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RESEARCH ARTICLE - CIVIL ENGINEERING

Shape Optimization of Trapezoidal Labyrinth Weirs Using Genetic Algorithm

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Abstract This paper describes a methodology for shape optimization of trapezoidal labyrinth weirs. The objective function is the volume of the weir. Different parameters of the weir are introduced as design variables including the total width of weir, width of upstream apex and actual length of side leg. Sensitivity analysis revealed that three design variables of weir width in one cycle, weir leg and number of cycles are the main parameters affecting weir optimization solution. The constraint conditions are the weir geometric shape and its different ratios. Genetic algorithm is applied to perform optimization process. The proposed method is applied successfully to Ute Dam labyrinth weir, and the results are compared with the real one. The results indicated that the volume of the trapezoidal labyrinth weir is reduced by an average of 21% which is obtained per 14 cycle numbers.

Keywords Labyrinth weir · Optimization · Genetic algorithm · Sensitivity analysis

1 Introduction

The spillway is one of the most important structures in dam construction projects. It provides the ability to release excess or flood water in a controlled or uncontrolled manner to ensure the safety of the dam during very major floods. Hence, it is very important the spillway facilities designed with sufficient capacity to prevent overtopping of the dam [1–4]. In dam construction, special attention should be paid to the shape optimization of spillways because their costs are very considerable in comparison with major parts of the dam's ones. For large and small dams, it is about 20 and 80% of the total dam construction costs, respectively [5,6].

According to the International Commission on Large Dams (ICOLD), the inadequate capacity of the spillway is the main cause of dam failures all over the world [5]. In order to satisfy this requirement, the labyrinth weirs are the best option to increase the discharge capacity because the increased crest length can provide much more unit discharge over conventional weirs for a given head [7,8]. Several numerical and experimental researches have been published on different issues of labyrinth weirs design [1–3,9–15], while their optimal design has been less considered contrary to other aspects. The optimal shape is the best design for a structure subjected to various constraints imposed by the restrictions placed on the design [16]. The labyrinth weir geometrical shape considered during the initial design phase is not always the best one from technical and economical points of view. The best geometrical shape should be determined by applying optimization methods, which employ a set of structural safety and minimal cost criteria [13]. The complicated flow pattern over the weir and the several geometrical parameters cause the optimum design of the labyrinth weirs has particular complexities.

Ghare et al. [6] established a mathematical optimization model for determining the optimum value of the discharge coefficient in labyrinth weirs based on the ratio of total upstream head to the weir height. The results indicated that by increasing this ratio, the discharge coefficient is considerably increased. Izadbakhsh et al. [17] investigated the hydraulic efficiency of the labyrinth weirs by use of discharge coefficient. A mathematical model and Flow3D CFD

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model are applied in this study. The results showed that the vertical aspect ratio increases in proportion to the head ratio, and after reaching the maximum point, it begins to decrease. Also, increasing the height of the weir leads to increase in the discharge coefficient. Crookston and Tullis [2] used the physical and numerical models to investigate the hydraulic performance of labyrinth weirs under high headwater ratios (greater than 1). The numerical results indicated that agreement between the physical and numerical modeling is about 5%. Also, in high headwater ratios, flow3D model is an acceptable technique to examine the performance of labyrinth weirs. A labyrinth weir design and analysis procedure were presented by Crookston and Tullis [1] based upon the results of physical modeling in a laboratory flume. The proposed method with experimental discharge coefficient data of this study was validated with other physical model studies.

In solving NP-hard problems with nonlinear and complicated objective functions and large number of variables, gradient-based optimization techniques often fail or reach local optimum [18]. To overcome these defects, the evolutionary-based algorithms have been presented for finding near-optimum solutions. The genetic algorithms (GAs) were the first evolutionary-based technique introduced in the literature [19]. GAs demonstrated abilities to reach near-optimum solutions in complex problems and are vastly used in science and engineering problems [20–23]. Diverse applications of genetic algorithm also can be found in different literatures.

An inverse problem is considered by Mera et al. [24] to identify the geometry of discontinuities in a conductive material with anisotropic conductivity from Cauchy data measurements taken on the boundary. In this regard, a real coded genetic algorithm in conjunction with a boundary element method is proposed to detect an anisotropic inclusion. It is found that the developed genetic algorithm is a robust, efficient method for detecting the size and location of subsurface inclusions.

Hacioglu and Ozkol [25] introduced the distribution strategies (DS) in evolutionary computations and their application to the inverse airfoil design problems. They developed new strategies combined with a real coded genetic algorithm to obtain a faster and more robust method. The performance of this new method is compared with classical and more commonly used genetic algorithms, and the considerably decreased number of computational fluid dynamics calculations showed the effectiveness of new proposed method. In research of Khan et al. [26], a genetic algorithm is employed to minimize the entropy generation rate in microchannel heat sinks. The results of optimization are compared with the existing results obtained by the Newton–Raphson method and concluded that the GA gives better overall performance of the microchannel heat sinks.

Different shape optimizations, such as arch dam optimization, were conducted by using genetic algorithms such as [27]; however, the shape optimization of labyrinth weirs was not considered with none of the evolutionary algorithms.

The present study describes a method for shape optimization of the trapezoidal labyrinth weir. In the optimization process, the total volume of the labyrinth weir is defined as the objective function. Design variables of processing are the geometric shape of trapezoidal labyrinth weir such as width, height and side angle, and the constraint conditions are several ratios of different design variables and design discharge. For optimization purposes, an optimum model of labyrinth weir is presented to make the cost of project minimal on the premise of meeting the structural and hydraulic needs. Generally, the cost of weir project is mainly dependent upon the volume of consumed concrete in weir. The total volume of the weir is the sum of the wall volume, the volume of head wall and the volume of the slab.

