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RESEARCH ARTICLE - ELECTRICAL ENGINEERING

Collocation Method for First Passage Time Problem of Power Systems Subject to Stochastic Excitations

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Abstract

Large penetration of renewable energies heavily threats the stable and reliable operation of power systems due to their randomness and intermittence characteristics. The first passage time problem is one of the critical issues in reliability assessment of new energy power systems. In this paper, we present and analyze the first passage time problem of power systems with stochastic excitation by collocation method. The power systems with stochastic excitations are modeled by stochastic differential equations. Then, the backward Kolmogorov equations and the generalized Pontryagin equations governing the conditional reliability function and the conditional moments of first passage time, respectively, are established based on the stochastic averaging method. The corresponding initial and boundary conditions are also provided. A numerical collocation method was proposed to solve the equations, and case studies were executed on a single-machine infinite-bus system under Gaussian excitation. Illustrations of the conditional reliability function and probability density functions for some cases are presented.

Keywords SDEs · First passage time · Stochastic averaging method · Backward Kolmogorov equation · Generalized Pontryagin equation · Collocation method

1 Introduction

Stability and reliability are two extremely important issues in the operation of power systems, especially when the systems were under stressed conditions or experienced (random) disturbances [1–3]. Along with the increasing penetration of the renewable energies like wind power and solar energy, the intermittent and random nature of the renewables exacerbate these problems [4–6]. It is necessary to develop better techniques to account for the uncertainty and stochastic perturbations caused by drifts of operating conditions and consequently to assess the reliability and stochastic stability of power systems.

The impact that the wind energy has on the power grids' operation is an ongoing area of research. A number of authors focus on to what extent more and more penetration of the wind power will affect the power systems. They have explored various approaches to cope with the impact of the wind power in the context of smart grids [7–12]. Dhople and his colleagues presented a stochastic hybrid system to analyze the impact of stochastic power injections on power system dynamics in [13]. Many simulations and/or analytical-based methods have been proposed to address related issues with integration of wind power [14–19].

The reliability of a system can be considered as the probability that there are no failures occurring in the system within a given time interval. One problem in this field is called the first passage time (FPT) problem. It has been widely applied in statistical physics, structure analysis and other related fields [20,21]. The FPT problem is directly related to solving of the Kolmogorov equations and Pontryagin equations with given initial and/or boundary conditions. Some strategies have been proposed to deal with FPT problems over the years. The FPT problems described by Hamiltonian systems

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with single or many degrees of freedom received extensively attention in the last three decades [20,21].

Qiu [3], Nwankpa et al. [22,23] analyzed the stochastic dynamic behavior of power systems by taking the stochastic perturbations of loads as Gaussian white noise and provided the relation between the FPT and the security of power systems. Their work seems to be the original application of FPT in the security and stability analysis fields of power systems. Unfortunately, little subsequent work has been done on the effect of random excitation on the dynamic behavior of power systems. In addition, the exact analytical solutions to the FPT problems, even for a simple system, are difficult to be found. Therefore, the numerical methods are needed.

In this paper, we take the variability of a large deployment of renewables as random perturbations. What we want to address is whether the power systems' operation can keep safe and stable during the stochastic excitation, to what probability the power grid can survive from these random disturbances. It is novel to study the FTP considering the effect of renewables, the mean first passage time (MFPT) will be used as a performance index to represent the dynamic behavior of the power systems with random excitations. The state evolution of the power systems will be described by stochastic differential equations (SDEs). The main contributions herein are as following:

1. Reducing the power systems' model with stochastic variations to the averaged Itô SDEs by the stochastic averaging method;
2. Deriving the corresponding backward Kolmogorov equations and the generalized Pontryagin equations;
3. Proposing a numerical technique based on orthogonal collocation method.

The validation of the method is verified by comparing the results to other methods through a single-machine infinite-bus system with stochastic excitation.

The remainder of this paper is arranged as follows. Section 2 provides a mathematical formulation of FPT problems and then gives a brief introduction of stochastic averaging method. The next section focuses on various methods to deal with FPT problems through a single-machine infinite-bus power system. Finally, in the last section, the conclusions on the FPT problem caused by stochastic excitations due to renewable energies and further potential studies are discussed.

