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A Linguistic Neutrosophic Multi-criteria Group Decision-Making Approach with EDAS Method

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Abstract

This study develops an approach that incorporates power aggregation operators with the evaluation based on distance from average solution (EDAS) method under linguistic neutrosophic situations to solve fuzzy multi-criteria group decision-making problems. Firstly, the existing operational laws and comparison methods of linguistic neutrosophic numbers (LNNs) are analysed. Secondly, the distance measurement between two LNNs is defined. Thirdly, the power-weighted averaging operator and the power-weighted geometric operator with LNNs are developed to support the decision makers' evaluation information. The models to derive the criteria weights are also constructed based on the proposed distance measurements. Finally, the EDAS method is extended to resolve group decision-making problems in the linguistic neutrosophic environment. An illustrative example of the property management company selection is given to verify the effectiveness and practicality of the proposed approach.

Keywords Multi-criteria group decision-making · Linguistic neutrosophic numbers · Distance measurements · Power aggregation operator · Evaluation based on distance from average solution method

1 Introduction

Decision-making is a common activity in various aspects of our daily lives, and it is generally defined as the act of seeking the best alternative from a set of alternatives (options, candidates) based on the judgments of one or several decision makers (DMs) (experts, judges). Considered as an important branch of decision-making, the method known as multi-criteria group decision-making (MCGDM) involves the generation of decisions by DMs based on the information evaluated based on a set of feasible alternatives, in which multiple criteria are used to find a common solution [1]. In practice, the subjective evaluation information given by DMs is usually vaguely qualitative owing to the nature of the information and the unavailability of precise quantitative information [2]. In this sense, the linguistic variables proposed by Zadeh [3] can be used to enhance the practicability, flexibility and reliability of decision processes when the decision problems are complex or ill-defined (i.e. an exact description of the problem by conventional quantitative expressions cannot be realised). However, the major disadvantage of using linguistic variables is that it can only express imprecise information but cannot present inconsistent information [4,5]. The neutrosophic sets (NSs) proposed by Smarandache [6] can be used as effective tools to address issues on inconsistent information. The theories related to NSs are introduced in the succeeding paragraphs.

As extension of intuitionistic fuzzy sets (IFSs) [7], NSs simultaneously consider truth-membership, indeterminacy-membership and falsity-membership to flexibly address uncertain, incomplete and inconsistent information. Wang et al. [8] proposed single-valued neutrosophic sets to overcome the difficulty of applying NSs in actual engineering and economic management situations. Subsequent studies mainly focused on other extensions of NSs, such as simplified neutrosophic sets (SNSs) [9,10], interval neutrosophic sets [11], multi-valued neutrosophic sets [12,13], and the distance measures [14,15], similarity measures [16,17], correlation coefficients [9,18] and cross-entropy [19,20] of the sets. One of these studies discusses linguistic neutrosophic sets (LNSs) [21] which combine the advantages of linguis-

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tic variables and NSs. Li et al. [21] proposed the concept of LNSs and linguistic neutrosophic-prioritised operators. Fang and Ye [22] developed the LNN-weighted arithmetic averaging operator and the LNN-weighted geometric averaging operator based on the newly proposed operations and comparison methods. On the one hand, LNSs can take advantage of the linguistic approach to address imprecise information. On the other hand, LNSs can maximise the use of NSs to express inconsistent information, thus fully maximising the potentials of both sets. Correlation papers [21,22] that discuss the existing operations and comparison laws are also presented here. Given the advantages of LNNs, the present study can be utilised in the linguistic neutrosophic context. Consequently, a universal decision-making approach can be developed to address MCGDM problems.

Information aggregation operators are frequently used for decision-making models, and they play significant roles in aggregating the information received by DMs. The most common types of operators are the arithmetic- and geometric-weighted operators [23,24], order-weighted operators [25,26], generalised operators [27,28], Bonferroni mean operators [29,30], power aggregation (PA) operators [31,32], Heronian mean operators [33,34], prioritised aggregation operators [35,36], and so on. Each of these operators has its own characteristics and has been applied to different sets, such as fuzzy sets (FSs), IFSs and NSs and their extended forms. In this study, we adopt the PA operator proposed by Yager [31] to adjust the linguistic neutrosophic environment. PA operators can manage information about the interrelationship of aggregated values, and they enable the values to be mutually reinforced. We utilise the PA operator to aggregate the DMs' evaluation information, thus relieving the influences of unreasonable information received by biased DMs.

Various decision-making methods have contributed to the development of multi-criteria decision-making (MCDM) research. Classical methods mainly include the following: TODIM [37,38], TOPSIS [39], PROMETHEE [40], ELECTRE [41], VIKOR [42] and other correlation methods [43–45]. Even at present, these classical methods are extensively investigated. The evaluation based on distance from average solution (EDAS) method proposed by Ghorabaee et al. [46], a novel decision-making approach, has been revised in the present work to address MCGDM problems. EDAS is a useful and an easy-to-calculate method for conflicting criteria scenarios, and it has been applied in multi-criteria inventory classification [46] and supplier selection [47]. Although EDAS has not been extensively studied in recent years, many users have proven that its results are highly consistent with other methods, such as TOPSIS, simple additive weighting, VIKOR and so on [46]. Even if the research is limited, we use the EDAS tool to manage the MCDM problems and overcome the issues related to the originality and simplicity of traditional compromise methods, such as TOPSIS and VIKOR [48]. Then, based on EDAS, we propose the extended EDAS. The evaluation information is displayed in the form of linguistic neutrosophic numbers (LNNs), and the expected functions are considered.

The acquisition of the criteria weights is a crucial step in the MCGDM problem-solving process. In ideal cases, the criteria weights are fully known. However, it is often difficult in reality to obtain the accurate values of the criteria weights due to the constraints of the DM's knowledge and the complexity of the environment. Thus, the weight information is often partially known or completely unknown. The correct derivation of the criteria weights is therefore of great significance. In this study, we establish a single-objective programming model based on linguistic neutrosophic positive ideal solution to derive the objective weight information.

On the basis of the above observations, a universal and convenient approach is proposed to find the best alternative from the evaluation information in relation to the criteria provided by the DMs for several alternatives. In other words, the present study aims to effectively solve MCGDM problems. The rest of this paper proceeds as follows. Section 2 presents a brief review of the linguistic scale functions, LNNs and their relevant concepts. Section 3 presents the distance measurement of LNNs and the two types of PA operators for LNNs, namely the linguistic neutrosophic power-weighted averaging (LNPWA) operator and the linguistic neutrosophic power-weighted geometric (LNPWG) operator. Section 4 presents the models to derive the criteria weights and the MCGDM approach based on the EDAS method. Section 5 gives an example of a property management company selection to verify the desirability of the proposed approach. The conclusions are discussed in the last section.

2 Background

In this section, the definitions and operations related to LNNs, including the linguistic term sets, linguistic scale functions, operations and expected functions of LNNs proposed by Fang [22] and Li [21], are briefly reviewed.

The decision-making information is usually qualitative and difficult to depict by numerical values in real life. Thus, Zadeh [3] proposed the use of linguistic variables that can be defined and described by linguistic term sets.

Let $S = \left\{ s_j \,|\, j = 0, 1, \ldots, 2l \right\}$ be a finite and fully ordered discrete linguistic term set, where l is a positive integer and s_j is a possible linguistic term for a linguistic variable. Then, s_μ and s_ν ($s_\mu, s_\nu \in S$) represent an ordered $s_\mu < s_\nu$ if and only if $\mu < \nu$. If a negation operator exists, then $Neg(s_j) = s_{2l-j}$ $\quad (\mu, \nu = 0, 1, \ldots, 2l)$.

Xu [49] extended S into the continuous term set $\bar{S} = \left\{ s_j \,|\, s_0 \le s_\alpha \le s_{2l}, \alpha \in [0, 2l] \right\}$ to preserve all the processing information. If $s_\alpha \in S$, then s_α is called the original linguistic term; otherwise s_α is called the virtual linguistic term.

2.1 Linguistic Scale Functions

The linguistic scale function is a popular tool for enabling the flexible use of dates and semantic expressions [38]. Different semantic values are assigned to linguistic terms in different situations to derive the LNNs operations.

Definition 1 [3]. Let $s_j \in S$ be a linguistic term. If $\theta_j \in [0, 1]$ is a numerical value, then the linguistic scale function f that conducts the mapping from s_j to θ_j $(j = 0, 1, \ldots, 2l)$ can be defined as follows:

$$f : s_j \to \theta_j,$$

where $0 \le \theta_0 < \theta_1 < \cdots < \theta_{2l} \le 1$.

