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RESEARCH ARTICLE - CIVIL ENGINEERING

Modelling the Pull-out Capacity of Ground Anchors Using Multi-objective Feature Selection

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Abstract Pull-out capacity of ground anchors is analogous to axial capacity of piles as both of them apply same methods. These available methods are mostly empirical; therefore, in this paper efficient prediction models for determining the uplift capacity of small ground anchors have been presented using recently developed artificial intelligence (AI) techniques. Multi-objective feature selection (MOFS) has been utilised to find the subset of influential parameters responsible for the pull-out capacity of ground anchors along with the development of prediction equations. MOFS has been applied with artificial neural network and non-dominated sorting genetic algorithm. Prediction models are also presented using two other AI techniques: functional network and multi-variate adaptive regression spline. AI models were compared in terms of different statistical parameters such as mean absolute error, root-mean-square error, correlation coefficient and ranking criterion approach have been implemented to assess the performance of different prediction models.

Keywords Ground anchors · Pull-out capacity · ANN · MARS · FN · NSGA II · Multi-objective feature selection

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1 Introduction

Short-term light constructions and marquees are usually made stable by the help of small ground anchors. Ground anchors are basically tensile resisting structures, which transfer the tension forces built on the superstructures to the ground due to the shear resistance of the soil. They are made of steel of different shapes and diameters with length around 1 m. The various parameters on which the pull-out capacity of ground anchors depend are: diameter of anchors, adhesion between the surrounding soil and anchors, physical properties of the soil, length of the anchors up to which it is embedded, installation methods, etc. There are several conventional methods available to design the ground anchors. Out of these available methods, three methods which are in recent use are Laboratoire Central des Pontset Chaussées (LCPC) [1], Das [2] and Bowels [3]. All the available prediction methods are purely based on the axial compression of piles. Axial piles are based on compressive loads, whereas ground anchors are based on tension. As the actual approaches to determine the pull-out capacity of ground anchors are underdeveloped, therefore proper assumptions have to be made during the design process. One such assumption is simulating the behaviour of ground anchors with that of micro-piles taking into consideration the scaling effect of dimensions. But still these methods are empirical and inadequate.

In general, geotechnical modelling is difficult due to the heterogeneous nature of soil. Moreover, apart from the empirical equations as stated above there are no available methods, which can be used to directly determine the pull-out capacity of marquee ground anchors. For overcoming the difficulties as discussed above, artificial intelligence (AI) techniques came to the discussion. Out of several AI methods, artificial neural network (ANN) is the most widely used technique to be ever used in the field of geotechnical engineering. ANN has been applied to a variety of problems in civil engineering [4–9]. Apart from ANN, other AI techniques, which are widely utilised, are genetic programming (GP), multi-variate adaptive regression splines (MARS), functional networks (FN), support vector machines (SVM) etc.

Research articles related to the pull-out capacity of ground anchors are limited [10]. Shahin and Jaksa [11] conducted a sequence of 119 in situ anchor pull-out tests and made a comparison with conventional methods. The in situ tests were conducted on six different types of soil namely, alluvial silt and sand; clay with some gravel; medium-grained sand; fine-grained sand; highly plastic black clay; and red, brown clay, dry, hard (Adelaide, South Australia). The anchors were of circular, hexagonal and star dropper shaped and varied in embedment length (400–800mm). From the comparison study, they found that the predictive accuracy of conventional methods in predicting the uplift capacity of anchors was inconsistent. Based on the same data set, AI models [12,13] are far better in estimating the pull-out capacity of anchors in comparison with traditional techniques. Shahin and Jaksa [13] also presented a model equation for hand calculation and found that the average cone tip resistance of the ground anchor had almost negligible effect on the pull-out capacity. Later on Samui et al. [14] and Shahin [15] developed least square support vector machine (LSSVM) and evolutionary polynomial regression (EPR) prediction models, respectively, for estimating the pull-out capacity of ground anchors using the same database.

Often in data-driven modelling, separation of the data set (subsets of training and testing data samples) is important [16]. In machine learning, major portion of the data are used for training and a smaller portion for testing by random sampling of data to ensure that the testing and training sets are alike for minimising the effects of data discrepancies and to better understand the characteristics of the model. So, the model is trained for several times to reduce the effect of overfitting. After a model is developed by using the training set, the model is tested by making predictions against the test set and through various ways the performance of the model is evaluated. Performance indicators such as coefficient of correlation (R), root-mean-square error (RMSE) and mean absolute error (MAE) show the average variation in the model.

Due to the complex nature of soil, it has been found that selection of influential parameters for data-driven modelling is important as a result, the performance of the model varies, so identification of the controlling factors is a must. Similarly, it is imprudent to include all the features of a model as it unnecessarily increases the complexity of the model giving a very little benefit in terms of the predictive capability of the developed model [16]. Also, many different combinations of features may give similar predictions. Thus, researchers are on constant lookout for reliable predictive models, which are not only less complex but also high in its predictive capability. One such algorithm, feature selection (FS) algorithm not only minimises the number of features but also maximises the predictive accuracy (minimisation of error) of the model. But the above-described objectives are mutually conflicting in nature; a decrease in number of features also decreases the prediction accuracy. Therefore, multi-objective evolutionary algorithms (MOEA) can be implemented which can simultaneously minimise all the objective functions. MOEA produces a Pareto front for the multi-objective minimisation problem from which one can find a trade-off solution between conflicting objectives. The Pareto front is defined as the set of non-dominated solutions, where each objective is considered as equally good. MOEA and ANN in conjunction with evolutionary algorithm have been applied in several fields with great degree of success [17,18].