The proposed method is successfully applied to a labyrinth weir, where good results are achieved. The results showed that the concrete volume of the optimized labyrinth weir is reduced by an average of 21% in comparison with the initial shape. In this model, the convergence time is very short and the method is very effective. So, it can be applied to other practical engineering problems. This paper is organized into four sections. Materials and methods for the analysis are described in Sect. 2, while Sect. 3 is devoted to the results and discussion, and the last section contains a summary and conclusion.

2 Labyrinth Weirs

Labyrinth weirs with a trapezoidal shape in plan are schematically presented in Fig. 1 [5]. The main parameters affecting performance of the weir include the total width of weir W, the width of the weir in one cycle w, the width of upstream apex A, the actual length of side leg l, the effective length of side leg l_C, the labyrinth weir leg B, the wall thickness t_w, the angle of the side wall α and the weir height relative to the canal bed P (see Fig. 1).

Various experimental relations are presented for describing the relation of discharge head in labyrinth weirs. Different definitions of the effective length and head parameters leading to numerous discharge-head relations are introduced. However, the simple relation of discharge head in labyrinth weirs is mostly used. This relation is presented by a general equation as [2]:

$$Q = \frac{2}{3} C_\mathrm{d} L_\mathrm{e} \sqrt{2g} H_\mathrm{T}^{1.5} \tag{1}$$

where Q is the discharge over the weir, C_d is dimensionless discharge coefficient, L_e denotes the effective length of the

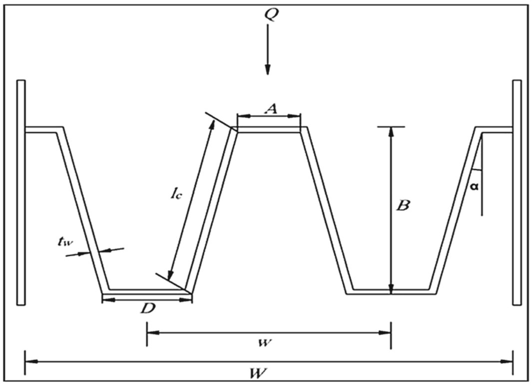


Fig. 1 Schematics of trapezoidal plans of the labyrinth weir

weir, g is the constant of gravitational acceleration, and H_T is the total head over weir ($H_T = H + V^2/2g$), where V and H, respectively, denote the depth- averaged velocity and piezometric head at the weir upstream relative to the crest elevation [4,7,28,29].

3 The Optimal Method of Labyrinth Weirs

The aim of shape optimization is to minimize the volume of consumed concrete during labyrinth weir construction while enhancing hydraulic criteria. The shape optimization problem is to find the design variable X while minimizing the objective function $F(X)$ under the defined constraint functions $g_j(X)$ that can be stated mathematically as [16]:

$$\text{Find} \quad X = [X_1, X_2, X_3, \ldots, X_n]^T,$$
$$a_i \le X \le b_i \quad (i = 1, 2, \ldots, n)$$

To minimize $F(X)$

$$g_j(X) \le 0 \quad (j = 1, 2, \ldots, m) \tag{2}$$

The subscripts n and m show the number of design variables and inequality constraints, respectively, where a_i and b_i are allowable lower and upper bounds of each design variable, which is defined according to requirements of the problem.

3.1 Design Variables

Increasing the number of defined design variables can improve the shape optimization of labyrinth weirs; however, it raises the problem complexities and the cost of calculations. According to the geometric model of labyrinth weir shown in Fig. 2, the design variables can be classified into two major groups: decision variables and design variables. Decision variables are the variables that directly can be controlled by the decision makers. In optimization of trapezoidal labyrinth weirs, three decision variables can be introduced: number of cycles in labyrinth weir (N) that must be selected as an integer number; crest shape in which quarter-round shape is considered; and the design rate flow (Q) of labyrinth weir.

Also 10 design variables are considered, which will be entered in the optimization process. The design variables in shape optimization of labyrinth weirs can be divided into dependent and independent variables, in which the dependent variables will be defined based on the independent ones. The independent and dependent design variables are introduced in Table 1.

3.2 Objective Function

The purpose of shape optimization is to present optimum geometric shape of labyrinth weir to make the cost of project minimal on the premise of meeting the structural and hydraulic needs. Generally, the cost of labyrinth weir project is mainly dependent upon the volume of weir body concrete. So, the objective function is defined as the total volume of weir body. The total volume of the weir body V_T is the sum of the wall volume V_w, the volume of head wall V_e and the volume of the slab V_s.

$$V_T = V_w + V_e + V_s$$

in which

$$V_w = N((2B/\cos\alpha) + 2A)Pt_w$$
$$V_e = 2(P + H_T + F_b)(B + H_T)t_w$$
$$V_s = (B + 2H_T)Wt_s \tag{3}$$