Thus, the function f monotonically increases relative to the subscript j. The symbol θ_j $(j = 0, 1, \ldots, 2l)$ reflects the preference of DMs when the linguistic terms $s_j \in S$ $(j = 0, 1, \ldots, 2l)$ are used. Subsequently, the function denotes the semantics of the linguistic terms. The following three functions can act as linguistic scale functions:

(1) $f_1(s_j) = \theta_j = \frac{j}{2l}(j = 0, 1, 2, \ldots, 2l)$,
In function (1), the evaluation scale for the linguistic information is divided by the average.

(2) $f_2(s_j) = \theta_j = \begin{cases} \frac{a^l - a^{l-j}}{2a^l - 2} & (j = 0, 1, 2, \ldots, l) \\ \frac{a^l + a^{l-j} - 2}{2a^l - 2} & (j = l, l+1, l+2, \ldots, 2l) \end{cases}$

In function (2), the linguistic term set extends from the middle to both ends, and the semantic deviation between the adjacent linguistic terms increases.

(3) $f_3(s_j) = \theta_j = \begin{cases} \frac{l^\alpha - (l-j)^\alpha}{2l^\alpha} & (j = 0, 1, 2, \ldots, l) \\ \frac{l^\beta + (j-l)^\beta}{2l^\beta} & (j = l, l+1, l+2, \ldots, 2l) \end{cases}.$

In function (3), the linguistic term set extends from the middle to both ends, and the semantic deviation between the adjacent linguistic terms decreases.

To preserve all the provided information and realise easy calculation [50], the functions given above can be expanded to $f^* : \overline{S} \to R^+ (R^+ = \{r \,|r \ge 0, r \in R\})$ which satisfies $f^*(s_j) = \theta_j$. It is also a strictly monotonically continuous and increasing function. An inverse function of f^* exists, and it is denoted by f^{*-1}. Furthermore, $f^*(s_{2l}) = 1$ can always be satisfied by adopting the functions f_1^*, f_2^* and f_3^* which are the expanded functions of f_1, f_2 and f_3, respectively.

2.2 LNNs

Definition 2 [21]. Let X be a universe of discourse and $\overline{S} = \{s_\alpha \,|s_0 \le s_\alpha \le s_{2l}, \alpha \in]0, 2l[\}$, then the LNSs can be defined as follows:

$$A = \{< x, s_\alpha(x), s_\beta(x), s_\gamma(x) > |x \in X\},$$

where $0 \le \alpha + \beta + \gamma \le 6l$, whilst $s_\alpha(x)$, $s_\beta(x)$, and $s_\gamma(x) \in \overline{S}$ represent the truth-membership function, the indeterminacy-membership function, and the falsity-membership function of element $x \in X$ to set A. Moreover, $< s_\alpha(x), s_\beta(x), s_\gamma(x) >$ is an LNN, and thus, A can be reviewed as a collection of LNNs.

Definition 3 [21]. Let f^* be the linguistic scale functions, and let f^{*-1} be the inverse function of f^*. If $a =< s_{\alpha_1}, s_{\beta_1}, s_{\gamma_1} >$ and $b =< s_{\alpha_2}, s_{\beta_2}, s_{\gamma_2} >$ are two LNNs, where $\xi > 0$, then the operations for the LNNs can be defined (see below), and the calculation results are still LNNs.

(1) $a \oplus b = \langle f^{*-1}(f^*(s_{\alpha_1}) + f^*(s_{\alpha_2}) - f^*(s_{\alpha_1})f^*(s_{\alpha_2})), f^{*-1}(f^*(s_{\beta_1})f^*(s_{\beta_1})), f^{*-1}(f^*(s_{\gamma_1})f^*(s_{\gamma_1}))\rangle$;
(2) $a \otimes b = \langle f^{*-1}(f^*(s_{\alpha_1})f^*(s_{\alpha_1})), f^{*-1}(f^*(s_{\beta_1}) + f^*(s_{\beta_2}) - f^*(s_{\beta_1})f^*(s_{\beta_2})), f^{*-1}(f^*(s_{\gamma_1}) + f^*(s_{\gamma_2}) - f^*(s_{\gamma_1})f^*(s_{\gamma_2}))\rangle$;
(3) $\xi a = \langle f^{*-1}(1 - (1 - f^*(s_{\alpha_1}))^\xi), f^{*-1}((f^*(s_{\beta_1}))^\xi), f^{*-1}((f^*(s_{\gamma_1}))^\xi)\rangle$;
(4) $a^\xi = \langle f^{*-1}((f^*(s_{\alpha_1}))^\xi), f^{*-1}(1 - (1 - f^*(s_{\beta_1}))^\xi), f^{*-1}(1 - (1 - f^*(s_\gamma))^\xi)\rangle$;
(5) $Neg(a) = \langle s_{\gamma_1}, s_{\beta_1}, s_{\alpha_1}\rangle$.

Definition 4 [22]. Let f^* be the linguistic scale functions, and let f^{*-1} be the inverse function of f^*. If $a =< s_{\alpha_1}, s_{\beta_1}, s_{\gamma_1} >$ and $b =< s_{\alpha_2}, s_{\beta_2}, s_{\gamma_2} >$ are two LNNs, where $\xi > 0$, then the operations for LNNs can be defined (see below), and the calculation results are still LNNs.

(1) $a \oplus b = \left\langle s_{\alpha_1 + \alpha_2 - \frac{\alpha_1\alpha_2}{2l}}, s_{\frac{\beta_1\beta_2}{2l}}, s_{\frac{\gamma_1\gamma_2}{2l}} \right\rangle$;
(2) $a \otimes b = \left\langle s_{\frac{\alpha_1\alpha_2}{2l}}, s_{\beta_1 + \beta_2 - \frac{\beta_1\beta_2}{2l}}, s_{\gamma_1 + \gamma_2 - \frac{\gamma_1\gamma_2}{2l}} \right\rangle$;
(3) $\xi a = \left\langle s_{2l - 2l(1 - \frac{\alpha_1}{2l})^\xi}, s_{2l(\frac{\beta_1}{2l})^\xi}, s_{2l(\frac{\gamma_1}{2l})^\xi} \right\rangle$;
(4) $a^\xi = \left\langle s_{2l(\frac{\alpha_1}{2l})^\xi}, s_{2l - 2l(1 - \frac{\beta_1}{2l})^\xi}, s_{2l - 2l(1 - \frac{\gamma_1}{2l})^\xi} \right\rangle$.

Definition 5 [21]. Let f^* be a linguistic scale function. If $a =< s_\alpha, s_\beta, s_\gamma >$ is an LNN, then the expected function $E(a)$, accuracy function $H(a)$ and certainty function $C(a)$ can be defined as follows:

$$E(a) = \frac{1}{3}(2 + f^*(s_\alpha) - f^*(s_\beta) - f^*(s_\gamma)), \quad (1)$$

$$H(a) = f^*(s_\alpha) - f^*(s_\gamma), \quad (2)$$

$$C(a) = f^*(s_\alpha). \quad (3)$$

Assume that a_i and a_j are two LNNs. Thus, they can be ranked according to the following rules:

(1) If $E(a_i) > E(a_j)$, then $a_i > a_j$;
(2) If $E(a_i) = E(a_j)$ and $H(a_i) > H(a_j)$, then $a_i > a_j$;
(3) If $E(a_i) = E(a_j)$, $H(a_i) = H(a_j)$ and $C(a_i) > C(a_j)$, then $a_i > a_j$;
(4) If $E(a_i) = E(a_j)$, $H(a_i) = H(a_j)$ and $C(a_i) = C(a_j)$, then $a_i = a_j$.

Definition 6 [22]. Let $a =< s_\alpha, s_\beta, s_\gamma >$ be an LNN, then the expected function $E(a)$ accuracy function $H(a)$ can be defined as follows:

$$E(a) = \frac{4l + \alpha - \beta - \gamma}{6l},$$
$$H(a) = \frac{\alpha - \gamma}{2l}.$$

Assume that a_i and a_j are two LNNs. Then, they can be ranked according to the following rules:

(1) If $E(a_i) > E(a_j)$, then $a_i > a_j$;
(2) If $E(a_i) = E(a_j)$ and $H(a_i) > H(a_j)$, then $a_i > a_j$;
(3) If $E(a_i) = E(a_j)$ and $H(a_i) = H(a_j)$, then $a_i = a_j$.