2 Problem Statement

The stochastic nature and uncertainties of real power systems come from the randomness of some electric sources, chargeable loads, storage cells, etc. The random factors can be generally divided into three categories: randomness of initial values caused by the power flow or disturbance; randomness of parameters caused by the structure and operation states of the power systems; and randomness of outside excitations comes from the integration of renewable energy sources and/or chargeable devices. All of these factors should be taken into consideration in analysis of power systems. However, only some specific scenarios were considered and most of the methods employed were based on deterministic frameworks in the traditional power system analysis. Fortunately, the mature theory in stochastic mathematics has facilitated effective and efficient analysis of power systems.

2.1 Stochastic Differential Equations Formulation

The idea of nonlinear differential algebraic equations (DAEs) [1] model to describe the dynamic properties of the power systems is unsuitable for the desired application considering the emerging intermittent and stochastic factors of new energies [13–19]. In such context, the stochastic modeling of power systems based on SDEs is constructed to reflect more accurate and more objective of the power systems nowadays.

There are several general approaches proposed in the number of papers on how to account uncertainties in power system analysis. The SDEs

$$\begin{aligned} \dot{X}(t) &= F\left(X(t), Y(t), t\right) \quad t \in [t_0, T] \\ X(t_0) &= X_0 \end{aligned} \tag{1}$$

can be used to represent the state evolution of the power systems. In Eq. (1), $F(\cdot)$ is an n-dimensional smooth vector function, $X(t)$ and $Y(t)$ are vectors of n-dimensional and m-dimensional random process, respectively. The DAEs model is a special case of (1) in which $X(t)$ and $Y(t)$ are not random process.

Most commonly used mathematical model is based on the Itô SDEs

$$\mathrm{d}X(t) = AX(t)\mathrm{d}t + Q\mathrm{d}B(t) \tag{2}$$

where $X(t)$ is the vector of the state variables; A is the drift coefficient matrix and Q is the diffusion coefficient matrix; and an n-dimensional Wiener process $B(t)$ represents the stochastic disturbance. In some ideal condition, the power injected by renewables can be modeled by the Gaussian random excitation for a given time frame.

The aforementioned Itô SDEs scheme is valuable in calculating FPT indices, so it is beneficial to the reliability and stability assessment of the power systems. In the past, classic perturbation methods were frequently used to approximate the solution of SDEs. Hereafter, we will extend and apply the stochastic averaging method to reduce the dimension and then handle the FPT problems.

2.2 Stochastic Averaging Method

The stochastic averaging method constitutes a potent and elegant analytical framework in many fields of mechanics and control [24]. The main idea is to approximate a complicated nonlinear system by a simpler one via averaging the rapidly fluctuating functions. It has received intensive interests in approximately determining the probability density functions (PDF) for the response of nonlinear systems. The stochastic averaging method for the Hamiltonian systems developed recently [20] is briefly reviewed here.

Consider the following n-degree-of-freedom randomly excited Hamiltonian systems

$$\begin{aligned}\dot{q}_i &= \frac{\partial H}{\partial p_i}\\ \dot{p}_i &= -\frac{\partial H}{\partial q_i} - c_{ij}\frac{\partial H}{\partial p_j} + f_{ik}\xi_k(t)\end{aligned} \tag{3}$$

where the subindices $i, j = 1, 2, \ldots, n$; $k = 1, 2, \ldots, m$; the vectors of state variables $q = (q_1, q_2, \ldots, q_n)^{\mathrm{T}}$ and $p = (p_1, p_2, \ldots, p_n)^{\mathrm{T}}$ are generalized displacement and momentum vector, respectively; $H = H(q, p)$ is the Hamiltonian function with continuously second derivatives. c_{ij} is the quasi-linear damping coefficient; $f_{ik}$is the amplitude of the stochastic excitation; $\xi_k(t)$ is a random process whose correlation functions are defined by

$$E[\xi_k(t)\xi_{k+l}(t+\tau)] = 2D_{kl}\delta(\tau) \tag{4}$$

where $D_{kl}$is the covariance matrix and $\delta(\tau)$ is the Dirac function.