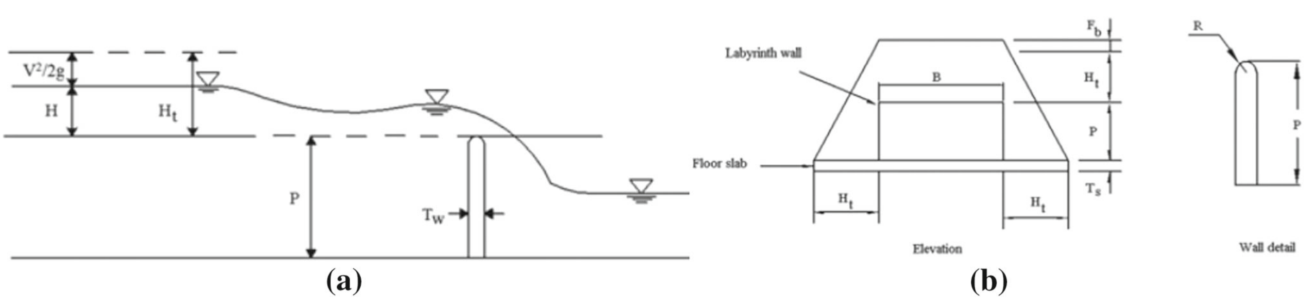


Fig. 2 Illustration of labyrinth weir geometric parameters in **a** longitudinal, **b** lateral sections [5]

Table 1 Definition of independent and dependent design variables

Design variable	Symbol	Definition
Independent variables		
Leg of labyrinth weir	B	–
Side wall angle	α	–
The upstream apex width	A	–
Weir height	P	–
Wall thickness	t_w	–
Total upstream head over labyrinth weir	H_T	$H + V^2/2g$
Crest shape	Quarter-round	
Dependent variables		
Width of the weir in one cycle	w	$= 2(B \tan\alpha + A)$
Total width of the weir	W_c	$= 2N(B \tan\alpha + A)$
Effective length of the weir	l_c	$= 2(B/\cos\alpha + A)$
Actual length of the side leg	l	$= 2N(B/\cos\alpha + A)$

where t_s is the thickness of the slab and F_b denotes the considered free board.

3.3 Constraint Functions

In shape optimization of labyrinth weir, only one type of constraint should be satisfied, as required by the hydraulic demands:

Headwater Ratio (H_t/P) The ratio of total head over the weir to weir height is called the headwater ratio. Unlike typical linear weirs, discharge coefficient in labyrinth weirs increases with decreasing headwater ratio, and as a result, the labyrinth weir indicates good performance in low headwater ratios. To achieve the high performance of labyrinth weir in the optimization process, this ratio must be limited by 0.9 and 0.05 as upper and lower limits, respectively [1].

$$0.05 \le \frac{H_t}{P} \le 0.9 \quad \rightarrow \begin{cases} \frac{H_t}{0.9P} - 1 \le 0 \\ 1 - \frac{H_t}{0.05P} \le 0 \end{cases} \tag{4}$$

Cycle Width Ratio (w/P) The ratio of each cycle width over the weir height is described as the cycle width ratio. Decreasing the cycle width ratio causes the discharge coefficient to reduce. Taylor [30] proposed that this ratio must be greater than 2. Lux and Hinchliff [31] recommended this ratio to be 2.5 for trapezoidal shapes weirs for design purposes. The upper bound for this ratio is proposed to be 4 by Tullis et al. [28] and Lux [32]. The minimum and maximum proposed values are selected as lower and upper bands, respectively:

$$2 \le \frac{(2B\tan\alpha + 2A)}{P} \le 4 \quad \rightarrow \begin{cases} \frac{(2B\tan\alpha+2A)}{4P} - 1 \le 0 \\ 1 - \frac{(2B\tan\alpha+2A)}{2P} \le 0 \end{cases} \tag{5}$$

Apex Width Ratio (A/w) The crossing length of weir apex, A, is the most effective parameter on discharge coefficient in trapezoidal labyrinth weirs. Increase in apex width leads to reduce the net length and the capacity of the labyrinth weir. Hence, apex width must be considered as small as possible. In trapezoidal shape weirs, this ratio is limited to 0.08 [28].

$$\frac{A}{0.08(2B\tan\alpha + 2A)} - 1 \le 0 \tag{6}$$

Magnification Ratio (L/w) The ratio of one cycle length over the one cycle width is described as the magnification ratio. For optimum performance, this ratio is limited between 3 and 9.5 [28].

$$3 \le \frac{(2B\cos\alpha + 2A)}{(2B\tan\alpha + 2A)} \le 9.5 \quad \rightarrow \begin{cases} \frac{(2B\cos\alpha+2A)}{9.5(2B\tan\alpha+2A)} - 1 \le 0 \\ 1 - \frac{(2B\cos\alpha+2A)}{3(2B\tan\alpha+2A)} \le 0 \end{cases} \tag{7}$$

Wall Thickness Ratio (A/t_w) Wall thickness ratio is described as the ratio of apex width over the wall thickness. For the best performance, this ratio is limited to 1 and 2 [32].

$$1 \le \frac{A}{t_w} \le 2 \quad \rightarrow \begin{cases} \frac{A}{2t_w} - 1 \le 0 \\ 1 - \frac{A}{t_w} \le 0 \end{cases} \tag{8}$$

Total Head $(H_T + P)$ Total head shows the water level compared with the canal bed level. Total head depends on the crossring flow over the weir and has different values among various weirs. So, this parameter must be determined for each special case study regarding the crossring flow over the weir.