On the basis of the correlation research on LNNs, two existing operational laws and comparison methods can be linked.

The operational laws of Fang [22] are the special case versions of the operational laws of Li [21]. If $f^*(s_j) = \frac{j}{2l} (j = 0, 1, 2, \ldots, 2l)$ (i.e. from Li's operational laws), then the distance between adjacent semantics is equal and the rules can be simplified (i.e. similar to those by Fang). The comparison rules of LNNs have similar problems. The comparison rules of Fang can be obtained by taking $f^*(s_j) = \frac{j}{2l} (j = 0, 1, 2, \ldots, 2l)$ from Li's comparison method. However, the comparison rules of Fang do not have a certainty function, and thus, they may lead to improper results. For example, when $a_1 =<s_4, s_2, s_2>$ and $a_2 =<s_3, s_2, s_1>$, then we can obtain $a_1 = a_2$ by using Fang's rules. Subsequently, $a_1 \neq a_2$. By contrast, Li's comparison rules will never reach the same relationship. Thus, we use Li's rules in the present work.

3 PA Operators with LNNs

In this section, the traditional PA operator is extended as LNPWA and LNPWG operators. The distance measurements between two LNNs can then be developed.

Definition 7 [31]. Let a_i $(i = 1, 2, \ldots, n)$ be a collection of values and Λ be the set of all given values. The PA operator is the $PA : \Lambda^n \to \Lambda$ defined as follows:

$$PA(a_1, a_2, \ldots, a_n) = \sum_{i=1}^{n} \frac{1 + T(a_i)}{\sum_{i=1}^{n} (1 + T(a_i))} a_i, \tag{4}$$

where $T(a_i) = \sum_{j=1, j\neq i}^{n} sup(a_i, a_i)$ and $sup(a_i, a_j)$ denote the supports for a_i and a_j which satisfy the following properties:

(1) $sup(a_i, a_j) \in [0, 1]$;
(2) $sup(a_i, a_j) = sup(a_j, a_i)$;
(3) If $d(a_i, a_j) < d(a_p, a_q)$, then $sup(a_i, a_j) \geq sup(a_p, a_q)$, where $d(a_i, a_j)$ is the distance between a_i and a_j.

3.1 Distance Measurement Between Two LNNs

Definition 8 Let $a_1 =<s_{\alpha_1}, s_{\beta_1}, s_{\gamma_1}>$ and $a_2 =<s_{\alpha_2}, s_{\beta_2}, s_{\gamma_2}>$ be any two LNNs, and let f^* be a linguistic scale function $\kappa \geq 0$. The generalised distance measure between a_i and a_j can be defined as

$$d(a_1, a_2) = \Big(\frac{1}{3}\big(\left|f^*(s_{\alpha_1}) - f^*(s_{\alpha_2})\right|^\kappa + \left|f^*(s_{\beta_1}) - f^*(s_{\beta_2})\right|^\kappa + \left|f^*(s_{\gamma_1}) - f^*(s_{\gamma_2})\right|^\kappa\big)\Big)^{\frac{1}{\kappa}}, \tag{5}$$

where $\kappa = 1$ or $\kappa = 2$, and Eq. (5) is reduced to the Hamming distance or the Euclidean distance, respectively.

Theorem 1 *Let $a_1 =< s_{\alpha_1}, s_{\beta_1}, s_{\gamma_1} >$, $a_2 =< s_{\alpha_2}, s_{\beta_2}, s_{\gamma_2} >$, and $a_3 =< s_{\alpha_3}, s_{\beta_3}, s_{\gamma_3} >$ be any three LNNs, and let f^* be a linguistic scale function. The distance measurement presented in Definition 8 satisfies the following three properties:*

(1) $d(a_1, a_2) \geq 0$;
(2) $d(a_1, a_2) = d(a_2, a_1)$;
(3) *If $s_{\alpha_1} > s_{\alpha_2} > s_{\alpha_3}$, $s_{\beta_1} < s_{\beta_2} < s_{\beta_3}$, and $s_{\gamma_1} < s_{\gamma_2} < s_{\gamma_3}$, then $d(a_1, a_3) > d(a_1, a_2)$, and $d(a_1, a_3) > d(a_2, a_3)$.*

Proof Properties (1) and (2) are clearly correct. The proof of property (3) can then be obtained.

$s_{\alpha_1} > s_{\alpha_2} > s_{\alpha_3}$, $s_{\beta_1} < s_{\beta_2} < s_{\beta_3}$, $s_{\gamma_1} < s_{\gamma_2} < s_{\gamma_3}$ and f^* are strictly monotonically increasing and continuous functions. Thus, $f^*(s_{\alpha_1}) > f^*(s_{\alpha_2}) > f^*(s_{\alpha_3})$, $f^*(s_{\beta_1}) > f^*(s_{\beta_2}) > f^*(s_{\beta_3})$ and $f^*(s_{\gamma_1}) > f^*(s_{\gamma_2}) > f^*(s_{\gamma_3})$. The following inequalities can then be obtained:

$$\left|f^*(s_{\alpha_1}) - f^*(s_{\alpha_3})\right|^\kappa > \left|f^*(s_{\alpha_1}) - f^*(s_{\alpha_2})\right|^\kappa,$$
$$\left|f^*(s_{\beta_1}) - f^*(s_{\beta_3})\right|^\kappa > \left|f^*(s_{\beta_1}) - f^*(s_{\beta_2})\right|^\kappa,$$
$$\left|f^*(s_{\gamma_1}) - f^*(s_{\gamma_3})\right|^\kappa > \left|f^*(s_{\gamma_1}) - f^*(s_{\gamma_2})\right|^\kappa.$$

Then,

$$\Big(\frac{1}{3}\big(\left|f^*(s_{\alpha_1}) - f^*(s_{\alpha_3})\right|^\kappa + \left|f^*(s_{\beta_1}) - f^*(s_{\beta_3})\right|^\kappa + \left|f^*(s_{\gamma_1}) - f^*(s_{\gamma_3})\right|^\kappa\big)\Big)^{\frac{1}{\kappa}}$$

$$> \left(\frac{1}{3}\left(\left|f^*(s_{\alpha_1}) - f^*(s_{\alpha_2})\right|^{\kappa} + \left|f^*(s_{\beta_1}) - f^*(s_{\beta_2})\right|^{\kappa} + \left|f^*(s_{\gamma_1}) - f^*(s_{\gamma_2})\right|^{\kappa}\right)\right)^{\frac{1}{\kappa}}.$$

Thus, $d(a_1, a_3) > d(a_1, a_2)$. The inequality $d(a_1, a_3) > d(a_2, a_3)$ can also be proven in a similar manner. This part concludes the proof of Theorem 1. □

Example 1 Assume that $a_1 = <s_3, s_2, s_1>$, $a_2 = <s_5, s_4, s_2>$, and $a_3 = <s_6, s_2, s_2>$ are three LNNs. In addition, let $\kappa = 2$, $l = 4$ and $f^* = f_1^*$. Thus, $d(a_1, a_2) = 0.125$, $d(a_1, a_3) = 0.132$ and $d(a_2, a_3) = 0.093$. Moreover, $d(a_1, a_3) > d(a_1, a_2)$ and $d(a_1, a_3) > d(a_2, a_3)$.

3.2 Power-Weighted Averaging Operator with LNNs

Definition 9 Let $a_i = < s_{\alpha_i}, s_{\beta_i}, s_{\gamma_i} >$ $(i = 1, 2, \ldots, n)$ be a collection of LNNs, and let Γ be the set of all LNNs and ω_i the weight of a_i $(i = 1, 2, \ldots, n)$, where $\omega_i \geq 0$ and $\sum_{i=1}^{n} \omega_i = 1$. The LNPWA operator is the mapping $\text{LNPWA} : \Gamma^n \to \Gamma$ defined as follows:

$$\text{LNPWA}(a_1, a_2, \ldots, a_n) = \mathop{\oplus}_{i=1}^{n} \frac{(1 + T(a_i))\omega_i a_i}{\sum_{i=1}^{n} (1 + T(a_i))\omega_i}, \tag{6}$$

where $T(a_i) = \sum_{j=1, j\neq i}^{n} \omega_j sup(a_i, a_j)$ and $sup(a_i, a_j)$ denote the supports for a_i and a_j, which satisfy the following properties:

(1) $sup(a_i, a_j) \in [0, 1]$;
(2) $sup(a_i, a_j) = sup(a_j, a_i)$
(3) If $d(a_i, a_j) < d(a_p, a_q)$, then $sup(a_i, a_j) \geq sup(a_p, a_q)$, where $d(a_i, a_j)$ is the distance measurement between a_i and a_j, as defined in Sect. 3.1.