Feature selection (FS) algorithm is of three types: wrapper, filter and embedded. In wrapper technique, a predictive model is used to evaluate each feature subset. Each new subset is used to train a model and tested and then ranked based on their accuracy rate or error rate. In filter technique, a proxy measure is used which is fast to compute. Some of the measures used in filter technique are mutual information [19], pointwise mutual information [20], Pearson product-moment correlation coefficient, inter-/intra-class distance or the scores of significance tests for each class/feature combinations [20,21]. Filter selects a feature set which is not tuned to a particular type of model, thus resulting in to be more general as compared to wrapper technique. Embedded technique uses a catch-all group method performing feature selection as a part of the modelling process. LASSO algorithm [22,23] is one such technique where during linear modelling the regression coefficients are penalised with an $L1$ penalty, shrinking many of them to zero. Embedded technique is in between filters and wrappers in terms of computational complexity. FS has also been used in text classification [24]. Implementation of evolutionary algorithms for FS has been made using differential evolution (DE) [25], genetic algorithms (GA) [26], genetic programming (GP) [27] and particle swarm optimisation (PSO) [28–30].

In the present study, a novel type of algorithm known as multi-objective feature selection (MOFS) is proposed to solve the above-described pitfalls. Using MOFS, not only the model is much more generalised but also it reduces the computation time along with better convergence. In this proposed MOFS (wrapper-type approach) algorithm, artificial neural network (ANN) is combined with non-dominated sorting genetic algorithm (NSAG II), where ANN acts as the learning algorithm and NSGA II performs the feature subset selection and minimises the errors for the developed prediction model simultaneously. By using three objectives for minimisation (a subset of features, training error and testing error), a variant of MOEA (modified non-dominated sorting genetic algorithm or NSGA II) is applied to investigate if a subset of features exists with cent per cent correct predictions for both training and testing data sets. The features fed to the MOFS are represented in binary form where 1 indicates selection of the feature and 0 indicates its non-selection. The performance of the proposed model is evaluated in terms of mean square error in which NSGA II minimises during the multi-objective optimisation process.

Along with the implementation of MOFS algorithm, two recent AI methods, functional network (FN) and multivariate adaptive regression spline (MARS) have also been implemented to find the pull-out capacity of ground anchors. Once the identification of the controlling features responsible for the pull-out capacity of ground anchors has been done via MOFS algorithm, FN and MARS have been implemented on the same subset of influential features to develop prediction models for the ground anchor for comparison.

2 Methodology

2.1 Multi-objective Feature Selection (MOFS)

2.1.1 Non-dominated Sorting Genetic Algorithm (NSGA II)

NSGA II [31] is an elitist non-dominated sorting genetic algorithm and is very popular in the application of multi-objective optimisation. Not only does it adopt an elite preservation strategy but also uses explicit diversity preservation technique. In this first, the parent population is initialised, from which the offspring population is created, and then, both the population are combined and finally classified based on non-dominated sorting. After the completion of non-dominated sorting, filling of new population starts with the best non-dominated front with assignment of rank as 1 and this continues for successive fronts and assignment of ranks simultaneously. Along with the non-dominated sorting, another niching strategy adopted is the crowding distance sorting in which the distance reflects the closeness of a solution to its neighbours, greater the distance better is the diversity of the Pareto front. Offspring population are created from parent population by using crowded tournament selection, crossover and mutation operators and this whole operation continues until a termination criterion is met. More details of the algorithm can be found in Deb et al. [31].

2.1.2 NSGA II with ANN for Feature Selection

In this study to solve the feature selection problem, wrapper-type approach is implemented where binary chromosomes are used to represent the features with a value of 0 and 1, 0 indicating that the required feature is not selected and 1 indicating that the required feature is selected. Three objectives are defined in the NSGA II algorithm, first being the minimisation of the number of selected features, second being the minimisation of training error rate and third being the minimisation of testing error rate in the learning algorithm. The training error and testing error are calculated based on mean square error. Learning algorithm used is feed-forward artificial neural networks (ANNs). Basic flow chart of the MOFS algorithm is presented in Fig. 1. Population size, number of generations, crossover probability and mutation probability are the parameters to be fine-tuned to get the optimal model.

