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RESEARCH ARTICLE - CIVIL ENGINEERING

Experimental Study on the Effect of an Expanding Conjunction Between a Spilling Basin and the Downstream Channel on the Height After Jump

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Abstract In order to study the impact of an expanding conjunction (with a divergent conjunction or with an abruptly expanding conjunction) between a stilling basin and the downstream channel on the height after jump, systematic experimental studies were carried out with different divergent angles and different abrupt expansion ratios at the conjunction. The link between the conjunction type and the height after jump is explored. The height after jump without an expanding conjunction is larger than that with a divergent conjunction, while the height after a jump without an expanding conjunction is larger than that with the abrupt expanding conjunction. Furthermore, with a larger divergent angle or larger expansion ratio, the impact on the height after jump becomes more variable. It is proposed that the existing formula which is used to calculate the height after jump in a straight channel is not appropriate for a jump with an expanding conjunction. This paper proposes a corrected formula to calculate the height after jump with an expanding conjunction, which is suggested for application in the engineering design process.

Keywords Stilling basin · Height after jump · Expanding conjunction · Divergent angle · Abrupt expansion ratio

1 Introduction

A stilling basin is one of the most commonly used energy dissipation structures, and it dissipates excessive energy downstream of a spill way, sluices or other outlet hydraulic structures by using a hydraulic jump in the stilling basin. This has been a field of study for more than a hundred years, with systematic research in the study of the hydraulic jump in a horizontal rectangular channel developing the understanding.

Belanger [1] was the first researcher to propose the application of the momentum principle to the hydraulic jump, and the famous Belanger equation is still widely used to calculate the height after jump. Based on that, Rouse [2] introduced the dimensionless Froude number to deal with the hydraulic jump. Furthermore, the Belanger equation was amended by using experimental data of the hydraulic jump in a horizontal sluice by Sarma and Newnham [3]. Hager [4] proposed a rational prediction method for the hydraulic jump ratio in terms of the inflow Froude number the channel width ratio, the relative energy dissipation, the length of the roller area and the length of jump area. Based on the study of a hydraulic jump with a sloping apron, Wang [5] has developed a formula for the hydraulic jump under the Kindsvater assumption. Then, Hager et al. [6,7] realized that the shearing stress has nearly no impact on the conjugate depth ratio with a smooth floor, but this influence enhanced and had a nonlinear effect with $Fr_1 > 8, h_1/b < 0.7$. Hager also obtained the relationship between the length of the hydraulic jump and Fr_1. Ohtsu and Yasuda [8] deduced an experimental formula for the length of the hydraulic jump by carrying

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out experiments with various channel slopes. For the horizontal rectangular channel, Bremen and Hager [9] derived a relationship between the length of the hydraulic jump and the abrupt expansion ratio channel with the T-jump in the abruptly expanding channel. Zhang et al. [10] conducted a series of experiments with a U-shape trough and proposed a method to calculate the conjugate depth, the length of the hydraulic jump and the level of the energy dissipation after the hydraulic jump. With a low Froude number, Zhou et al. [11] indicated that most energy was dissipated after the hydraulic jump, and the author also proposed some engineering techniques. Ni and Liu [12] developed the formula which can help to calculate the ratio of the conjugate depth and the length of the hydraulic jump based on the former researchers' achievements. Gu and Lian [13] studied the fluctuating pressure distribution of the hydraulic jump with various Froude numbers, and the probability distribution characteristics and the maximum amplitude along the river flow in the hydraulic jump area were discussed. Murzyn and Chanson [14] applied a phase-detection probe to record the flow properties with Froude numbers between 3.1 and 8.5. Acoustic displacement meters and time-averaged depth measurements were used to monitor the dynamic free surfaces. This made the monitoring of the free surfaces much more convenient. Mignot and Chenfuegos [15] studied the characteristics of the energy dissipation and the turbulence in undeveloped and partially developed hydraulic jumps. Rao and Zhang [16] proposed an approximate method to estimate the conjugate water in the slope following outlet structures. Zhang et al. [17] induced a jump parameter G for adverse slopes based on investigations of hydraulic jump and the adverse slope, and the relationship between G and the upstream Froude number. Ma et al. [18] deduced an hydraulic jump equation for an unpressured pipe based on the study of different types and sizes of cross sections and the hydraulic jump equation for horizontal rectangular channels. Based on the assumption of a linear variation of the water surface and the trapezoidal pressure distribution on the side wall of the hydraulic jump area, in an expanding channel, Ning et al. [19] deduced an equation for the hydraulic jump by using the momentum principle. Guo et al. [20] proposed an iterative method to calculate the hydraulic jump equation for a circular cross section. Zhang and Zhao [21] carried out a series of experiments on the hydraulic jump, velocity distribution, wall shear stress in a rectangular channel and local head loss. The total head losses along the flow direction were measured, and the relationship between the head loss and the Froude number was discussed.

From an economic point of view, generally, the downstream channel should be designed wider than the width of the stilling basin. So the conjunction section between the stilling basin and the downstream channel is usually designed to be divergent or abruptly expanding. In this paper, for these two conjunction types, a series of experiments were carried out and the impact of the expanding channel on the height after jump is discussed. Furthermore, the method of calculation of the height after jump is proposed which can be included in the engineering design process.