We can write Eq. (3) in its equivalent Itô form

$$\begin{aligned}\mathrm{d}q_i &= \frac{\partial H}{\partial p_i}\mathrm{d}t\\ \mathrm{d}p_i &= -\left[\frac{\partial H}{\partial q_i} + m_{ij}(q, p)\frac{\partial H}{\partial p_j}\right]\mathrm{d}t + \sigma_{ik}(q, p)\mathrm{d}B_k(t)\end{aligned} \tag{5}$$

where $m_{ij}(q, p) = c_{ij}, \sigma_{ik}(q, p) = (fL)_{ik}, f = (f_{ik})$, $LL^T = 2D = 2(D_{kl}), B_k(t)$ are the Wiener processes, $i, j = 1, 2, \ldots, n$, and $k = 1, 2, \ldots, m$.

Then the following averaged Itô SDEs can be obtained by Itô differential rule and the stochastic averaging method

$$\mathrm{d}H = m(H)\mathrm{d}t + \sigma(H)\mathrm{d}B(t) \tag{6}$$

in which the drift and diffusion coefficients are

$$\begin{aligned}m(H) = \frac{1}{T}\int_\Omega \left(-m_{ij}\frac{\partial H}{\partial p_i}\frac{\partial H}{\partial p_j} + \frac{1}{2}\sigma_{ik}\sigma_{jk}\frac{\partial^2 H}{\partial p_i \partial p_j}\right)\\ \Big/\left(\frac{\partial H}{\partial p_1}\right)\mathrm{d}q_1\ldots\mathrm{d}q_n\mathrm{d}p_2\ldots\mathrm{d}p_n\end{aligned} \tag{7}$$

and

$$\sigma^2(H) = \frac{1}{T}\int_\Omega \frac{\sigma_{ik}\sigma_{jk}\frac{\partial H}{\partial p_i}\frac{\partial H}{\partial p_j}}{\partial H/\partial p_1}\mathrm{d}q_1\ldots\mathrm{d}q_n\mathrm{d}p_2\ldots\mathrm{d}p_n, \tag{8}$$

respectively. In which

$$T = T(H) = \int_\Omega \frac{1}{\partial H/\partial p_1}\mathrm{d}q_1\ldots\mathrm{d}q_n\mathrm{d}p_2\ldots\mathrm{d}p_n \tag{9}$$

and the integral region is

$$\begin{aligned}&\Omega = \{(q_1, \ldots, q_n; p_2, \ldots, p_n)\\ &|H(q_1, \cdots, q_n, 0, p_2, \ldots, p_n) \le H\}\end{aligned}$$

2.3 First Passage Time Problems

The FPT is one of the important performance indices in reliability assessment of stochastic dynamics. It usually means finding the PDF of the time when the stochastic state process first crosses a designated threshold. The general approach to deal with FPT problems is based on stochastic process and PDEs. Since that the random process is determined by Fokker–Planck–Kolmogorov (FPK) equation. The PDEs governing the probability of FPT can be derived from the FPK equation. Both of them need resort to analytic and numerical techniques.

The conditional reliability function (CRF) is defined by

$$\begin{aligned}&R(t|H_0) = \mathrm{P}\{H(s) \in (H_{\min}, H_c), s \in (0, t]|\\ &H_0 \in (H_{\min}, H_c)\}\end{aligned} \tag{10}$$

It should satisfy the so-called backward Kolmogorov equation

$$\frac{\partial R}{\partial t} = m(H_0)\frac{\partial R}{\partial H_0} + \frac{1}{2}\sigma^2(H_0)\frac{\partial^2 R}{\partial H_0^2} \tag{11}$$

and corresponding boundary values

$$\begin{aligned}R(0|H_c) &= 0\\ R(0|H_{\min}) &= \text{finite}\end{aligned} \tag{12}$$

and initial values

$$R(0|H_0) = 1, \quad H_0 \in [H_{\min}, H_c) \tag{13}$$

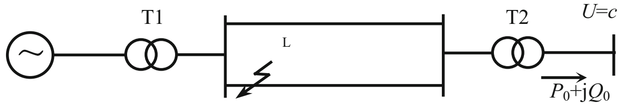

Fig. 1 Single-machine infinite-bus system

in which the initial value $H_0 = H(0)$, H_c is the critical value of the given security region $[H_{\min}, H_c)$.