Theorem 2 *Let $a_i = <s_{\alpha_i}, s_{\beta_i}, s_{\gamma_i}>$ $(i = 1, 2, \ldots, n)$ be a collection of LNNs and ω_i the weight of a_i $(i = 1, 2, \ldots, n)$, where $\omega_i \geq 0$ and $\sum_{i=1}^{n} \omega_i = 1$. Then, the result aggregated from Definition 9 is still an LNN. Moreover,*

$$\begin{aligned}
&\text{LNPWA}(a_1, a_2, \ldots, a_n) \\
&= \Bigg\langle f^{*-1}\left(1 - \prod_{i=1}^{n}(1 - f^*(s_{\alpha_i}))^{(1+T(a_i))\omega_i/\sum_{i=1}^{n}\omega_i(1+T(a_i))}\right), \\
&\quad f^{*-1}\left(\prod_{i=1}^{n}(f(s_{\beta_i}))^{(1+T(a_i))\omega_i/\sum_{i=1}^{n}\omega_i(1+T(a_i))}\right), \\
&\quad f^{*-1}\left(\prod_{i=1}^{n}(f(s_{\gamma_i}))^{(1+T(a_i))\omega_i/\sum_{i=1}^{n}\omega_i(1+T(a_i))}\right)\Bigg\rangle,
\end{aligned} \tag{7}$$

where $T(a_i) = \sum_{j=1, j\neq i}^{n} \omega_j sup(a_i, a_j)$, which satisfies the conditions presented in Definition 9.

Proof For convenience, let $\varphi_i = \frac{(1+T(a_i))\omega_i}{\sum_{i=1}^{n}\omega_i(1+T(a_i))}$ for the purpose of this proof. Then, Eq. (7) can be proven via the mathematical induction of n.

(1) When $n = 2$, we have

$$\begin{aligned}
&\text{LNPWA}(a_1, a_2) = \phi_1 a_1 \oplus \phi_2 a_2 \\
&= \Big\langle f^{*-1}(1 - (1 - f^*(s_{\alpha_1}))^{\phi_1}(1 - f^*(s_{\alpha_2}))^{\phi_2}), \\
&\quad f^{*-1}((f(s_{\beta_1}))^{\phi_1}(f(s_{\beta_2}))^{\phi_2}), \\
&\quad f^{*-1}((f(s_{\gamma_1}))^{\phi_1}(f(s_{\gamma_2}))^{\phi_2})\Big\rangle \\
&= \Bigg\langle f^{*-1}\left(1 - \prod_{i=1}^{2}(1 - f^*(s_{\alpha_i}))^{(1+T(a_i))\omega_i/\sum_{i=1}^{n}\omega_i(1+T(a_i))}\right), \\
&\quad f^{*-1}\left(\prod_{i=1}^{2}(f(s_{\beta_i}))^{(1+T(a_i))\omega_i/\sum_{i=1}^{n}\omega_i(1+T(a_i))}\right), \\
&\quad f^{*-1}\left(\prod_{i=1}^{2}(f(s_{\gamma_i}))^{(1+T(a_i))\omega_i/\sum_{i=1}^{n}\omega_i(1+T(a_i))}\right)\Bigg\rangle.
\end{aligned}$$

Therefore, when $n = 2$, Eq. (7) is true.

(2) Assume that when $n = k$, Eq. (7) is true. Thus,

$$\begin{aligned}
&\text{LNPWA}(a_1, a_2, \ldots, a_k) \\
&= \Bigg\langle f^{*-1}\left(1 - \prod_{i=1}^{k}(1 - f^*(s_{\alpha_i}))^{\phi_i}\right), \\
&\quad f^{*-1}\left(\prod_{i=1}^{k}(f(s_{\beta_i}))^{\phi_i}\right), f^{*-1}\left(\prod_{i=1}^{k}(f(s_{\gamma_i}))^{\phi_i}\right)\Bigg\rangle.
\end{aligned}$$

Then, when $n = k + 1$, we have

$$\begin{aligned}
&\text{LNPWA}(a_1, a_2, \ldots, a_k, a_{k+1}) \\
&= \text{LNPWA}(a_1, a_2, \ldots, a_k) \oplus \phi_{k+1} a_{k+1} \\
&= \Bigg\langle f^{*-1}\left(1 - \prod_{i=1}^{k}(1 - f^*(s_{\alpha_i}))^{\phi_i}\right), \\
&\quad f^{*-1}\left(\prod_{i=1}^{k}(f(s_{\beta_i}))^{\phi_i}\right), f^{*-1}\left(\prod_{i=1}^{k}(f(s_{\gamma_i}))^{\phi_i}\right)\Bigg\rangle \oplus \\
&\quad \Big\langle f^{*-1}(1 - (1 - f^*(s_{\alpha_{k+1}}))^{\phi_{k+1}}), \\
&\quad f^{*-1}((f(s_{\beta_{k+1}}))^{\phi_{k+1}}), f^{*-1}((f(s_{\gamma_{k+1}}))^{\phi_{k+1}})\Big\rangle \\
&= \Bigg\langle f^{*-1}\left(1 - \prod_{i=1}^{k}(1 - f^*(s_{\alpha_i}))^{\phi_i}(1 - f^*(s_{\alpha_{k+1}}))^{\phi_{k+1}}\right), \\
&\quad f^{*-1}\left(\prod_{i=1}^{k}(f(s_{\beta_i}))^{\phi_i}(f(s_{\beta_{k+1}}))^{\phi_{k+1}}\right), \\
&\quad f^{*-1}\left(\prod_{i=1}^{k}(f(s_{\gamma_i}))^{\phi_i}(f(s_{\gamma_{k+1}}))^{\phi_{k+1}}\right)\Bigg\rangle \\
&= \Bigg\langle f^{*-1}\left(1 - \prod_{i=1}^{k+1}(1 - f^*(s_{\alpha_i}))^{(1+T(a_i))\omega_i/\sum_{i=1}^{n}\omega_i(1+T(a_i))}\right),
\end{aligned}$$

$$f^{*-1}\left(\prod_{i=1}^{k+1}(f(s_{\beta_i}))^{(1+T(a_i))\omega_i/\sum_{i=1}^{n}\omega_i(1+T(a_i))}\right),$$
$$f^{*-1}\left(\prod_{i=1}^{k+1}(f(s_{\gamma_i}))^{(1+T(a_i))\omega_i/\sum_{i=1}^{n}\omega_i(1+T(a_i))}\right)\Bigg\rangle.$$

Thus, when $n = k + 1$, Eq. (7) is true.

According to (1) and (2), we can calculate Eq. (7) for any n. □

Theorem 3 *(Idempotency). Let* $a_i =< s_{\alpha_i}, s_{\beta_i}, s_{\gamma_i} >$ $(i = 1, 2, \ldots, n)$ *be a collection of LNNs. For all* $a_i =< s_\alpha, s_\beta, s_\gamma >= a$ $(i = 1, 2, \ldots, n)$*, there is* $\mathrm{LNPWA}(a_1, a_2, \ldots, a_n) = a$.

Proof Given that all $a_i = a$, we have

$$\begin{aligned}\mathrm{LNPWA}(a_1, a_2, \ldots, a_n) &= \bigoplus_{i=1}^{n}\frac{(1+T(a_i))\omega_i a_i}{\sum_{i=1}^{n}(1+T(a_i))\omega_i}\\ &= \bigoplus_{i=1}^{n}\frac{(1+T(a_i))\omega_i a}{\sum_{i=1}^{n}(1+T(a_i))\omega_i}\\ &= a\sum_{i=1}^{n}\frac{(1+T(a_i))\omega_i}{\sum_{i=1}^{n}(1+T(a_i))\omega_i} = a.\end{aligned}$$

□

Theorem 4 (Boundedness) *Let* $a_i =< s_{\alpha_i}, s_{\beta_i}, s_{\gamma_i} >$ $(i = 1, 2, \ldots, n)$ *be a collection of LNNs. Suppose that* $a =< s_{\max_i \alpha_i}, s_{\min_i \beta_i}, s_{\min_i \gamma_i} >$ *and* $b =< s_{\min_i \alpha_i}, s_{\max_i \beta_i}, s_{\max_i \gamma_i} >$*, then* $b \le \mathrm{LNPWA}(a_1, a_2, \ldots, a_n) \le a$.