2 Experimental Method

2.1 Experimental Arrangement

Experiments were carried out in the hydraulic laboratory in Shandong Agricultural University, and the whole system is composed of a pump, a head water pond, water supply pipelines, a measuring weir, a flow steadying grid, channels, a model test area, a tail water pond, a backwater channel and an underground reservoir, as shown in Fig. 1.

2.2 Experimental Model

The experimental model includes an upstream channel, a stilling basin and a downstream channel. The profile of the downstream channel is horizontal and rectangular,

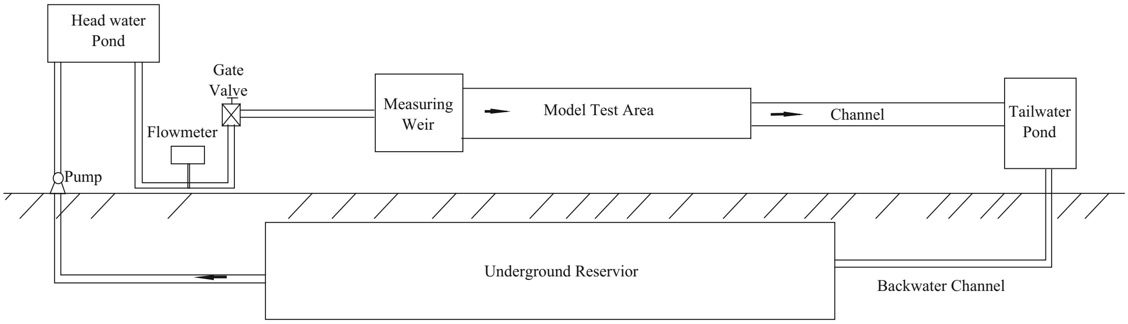

Fig. 1 Experimental system diagram

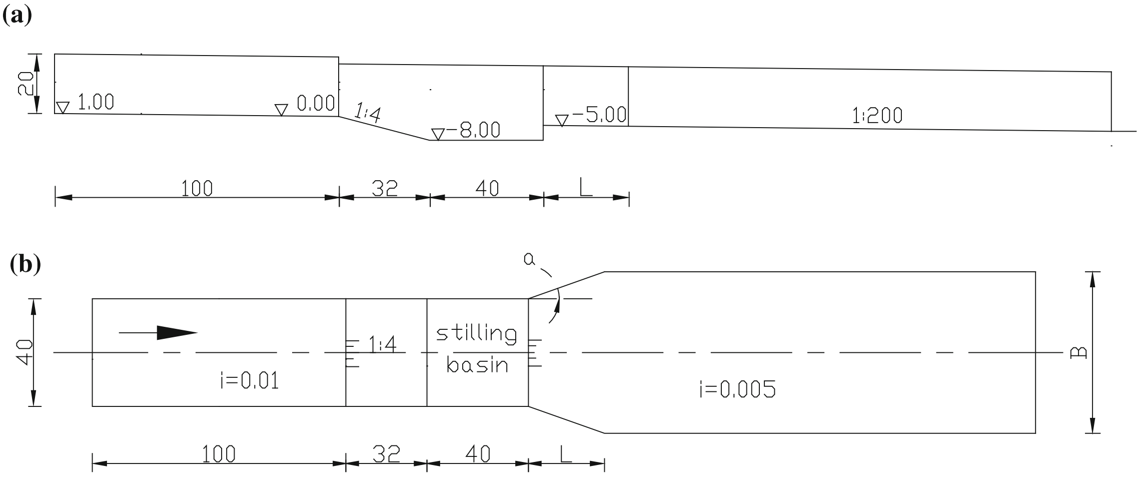

Fig. 2 Design diagram of experimental model. **a** Side view, **b** top view

Table 1 Experimental schemas and variables

Scheme no.	Conjunction type	Divergent angle α (°)	Abrupt expansion ratio β[a]	Length of expanding zone L (cm)	Width of downstream channel B (cm)
1	Without expanding	–	–	–	40
2	Divergent	18.43	–	10	60
3		26.57	–	20	60
4		45	–	30	60
5	Abruptly expanding	90	1.2	–	48
6			1.4	–	56
7			1.5	–	60

[a] Abrupt expansion ratio β is the ratio of the width of the downstream channel and the width of the stilling basin

while the significant measurements include: the width of the upstream channel (40 cm in this paper), the slope geometry in the stilling basin (32 cm in length with the slope ratio 0.25 in this paper), the length and width of the stilling basin (both are 40 cm in this paper), and the depth of the stilling basin (3 cm in this paper). The design diagram of the experimental model is shown in Fig. 2.

In Fig. 2, B is the width of the downstream channel, L is the length of the expanding zone, and α is the divergent angle.

2.3 Experimental Schemas

Seven schemas were used, and experiments were divided into three groups based on various sizes of the conjunction section between the stilling basin and the downstream channel (without an expanding conjunction, with a divergent conjunction and with an abruptly expanding conjunction). Schemas are shown in Table 1.