The conditional probability density function (CPDF) of the FPT corresponds to the negative partial derivative of CRF

$$f(\tau | H_0) = -\left.\frac{\partial R(t | H_0)}{\partial t}\right|_{t=\tau} \tag{14}$$

According to the definition in mathematical statistics, the conditional moments of the FPT are

$$\mu_k(H_0) = \int_0^{+\infty} \tau^k f(\tau | H_0)\mathrm{d}\tau, \quad k = 1, 2, \ldots \tag{15}$$

If only the moments of FPT are concerned, a set of simpler PDEs can be obtained. These equations governing the moments are so-called generalized Pontryagin equations

$$\frac{1}{2}\sigma^2(H_0)\frac{\partial^2 \mu_k}{\partial H_0^2} + m(H_0)\frac{\partial \mu_k}{\partial H_0} = -k\mu_{k-1}, \ (k = 1, 2, \ldots) \tag{16}$$

and the boundary conditions are

$$\mu_{\mathrm{k}}(H_c) = 0 \tag{17}$$

$$\mu_k(H_{\min}) = \text{finite} < +\infty \tag{18}$$

These constitute an ellipse boundary value problem.

3 Case Study

The single-machine infinite-bus (SMIB) system is a typical example in power system analysis. Most of the power systems can be equivalent to a SMIB system in essence. Herein we take a simple model, the SMIB system with stochastic excitation, for the purpose to carry out FPT issues as described in the previous sections.

3.1 SMIB System with Random Excitation

The deterministic differential equation model for a SMIB power system described in Fig. 1 is

$$M\frac{\mathrm{d}^2\delta}{\mathrm{d}t^2} + D\frac{\mathrm{d}\delta}{\mathrm{d}t} = P_{\mathrm{m}} - P_{\mathrm{e}} \tag{19}$$

where M is the combined inertia constant. The damping coefficient D accounts for the mechanical rotational loss due to damper winding and all other forms of damping in the electromechanical system. The angle of the voltage behind transient reactance δoften indicates the rotor position of generator. The input mechanical power P_{m} is assumed to be constant, i.e.,

$$P_{\mathrm{m}} = \frac{E'U}{X_\Sigma}\sin\delta = P_{\max}\sin\delta_0 \tag{20}$$

and the electrical power P_{e} is as following

$$P_{\mathrm{e}} = \frac{E'U}{X_\Sigma}\sin\delta = P_{\max}\sin\delta \tag{21}$$

Given that the interaction between a wind-power-like renewable generation and the power grid is predominantly through its power injection, the underlying problem is how to model the effect of the random input. Usually, the stochastic excitations of the system are all treated as stochastic processes and satisfy some properties in mathematics. As proposed formulation in the literature [15,16], the wind power penetration is taken as Gaussian random process under normal operating conditions. Then, the SMIB power system with stochastic injections of wind can be modeled as

$$M\frac{\mathrm{d}^2\delta}{\mathrm{d}t^2} + D\frac{\mathrm{d}\delta}{\mathrm{d}t} = P_{\mathrm{m}} - P_{\mathrm{e}} + \sigma_0 W(t) \tag{22}$$

where σ_0 is the intensity of random excitation, $W(t)$ is the Gaussian process. The merit of Gaussian random excitation means that the response of the power systems will be a diffusion Markovian process.

Equation (22) can be transformed into

$$\begin{aligned}\frac{\mathrm{d}\delta}{\mathrm{d}t} &= \omega \\ \frac{\mathrm{d}\omega}{\mathrm{d}t} &= -\frac{D}{M}\omega + \frac{1}{M}(P_{\mathrm{m}} - P_{\mathrm{e}}) + \frac{\sigma_0}{M}W(t)\end{aligned} \tag{23}$$

where ω is the rotor speed.