2.2 Functional Network (FN)

Functional network (FN) proposed by Castillo et al. [32,33] is a recent technique. The preliminary topology of FN network is centred on the domain knowledge of problem to be solved. Functional equations are used to simplify the preliminary topology of FN. The advantage of FN over ANN is that it uses both the knowledge's of domain and data simultaneously. FN uses random multi-argument and vector-valued functions, whereas ANNs use sigmoidal functions. The functions by the help of structural learning and parametric learning are learned and estimated, respectively. But in case of artificial neural networks, the neural functions are predetermined and fixed. By the use of intermediate layers in FN, several neuron outputs can be connected to a same unit, which in case of ANN is not possible. Based on the learning method, functional networks are of two types viz. structural learning and parametric learning. In structural learning, initial topology of the network is built on the assets obtainable to the designer. Further simplification is undertaken with the help of functional equations, whereas in parametric learning, estimation of the neuron function is based on the combination of functional families. Associated parameters are estimated from available data. A functional network is a combination of three types of units/elements. They are storing units (input layer, output layer and processing layers), computing units and directed link sets. The arrangement of the neural functions $f_i(x)$ can be done as per the equation given below.

$$f_i(x) = \sum_{j=1}^{m} a_{ij}\phi_{ij}(X) \tag{1}$$

where ϕ is the shape function, having algebraic expressions, exponential functions and/or trigonometry functions. A set of linear or nonlinear algebraic equations are obtained by the help of associative optimisation functions. Previous information about the functional equation is vital for working with functional network. Cauchy's functional equation is the most common instance for the functional equations. The efficiency of the FN depends upon the type of basic function to be used (exponential/polynomial/sine/cosine/tangent function) and degree/order of the function. This study applies the use

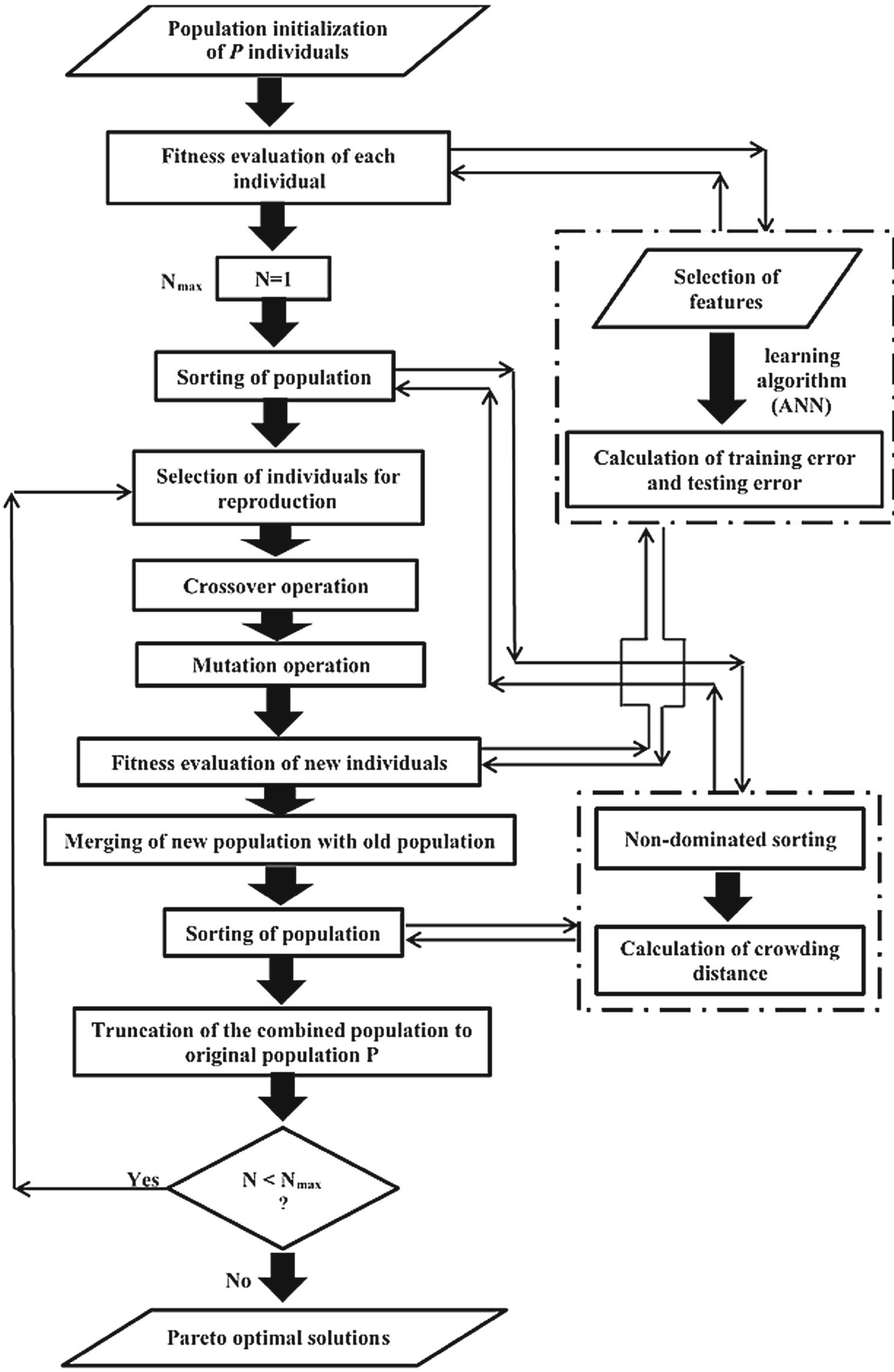

Fig. 1 Flow chart of MOFS algorithm

of associativity FNs. In-depth discussions can be found in Das and Suman [34].