2.4 Flow Conditions and Experimental Measurements

2.4.1 Flow Conditions

Based on the design standard for the stilling basin, the submerged jump should happen in the stilling basin. In this paper, with the same flow conditions in both the upstream and downstream channel, the submerged jump was in the stilling basin which was controlling the flow discharge.

2.4.2 Experimental Measurements

The objective of these experiments is to study the impact of different expanding conjunction sizes on the height after jump in the stilling basin. So the experimental measurements included the flow discharge, the water depth and the velocity along the channel. The observation of the flow characteristics in the stilling basin and in the downstream channel was also important for this study. In detail, the water depth upstream of the stilling basin and the height after the jump (including the depth in the stilling basin) were measured. Each measuring section had three measuring lines (center, left bank, right bank), and the average value represented the average water depth of each section.

3 Results and Analysis

3.1 Equation of the Hydraulic Jump in the Horizontal Stilling Basin

Figure 3 illustrates the behavior of a hydraulic jump in the stilling basin. Section 1–1 is the section before the hydraulic jump. The water depth of this section is called the height before jump (h_1). For the horizontal stilling basin, the depth increases before the hydraulic jump due to fluid friction slowing the water. The section 2–2 is the section after the hydraulic jump. The water depth of this section is called height after jump (h_2).

The momentum equation for section 1–1 and 2–2 can be expressed as

$$\frac{1}{2}\gamma h_1^2 - \frac{1}{2}\gamma h_2^2 - F = \gamma\alpha q\frac{(v_2 - v_1)}{g} \tag{1}$$

where h_1 is the height before jump; h_2 is the height after jump; $q(q = Q/B)$ is the unit flow discharge; Q is the total flow discharge; B is the width of the stilling basin; $v_1(v_1 = q/h_1)$ is the velocity before jump (section 1–1); $v_2(v_2 = q/h_2)$ is the velocity after jump (section 2–2); F is the friction of the stilling basin; α is the momentum correction factor; γ is the density of water.

Belanger [1] excluded the factor of friction in the stilling basin and deduced the classical hydraulic jump equation

$$\frac{h_2}{h_1} = \frac{1}{2}\left(\sqrt{1 + 8\alpha Fr_1^2} - 1\right) \tag{2}$$

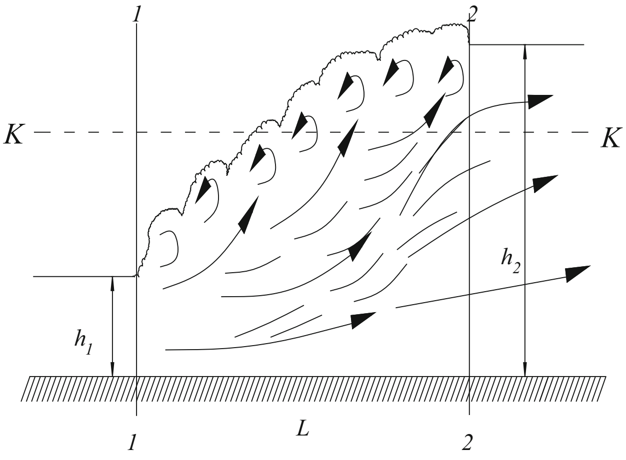

Fig. 3 Illustration of behavior in a hydraulic jump

where $Fr_1(Fr_1 = q/\sqrt{gh_1^3})$ is the Froude number before hydraulic jump. h_1 and h_2 are called conjugate depths. It is obvious that with the known value of h_1 or h_2, the other one can be calculated by Eq. (2).

In engineering practice, the depth of the stilling basin is unknown and should be designed for the known height after the jump. So, with the exact value of the height before jump, the height after jump can be calculated and then the depth of the stilling basin can be determined. Besides, under submerged flow conditions, the height before jump is not easy to measure and the water depth of the vena contraction section should be used instead.

3.2 Calculation of the Water Depth of the Vena Contraction Section

Figure 4 presents the water depth of the vena contraction section in a horizontal stilling basin.

The water depth shows a minimum on section 2–2, which is called the vena contraction section, and the water depth of this section is called h_c. Making hydraulic assumptions the vena contraction section, and h_c can be referred to as h_1. Using energy equation on section 1–1 and section 2–2 in Fig. 4, the water depth on vena contraction section can be expressed as (Hydraulics [22])

$$T_0 = h_c + \frac{q^2}{2g\varphi^2 h_c^2} \tag{3}$$

where $T_0 \left(T_0 = p + h_0 + \frac{\alpha v_0^2}{2g}\right)$ is the water head on section 1–1; p is the height difference between upstream channel and the bottom of the stilling basin; h_0 and v_0 are the water depth and velocity on section 1–1; h_c is the water depth on the vena contraction section; φ is a parameter of velocity, and it has a relationship with the type of inlet, the friction of the dam surface, the height of the dam and the water head. The recommended range of φ is from 0.8 to 0.9 based on the book of Hydraulics [22].

3.3 Experimental Results

Heights after the jump in different schemas were measured under the submerged hydraulic jump condition described in this paper. The heights after jump with various conjunction sizes are shown in Table 2.

3.4 Analysis

The plotted heights after jump versus flow discharges based on the various schemas and other results are shown in Figs. 5 and 6.