Substitute (20) and (21) into (23) and let $q = \delta$, $p = \omega$, the ratio of damping coefficients to inertia constant D/M, denoted by β, then

$$\begin{aligned}\dot{q} &= p \\ \dot{p} &= -\beta p + \frac{1}{M}(P_{\mathrm{m}} - P_{\max}\sin q) + \frac{\sigma_0}{M}W(t)\end{aligned} \tag{24}$$

The Hamiltonian is

$$H = H(q, p) = \frac{p^2}{2} - \frac{P_{\mathrm{m}}}{M}q - \frac{P_{\max}}{M}\cos q \tag{25}$$

3.2 Stochastic Averaging Method

Applying the stochastic averaging method of Hamiltonian system, we can reduce two-dimensional dynamic system (24) to the one-dimensional averaged Itô SDEs of the diffusion process as in (6). It has been demonstrated that the approximation based on stochastic averaging method can lead to considerably accurate results due to Khasminskii theorem [24]. The corresponding drift and diffusion coefficients are

$$m(H) = \frac{1}{T}\int_{\Omega} \frac{-\beta p^2 + \frac{1}{2}\sigma_0^2}{p} \mathrm{d}q \tag{26}$$

and

$$\sigma^2(H) = \frac{1}{T}\int_{\Omega} \sigma_0^2 p \mathrm{d}q, \tag{27}$$

respectively, and

$$T = T(H) = \int_{\Omega} \frac{1}{p} \mathrm{d}q \tag{28}$$

The integration region is

$$\Omega = \{q \mid H(q,0) \le H\} = \left\{ q \left| -\frac{P_{\mathrm{m}}}{M} q - \frac{P_{\max}}{M} \cos q \le H \right. \right\}.$$

The associated backward Kolmogorov equation as (11), governing the CRF, can be constructed with appropriate initial value (13), and we shall take the absorbing boundary conditions, which means that the right hand systems of (12) are all zero. Generalized Pontryagin Eq. (16) which governs the conditional moments can also be derived. The MFPT is the solution of singularly perturbed boundary value problem (16)–(18).

Since it is difficult to find the solutions of these PDEs analytically, especially in high dimensions case, the FPT problem is one of the difficult branches in theory of stochastic dynamics. The analytical solutions of backward Kolmogorov equations and generalized Pontryagin equations are available only for some simpler special cases. Therefore, some numerical methods or simulation procedures have been resorted to carry out the related studies [25,26].

Fortunately, the FPT problem with Gaussian excitation considered herein has an infinite series-based approximation for the CRF. After complex routine of separation of variables in PDEs, we can get the series-based solution about the CRF

$$\begin{aligned} R(t|H_0) = {} & \frac{4}{\pi} \exp\left(-\frac{m^2(H_0)}{2\sigma^2(H_0)} t\right) \\ & \cdot \sum_{i=1}^{\infty} \frac{(-1)^{n+1}}{n} \cdot \exp\left(-\frac{n^2\pi^2\sigma^2(H_0)}{2(H_c - H_{\min})^2} t\right) \\ & \cdot \sin\left(\frac{n\pi(H_c + H_{\min})}{2(H_c - H_{\min})}\right) \\ & \cdot \sin\left(\frac{n\pi(H_0 - H_{\min})}{H_c - H_{\min}}\right) \end{aligned} \tag{29}$$

Hereafter, we shall not dwell on the technique of separation of variables for the analytical solutions of the CRF but only show the expression and compare the numerical solutions founded by MATLAB.

3.3 Collocation Method

The collocation method is one kind of spectral method. It is a numerical technique to deal with solutions of differential equations. The main idea of spectral collocation methods is interpolating the given discrete data on a discretized grid globally and then evaluating the derivative of the interpolant on the grid. It can often achieve higher accuracy and demands less computer memory than other methods like the finite difference and finite element method [26]. It has been widely spread in some engineering applications, particularly for chemical engineering.