2.3 Multi-variate Adaptive Regression Splines (MARS)

MARS proposed by Friedman [35] follows a nonlinear, nonparametric approach. It creates relationships between different input variables by the help of coefficients and basis functions. MARS algorithm follows a divide and conquer strategy. It efficiently handles both continuous and categorical data samples. Preliminary data preparation is almost negligible, when using MARS algorithm. Like recursive partitioning, MARS algorithm also uses automatic variable selection to find the important variables in the data set. MARS

Table 1 Statistical parameters of the data samples related to the pullout capacity of ground anchors

	Maximum	Minimum	Average	Standard deviation
D_{eq} (mm)	44.60	25.00	30.81	7.71
L (mm)	800.00	400.00	579.83	120.44
$q_{c\text{-tip}}$ (MPa)	3.55	0.95	1.93	0.57
f_s (kPa)	179.71	12.22	57.59	40.45
I	2.00	1.00	1.59	0.49
$Q_{u(m)}$ (kN)	3.80	0.29	1.75	0.77

models are flexible enough to model nonlinearity and variable interactions. Data sets large in size can be easily handled by MARS to develop models and make predictions in very less amount of time. A MARS model contains a number of piece-wise linear/cubic functions and knots (end points of splines). Creation of models in MARS follows a two-step process. In the first step, the basis functions, which has the lowest training error, are added sequentially. This process continues until the maximum number of basis functions are reached. Then in the second step, the best viable sub-model is obtained by removing the least effective terms. Once the pruning of model is complete, it is validated by generalised cross-validation (GCV) process. Elaborate discussion about MARS algorithm can be found in Das and Suman [34]. For the sake of creating simple linear models, only piece-wise linear basis functions have been used in this study. User needs to fix the maximum number of basis functions, generalised cross-validation (GCV) penalty per knot, type of function (piece-wise linear function or piece-wise cubic function), maximum degree of interaction between input variables and maximum number of basis functions in the pruned model to the best MARS model. For better understanding of MARS algorithm refer Das and Suman [34]. Models created by MARS are more flexible in comparison with models developed by linear regression method. MARS models have a good bias and variance trade-off. For numeric type of data, MARS is found to be more efficient than recursive partitioning as hinges are more suitable for numeric variables than the piece-wise constant segmentation used by recursive partitioning.

The MOFS, FN and MARS methodologies have been implemented using MATLAB.

3 Database and Pre-processing

In this study, data set of Shahin and Jaksa [13] was utilised. The database consists of 119 in situ results conducted on small (embedment depth 400, 600 and 800 mm) mild steel ground anchors at six different locations in Adelaide, South Australia, on alluvial soils with silt and sand, clay with some gravel, fine-grained sand, medium-grained sand, highly plastic black clay, and red, brown medium plastic clay. It contains in situ cone penetration test results; average resistance of the tip of cone ($q_{c\text{-tip}}$), average sleeve friction and equivalent anchor dia (D_{eq}), embedment length (L), along the embedment depth and the installation technique (I) (1 is for static installation and 2 is for dynamic installation). The ultimate pull-out capacity ($Q_{u(m)}$) of the ground anchor measured using drilling rig and data acquisition system. Statistical parameters of the data set are presented in Table 1.

In the MOFS algorithm, ANN training function used was Levenberg–Marquardt type consisting of 2 hidden neurons and performance of the neural network was based on MSE. Out of the total data set, 70% of the samples were used for training and the remaining 30% for testing. Data were normalised in the range [0, 1]. In NSGA II, uniform crossover technique was applied where replacement of the genetic material of the two selected parents takes place uniformly at several points. Conventional mutation operator was used on each bit separately and changing randomly its value. After the identification of the influential parameters responsible for the pull-out capacity of the ground anchors by MOFS algorithm, FN and MARS algorithm were applied on the same set of features/input variables. For FN and MARS algorithm, training (95 data samples) and testing (24 data samples) data set were normalised between 0 and 1.

4 Results and Discussions

Statistical comparison of all the AI models developed in this study along with the available AI models from the literature was done in terms of mean absolute error (MAE), root-mean-square error (RMSE), correlation coefficient (R) and is presented in Table 6. The overfitting ratio, which is the ratio between the RMSE of testing to training, was found out and also presented in Table 6. Overfitting ratio indicates the generalisation of the prediction models. Residual plots (residual error between the measured and the predicted values) of all the AI models developed in this research has been presented in Figs. 3, 4, 5 and 6 for the training and testing data set. If the residuals appear to behave randomly (equally distributed on both sides of the zero line), it suggests that the model fits the data well otherwise it is a poorly fitted model.

4.1 MOFS Model

Pareto optimal solutions given by MOFS algorithm are presented below with the number of input parameters used for modelling and the error rate of training and testing in terms of MSE. The optimum results obtained with NSGA II parameters as: population size = 50, crossover probability = 0.95, mutation probability = 0.1 and mutation rate = 0.1. Results of the multi-objective optimisation are presented in Fig. 2, and the details of the Pareto front are given in Table 2. Figure 2 clearly shows that MSE for training and testing decreases with increase in the number of input variables/features. Also, the difference between the training error and testing error which indicates the generalisation of a model (small difference means more generalised is the model) is least when number of input features is 4.