Design Discharge (Q_d) Q_d describes the maximum probable flooding discharge that should be passed to the downstream region by labyrinth weir. Hence, the passing flow over the weirs (Q) which may be calculated by equation (1) must be equal or larger than the design discharge Q_d [33].

$$Q_d - \left(2/3\sqrt{2g}C_d H_T^{1.5} N\ (2B/\cos\alpha + 2A)\right) \le 0 \tag{9}$$

3.4 Fitness Function

Penalty function method causes a constrained optimization problem to be converted to an unconstrained one. To reduce the number of penalty parameters, often the constraints are normalized (generally between [−1, 1]) and only one penalty parameter is used [21,22]. Hence, all constraints are normalized manually in the similar way to make each constraint violation approximately on the same scale. The scaled constraints are listed as follows:

$$g_1 = \begin{cases} [(H_\mathrm{T}/P) \times (1/0.9)] - 1 \\ 1 - [(H_\mathrm{T}/P) \times (1/0.05)] \end{cases} \tag{10}$$

$$g_2 = \begin{cases} [((2B\tan\alpha + 2A)/P) \times (1/4)] - 1 \\ 1 - [((2B\tan\alpha + 2A)/P) \times (1/2)] \end{cases} \tag{11}$$

$$g_3 = [(A/(2B\tan\alpha + 2A) \times (1/0.08))] - 1 \tag{12}$$

$$g_4 = \begin{cases} [((2B/\cos\alpha + 2A)/(2B\tan\alpha + 2A)) \times (1/9.5)] - 1 \\ 1 - [((2B/\cos\alpha + 2A)/(2B\tan\alpha + 2A)) \times (1/3)] \end{cases} \tag{13}$$

$$g_5 = \begin{cases} [(A/t_\mathrm{w}) \times (1/2)] - 1 \\ 1 - [(A/t_\mathrm{w}) \times (1/1)] \end{cases} \tag{14}$$

$$g_6 = \begin{cases} [(H_\mathrm{T} + P) \times (1/\beta_1)] - 1 \\ 1 - [(H_\mathrm{T} + P) \times (1/\beta_2)] \end{cases} \tag{15}$$

$$g_7 = 1 - \left(\frac{2}{3}\sqrt{2g}C_\mathrm{d}H_\mathrm{T}^{1.5}N\,(2B/\cos\alpha + 2A)\right) \times (1/Q_\mathrm{d}) \tag{16}$$

where β_1 and β_2 are coefficients that varied in dealing with various requirements of each special weir and must be determined based on meeting the design needs. By applying a penalty function to the objective function, the fitness function is obtained as follows:

$$V = V_\mathrm{T} + \sum_{i=1}^{n}\left[R_{(i)}\left|g_{(i)}\right|^{\mathrm{Pow}}\right] \tag{17}$$

where V presents the fitness function, V_T denotes the objective function, $R_{(i)}$is the penalty parameter of the ith constraint, n is the number of constraints, $g_{(i)}$ shows the normalized constraints, and pow is the power of the constraint. The power value strongly depends on the type of optimization problem and should be determined by the trial-and-error process. In this paper, different values between 1 and 2 are examined, and finally, the value of 1.1 is obtained as the best option.

4 Optimization Algorithm

GAs are inspired by the evolutionist theory explaining the origin of species [19]. Many versions of genetic algorithm are available. Using an appropriate version depends on the type of problem constraints. The version of adaptive genetic algorithm is applied in this paper. Applying the adaptive strategy helps to develop the adaptive genetic algorithm in which the method operators are systematically adapted with respect to the problem constraints. The adaptive operators always keep the GA in feasible regions of the decision space and consequently improve the optimum performance in terms of speed and reliability [34].

GAs have different components in that various options can be considered for them. The main components are population size, selection, probability of crossover and mutation. Functioning and running speed of the GA program strongly affected the accurate selection of these components. Hence, in this study, in order to utilize the best option for each component of GA, different options have been considered and their performance was examined to reach the best solution. The examined options for each component are listed in Table 2.

In order to investigate the efficiency and performance of the optimization method for the shape optimization of trapezoidal labyrinth weirs, the Ute Dam labyrinth weir is chosen as a real work structure. Ute Dam is a 40-meter-high earthen embankment dam on the Canadian River and is located 32 km west of Logan, New Mexico. The dam and its weir were constructed in 1963 and then were substantially modified in 1984 to increase the storage capacity of the reservoir [35]. The weir modification consisted of constructing a 14-cycle labyrinth weir to pass the 12177 (m^3) 430,000 cfs design flood. The length of the weir is about 1024 meters that can pass down the design flood under a hydraulic head of 5.8 meters. Figure 3 shows the constructed labyrinth weir of Ute Dam.

4.1 Sensitivity Analysis of Optimization Algorithm

Sensitivity analysis is one of the most preoccupying and interesting areas in optimization. Many attempts are made

Table 2 Options for each component of genetic algorithm

Component	Options					
Population	60	90	120	150	–	–
Crossover	Scattered	Single point	Two points	Intermediate	Heuristic	Arithmetic
Similarity coefficient	1.1	1.2	1.3	–	–	–
Selection	Stochastic uniform	Reminder	Uniform	Roulette	Tournament	–
Mutation	Gaussian	Uniform	Adaptive feasible	Constraint dependent	–	–

Fig. 3 Constructed labyrinth weir of Ute Dam

to investigate the problem's behavior when the input data change. A sensitivity analysis may be performed in a general optimization problem, including sensitivities of the objective function and the design variables with respect to the data. In order to preset more accurate, optimal design for labyrinth weir, sensitivity analysis has been conducted in different parameters of the algorithm. So, the variations of different parameters including objective function, design variables and decision variables with respect to the different options of each parameter of genetic algorithm are presented in the following section:

Population Size In running genetic algorithm, the population distribution is one of the most important parameters that are controlled by regulating the initial size of the population. Sensitivity analysis of the design variables and fitness function to the different values of population size is presented in Fig. 4. The best value of population size is resulted to be 90, mostly corresponding to the minimum value of the fitness function.