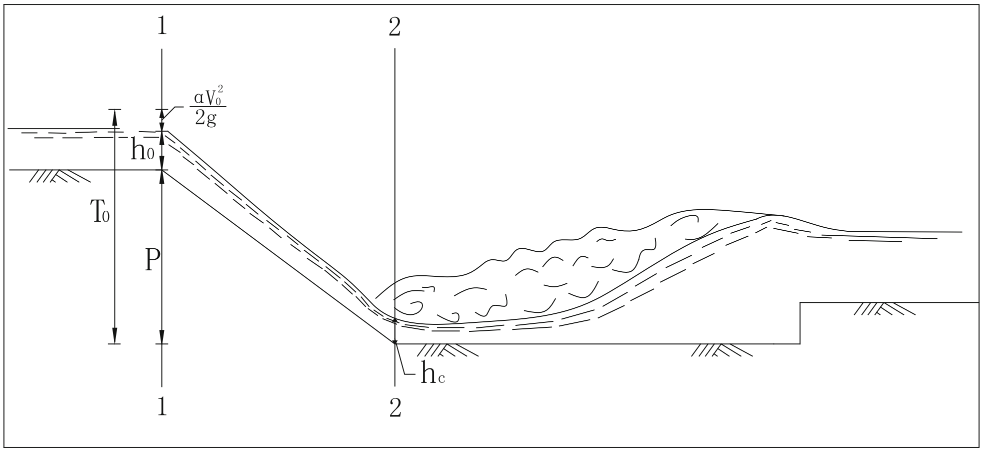

Fig. 4 Illustration of water depth of the vena contraction section in a horizontal stilling basin (submerged jump)

Table 2 The calculated and measured heights after jump in various schemas

Flow discharge (m^3/h)	Without expanding conjunction schema (mm)	Abruptly expanding conjunction schemas (mm)			Divergent conjunction schemas (mm)				Calculated value (mm)
		1.2	1.4	1.5	90°	45°	26.57°	18.43°	
50	111.7	106.3	105.4	103.9	103.9	107.2	106.3	109.1	93.1
60	120.0	114.1	111.3	112.5	112.5	113.6	114.8	116.3	101.7
70	126.3	118.6	122.1	118.0	118.0	119.9	120.6	121.1	110.4
80	136.8	122.9	122.5	121.8	121.8	123.2	122.5	124.8	120.3
90	138.4	129.3	126.0	125.0	125.0	127.0	127.9	127.7	125.3
100	142.5	132.5	131.3	128.1	128.1	130.6	131.1	132.8	135.5
110	146.8	137.5	131.8	131.0	131.0	131.3	134.3	140.2	140.0
120	158.5	150.0						141.0	148.8

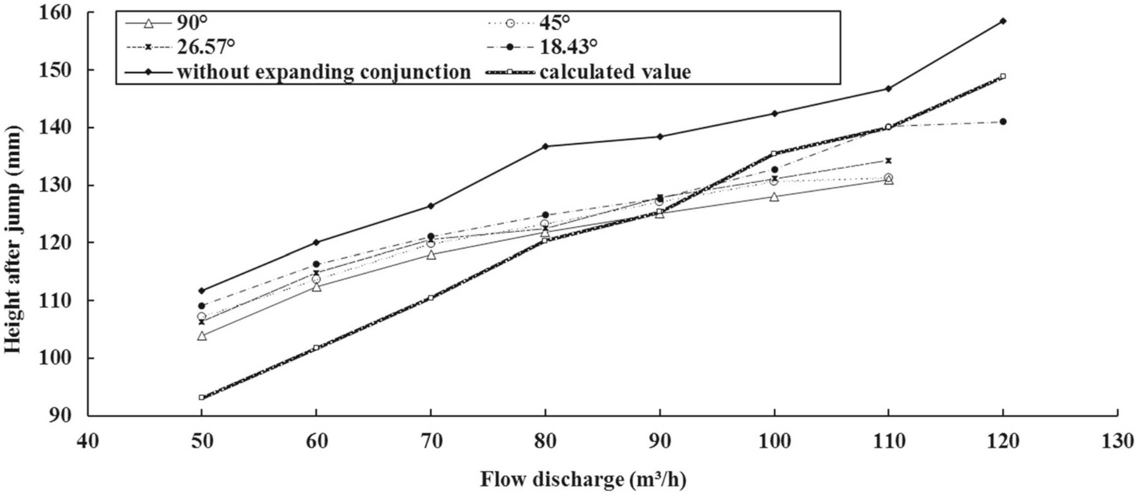

Fig. 5 Relationship between height after jump and flow discharge (with various divergent angles)