From Table 2, it can be inferred that the most influential features responsible for the pull-out capacity of ground anchors are average sleeve friction (f_s) along the embedment length and embedment length (L), as these two parameters are selected for a maximum number of times. The least influential parameter is average cone tip resistance ($q_{\text{c-tip}}$) which agrees well with previous study [12]. It can also be seen that MSE values with four and five selected features are comparable. Hence, two prediction models for the pull-out capacity of ground anchors corresponding to 4 number of parameters (D_{eq}, L, f_s, I) (MOFS1) and 5 number of parameters ($D_{\text{eq}}, L, q_{\text{c-tip}}, f_s, I$) (MOFS2), respectively, are presented as follows.

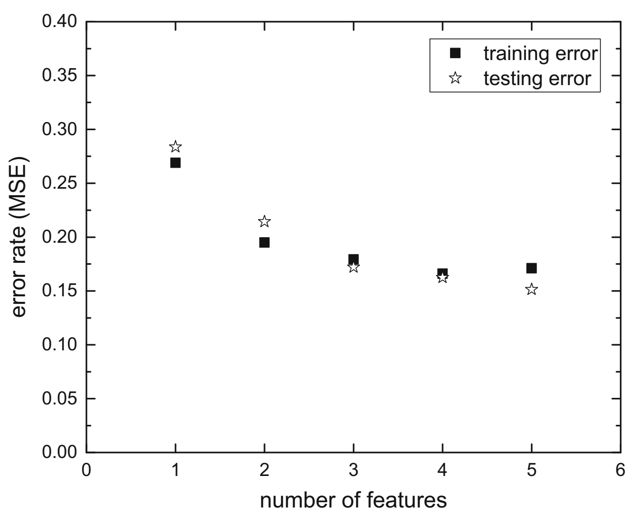

Fig. 2 Pareto front obtained from MOFS algorithm

Table 2 Details of the Pareto front obtained from the MOFS algorithm

Selected features						MSE
–	–	–	f_s	–	Training	0.269
					Testing	0.284
–	L	–	f_s	–	Training	0.195
					Testing	0.214
D_{eq}	L	–	f_s	–	Training	0.179
					Testing	0.172
D_{eq}	L	–	f_s	I	Training	0.166
					Testing	0.163
D_{eq}	L	$q_{\text{c-tip}}$	f_s	I	Training	0.171
					Testing	0.151

4.1.1 MOFS1

For four numbers of features, the developed model is named as MOFS1 and its mathematical form is presented below (weights and biases of the MOFS1 model are given in Table 3).

$$Q_{u(p)} = 3.51\,[325.129 + 0.134 \tanh(A_1) - 325.664 \tanh(A_2)] + 0.29 \tag{2}$$

where for static installation:

$$A_1 = -5.258 - 10^{-3}\left[5.72 D_{\text{eq}} + 11.997L - 564.67 f_s\right] \tag{3}$$
$$A_2 = 3.729 - 10^{-3}\left[3.2 D_{\text{eq}} + 0.474L + 0.884 f_s\right] \tag{4}$$

and for dynamic installation:

$$A_1 = -5.235 - 10^{-3}\left[5.72 D_{\text{eq}} + 11.997L - 564.67 f_s\right] \tag{5}$$
$$A_2 = 3.755 - 10^{-3}\left[3.2 D_{\text{eq}} + 0.474L + 0.884 f_s\right] \tag{6}$$

It is evident from Table 6 that model is well generalised as the R values for both training and testing are nearly same (training = 0.856 and testing = 0.834). The MAE and RMSE of the MOFS1 model are 0.294, 0.408 and 0.283, 0.403 kN for training and testing data set, respectively (Table 6). From Fig. 3, it can be observed that the MOFS1 model is a good fit model with a maximum residual error of approximately 1 and 1.25 kN for training and testing, respectively, on either side of the zero error line. Overfitting ratio of the model is 0.988 (close to 1.0), which indicates that the developed model is well fitted.

4.1.2 MOFS2

MOFS2 model has been developed taking into account all the features of the data set, i.e. for five number of variables ($D_{\text{eq}}, L, q_{\text{c-tip}}, f_s, I$). Model equation for predicting the pull-out capacity of ground anchors along with the connection weights and biases (Table 4) of MOFS2 model is presented below.

Table 3 Connection weights and biases of the MOFS1 model

Neuron (hidden)	Weights (w_{ik})					Biases	
	Input				Output		
	D_{eq}	L	f_s	I	Q_u	b_{hk}	b_0
k_1	−0.112	−4.799	94.577	0.023	0.134	−3.299	325.129
k_2	−0.063	−0.189	−0.148	0.027	−325.664	3.448	–