Proof Given that $s_{\max_i \alpha_i} \ge s_{\alpha_i} \ge s_{\min_i \alpha_i}$, $s_{\min_i \beta_i} \le s_{\beta_i} \le s_{\max_i \beta_i}$ and $s_{\min_i \gamma_i} \le s_{\gamma_i} \le s_{\max_i \gamma_i}$, according to Theorem 3, we have

$$\begin{aligned}b &= \mathrm{LNPWA}(b, b, \ldots, b) \le \mathrm{LNPWA}(a_1, a_2, \ldots, a_n)\\ &\le \mathrm{LNPWA}(a, a, \ldots, a) = a\end{aligned}$$

Thus, $b \le \mathrm{LNPWA}(a_1, a_2, \ldots, a_n) \le a$. □

3.3 Power-Weighted Geometric Operator with LNNs

Definition 10 Let $a_i = \langle s_{\alpha_i}, s_{\beta_i}, s_{\gamma_i}\rangle$ $(i = 1, 2, \ldots, n)$ be a collection of LNNs, Γ the set of all LNNs, and ω_i the weight of a_i $(i = 1, 2, \ldots, n)$, where $\omega_i \ge 0$ and $\sum_{i=1}^{n}\omega_i = 1$. Then, the LNPWG operator is the mapping $\mathrm{LNPWA} : \Gamma^n \to \Gamma$ defined as follows:

$$\mathrm{LNPWG}(a_1, a_2, \ldots, a_n) = \prod_{i=1}^{n} a_i^{\omega_i(1+T(a_i))/\sum_{i=1}^{n}\omega_i(1+T(a_i))}, \tag{8}$$

where $T(a_i) = \sum_{j=1, j\ne i}^{n}\omega_j sup(a_i, a_j)$ and $sup(a_i, a_j)$ denote the supports for a_i and a_j which satisfy the following three properties:

(1) $sup(a_i, a_j) \in [0, 1]$;
(2) $sup(a_i, a_j) = sup(a_j, a_i)$;
(3) If $d(a_i, a_j) < d(a_p, a_q)$, then $sup(a_i, a_j) \ge sup(a_p, a_q)$, where $d(a_i, a_j)$ is the distance measurement between a_i and a_j, defined in Sect. 3.1.

Theorem 5 *Let* $a_i = \langle s_{\alpha_i}, s_{\beta_i}, s_{\gamma_i}\rangle$ $(i = 1, 2, \ldots, n)$ *be a collection of LNNs and* ω_i *the weight of* a_i $(i = 1, 2, \ldots, n)$*, where* $\omega_i \ge 0$ *and* $\sum_{i=1}^{n}\omega_i = 1$*. Then, the result aggregated from Definition 10 is still an LNN. Moreover,*

$$\begin{aligned}&\mathrm{LNPWG}(a_1, a_2, \ldots, a_n)\\ &= \Big\langle f^{*-1}\left(\prod_{i=1}^{n}(f^*(s_{\alpha_i}))^{\omega_i(1+T(a_i))/\sum_{i=1}^{n}\omega_i(1+T(a_i))}\right),\\ &\quad f^{*-1}\left(1-(\prod_{i=1}^{n}(1-f^*(s_{\beta_i}))^{(1+T(a_i))\omega_i/\sum_{i=1}^{n}\omega_i(1+T(a_i))}\right),\\ &\quad f^{*-1}\left(1-(\prod_{i=1}^{n}(1-f^*(s_{\gamma_i}))^{(1+T(a_i))\omega_i/\sum_{i=1}^{n}\omega_i(1+T(a_i))}\right)\Big\rangle,\end{aligned}$$

where $T(a_i) = \sum_{j=1, j\ne i}^{n}\omega_j \sup(a_i, a_j)$ *which satisfies the conditions presented in Definition 10.*

Proof Theorem 5 can be proven via mathematical induction, but the full discussion of the process is omitted here.

Similarly, the LNPWG operator has the characteristics of idempotency and boundedness. □

4 Method

In this section, novel models based on the linguistic neutrosophic positive ideal solution of deriving the criteria weights are constructed. The weight information of the proposed models is either partially known or completely unknown. Subsequently, the LN-EDAS decision-making method is introduced. The extended EDAS and the PA operator are also combined to effectively manage the LNNs. Consequently, a novel linguistic neutrosophic MCGDM approach is developed.

4.1 Models of Deriving Criteria Weights

Assume that $R = [r_{ij}]_{m\times n}$ $(i = 1, 2, \ldots, m; j = 1, 2, \ldots, n)$ is a linguistic neutrosophic decision matrix, where $r_{ij} = \langle s_{\alpha_{ij}}, s_{\beta_{ij}}, s_{\gamma_{ij}}\rangle$ is expressed in the form of LNNs. Then, the linguistic neutrosophic positive ideal solution can be derived as

$$\begin{aligned}\mathrm{LN-PIS} = R^+ &= \left\{r_1^+, r_2^+, \ldots, r_n^+\right\}\\ &= \left\{\max_i r_{ij} \,|\, j = 1, 2, \ldots, n\right\}.\end{aligned} \tag{9}$$

Assume that $\omega_j \quad (j=1,2,\ldots,n)$ is the weight of the j-th criterion. Then, $\omega_j \geq 0$ and $\sum_{j=1}^{n}\omega_j = 1$. The deviation degree between each alternative $a_i \ (i=1,2,\ldots,m)$ and the linguistic neutrosophic positive ideal solution R^+ can be defined as

$$\begin{aligned}\Delta(a_i, R^+) &= \textstyle\sum_{j=1}^{n}\omega_j^2 d^2(r_{ij}, r_j^+)\\ &= \textstyle\sum_{j=1}^{n}\omega_j^2(\frac{1}{3}((f^*(s_{\alpha_1}) - f^*(s_{\alpha_2}))^2\\ &\quad + \left(f^*(s_{\beta_1}) - f^*(s_{\beta_2})\right)^2\\ &\quad + \left(f^*(s_{\gamma_1}) - f^*(s_{\gamma_2})\right)^2)\end{aligned}, \tag{10}$$

where $d(r_{ij}, r_j^+)$ is the distance measurement between r_{ij} and r_j^+, as defined in Sect. 3.1.

The smaller the deviation degree between each alternative and linguistic neutrosophic positive ideal solution is, the better the alternative will be. Therefore, we construct the following single-objective programming model:

$$\cdot\begin{cases}\min g(x) = \sum_{i=1}^{m}\sum_{j=1}^{n}\omega_j^2 d^2(r_{ij}, r_j^+)\\ \sum_{j=1}^{n}\omega_j = 1, \omega_j \geq 0, j=1,2,\ldots,n\end{cases}. \tag{11}$$

We also construct the Lagrange function to solve this model.

$$L(\omega,\lambda) = \sum_{i=1}^{m}\sum_{j=1}^{n}\omega_j^2 d^2(r_{ij}, r_j^+) + 2\lambda\left(\sum_{j=1}^{n}\omega_j - 1\right),$$

where λ is the Lagrange multiplier. By calculating the partial derivatives with respect to ω_j and λ, the optimal weight ω_j can be obtained as

$$\omega_j = \frac{1}{\left(\sum_{j=1}^{n}\frac{1}{\sum_{i=1}^{m}d^2(r_{ij}, r_j^+)}\right)\left(\sum_{i=1}^{m}d^2(r_{ij}, r_j^+)\right)}. \tag{12}$$

The above result is applied to situations where the weight information is completely unknown. For the situation where the information is partly known, the criteria weights can be obtained by using the following model, in which the set of known information for the criteria weights is ψ:

$$\begin{cases}\min g(x) = \sum_{i=1}^{m}\sum_{j=1}^{n}\omega_j^2 d^2(r_{ij}, r_j^+)\\ \omega_j \in \psi, \sum_{j=1}^{n}\omega_j = 1, \omega_j \geq 0, j=1,2,\ldots,n\end{cases}. \tag{13}$$

4.2 LN-EDAS Decision-Making Method

EDAS is a useful method for multi-criteria inventory classification [46] and supplier selection [47], and it can be effectively utilised for some conflicting criteria. In this method, two necessary measures are considered, namely the positive distance from average (PDA) and the negative distance from average (NDA). These measures can express the difference between each alternative and average solution. The higher values of PDA and the lower values of NDA are, the best alternative selected will be. Subsequently, based on EDAS, we propose the extended EDAS whose evaluation information is displayed in the form of LNNs and the expected functions are considered.