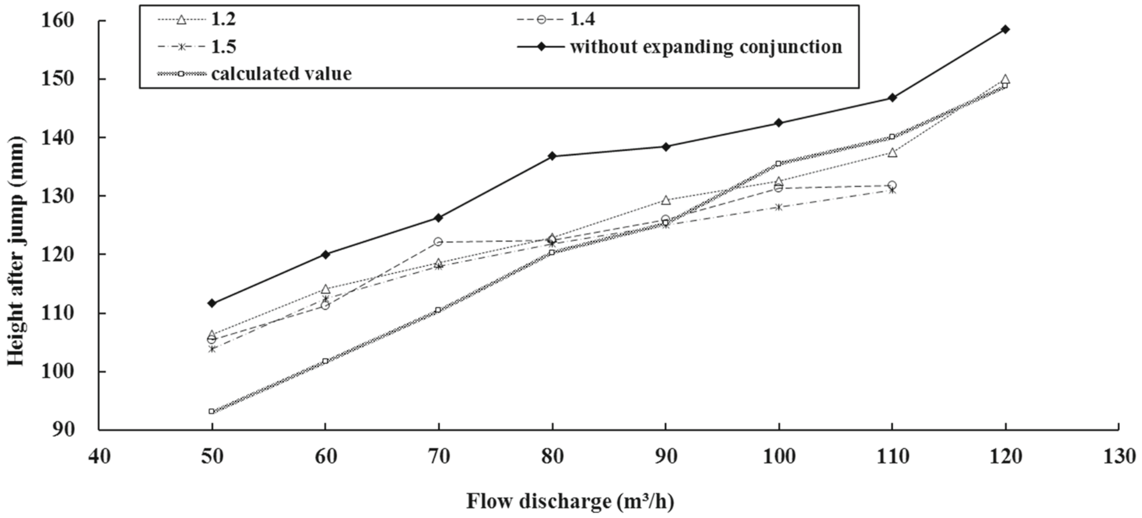

Fig. 6 Relationship between height after jump and flow discharge (with various abrupt expansion ratios)

3.4.1 The Impact of the Divergent Angle

It is shown in Fig. 5 that the line representing the height after the jump without expanding the conjunction is above other lines. That means the height after the jump is smaller with a divergent conjunction than that without an expanding conjunction. Furthermore, under the same flow discharge, the decreases of the height after the jump as the divergent angle increases can be easily seen. The reason for this phenomenon can be explained as follows: when water moves forward at the end of the stilling basin, comparing the channel without an expanding conjunction, the velocity will decrease at the divergent conjunction, and this will cause an extra loss of energy. Meanwhile, the velocity distribution at the section 2–2 (in Fig. 3) is changed while the momentum correction factor in the momentum equation has changed as well. All these cause the height after the jump to decrease.

3.4.2 The Impact of the Abrupt Expansion Ratio

Comparing different lines in Fig. 6, it is obvious that the line representing the height after jump without an expanding conjunction is above other lines, which means the height after the jump without an expanding conjunction is bigger than that with an abruptly expanding conjunction. Furthermore, excluding one scheme (with flow discharge 70 m^3/h) while under the same flow discharge conditions, the decrease in the height after jump as the abrupt expansion ratio increases can be easily seen. It is proposed that the reason for this phenomenon is that when water moves forward at the end of the stilling basin, compared to the channel without expanding conjunction, the velocity will decrease at the abruptly expanding conjunction, and this will cause an extra loss of energy. Observation also showed that the flow regime in the abruptly expanding conjunction was variable approaching an irregular vortex. At the same time, the velocity distribution at the section 2–2 (in Fig. 3) is changed while the momentum correction factor in the momentum equation also changed. So the height after the jump is different for different abruptly expanding ratios.

In total, the types of conjunction between the stilling basin and the downstream channel affect the height after the jump, and this impact should not be neglected. In particular, with a larger divergent angle or with a larger abrupt expansion ratio the influence is more obvious.

3.4.3 The Decreasing Ratio of the Height After Jump

Using a decreasing ratio to describe the influence of expanding the conjunction, it can be expressed as

$$r = \frac{h_2^n - h_2^k}{h_2^n} \times 100 \tag{4}$$

where r is the decreasing ratio (%); h_2^n is the height after jump without an expanding conjunction (m); h_2^k is the height after jump with an expanding conjunction (m). The decreasing ratio in the schemas is shown in Figs. 7 and 8.

It is shown in Fig. 7 that the decreasing ratio of the height after jump in the stilling basin was different with different divergent angles: the decreasing ratio in the range of 2.3–11.04% for 18.43°, while the average decreasing ratio is 6.42%; for a 26.57° divergent angle, the range of the decreasing rate is 4.33–10.48%, and the average percentage is 6.89%; for a 45° divergent angle, the decreasing ratio is between 4.0 and 10.54%, while the average ratio is 7.36%. With the larger divergent angle, the decreasing ratio is larger as well (except for when the divergent angle = 26.57° with the flow discharge = 50 or 80 m^3/h).