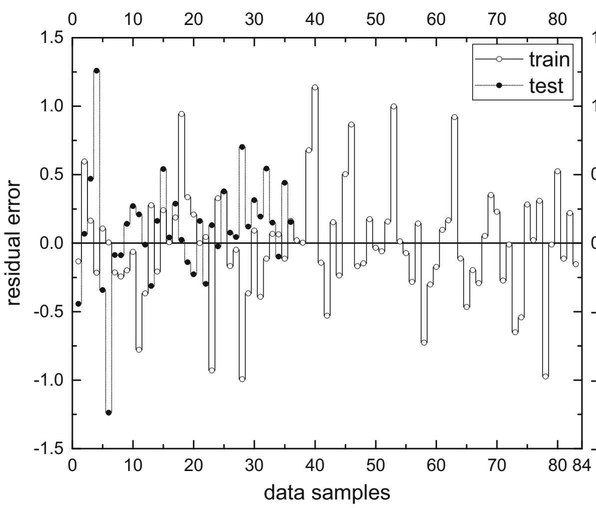

Fig. 3 Residual Error of MOFS1 Model

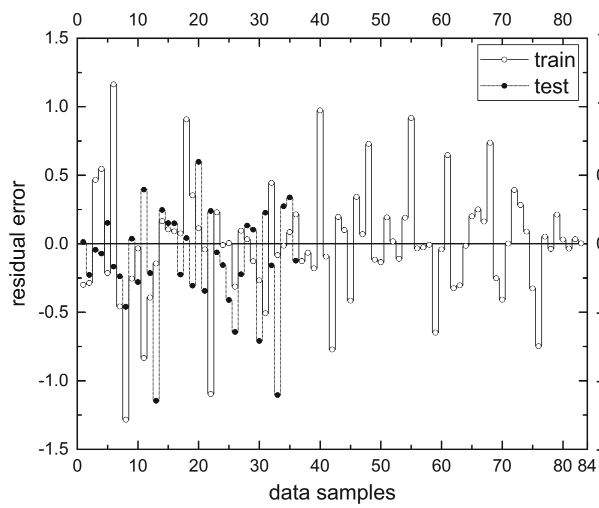

Fig. 4 Residual error of MOFS2 model

$$Q_{u(p)} = 3.51\left[40.189 + 41.814\tanh(A_1) + 0.158\tanh(A_2)\right] + 0.29 \tag{7}$$

where for static installation:

$$A_1 = -2.083 + 10^{-3}\left[1.339D_{\mathrm{eq}} + 0.252L + 6.889q_{\text{c-tip}} + 0.404f_s\right] \tag{8}$$

$$A_2 = 2.061 - 10^{-3}\left[25.076D_{\mathrm{eq}} + 8.227L + 937.864q_{\text{c-tip}} - 275.951f_s\right] \tag{9}$$

and for dynamic installation:

$$A_1 = -2.101 + 10^{-3}\left[1.339D_{\mathrm{eq}} + 0.252L + 6.889q_{\text{c-tip}} + 0.404f_s\right] \tag{10}$$

$$A_2 = 1.366 - 10^{-3}\left[25.076D_{\mathrm{eq}} + 8.227L + 937.864q_{\text{c-tip}} - 275.951f_s\right] \tag{11}$$

Figure 4 shows that the maximum error in prediction for the MOFS2 model is around 1.25 kN for training on either side of the zero error line and for testing it is approximately −1.15 kN. The correlation coefficient between measured and predicted values of pull-out capacity for the MOFS2 model as presented in Table 6 is 0.854 and 0.861, respectively, for training and testing. Also from Table 6, MAE and RMSE are given as 0.287, 0.414 and 0.289, 0.389 kN, respectively, for training and testing. The value of overfitting ratio as indicated in Table 6 is 0.940.

4.2 FN Model

A FN model with degree four and *polynomial* BF (associative type) found to give optimum result. The corresponding prediction equation is given by:

$$Q_{u(p)} = 0.364D^3 + 2.883L^2 - 2L^3 + 11.837f_s - 45.222f_s^2 + 80.789f_s^3 - 45.701f_s^4 - 0.277I^4 + 0.331 \tag{12}$$

Table 4 Connection weights and biases of the MOFS2 model

Neuron (hidden)	Weights (w_{ik})						Biases	
	Input					Output		
	D_{eq}	L	$q_{\text{c-tip}}$	f_s	I	Q_u	b_{hk}	b_0
k_1	0.026	0.101	0.018	0.068	−0.017	41.814	−1.937	40.189
k_2	−0.491	−3.291	−2.438	46.219	−0.695	0.158	0.624	–