Firstly, the average solution is calculated according to all presented criteria.

$$\mathrm{AV} = [\mathrm{AV}_j]_{1\times n} = \left[\frac{\sum_{i=1}^{m} r_{ij}}{m}\right]_{1\times n}, \tag{14}$$

where $r_{ij} \ (i=1,2,\ldots,m;\ j=1,2,\ldots,n)$ is the linguistic neutrosophic evaluation information of the i-th alternative on j-th provided by the DMs. Then, the PDA and NDA matrices are calculated as

$$\begin{aligned}\mathrm{PDA} &= [P_{ij}]_{m\times n} = \left[\frac{\max(0, E(r_{ij}) - E(\mathrm{AV}_j))}{E(\mathrm{AV}_j)}\right]_{m\times n},\\ \mathrm{NDA} &= [N_{ij}]_{m\times n} = \left[\frac{\max(0, E(\mathrm{AV}_j) - E(r_{ij}))}{E(\mathrm{AV}_j)}\right]_{m\times n}\end{aligned} \tag{15}$$

where $E(r_{ij})$ and $E(\mathrm{AV}_j)$ represent the expected values of r_{ij} and AV_j which can be obtained by Definition 5. Then, the weighted sums of PDA and NDA of all property companies are obtained by

$$\begin{aligned}\mathrm{SP}_i &= \sum_{j=1}^{n} w_j P_{ij},\\ \mathrm{SN}_i &= \sum_{j=1}^{n} w_j N_{ij},\end{aligned} \tag{16}$$

and normalised as

$$\begin{aligned}\mathrm{NSP}_i &= \frac{\mathrm{SP}_i}{\max_i \mathrm{SP}_i},\\ \mathrm{NSN}_i &= 1 - \frac{\mathrm{SN}_i}{\max_i \mathrm{SN}_i}.\end{aligned} \tag{17}$$

Finally, the appraisal score (AS) of all property companies is calculated by

$$\mathrm{AS}_i = \frac{1}{2}(\mathrm{NSP}_i + \mathrm{NSN}_i). \tag{18}$$

The higher the value of AS_i is, the better the property company will be.

4.3 MCGDM Approach Based on LN-EDAS and PA Operators

From the preparatory phase, an MCGDM approach based on EDAS and PA operators with LNNs is proposed. Let $A =$

$\{a_1, a_2, \ldots, a_m\}$ be a set of alternatives, $C = \{c_1, c_2, \ldots, c_n\}$ the criteria set and $E = \{e_1, e_2, \ldots, e_p\}$ the DM set. Assume that the weight of the criteria is w_j $(j = 1, 2, \ldots, n)$, where $w_j \geq 0$ and $\sum_{j=1}^{n} w_j = 1$, and the weight of the DMs is ω_k $(k = 1, 2, \ldots, p)$, where $\omega_k \geq 0$ and $\sum_{k=1}^{p} \omega_k = 1$. Let $H^k = [h_{ij}^k]_{m\times n}$ be the decision matrices, where $h_{ij}^k = < s_{\alpha_{ijk}}, s_{\beta_{ijk}}, s_{\gamma_{ijk}} >$ $(i = 1, 2, \ldots, m; j = 1, 2, \ldots, n; k = 1, 2, \ldots, p)$ is the linguistic neutrosophic evaluation information of i-th alternative on j-th provided by DMs e_k.

The process of the proposed approach includes the following steps:

Step 1 Normalise the decision matrix.

Two criteria types, namely benefit criteria and cost criteria, are included in the decision matrices. Thus, the negation operator in Definition 3 is used to unify both criteria and transform the cost-type criteria values into benefit-type criteria values. Assume that the transformed standardised matrix is $R^k = [r_{ijk}]_{m\times n}$ $(k = 1, 2, \ldots, p)$. Then, the original matrix H^k can be transformed into R^k as follows:

$$r_{ijk} = \begin{cases} h_{ijk}, & \text{for benifit criterion } c_j \\ Neg(h_{ijk}), & \text{for cost criterion } c_j \end{cases}. \tag{19}$$

Step 2 Calculate the supports.

Obtain the supports with the formula

$$sup(r_{ijk_1}, r_{ijk_2}) = 1 - d(r_{ijk_1}, r_{ijk_2})$$
$$(i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n;\ k_1, k_2 = 1, 2, \ldots, p), \tag{20}$$

where $d(r_{ijk_1}, r_{ijk_2})$ is the distance measurement between r_{ijk_1} and r_{ijk_2}, as defined in Sect. 3.1.

Step 3 Calculate the weights φ_{ijk} associated with r_{ijk} $(k = 1, 2, \ldots, p)$.

$$\varphi_{ijk} = \frac{(1 + T(r_{ijk}))\omega_k}{\sum_{k=1}^{p} \omega_k (1 + T(r_{ijk}))}, \tag{21}$$

where $T(r_{ijk}) = \sum_{\widetilde{k}=1, \widetilde{k}\neq k}^{p} \omega_{\widetilde{k}} sup(r_{ijk}, r_{ij\widetilde{k}})$ and $\omega_{\widetilde{k}}$ is the weight of DM $e_{\widetilde{k}}$.

Step 4 Obtain the comprehensive evaluation matrix.

Utilise either the LNPWA operator or the LNPWG operator to aggregate the evaluation information provided by the DMs to determine the comprehensive evaluation matrix $R = [r_{ij}]_{m\times n}$.

Step 5 Obtain the criteria weights.

Utilise Eq. (12) to derive the weight information if the criteria weights are completely unknown or utilise model Eq. (13) to derive the weight information if the criteria weights are partly known.

Step 6 Obtain the average solution according to Eq. (14).

Step 7 Obtain the PDA and NDA matrices according to Eq. (15).

Step 8 Obtain the weighted sum of the PDA SP_i and the NDA SN_i according to Eq. (16).

Step 9 Normalise the values of SP and SN according to Eq. (17).

Step 10 Obtain the AS according to Eq. (18).

Step 11 Rank the alternatives.

The alternatives are ranked according to the decreasing values of AS. The alternative with the highest AS is the best choice.

5 Illustrative Example of Property Management Company Selection

In this section, we used the proposed MCGDM approach to help a developer select a suitable property management company in Zhengzhou in the Province of Henan. The developer invited three experts, who are denoted here as e_1, e_2 and e_3. Two of the experts are members of the Zhengzhou Property Management Association. Furthermore, all of the experts play important roles in the real estate sector in Henan. The main purpose of the developer was to construct residences, and thus, five satisfactory property management companies that perform well in residential property construction were selected. The companies were Jianye Property, Xinyuan Property, Lvdu Property, Vanke Property and Zhengshang Property. For convenience, the names of these candidate property management companies were replaced by a_1, a_2, a_3, a_4 and a_5. After a discussion between the developer and the experts, five vital criteria were selected: staff quality c_1, service level c_2, customer rights protection c_3, customer satisfaction level c_4 and emergency capability c_5. The specific condition of the criteria weights was $w_4 > w_5$. The evaluation provided by the experts was all equal, and the weight of the experts was $\omega_k = \frac{1}{3}$ $(k = 1, 2, 3)$. In addition, the linguistic term set $S = \{s_0 = \text{extremely low}, s_1 = \text{very low}, s_2 = \text{low}, s_3 = \text{slightly low}, s_4 = \text{medium}, s_5 = \text{slightly high}, s_6 = \text{high}, s_7 = \text{very high}, s_8 = \text{extremely high}\}$ was employed here. After interviewing the experts, the sets of linguistic neutrosophic evaluation information were transformed into LNNs. The results are shown in Tables 1, 2 and 3.

5.1 Illustration of the Proposed Model

The procedure of the case study can be summarised by the following steps:

Step 1 Normalise the decision matrix.

Considering that all of the criteria are benefit-type criteria, a normalisation is not needed. Therefore, $R^k = H^k$ $(k = 1, 2, 3)$.

Step 2 Calculate the supports.