When an analytic solution is known for a CRF of MFPT problem, any numerical scheme can be tuned to provide very high accuracy for the CRF. However, this does not mean that the same accuracy will be attained for other options using the same scheme. We first briefly discuss the collocation method and then make some changes to get the orthogonal collocation method.

Suppose the differential equation is $G(y) = 0$, and expanding the dependent variable as a series

$$y(x) = \sum_{i=1}^{N+2} a_i y_i(x) \tag{30}$$

then we can formulate the residual Res by substituting the expansion into the differential equation

$$\mathrm{Res} = \mathrm{G}\left(\sum_{i=1}^{N+2} a_i y_i(x)\right) \tag{31}$$

The residual Res will be set to zero at a set of points $\{x_j\}(j = 2, 3, \ldots, N+1)$, i.e.,

$$\mathrm{G}\left(\sum_{i=1}^{N+2} a_i y_i(x_j)\right) = 0, \quad j = 2, 3, \ldots, N+1 \tag{32}$$

These points $x_j (j = 2, 3, \ldots, N+1)$ are so-called collocation points.

There are totally $N+2$ equations for $N+2$ unknowns because Eq. (32) can generate N equations and two more equations coming from the boundary conditions. We can find the coefficients in the expansion and so the solution. This procedure is especially useful when the series expansion is based on orthogonal polynomials, and the collocation points are roots of an orthogonal polynomial.

We next apply this method to the convective diffusive equation

$$\frac{\partial u}{\partial t}=a\frac{\partial^2 u}{\partial x^2}+b\frac{\partial u}{\partial x} \tag{33}$$

The dimensionless length is 1.0 after the non-dimensional transformation. The corresponding boundary and initial values are

$$u(0,t)=u(1,t)=0 \tag{34}$$

and

$$u(x,0)=1 \tag{35}$$

In the orthogonal collocation method, we expand the solution in a series involving Legendre orthogonal polynomials. Other orthogonal polynomials can also work. The collocation points are N interior points plus one at each end, and the domain is always transformed to lie on 0 to 1. To define the coefficient matrices, we evaluate the expression and its derivatives at these collocation points.

Let

$$u(x_j,t)=\sum_{i=1}^{N+2}\lambda_i(t)x_j^{i-1} \tag{36}$$

then

$$\frac{\partial u}{\partial x}(x_j,t)=\sum_{i=1}^{N+2}\lambda_i(t)\cdot(i-1)x_j^{i-2} \tag{37}$$

and

$$\frac{\partial^2 u}{\partial x^2}(x_j,t)=\sum_{i=1}^{N+2}\lambda_i(t)\cdot(i-1)(i-2)x_j^{i-3} \tag{38}$$

Put these formulas in matrix notation,

$$u=Q\lambda,\quad \frac{\partial u}{\partial x}=C\lambda,\quad \frac{\partial^2 u}{\partial x^2}=\Psi\lambda \tag{39}$$

where Q, C, and Ψ are $(N+2)\times(N+2)$ matrices and

$$Q_{ji}=x_j^{i-1} \tag{40}$$

$$C_{ji}=(i-1)x_j^{i-2} \tag{41}$$

$$\Psi_{ji}=(i-1)(i-2)x_j^{i-3} \tag{42}$$

Solving the first equation in (39) for λ, we can rewrite the first and second derivatives as

$$\lambda=Q^{-1}u \tag{43}$$

$$\frac{\partial u}{\partial x}=CQ^{-1}u=Au \tag{44}$$

$$\frac{\partial^2 u}{\partial x^2}=\Psi Q^{-1}u=Bu \tag{45}$$

Thus, the derivative at any collocation point can be determined in terms of the solution at the collocation points. The same property is enjoyed by the finite difference method and the finite element method, and this property accounts for some of the popularity of the orthogonal collocation method. If we wish to find the solution at a point that is not a collocation point, then we use (36). Once we know the solution at all collocation points, we can find λ, and then we can find the solution for any x. In order to evaluate integrals accurately, we use the Gaussian quadrature.