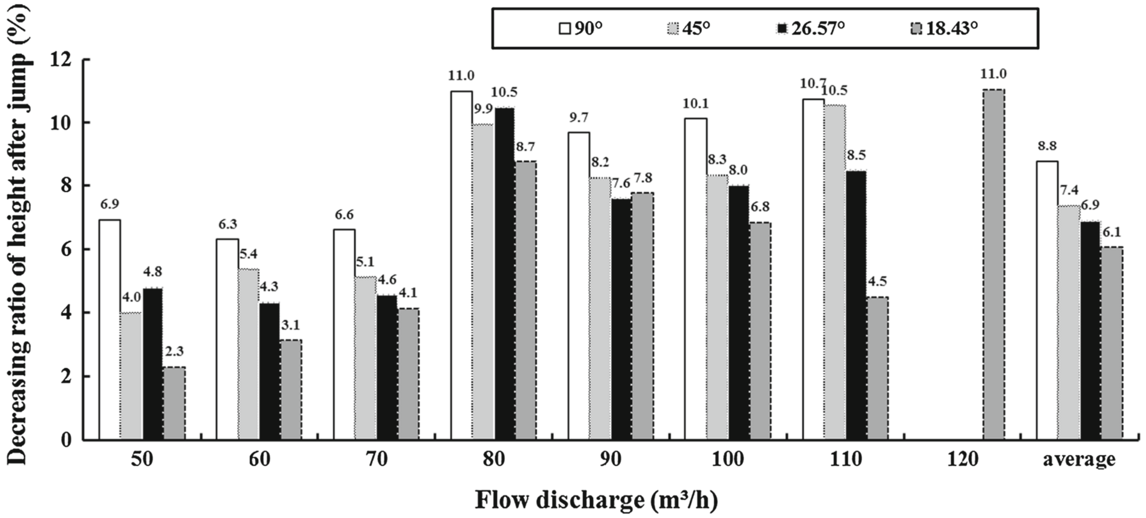

Fig. 7 Histogram of the decreasing ratio of the height after the jump in various flow discharges (with various divergent angles)

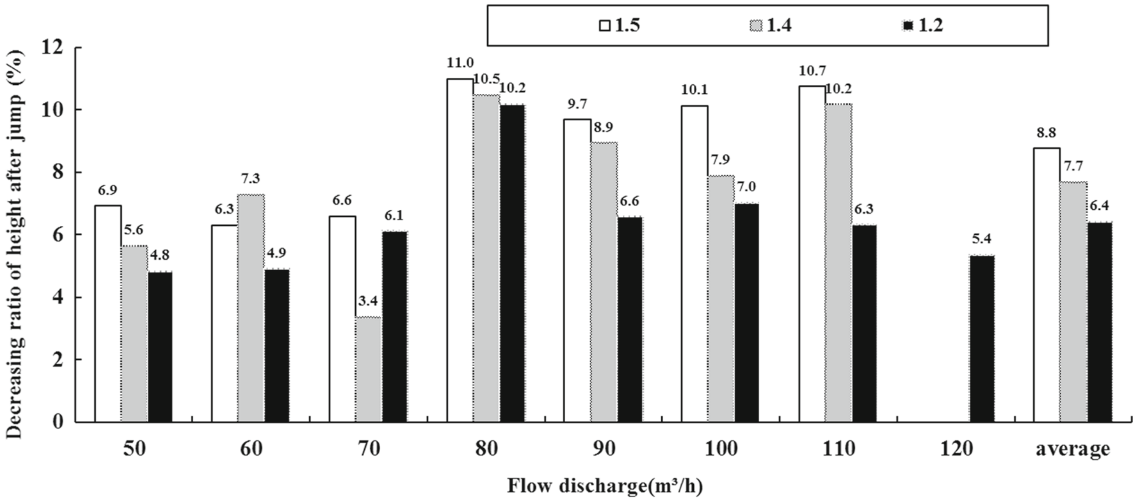

Fig. 8 Histogram of the decreasing ratio of the height after the jump in various flow discharges (with various abrupt expansion ratios)

Similarly, in Fig. 8, under the same flow discharge, with the larger abrupt expansion ratio, the decreasing ratio of the height after jump becomes larger (except for the abrupt expansion ratio = 1.4 with the flow discharge = 60 m^3/h, 70 m^3/h). The decreasing ratio is changing in the range of 4.84–10.19% when the abrupt expansion ratio is 1.2, while the average decreasing ratio is 6.42%; for a 1.4 abrupt expansion ratio, the ratio ranges between 5.64 and 10.48%, and the average is 7.68%; for a 1.5 abrupt expansion ratio, the ratio changes from 6.30 to 10.99%, and the average is 8.77%.

3.5 Calculation of the Height After Jump

3.5.1 Verification of the Existing Formula

Using Eq. (2) to calculate the height after jump and to compare with the observed one, the error histogram is drawn as shown in Figs. 9 and 10.

Based on Figs. 9 and 10, when the flow discharge is below 90 m^3/h, the calculated value is smaller than the observed one. With smaller discharges, the error gets bigger, especially when flow discharge is below 60 m^3/h, when the error is nearly 10%. For flow discharges larger than 90 m^3/h, the calculated value is larger than observed value. With smaller discharges, the error rises more rapidly.