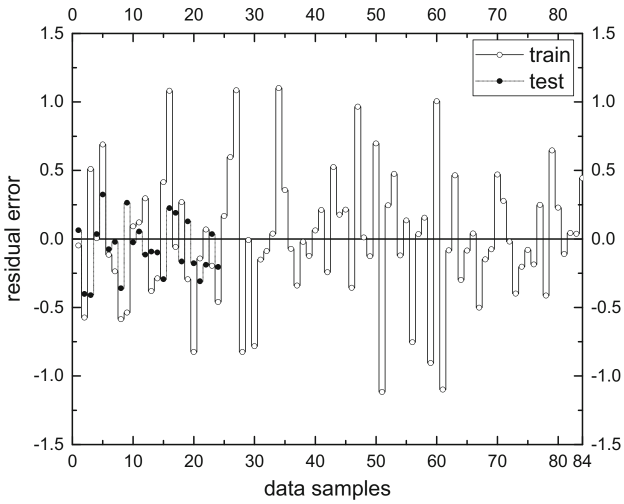

Fig. 5 Residual error of FN model

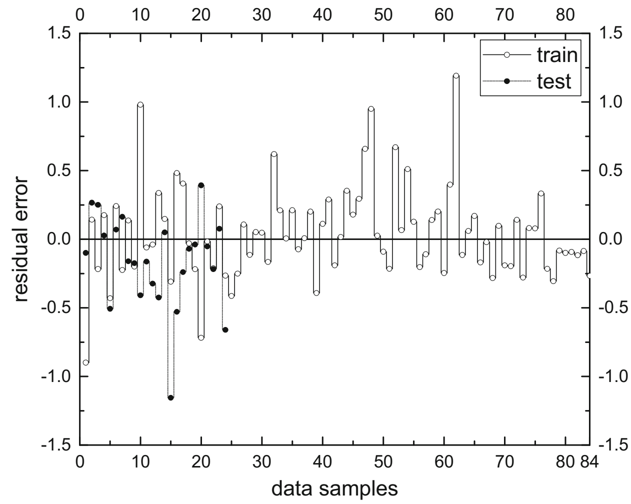

Fig. 6 Residual error of MARS model

In Eq. 12, the values of the inputs to be used are their normalised values between 0 and 1. Figure 5 shows the residual error plot between the measured and the predicted pull-out capacity of ground anchors. It can be seen that the model fits well along with a maximum deviation of 1 kN on both sides of the zero line for training phase and for testing phase it is approximately 0.5 kN. The values of R in training and testing for the FN model are 0.832 and 0.935, respectively, as indicated in Table 6. MAE and RMSE for the FN model as shown in Table 6 are 0.328, 0.444 and 0.177, 0.214 kN for training and testing, respectively. The overfitting ratio (Table 6) for the FN model is 0.482, which indicates that the FN model developed is under fitted.

4.3 MARS Model

In MARS modelling, the best model was obtained corresponding to 11 basis functions and the equivalent model equation is given below.

$$\begin{aligned} Q_{u(p)} = {} & 1.97 + 4.967 \times BF1 - 9.209 \times BF2 - 0.856 \\ & \times BF3 - 6.151 \times BF4 - 10.927 \times BF5 \\ & - 441.712 \times BF6 + 10.769 \times BF7 + 29.603 \\ & \times BF8 + 2382.71 \times BF9 - 288.274 \times BF10 \\ & - 7.096 \times BF11 \end{aligned} \tag{13}$$

Details of the respective BFs are presented in Table 5. Normalised input values have to be used in Eq. 13. The residual error plot of the MARS model is shown in Fig. 6 and from it can be observed that the scatter of the error around the zero line is random with a maximum error of approx. 1.25 kN from the measured value during training and −1.20 kN for testing. It can be seen from Table 6 that the values of R in training and testing are same, i.e. 0.894, which indicates a strong correlation between the predicted and observed values [36]. MAE and RMSE for training and testing (Table 6) are 0.254, 0.347 and 0.272, 0.371 kN, respectively. From Table 6, it can be inferred that the MARS model has better generalisation (overfitting ratio = 1.069).

4.4 Evaluation of AI Models

As it can be seen from Table 6, different AI models perform differently based on different criteria. Hence, a ranking system as suggested by Abu-Farsakh and Titi [37] has been employed to assess the overall performance of the prediction models taking into account the total number of data samples, which in this case is 119. Three different evaluation criterions has been considered in this study. First ranking

Table 5 Details of the BFs for the MARS model

BF1	$\max(0, f_s - 0.1415)$	BF7	$\max(0, L - 0.5) \times \max(0, f_s - 0.2515)$
BF2	$\max(0, 0.1415 - f_s)$	BF8	$\max(0, L - 0.5) \times \max(0, 0.2515 - f_s)$
BF3	$\max(0, 0.5 - L)$	BF9	BF1 $\times \max(0, 0.3129 - f_s) \times \max(0, q_{\text{c-tip}} - 0.3615)$
BF4	BF1 $\times \max(D - 0.4336)$	BF10	BF1 $\times \max(0, 0.3129 - f_s) \times \max(0, 0.3615 - q_{\text{c-tip}})$
BF5	BF1 $\times \max(0.4336 - D)$	BF11	$\max(0, L - 0.5) \times \max(0, q_{\text{c-tip}} - 0.2807)$
BF6	BF5 $\times \max(0, D + 0)$		

Table 6 Statistical comparison of different AI models

	R	MAE (kN)	RMSE (kN)	Overfitting ratio
ANN [12]				
Multi-layer perceptron				
Training	0.830	0.320	0.430	1.070
Testing	0.850	0.350	0.460	
B-spline neuro-fuzzy network				
Training	0.830	0.310	0.420	0.929
Testing	0.890	0.300	0.390	
EPR [15]				
Training	0.789	0.340	0.460	0.935
Testing	0.872	0.370	0.430	
FN (present study)				
Training	0.832	0.328	0.444	0.482
Testing	0.935	0.177	0.214	
MARS (present study)				
Training	0.894	0.254	0.347	1.069
Testing	0.894	0.272	0.371	
MOFS (present study)				
MOFS1				
Training	0.856	0.294	0.408	0.988
Testing	0.834	0.283	0.403	
MOFS2				
Training	0.854	0.287	0.414	0.940
Testing	0.861	0.289	0.389	

Table 7 Evaluation of performance of different prediction models and their ranking based on rank index proposed by Abu-Farsakh and Titi [34]