3.4 Numerical Results

The system parameter values for numerical calculation are taken as σ_0= 0.8, M = 2.0, P = 1.0, c = 0.5, D = 0.25, E = 1.2 p.u., U = 1.0 p.u., G_1 = 0.5, G_2 = 0.1, B_2 = 0.6, δ_0= 0.618, $H_{\min}$ = 5 and H_c=100.

We address the numerical techniques for FPT problems in this subsection. The technique of collocation method is employed to treat the solution of PDEs include both backward Kolmogorov equation (11) and generalized Pontryagin equation (16). The numerical routine comes out MFPT = 22.4. And corresponding analytical value MFPT = 22.9 if we calculate by the formula in [3]

$$\mathrm{MFPT}=\frac{2\pi \mathrm{M}F}{\left[\gamma^2-4\mathrm{M}C_2\cos(\sin^{-1}(C_1/C_2))\right]^{\frac{1}{2}}-\gamma} \tag{46}$$

in which

$$\begin{aligned}F=\exp\{&A\left[\pi+2\sin^{-1}(C_1/C_2)\right]\\&+2(C_1/C_2)\cos\left(\sin^{-1}(C_1/C_2)\right)\}\end{aligned} \tag{47}$$

where

$$A=\left[2C_1\gamma/\left(\varepsilon G_1E^2\right)\right]^2 \tag{48}$$

and C_1 is the power transmission through the transmission line, C_2 is the maximum power transferred. These two values agree so well.

The numerical results for the CRF, CPDF and MFPT are shown in Figs. 2, 3, 4 and 5. The behavior of CRF is shown in Fig. 2, and the conditional PDF is in Fig. 3. Figures 4 and 5