Crossover In GAs, crossover occurs during evolution according to a user-definable crossover probability. In the present study, six of the most widely used algorithms are considered and the sensitivity analysis was conducted to select the best option. The results are presented in Fig. 5.

The graphs show that using the heuristic crossover leads in achievement of minimum fitness function. The performance of the heuristic crossover, however, mostly depended on the selection of optimum value for similarity coefficient. Hence, three values of similarity coefficient that are equal to 1.1, 1.2 and 1.3 are selected and more investigated by comparison of the fitness function. Figure 6 shows the results.

Selection In evolutionary algorithms, selection of the best individuals is performed based on evolution of fitness function. Figure 7 demonstrates the sensitivity analysis of fitness function and other design variables to different options of selection parameter.

As shown in Fig. 7, applying the uniform option results in minimum fitness function and is a very convenient option for selection. However, it should be noted that using the tournament option convergence is not established, and as a result, the results of this option are not presented in Fig. 7.

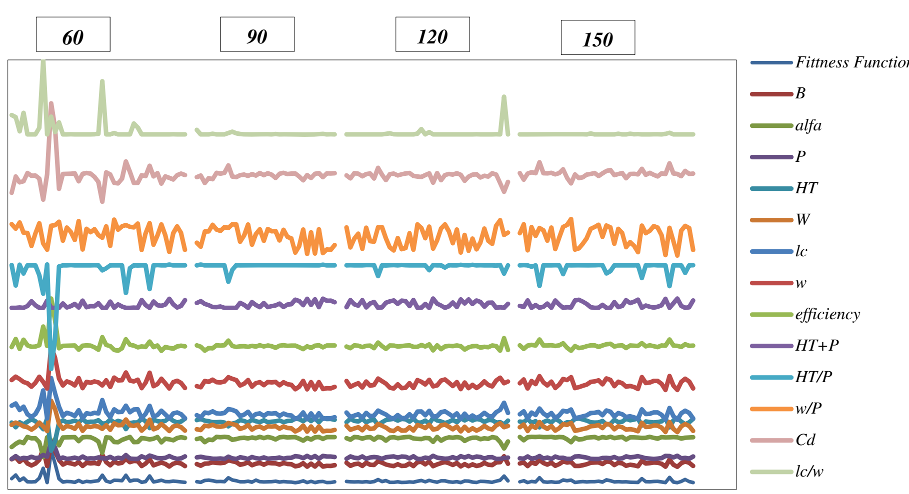
Mutation In GAs, mutation helps escape from local minima's trapped and maintains diversity in the population. Figure 8 shows the sensitivity of fitness function to different options of mutation. As obvious in Fig. 8, the minimum value of fitness function is achieved by using Constraint Dependent.

5 Results and Discussion

The optimization process of labyrinth weir according to the mentioned methodology converged after 1000 iterations. The convergence rate of the fitness function in the optimization process is presented in Fig. 9.

In shape optimization of labyrinth weir, in initial step, the number of 9–14 cycles is considered and the values of design variables and objective function are obtained for each cycle number. The initial and optimum values of design

