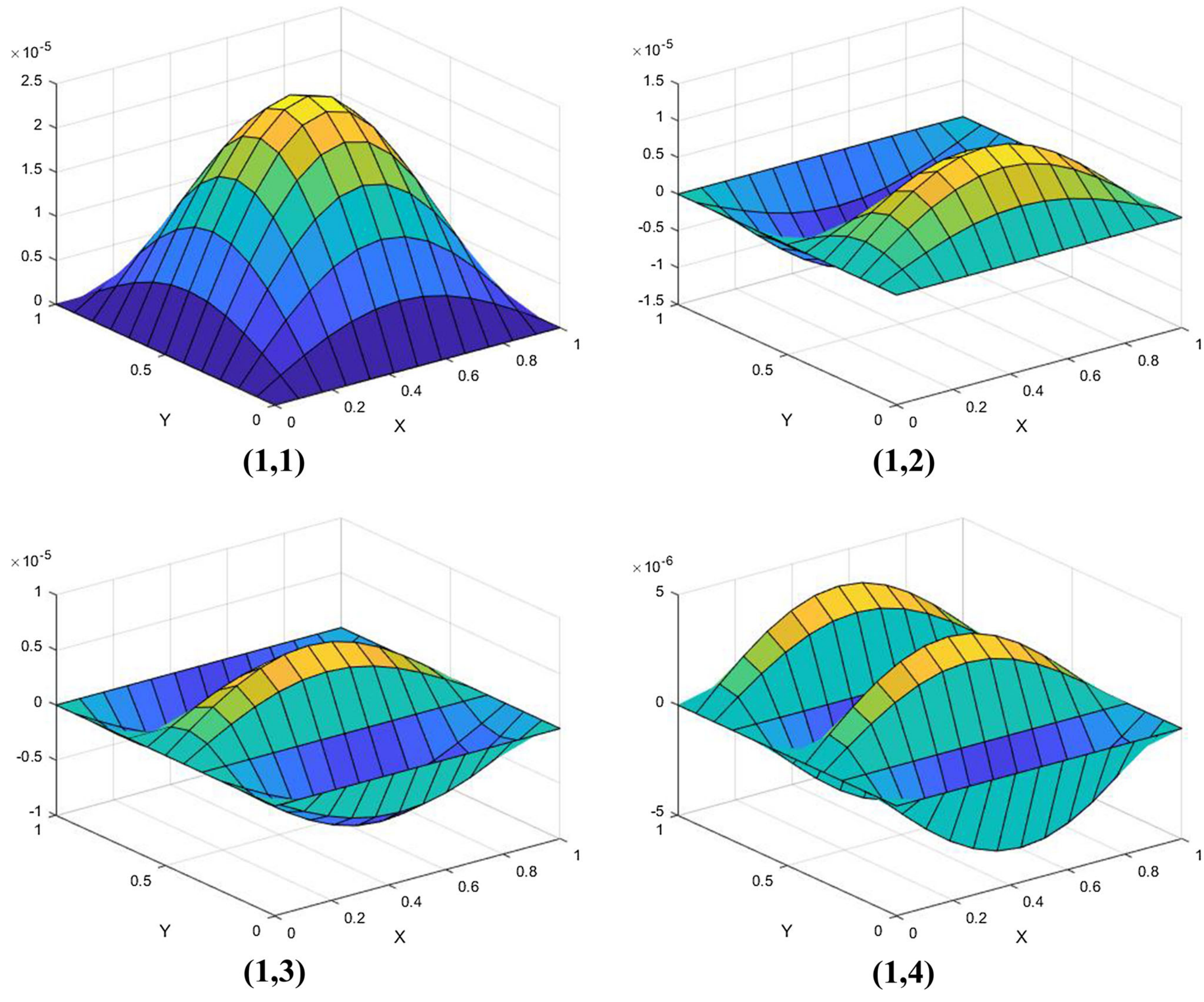

Fig. 7 Mode shapes for simply supported $(0°/C/0°)$ spherical shell panel

support conditions are well-known design parameter which affects the static deflection largely because it may vary the stiffness as the number of constraints increases or decreases. In this example, the responses are following as expected, i.e. maximum and minimum for all-side simply supported and clamped, whereas the deflection values in between for CSCS support.

Similarly, the frequency responses computed for the skew sandwich (0°/C/0°), $(0°/90°/C/90°/0°)$ and $(0°/90°/C/0°/90°)$ curved (cylindrical, spherical, elliptical and hyperboloidal) shell panels using the material properties as M2 of Case II with the curvature ratios ($R/a = 10$ and 100) are presented in Tables 13, 14, 15 and 16, 17, 18, respectively. It is observed from the frequency responses that as thickness ratio and curvature ratio increase, the frequency values decrease. It is very well known that the stiffness of the structure changes as the thickness ratios and the curvature ratios of the structure change. The lamination scheme also affects the frequency responses, and it indicates that as the number of layers increases, the frequency values increase, and the frequencies are slightly higher in the case of anti-symmetric lay-up in comparison with symmetric lay-up. The frequencies are showing higher and lower values for spherical and hyperboloidal panels. It is important to mention that the support conditions also affect the frequency responses significantly. Also, it is observed that the frequency is higher in the case of clamped supported structure, and it decreases as the constraints reduced, i.e. CCCC, CSCS and SSSS.

Now, an example is solved to compute the normal and the shear stress values for the doubly curved shell panel. For the computational purpose, a square simply supported three-layered ($0°/C/0°$) spherical sandwich shell panel ($a/h = 80$) problem under sinusoidal loading is solved. The non-dimensional stresses are obtained using M2 properties of the Case I for four different curvature ratios ($R/a = 5$, 10, 20 and 50) and presented in Table 19. It is observed from the table that the stress values are increasing while the curvature ratio (R/a) increases. This is because the panel flatness increases with the curvature ratio and the load-bearing capacity reduces subsequently.

Finally, the mode shapes are obtained for a square simply supported sandwich spherical shell panel and presented in Fig. 7. The mode shapes are obtained using the M1 properties (case II) for ($0°/C/0°$) spherical shell panel with $R/a = 10$, $h_c/h_f = 8$ and $a/h = 80$.

6 Conclusions

The static bending and the frequency responses of the skew sandwich shell panel with different core materials are investigated numerically in the framework of the HSDT kinematics. The computer code has been developed for the numerical analysis via current FE-higher-order model in MATLAB environment. The convergence and the validation behaviour of the presently obtained numerical results have been verified by the comprehensive testing. This is clear from the present study that the currently proposed HSDT model is capable of solving skew sandwich structural responses with ease and more efficient manner. Finally, the efficacy of the proposed model is demonstrated by solving a different kind of numerical examples for various design-related parameters and geometrical configurations. Based on the numerical study, the following valuable conclusions are drawn.

- The deflection responses of the curved sandwich skew shell panels are decreased with the increase in the thickness ratio values and decrease in the curvature ratios. Similarly, the frequency values follow a declining trend for the increase in both the thickness ratios and the curvature ratios.
- It is very well known that the structural stiffness changes when the geometrical parameters change and which in turn significantly affect the deflection including the frequency values for any variation in the curvature ratios and the thickness ratios.
- Also, the skew angles and the support conditions affect the bending and the frequency values of the skew sandwich shell panels considerably.
- It is interesting to note that the stress values of the panel structure follow an increasing trend while the curvature ratio increased due to the increment in the structural flatness.

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