Table 1 Evaluation information of e_1

	c_1	c_2	c_3	c_4	c_5
a_1	$\langle s_7, s_2, s_2\rangle$	$\langle s_7, s_1, s_1\rangle$	$\langle s_5, s_3, s_3\rangle$	$\langle s_6, s_2, s_3\rangle$	$\langle s_5, s_1, s_4\rangle$
a_2	$\langle s_5, s_3, s_1\rangle$	$\langle s_5, s_1, s_4\rangle$	$\langle s_5, s_1, s_1\rangle$	$\langle s_5, s_1, s_2\rangle$	$\langle s_4, s_0, s_2\rangle$
a_3	$\langle s_6, s_3, s_2\rangle$	$\langle s_4, s_2, s_3\rangle$	$\langle s_6, s_0, s_2\rangle$	$\langle s_4, s_4, s_1\rangle$	$\langle s_4, s_2, s_4\rangle$
a_4	$\langle s_7, s_3, s_3\rangle$	$\langle s_6, s_3, s_1\rangle$	$\langle s_7, s_3, s_2\rangle$	$\langle s_6, s_1, s_2\rangle$	$\langle s_4, s_1, s_1\rangle$
a_5	$\langle s_5, s_1, s_1\rangle$	$\langle s_5, s_1, s_3\rangle$	$\langle s_6, s_1, s_2\rangle$	$\langle s_6, s_2, s_1\rangle$	$\langle s_5, s_2, s_4\rangle$

Table 2 Evaluation information of e_2

	c_1	c_2	c_3	c_4	c_5
a_1	$\langle s_5, s_3, s_2\rangle$	$\langle s_6, s_2, s_1\rangle$	$\langle s_5, s_0, s_1\rangle$	$\langle s_6, s_1, s_1\rangle$	$\langle s_4, s_4, s_2\rangle$
a_2	$\langle s_6, s_2, s_3\rangle$	$\langle s_5, s_3, s_2\rangle$	$\langle s_4, s_3, s_0\rangle$	$\langle s_5, s_4, s_2\rangle$	$\langle s_4, s_0, s_3\rangle$
a_3	$\langle s_4, s_5, s_2\rangle$	$\langle s_5, s_1, s_0\rangle$	$\langle s_6, s_1, s_2\rangle$	$\langle s_6, s_2, s_3\rangle$	$\langle s_5, s_1, s_2\rangle$
a_4	$\langle s_6, s_2, s_1\rangle$	$\langle s_5, s_4, s_0\rangle$	$\langle s_6, s_2, s_1\rangle$	$\langle s_6, s_1, s_2\rangle$	$\langle s_6, s_1, s_2\rangle$
a_5	$\langle s_5, s_1, s_2\rangle$	$\langle s_4, s_2, s_1\rangle$	$\langle s_5, s_3, s_1\rangle$	$\langle s_5, s_3, s_2\rangle$	$\langle s_4, s_4, s_4\rangle$

Table 3 Evaluation information of e_3

	c_1	c_2	c_3	c_4	c_5
a_1	$\langle s_6, s_1, s_3\rangle$	$\langle s_6, s_3, s_1\rangle$	$\langle s_7, s_1, s_1\rangle$	$\langle s_6, s_2, s_2\rangle$	$\langle s_6, s_2, s_2\rangle$
a_2	$\langle s_7, s_3, s_3\rangle$	$\langle s_5, s_2, s_2\rangle$	$\langle s_5, s_2, s_2\rangle$	$\langle s_5, s_1, s_3\rangle$	$\langle s_4, s_3, s_2\rangle$
a_3	$\langle s_7, s_2, s_3\rangle$	$\langle s_6, s_1, s_4\rangle$	$\langle s_6, s_1, s_2\rangle$	$\langle s_7, s_1, s_0\rangle$	$\langle s_6, s_1, s_5\rangle$
a_4	$\langle s_7, s_3, s_2\rangle$	$\langle s_6, s_2, s_2\rangle$	$\langle s_5, s_4, s_1\rangle$	$\langle s_7, s_2, s_2\rangle$	$\langle s_6, s_0, s_2\rangle$
a_5	$\langle s_6, s_2, s_1\rangle$	$\langle s_5, s_3, s_3\rangle$	$\langle s_5, s_2, s_0\rangle$	$\langle s_6, s_1, s_1\rangle$	$\langle s_6, s_1, s_2\rangle$

Assume that $f^* = f_1^*(s_\delta) = \frac{\delta}{8}$ and $\kappa = 2$. The supports are as follows:

$$sup(r_{ij1}, r_{ij2}) = sup(r_{ij2}, r_{ij1}) = \begin{bmatrix} 0.907 & 0.941 & 0.850 & 0.907 & 0.844 \\ 0.898 & 0.882 & 0.898 & 0.875 & 0.958 \\ 0.882 & 0.862 & 0.958 & 0.856 & 0.898 \\ 0.898 & 0.928 & 0.928 & 1.000 & 0.907 \\ 0.958 & 0.898 & 0.898 & 0.928 & 0.907 \end{bmatrix},$$

$$sup(r_{ij1}, r_{ij3}) = sup(r_{ij3}, r_{ij1}) = \begin{bmatrix} 0.928 & 0.907 & 0.856 & 0.958 & 0.898 \\ 0.882 & 0.907 & 0.941 & 0.958 & 0.875 \\ 0.928 & 0.898 & 0.958 & 0.818 & 0.898 \\ 0.958 & 0.941 & 0.898 & 0.941 & 0.898 \\ 0.941 & 0.917 & 0.898 & 0.958 & 0.898 \end{bmatrix},$$

$$sup(r_{ij2}, r_{ij3}) = sup(r_{ij3}, r_{ij2}) = \begin{bmatrix} 0.898 & 0.958 & 0.907 & 0.941 & 0.882 \\ 0.941 & 0.958 & 0.898 & 0.868 & 0.868 \\ 0.818 & 0.828 & 1.000 & 0.862 & 0.868 \\ 0.928 & 0.875 & 0.907 & 0.941 & 0.958 \\ 0.928 & 0.898 & 0.941 & 0.898 & 0.828 \end{bmatrix}.$$

Step 3 Calculate the weights ϕ_{ijk} associated with r_{ijk} $(k = 1, 2, 3)$.

According to Eq. (21), the values of ϕ_{ijk} $(k = 1, 2, 3)$ are as follows:

$$\varphi_{ij1} = \begin{bmatrix} 0.334 & 0.332 & 0.331 & 0.333 & 0.333 \\ 0.331 & 0.330 & 0.334 & 0.336 & 0.336 \\ 0.337 & 0.336 & 0.331 & 0.332 & 0.335 \\ 0.333 & 0.336 & 0.334 & 0.335 & 0.331 \\ 0.334 & 0.334 & 0.331 & 0.335 & 0.337 \end{bmatrix},$$

$$\varphi_{ij2} = \begin{bmatrix} 0.332 & 0.335 & 0.334 & 0.332 & 0.332 \\ 0.335 & 0.334 & 0.331 & 0.329 & 0.335 \\ 0.330 & 0.331 & 0.334 & 0.335 & 0.333 \\ 0.331 & 0.332 & 0.334 & 0.335 & 0.335 \\ 0.333 & 0.332 & 0.334 & 0.331 & 0.332 \end{bmatrix},$$

$$\varphi_{ij3} = \begin{bmatrix} 0.334 & 0.333 & 0.335 & 0.335 & 0.335 \\ 0.334 & 0.336 & 0.334 & 0.335 & 0.329 \\ 0.333 & 0.333 & 0.334 & 0.333 & 0.333 \\ 0.335 & 0.332 & 0.332 & 0.331 & 0.334 \\ 0.332 & 0.334 & 0.334 & 0.333 & 0.331 \end{bmatrix}.$$

Step 4 Obtain the comprehensive evaluation matrix.

The linguistic neutrosophic evaluation information is aggregated by utilising the LNPWA operator. The comprehensive evaluation matrix R is

$$R=\begin{bmatrix} <s_{6.185},s_{1.816},s_{2.290}> & <s_{6.411},s_{1.819},s_{1.000}> & <s_{5.923},s_{0.000},s_{1.438}> & <s_{6.000},s_{1.589},s_{1.819}> & <s_{5.119},s_{1.998},s_{2.519}> \\ <s_{6.185},s_{2.619},s_{2.085}> & <s_{5.000},s_{1.821},s_{2.515}> & <s_{4.700},s_{1.814},s_{0.000}> & <s_{5.000},s_{1.579},s_{2.291}> & <s_{4.000},s_{0.000},s_{2.291}> \\ <s_{6.004},s_{3.102},s_{2.289}> & <s_{5.114},s_{1.262},s_{0.000}> & <s_{6.000},s_{0.000},s_{2.000}> & <s_{6.001},s_{1.999},s_{0.000}> & <s_{4.697},s_{1.261},s_{3.421}> \\ <s_{6.742},s_{2.623},s_{1.820}> & <s_{5.712},s_{2.884},s_{0.000}> & <s_{6.184},s_{1.819},s_{1.997}> & <s_{6.410},s_{1.258},s_{2.000}> & <s_{5.485},s_{0.000},s_{1.590}> \\ <s_{5.378},s_{1.259},s_{1.260}> & <s_{4.699},s_{1.817},s_{2.082}> & <s_{5.377},s_{1.820},s_{0.000}> & <s_{5.713},s_{1.816},s_{1.258}> & <s_{5.114},s_{2.001},s_{3.179}> \end{bmatrix}$$

Step 5 Obtain the criteria weights.