Fig. 4 Results of sensitivity analysis in determination of population size



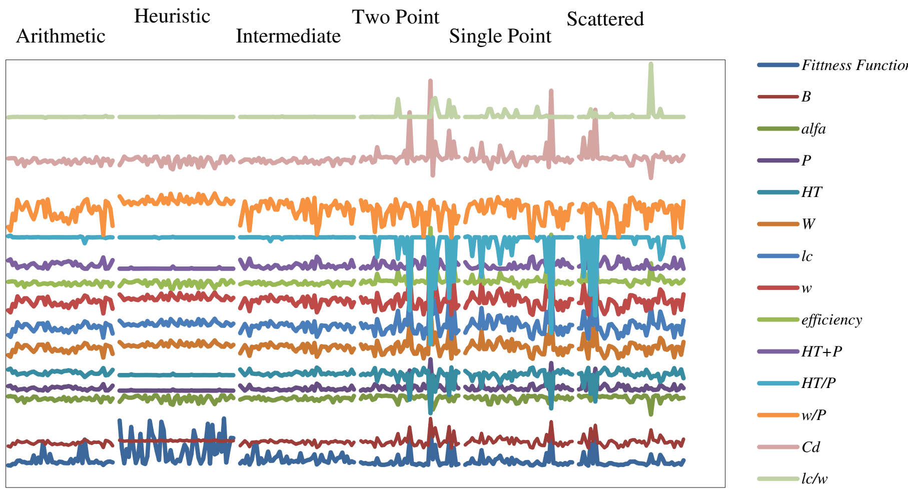


Fig. 5 Results of sensitivity analysis in determination of crossover

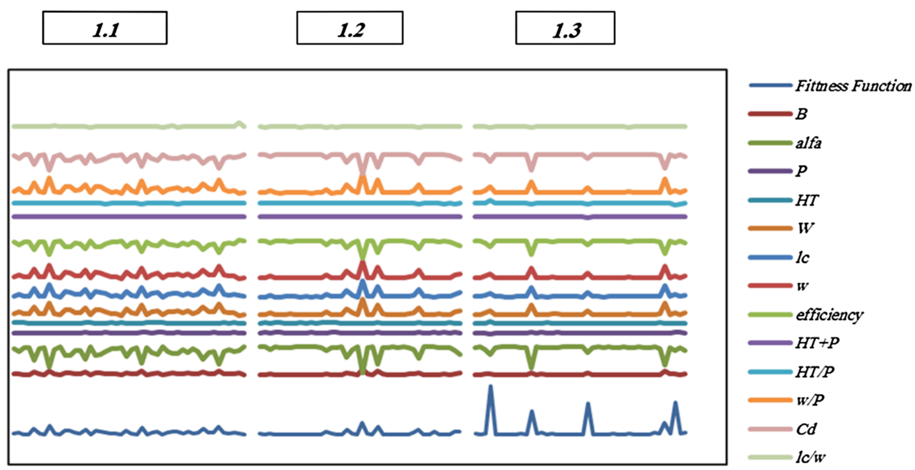


Fig. 6 Results of sensitivity analysis in determination of best value of similarity coefficient

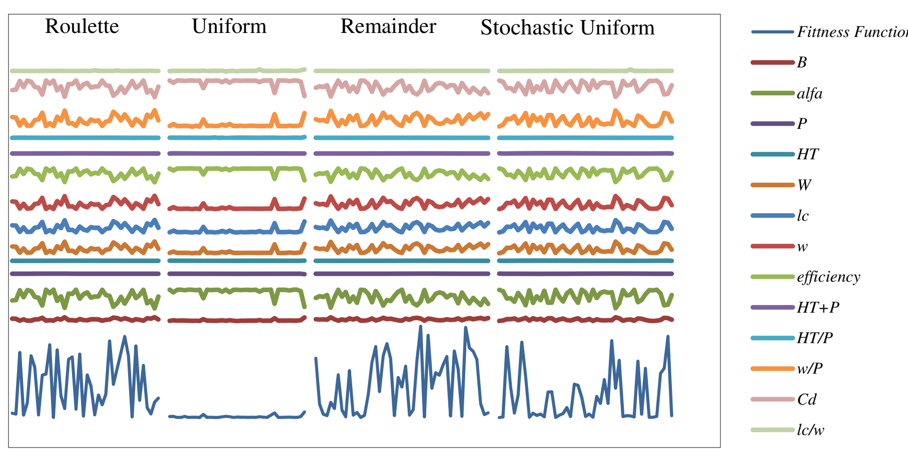


Fig. 7 Results of sensitivity analysis in determination of best option for selection

variables and objective function are given in Table 3 (all dimensions are in meters). The optimum volume of the labyrinth weir is 2640 m^3 less than the initial volume, i.e., 21.47% less.

As can be seen in table 3, minimum value of labyrinth weir volume defined by the present optimization method was achieved in applying 13 and 14 cycles. However, in applying cycle numbers of 9, 10, 11 and 12, low difference is achieved between the resulted weir volumes. So, regarding the low difference between the weir volumes using different cycles, other criteria should be considered to reach the optimum model. In this regard, the effect of cycle numbers on other parameters of labyrinth weirs such as B, A, t_w, H_T, P and W is studied. The variations of design vari-