3.5.2 The Correction of Formula with an Expanding Conjunction

Based on all the data from this research, Eq. (2) has an error that cannot be neglected when it is used to calculate the height after jump with an expanding conjunction. It is necessary to make a correction to the existing formula to get better agreement with experimental data. This is especially relevant

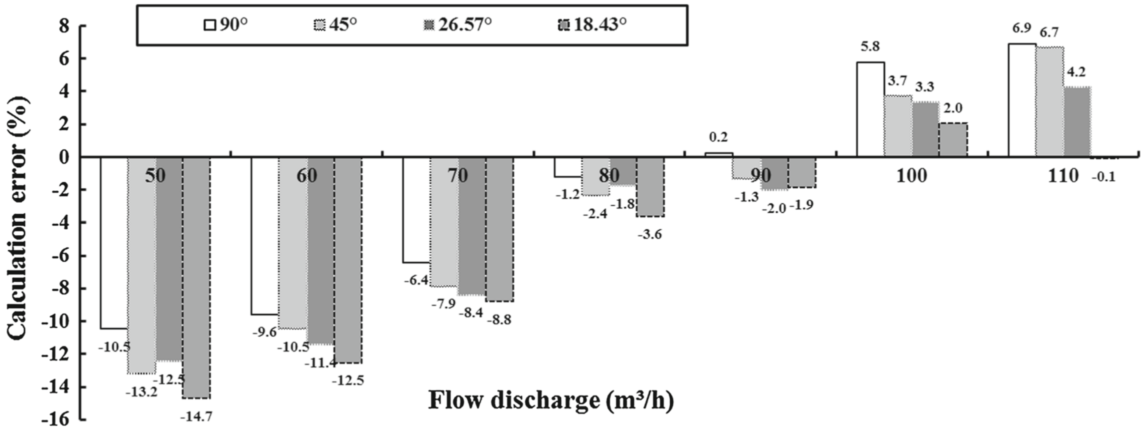

Fig. 9 Calculated error histogram of the height after the jump in the various flow discharges (with various divergent angles)

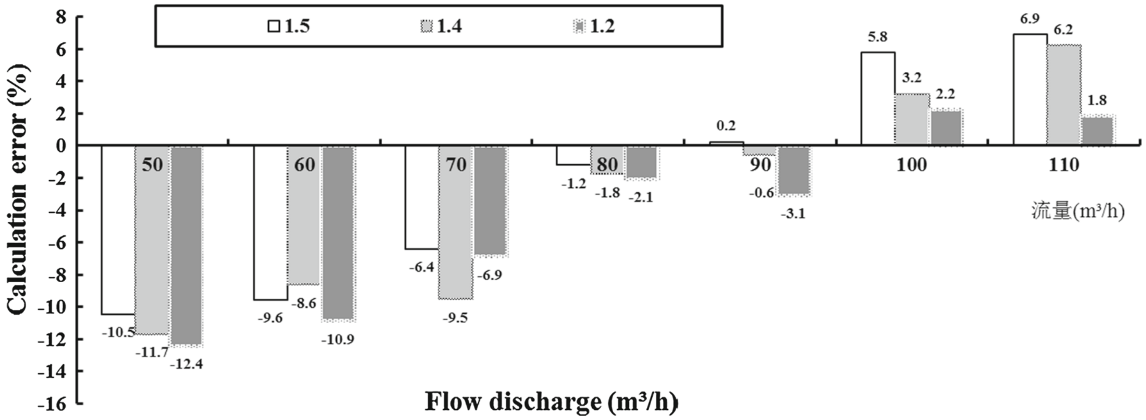

Fig. 10 Calculated error histogram of the height after the jump in the various flow discharges (with various abrupt expansion ratios)

in the circumstances considered in this paper which are quite often seen in engineering practice.

The experiments which were carried out in this research showed that the height after the jump has a relationship with the divergent angle, the abrupt expansion ratio and the Froude number in the upstream section. So the relationship between h_{2S}/h_{2J} and X, for divergent conjunction, $X = Fr_1 \times \alpha$ (α is the divergent angle in radians), and for an abruptly expanding conjunction, $X = Fr_1/\beta$ (β here is the abrupt expansion ratio). The relationships are shown in Figs. 11 and 12.

It is shown in Figs. 11 and 12 that the relative coefficient is larger than 0.89 and bigger than $r_5^{0.01} = 0.87$. So the relationship between h_{2S}/h_{2J} and X is obvious. The experimental formula including the expanding conjunction parameter

$$h_{2k} = aFr_1^b h_2 \tag{5}$$

where h_{2k} is the height after jump, h_2 is the calculated value based on the normal method [Eq. (2)] without the expanding conjunction, and Fr_1 is Froude number of the upstream section. Here, a is the coefficient and b is the index in the power function. The value of these two should be derived from Table 3. Data in Table 3 are obtained from fitting the curves in Figs. 11 and 12. The range of Eq. (5) within which these coefficients are applicable is: $0° < \alpha \leq 45°$ (with divergent conjunction), $1 < \beta \leq 1.5$ (with an abruptly expanding conjunction) in a rectangular channel.

The error between the calculated height after jump using Eq. (5) and the observed value is written in Table 4.

It can be seen from Table 4 that the error can be controlled below 5%. That means Eq. (5) has a high precision and can be used to predict the height after jump with an abruptly expanding conjunction or with a divergent conjunction.