The criteria weights w_j $(j = 1, 2, \ldots, 5)$ can be calculated by Eq. (13) as

$$w = (0.236, 0.191, 0.201, 0.204, 0.167).$$

Step 6 Obtain the average solution according to Eq. (14).

The average solution on the basis of all the criteria can be calculated by Eq. (14) as

$$\mathrm{AV}=\begin{bmatrix} <s_{6.150},s_{2.175},s_{1.905}> & <s_{5.469},s_{1.854},s_{0.000}> & <s_{5.693},s_{0.000},s_{0.000}> & <s_{5.872},s_{1.629},s_{0.000}> & <s_{4.922},s_{0.000},s_{2.511}> \end{bmatrix}$$

Step 7 Obtain the PDA and NDA matrices according to Eq. (15).

The PDA and NDA matrices can be calculated by Eq. (15) as follows:

$$\mathrm{PDA}=[P_{ij}]_{5\times5}=\begin{bmatrix} 0.001 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.012 & 0.000 & 0.000 & 0.000 \\ 0.013 & 0.000 & 0.000 & 0.000 & 0.081 \\ 0.044 & 0.000 & 0.000 & 0.000 & 0.000 \end{bmatrix},$$

$$\mathrm{NDA}=[N_{ij}]_{5\times5}=\begin{bmatrix} 0.000 & 0.001 & 0.056 & 0.082 & 0.098 \\ 0.033 & 0.150 & 0.129 & 0.154 & 0.038 \\ 0.081 & 0.000 & 0.078 & 0.012 & 0.130 \\ 0.000 & 0.040 & 0.153 & 0.054 & 0.000 \\ 0.000 & 0.144 & 0.098 & 0.079 & 0.135 \end{bmatrix}.$$

Step 8 Obtain the weighted sums of PDA SP_i and NDA SN_i according to Eq. (16).

The weighted sums of PDA and NDA for all the alternatives are calculated by Eq. (16) as follows: $\mathrm{SP}_1 = 0$, $\mathrm{SP}_2 = 0$, $\mathrm{SP}_3 = 0.002$, $\mathrm{SP}_4 = 0.017$, $\mathrm{SP}_5 = 0.010$, $\mathrm{SN}_1 = 0.045$, $\mathrm{SN}_2 = 0.1$, $\mathrm{SN}_3 = 0.059$, $\mathrm{SN}_4 = 0.05$ and $\mathrm{SN}_5 = 0.086$.

Step 9 Normalise the values of SP and SN according to Eq. (17).

The normalised values of SP and SN are calculated by Eq. (17) as follows: $\mathrm{NSP}_1 = 0.008$, $\mathrm{NSP}_2 = 0$, $\mathrm{NSP}_3 = 0.140$, $\mathrm{NSP}_4 = 1$, $\mathrm{NSP}_5 = 0.627$, $\mathrm{NSN}_1 = 0.556$, $\mathrm{NSN}_2 = 0$, $\mathrm{NSN}_3 = 0.412$, $\mathrm{NSN}_4 = 0.506$ and $\mathrm{NSN}_5 = 0.143$.

Step 10 Obtain the AS according to Eq. (18).

The AS is calculated by Eq. (18) as follows: $\mathrm{AS}_1 = 0.282$, $\mathrm{AS}_2 = 0$, $\mathrm{AS}_3 = 0.276$, $\mathrm{AS}_4 = 0.753$ and $\mathrm{AS}_5 = 0.385$.

Step 11 Rank the alternatives.

The results are obtained and ranked according to the decreasing values of AS as follows: $a_4 \succ a_5 \succ a_1 \succ a_3 \succ a_2$. Consequently, Vanke Property a_4 is the best choice for the developer.

5.2 Sensitivity Analysis and Comparative Analysis

To investigate the effects of the distance parameters and the operators on the final results, we take different values of κ and different operators defined in Sect. 3. The final ranking results are shown in Table 4.

The ranking results vary with the different parameters of distance measurement. Although slight differences across the final results are observed, the best alternatives are all the same. The variations are clearly caused by the different parameters of the distance measurement and the different operators, which suggests that distance measurements and the operators collectively influence the decision-making process. Beyond of that, the backgrounds of the DMs chosen by the developer should also be considered.

Then, the effectiveness of the proposed method is demonstrated and the proposed method is compared with relevant studies. The decision information used by Ghorabaee [47] involves triangular fuzzy numbers (TFNs) that are transformed by linguistic terms, whereas the decision information used in the proposed method include LNNs. The LNNs comprise three parts (i.e. truth-membership, indeterminacy-membership and falsity-membership) that are all expressed by linguistic variables. Therefore, the representation of LNNs is more suitable and feasible than those of real numbers or TFNs transformed by linguistic variables. Consequently, the approach based on LNNs can effectively solve complicated decision-making problems.

The methods based on LNNs proposed by Li [21] only consider a single DM, and it is difficult and unrealistic to make decisions by referring to only one DM in cases of complex decision-making problems. By contrast, the proposed approach can be used for group decision-making environments, in which each evaluation information is represented as a matrix in LNNs.

Table 4 Ranking results of different values of κ

Ranking results	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$
LNPWA	$a_4 \succ a_5 \succ a_3 \succ a_1 \succ a_2$	$a_4 \succ a_5 \succ a_1 \succ a_3 \succ a_2$	$a_4 \succ a_5 \succ a_1 \succ a_3 \succ a_2$
LNPWG	$a_4 \succ a_1 \succ a_5 \succ a_3 \succ a_2$	$a_4 \succ a_1 \succ a_5 \succ a_3 \succ a_2$	$a_4 \succ a_5 \succ a_1 \succ a_2 \succ a_3$

EDAS is considered an appropriate method for some conflicting criteria. Traditional compromise MCDM methods, such as VIKOR and TOPSIS, to obtain the best alternatives by calculating the distance from the ideal and nadir solution. By contrast, EDAS only needs to calculate the expected function from the average solution. A calculation of the ideal and nadir solution is not needed, and hence, EDAS is a relatively easier approach.

6 Discussions

In this study, we develop two linguistic neutrosophic PA operators to deal with the linguistic neutrosophic evaluation information provided by DMs. The MCGDM approach based on EDAS is also developed to obtain the best alternatives. In addition, the models commonly used to derive criteria weights are constructed to render these items more objective. In the present work, the proposed approach is applied to the selection of an appropriate company given that property management activities are necessary and urgent endeavours. LNNs, which combine the advantages of linguistic term sets and SNSs, are applied, and the distance measurement and semantic situations are also considered.

The case study shows that the proposed MCGDM method is practical and effective. The advantages of the proposed approach are as follows:

(a) The proposed approach is based on LNN which is suitable in real life situations. LNNs have the capacity to deal with imprecise and vague information. Each of the element's truth-membership, indeterminacy-membership and falsity-membership degree is expressed as set of linguistic variables rather than a set of crisp numbers. Thus, the DMs may find it more flexible and convenient to express their opinions as linguistic information. The existing operation rules and comparison rules are contrasted and discussed.

(b) The distance measurements of LNNs, including the Hamming distance and Euclidean distance, are determined. The properties of the distance measurement are also given.

(c) Models are established to obtain the criteria weights, and the evaluation results have become more reasonable. Consequently, the subjectivity of the traditional methods for determining weights is resolved.

(d) EDAS is a novel decision-making method, and it is applied to property management selection in this study. Furthermore, EDAS is not only easy to calculate, but it can also realise the reasonable and stable ranking of alternatives.

The proposed MCGDM method is applicable to many other decision-making or strategy-selection problems. In the future, a few more MCGDM methods based on LNNs may be developed, given that LNNs are widely applied to many domains in the real world. Moreover, the present study has some disadvantages and limitations. For instance, the decision matrices are obtained subjectively by DMs and the consistency of the evaluation information is not considered. Then, the proposed approach is based on linguistic neutrosophic situations, but they may not be properly expressed in complicated conditions. Expanded situations, such as hesitant linguistic neutrosophic and interval linguistic neutrosophic environments, should be considered. A detailed and comprehensive MCDM study will be conducted in the future.

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