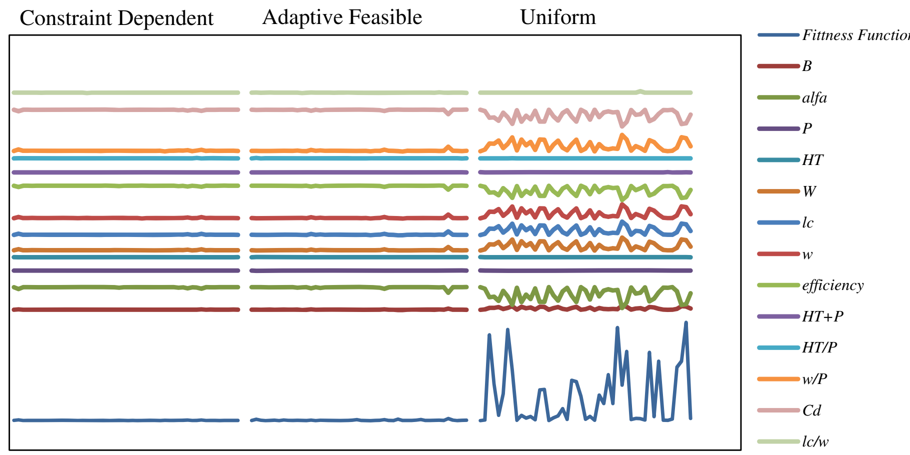


Fig. 8 Results of sensitivity analysis in determination of mutation option

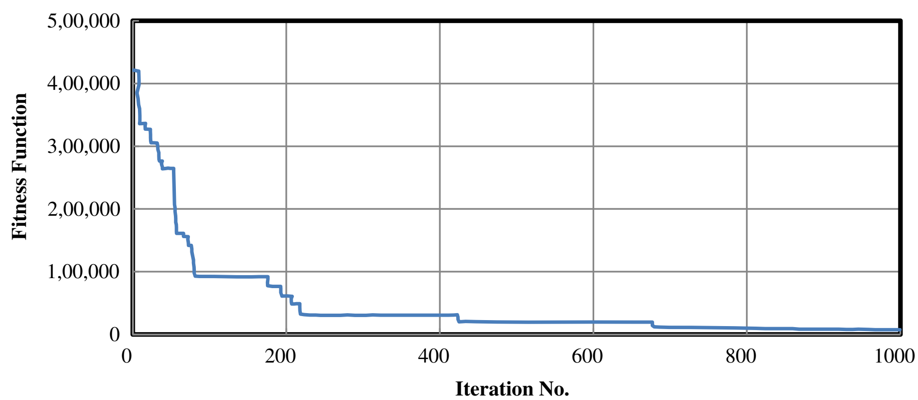


Fig. 9 Convergence rate of the fitness function

Table 3 Initial and optimum values of design variables and objective function

Variable	UTE	$N = 14$	$N = 13$	$N = 12$	$N = 11$	$N = 10$	$N = 9$
B	33.990	22.024	23.687	25.625	27.910	30.611	33.976
α	12.147	17.727	17.848	17.973	18.093	18.215	18.340
A	1.820	1.000	1.001	1.000	1.002	1.002	1.000
P	9.140	7.893	7.894	7.894	7.895	7.882	7.896
t_{w}	1.065	1	1.000	1.000	1.000	1.000	1.000
H_{T}	5.790	7.106	7.106	7.106	7.107	7.119	7.111
W	255.807	225.154	224.350	223.527	222.676	221.520	220.755
l_{c}	73.176	48.246	51.772	55.881	60.730	66.456	73.591
w	18.271	16.082	17.257	18.627	20.243	22.152	24.528
$H_{\mathrm{T}} + P$	14.930	15.000	15.000	15.000	15.002	15.001	15.007
H_{T}/P	0.633	0.900	0.900	0.900	0.900	0.903	0.900
w/P	1.999	2.037	2.186	2.359	2.564	2.810	3.106
A/w	0.099	0.062	0.058	0.053	0.049	0.045	0.040
Q	16181.880	15569.960	15569.140	15570.260	15568.860	15566.030	15559.260
C_d	0.383	0.412	0.413	0.415	0.416	0.417	0.419
l/w	4.004	2.999	2.999	2.999	3.000	3.000	3.000
A/t_{w}	1.708	1.000	1.001	1.000	1.002	1.002	1.000
V_{T}	**9972.353**	**7331.845**	**7977.882**	**7908.075**	**7876.164**	**7851.233**	**7831.125**
%	–	**21.47**	**21.47**	**21.02**	**20.71**	**20.21**	**19.45**

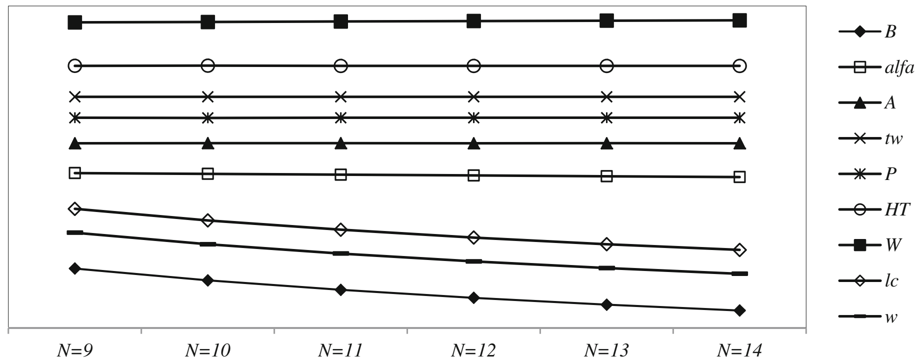


Fig. 10 Variations of design variables considering different cycle numbers

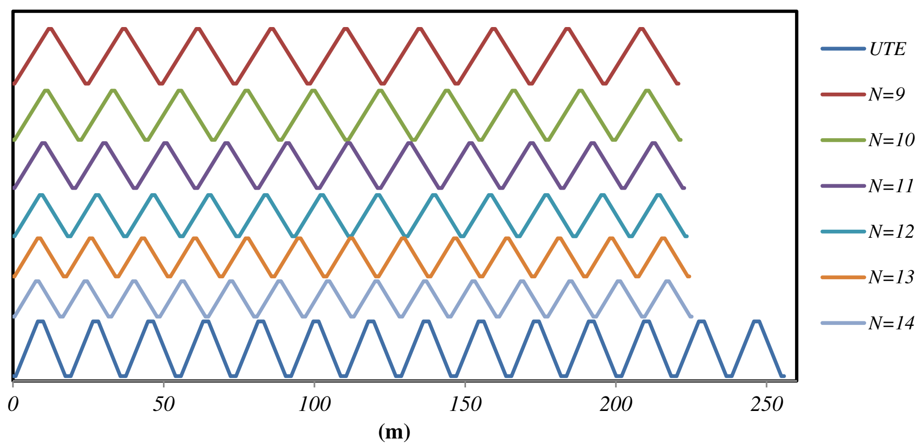


Fig. 11 Shape of labyrinth weir at initial and optimum designs with different cycle numbers

ables in using 9 to 14 cycle numbers are presented in Fig. 10. The difference between the initial and optimum design shapes due to different cycle numbers also be clearly seen in Fig. 11.

As can be seen in Fig. 10, some of the variables such as B, α, A, P, t_w, and H_T remained constant at different cycle numbers in which increase or decrease in cycle numbers has not considerable effect on these parameters. The three important and effective design variables in shape optimization of labyrinth weir will be the width of labyrinth weir in each cycle (w), leg of weir (B) and the effective length of side leg (l_c).

Comparison of the real and optimized values of weir's width and leg in one cycle for different cycle numbers is presented in Fig. 12. The real and optimum values of width and leg of the weir in one cycle are also summarized in Table 4.