	Best fit calculations			Arithmetic calculations of $Q_{u(p)}/Q_{u(m)}$			Cumulative probability of $Q_{u(p)}/Q_{u(m)}$			Overall rank	
	R	E	$R1$	μ	σ	$R2$	P_{50}	P_{90}	$R3$	RI	Final rank
ANN [12] (multi-layer perceptron)	0.830	0.684	5	1.128	0.374	6	1.045	1.597	6	17	6
EPR [15]	0.808	0.651	6	1.081	0.369	5	1.013	1.587	5	16	5
FN (present study)	0.846	0.716	4	1.065	0.332	3	1.028	1.465	4	11	4
MARS (present study)	0.890	0.789	1	1.081	0.318	4	1.028	1.440	3	8	3
MOFS1 (4) (present study)	0.849	0.719	3	1.036	0.311	1	0.987	1.276	2	6	2
MOFS2 (5) (present study)	0.851	0.719	2	1.074	0.310	2	1.009	1.304	1	5	1

(best fit calculations) consists of correlation coefficient (R) and Nash–Sutcliff coefficient of efficiency (E) [16]. Second (arithmetic calculations) consists of arithmetic mean (μ) and standard deviation (σ) of the $Q_{u(p)}/Q_{u(m)}$ ratio. And finally third ranking (cumulative probability distribution of $Q_{u(p)}/Q_{u(m)}$) contains the P_{50} and P_{90} values of $Q_{u(p)}/Q_{u(m)}$, details of which can be found in Abu-Farsakh and Titi [37] and Das and Suman [34]. Combining all the three ranking criteria, a ranking index (RI) (Table 7) was obtained and all the AI models were given an overall rank (lower RI means better the rank of the model). As per the best fit calculation criteria (Table 7) better rank was awarded to MARS model as its R (0.890) and E (0.789) values were generally closer to one as compared to others followed by the MOFS2 model ($R = 0.851$ and $E = 0.719$). Analysis of the arithmetic calculation of $Q_{u(p)}/Q_{u(m)}$ ratio (Table 7) indicated the best model to be MOFS1 ($\mu = 1.036, \sigma = 0.311$) and second best model to be MOFS2 ($\mu = 1.074, \sigma = 0.310$). In this the best model has μ closer to unity with least deviation, i.e. σ should be closer to zero. For the third criteria, graphical representation of the cumulative probability distribution of $Q_{u(p)}/Q_{u(m)}$ ratio of FN, MARS, MOFS1 and MOFS2 models are presented in Fig. 7. The P_{50} values of the developed prediction model define the model in terms of under prediction and over prediction. For the best model, P_{50} value should be to closer to 1.0 and the difference between P_{50} and P_{90} should be least. Among the developed models, MOFS2 model ($P_{50} = 1.009$) is the "best model". As per the P_{90} value also MOFS2 model found to be more efficient followed by MOFS1 model. The corresponding P_{50} and P_{90} values (Table 7) of MOFS2 and MOFS1 model are 1.009, 1.304 and 0.987, 1.276, respectively. Based on the RI values of different prediction models, the best performing model was found to be MOFS2 with a RI of 5 and second best model was MOFS1 with a RI of 6.

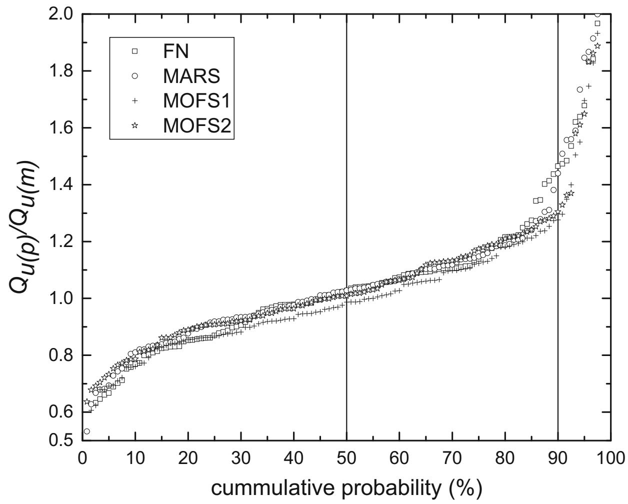

Fig. 7 Cumulative probability distribution of training and testing data

Hence, it can be easily concluded that out of all the AI models, MOFS2 model is best followed closely by MOFS1 model as indicated in the ranking criteria (Table 7). The performance of the model equations (based on the ranking criteria) of the present AI models is better than available AI models [12,15]. Also the MOFS2 and MOFS1 model equations are less complex and better comprehensible in comparison with the multi-layer perceptron model [12] equation.

5 Conclusion

This paper deals with the development of prediction model for pull-out capacity of ground anchor using MOFS technique, simultaneously considering minimum number of features (input variable) and minimising training and testing errors. Prediction models are also developed using FN and MARS using optimum features obtained as per MOFS. Identification of the subset of features responsible for the predictive capacity of the model is addressed here by considering it as a multi-objective optimisation problem. Statistical comparison was made between the developed AI models and the AI models available from the literature in terms of MAE, RMSE, R and overfitting ratio. Ranking of various AI models were also done. As per the ranking system, MOFS2 had the best overall performance tailed by MOFS1, MARS and FN. Model equations for all the AI models (MOFS1, MOFS2, FN and MARS) developed in this research have been presented, which can be implemented by the field professionals. Also, the prediction equations of MOFS2 and MOFS1 for determining the pull-out capacity of ground anchors in comparison with the prediction equation of multi-layer perceptron model are less complex and better comprehensible, which can be used for hand calculations as the MOFS2 and MOFS1 equations are simple and better in prediction as compared to others.

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