4 Conclusions and Suggestions

On the basis of an extensive experimental investigation, four conclusions are proposed in this paper. (1) The expanding type of conjunction between the end of the stilling basin and the downstream channel which have a strong impact on the height after jump, the height is smaller with an expanding conjunction than without an expanding conjunction. If it is possible to expand the conjunction, this can cause the height

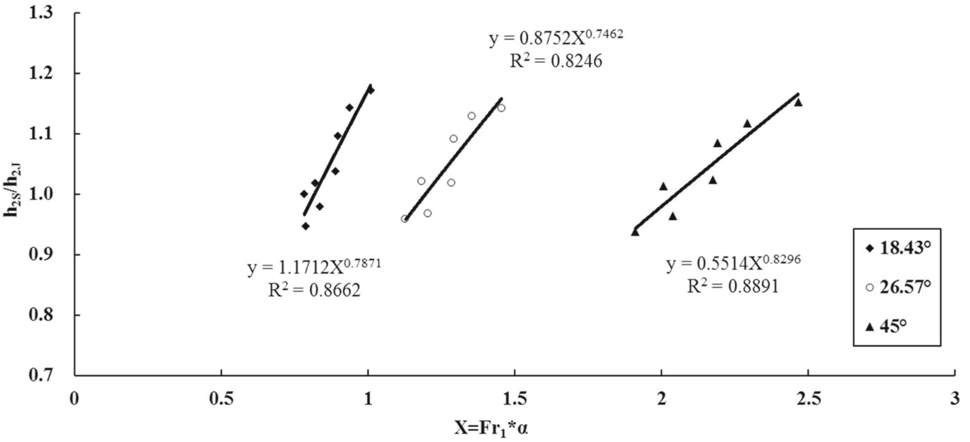

Fig. 11 The relationship between $h_{2S}/h_{2J} \sim X$ with divergent conjunction

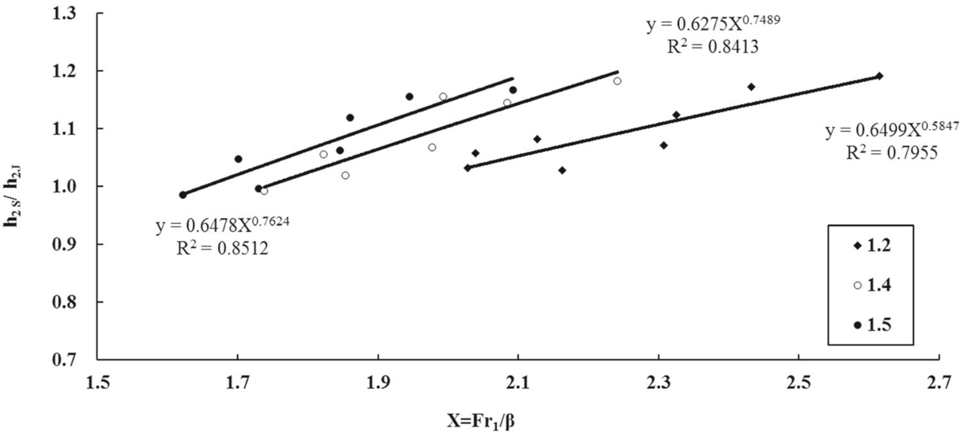

Fig. 12 The relationship between $h_{2S}/h_{2J} \sim X$ with an abrupt expanding conjunction

Table 3 Values of *a* and *b*

Expanding status		*a*	*b*
Divergent conjunction	18.430	0.4796	0.7871
	26.570	0.4933	0.7462
	450	0.4513	0.8296
Abruptly expanding conjunction	1.2	0.5841	0.5847
	1.4	0.4877	0.7489
	1.5	0.4755	0.7624

Table 4 The error between the calculated height after jump and the observed value (%)

Flow discharge (m^3/h)	Abrupt expansion ratios			Divergent angles		
	1.2	1.4	1.5	18.43°	26.57°	45°
50	−0.2	1.4	1.8	0.6	1.4	1.2
60	−2.6	−0.6	−2.7	−2.5	−2.8	−1.7
70	−0.9	−4.8	−2.7	−1.9	−2.8	−2.6
80	3.7	2.7	2.1	3.0	3.6	2.6
90	−2.1	−2.2	−2.6	−1.6	−2.7	−3.1
100	4.3	2.8	4.1	3.7	3.9	3.3
110	0.1	0.8	0.1	−3.5	−0.1	0.7
120	−2.2			2.3		

after jump to decrease. (2) For the condition of a divergent conjunction, the larger the divergent angle, the shorter the height after jump, and this impact obvious especially with a large divergent angle. The average decreasing ratios are 6.06, 6.89, 7.36, and 8.77% corresponding to 18.43°, 26.57°, 45°, and 90°. (3) For an abruptly expanding conjunction, the larger the abrupt expansion ratio results in a shorter height after jump, and this impact can be more violent especially for large ratios. The average decreasing ratios are 6.57, 7.68, and 8.77% corresponding to the abrupt expanding ratios of 1.2, 1.4, and 1.5. (4) The existing formula which was used to calculate the height after jump was not suitable for the condition with an expanding conjunction and a corrected formula was established [Eq. (5)].

Because of the limitation of the experiments reported in this paper, Eq. (5) can only be used under the conditions that the stilling basin and the downstream channel sections are rectangular. More experiments of various widths of the conjunction, different divergent angles and more types of conjunction will be the subject of further research.

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