It is obvious that increasing the cycle numbers of labyrinth weir leads to considerable reduction in width and leg of weir in one cycle, which causes significant decrease in total volume of labyrinth weir. In applying 9 cycles, although reduction of the total volume of the labyrinth weir is resulted, the width of weir in one cycle has increased about 25% regarding the real one. The width of the weir continues to increase, until in a cycle of 13, reduction in the weir width is initiated. Reduction of the weir leg in one cycle, however,

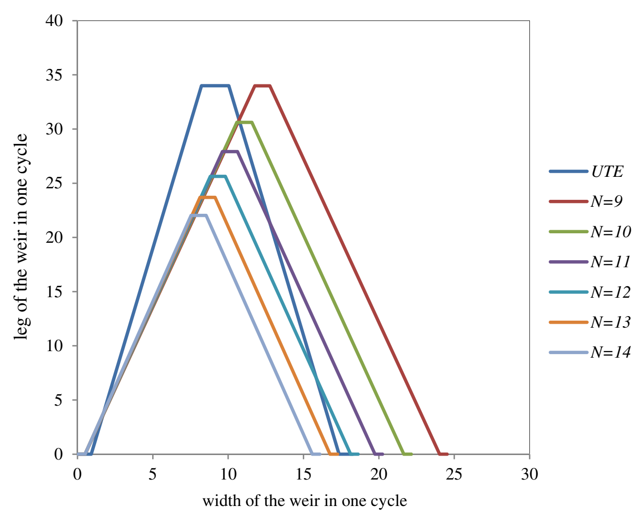


Fig. 12 Comparison of one cycle of the labyrinth weir for different cycle numbers

has been observed in all cycle numbers. It can be concluded that the labyrinth weir is mostly sensitive to the width of each cycle than that of the cycle leg. Another view of the labyrinth weir comparison in one cycle for different cycle numbers is given in Fig. 13.

Table 4 Real and optimum values of width and leg of the labyrinth weir in one cycle for different cycle numbers

Cycle numbers	Ute	$N = 9$	$N = 10$	$N = 11$	$N = 12$	$N = 13$	$N = 14$
Width of the weirs in one cycle (m)	18.271	24.528	22.152	20.243	18.627	17.257	16.082
Decreasing of weir width in one cycle (%)	–	−25.5	−17.5	−9.741	−1.91	+5.59	+11.98
Length of the weir in one cycle (m)	33.990	33.976	30.611	27.910	25.625	23.687	22.024
Decreasing of weir length in one cycle (%)	–	+0.041	+9.94	+17.88	+24.61	+30.31	+35.20

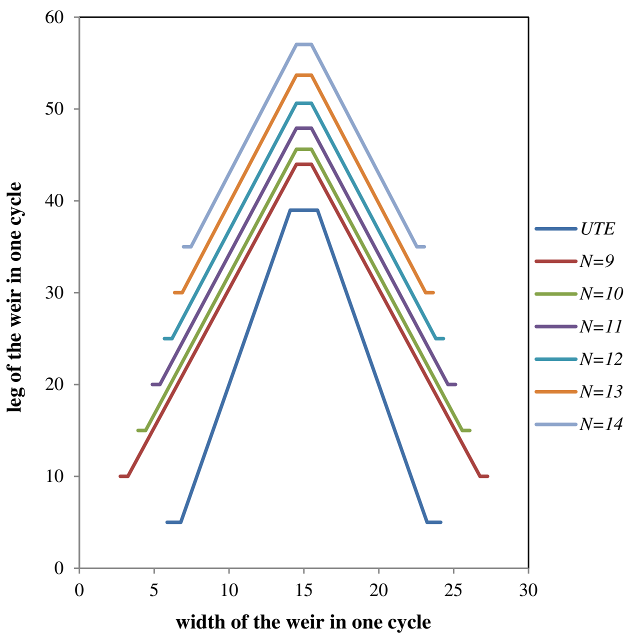


Fig. 13 Other comparison of labyrinth weir in one cycle for different cycle numbers

After performing the optimization process, the labyrinth weir total volume decreased by 21% in comparison with that in the initial design.

6 Conclusion

In this paper, a new methodology is developed for shape optimization of trapezoidal labyrinth weirs. The genetic algorithm was used to reach the optimal solution. Like other meta-heuristic optimization algorithms, GA uses a combination of randomness and exploitation of previously obtained favorable results to perform global optimization. The volume of the labyrinth weir was considered as the objective function. Among different parameters of labyrinth weir selected as design variables, only three parameters, weir width in one cycle, weir leg and the number of cycles, were determined as the most effective parameters in shape optimization of labyrinth weir. The quarter-round shape was also selected for the crest shape. In order to examine the effectiveness of the proposed methodology, optimal design of the Ute Dam labyrinth weir is performed. The optimized volume of labyrinth weir as well as the design variables values is obtained and compared with the real ones. The results indicated that increasing the cycle numbers of labyrinth weir leads to considerable reduction in width and leg of weir in one cycle and as a result to reduction the total volume of labyrinth weir. The optimum volume of the labyrinth weir is 2640 m^3 less than the initial volume, i.e., 21.47% less.

For future extension of this work and for more efficient optimal design, two procedures can be considered:

- Applying other effective parameters as design variable in optimization, such as discharge over weir, crest shape and weir height. It seems that using high number of design variable can lead to more accurate results.
- Using other meta-heuristic algorithms such as ACO, PSO or hybrid the GA with other algorithms (e.g., GA-ACO, GA-PSO or GA-ICA), to reach the new probable and possible results, and comparing them with the presented results in this paper.

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