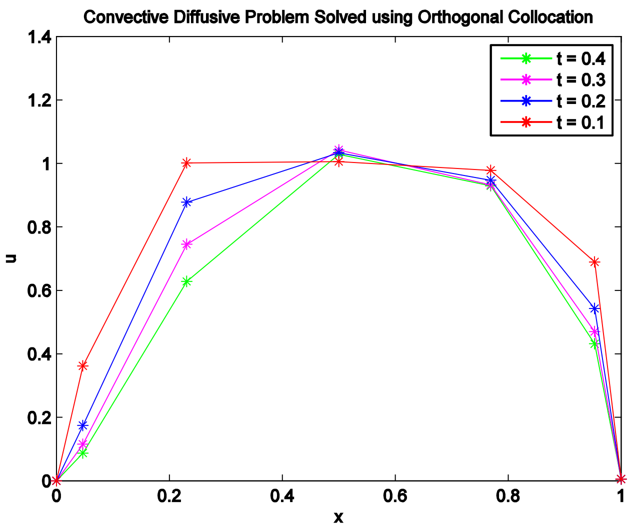

Fig. 2 Conditional reliability function with $H_0 = 10.0$

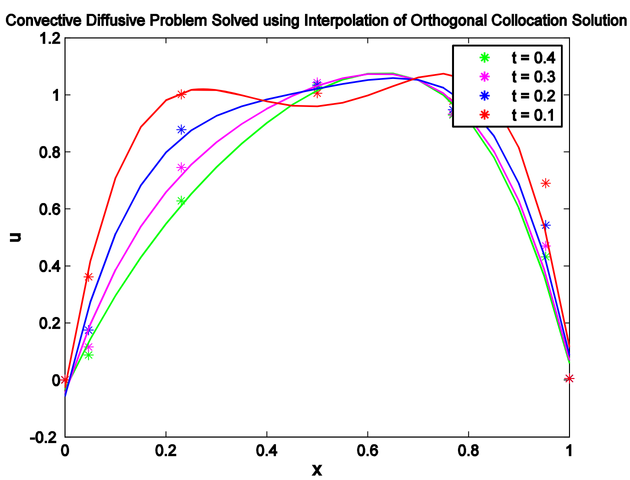

Fig. 3 Probability density function with $H_0 = 8.0$

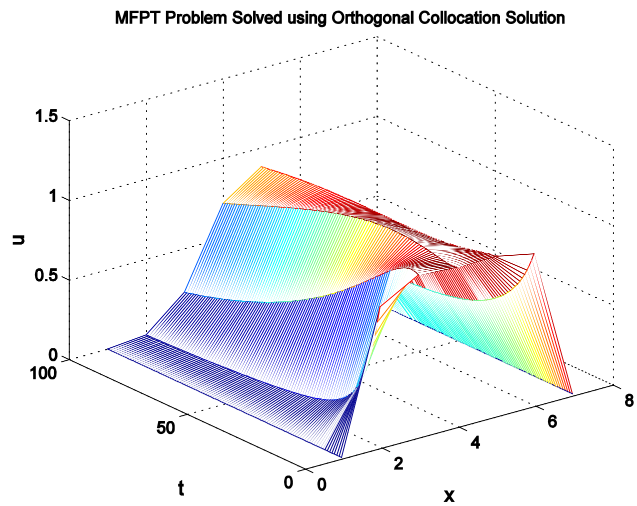

Fig. 4 Variations of MFPT with initial Hamiltonian H_0

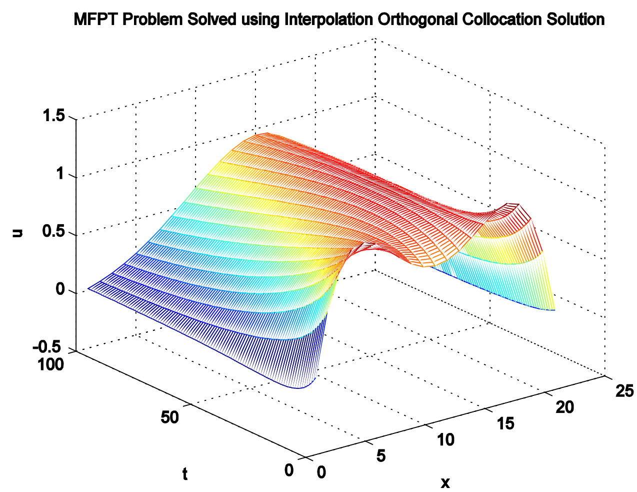

Fig. 5 Variations of MFPT with left boundary condition

provide a view on how the MFPT varies as the initial energy increases or the left boundary value goes up.

In real applications of the FPT problem, it is easier for us to deal with the CPDF than to deal with the CRF itself. Furthermore, the PDF of the FPT indeed contains sufficient information for the FPT problems. When the PDF of the FPT is obtained, one can perform stochastic reliability and stability analysis.

4 Conclusions

In order to investigate the effect of large penetration of renewables on the dynamical behavior of power systems, we have studied the FPT problems in stochastic dynamical power systems by the stochastic averaging method and numerical simulations based on collocation method. A two-dimensional SDEs-based stochastic model is employed to represent a single-machine infinite-bus system perturbed by a Gaussian random process. The results show that the CRF is an almost monotonously decreasing function of time and the MFPT varies with the initial energy function, the left boundary conditions and the strength of the stochastic excitation. For the cases investigated herein, the numerical result is well agreed with the corresponding analytical one within a few percent.

It is more realistic to study the dynamics of a multi-machine system, especially when the system components (generators, loads, transmission lines, etc.) are subjected to random noises. However, the state space model of the power systems is generally more than two-dimensional and the stochastic excitations pertaining to renewables are not usually Gaussian process. The algorithms and programs usually require large computer power and need a lot of time to give accurate estimation. The consequence of these facts says that it is difficult to use the theory of diffusion process for the

FPT problems directly. An alternative idea in this case is to reduce the system model to averaged Itô equations first by the stochastic averaging method. Then, the orthogonal collocation method and FPT problems proposed here allow us to study reliability of higher-dimensional power systems with stochastic disturbance.

At last, we should point out that some better numerical approaches need to be constructed. Both the accuracy and efficiency requirements should be taken into account. Furthermore, the FPT T should clearly indicate the sensitivity of system performance to the changes in the network configuration. We also need to correlate the FPT with the intensity of random noise (magnitude of σ).

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