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A Review of Modeling Thermal Displacement Processes in Porous Media

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Abstract The subject of heat transfer in oil reservoirs has gained huge attention, due to its diverse range of applications in petroleum reservoir management and thermal recovery for enhanced oil recovery. Thermal recovery methods entail the addition of heat energy into the reservoir through injection wells with the aim of reducing the in situ oil viscosity which is usually around several thousand centipoise cP (in S.I unit kg/m s) at reservoir conditions to very low values at steam temperatures. In addition, several other mechanisms are associated with thermal recovery methods. These include thermal expansion of oil, steam distillation, and relative permeability changes, which contribute to the ultimate recovery of the reservoir. In this article, a detailed review of non-isothermal modeling in an oil reservoir is presented. In addition, a few remarks regarding the momentum transport and the energy balance equations and its various modifications through the years are provided. Finally, a memory-based formulation is proposed to capture the alteration of rock and fluid properties with time as well as accounting for other phenomena not described by classic diffusion equations.

Keywords Heat transfer · Reservoir management · Enhanced oil recovery · Memory-based

List of symbols

a_{sf}	Specific surface area (fluid to solid contact) (m^2)
$A(t)$	Cumulative heated area (m^2)
c_F	Non-dimensional form-drag constant
c_p	Specific heat (J/kg K)
C	Component
C_c	Coke concentration (gmol/m^3)
d_p	Spherical particle diameter (m)
Da	Darcy number, κ/L^2, dimensionless
E_H	Heating efficiency, percentage
g	Acceleration due to gravity (m/s^2)
h	Pay thickness (m)
h_{sf}	Fluid to solid heat transfer coefficient (W/m^2 K)
H	Aquifer height (m)
H_o	Heat injection rate (J/s)
k	Thermal conductivity (W/m K)
K	Permeability (m^2)
P	Pressure (Pa)
q	Heat flux (W/m^2)
$Q(t)$	Heat stored in the pay zone (J)
Ra	Rayleigh number, dimensionless
Re_{dp}	Reynolds number based on particle diameter, $\rho u \frac{d_p}{\mu}$ dimensionless
Re_κ	Reynolds number based on permeability, $\rho\sqrt{\kappa}\frac{u}{\mu}$, dimensionless
r	Radial distance (m)
R	Thermal retardation factor
S	Saturation, fraction
t	Time (s)
T	Temperature (K)
$\vec{u}$	Velocity vector (m/s)
$U_{hz}(x, y, z, t)$	Heat flux in the vertical direction (J/s)
v	Heat velocity (m/s)
x_D	Dimensionless distance
z	Vertical distance (m)

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Greek alphabets

α Thermal diffusivity (m^2/s)
α_L Longitudinal dispersivity (m)
α_t Transverse dispersivity (m)
α' Overburden thermal diffusivity (m^2/s)
γ Fractional-order derivative
η Pseudo-diffusivity ($m^3\,s^{2-\gamma}/kg$)
μ Dynamic viscosity (kg/m s)
κ Thermal dispersion tensor (W/m K)
ν Kinematic viscosity (m^2/s)
ρ Fluid density (kg/m^3)
σ_r Mean square variance
τ Dimensionless time
ϕ Porosity, fraction
Γ Standard gamma function
Φ Fluid potential (Pa)

Subscripts

c Coke
e Effective
f Fluid
g Gas
o Oil
p Pore
r Reservoir
s Rock solid matrix
sf Solid-to-fluid interface
w Water
0 Initial
1 Reservoir

1 Introduction

Processes that involve the injection of a hot fluid into a cooler, fluid-saturated, subsurface rock or vice versa are well established and include thermal EOR, detection of water influx in production wells [1], groundwater transport, contaminant transport, heat scavenging, water re-injection in subsurface aquifers, and in geothermal reservoirs applications. Accurate prediction of the performance of any such process requires a model that addresses all heat transfer characteristics of the rock-to-fluid system. However, such mathematical models could be very complex to handle analytically.

Numerous formulations have been presented to model these thermal displacement processes, each focusing on one or more aspects of the problem and adopting different assumptions.

The classical equation describing the flow of single phase or multiple phases through an oil reservoir has been expressed

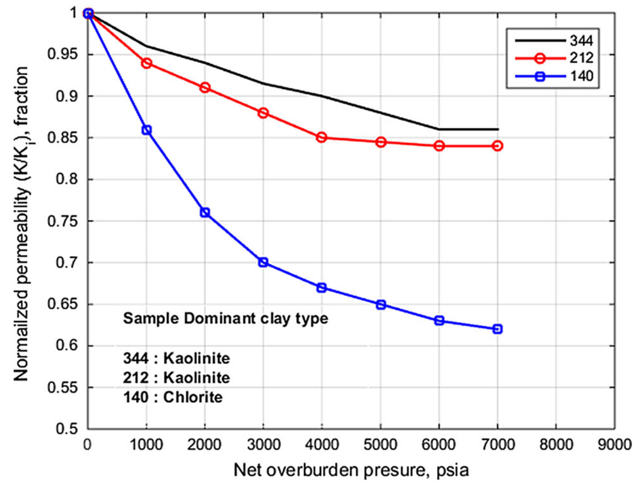


Fig. 1 Effect of net confining stress on permeability. (*Data source*: Ref. [24])

in several publications [2–5] which are based on Darcy's law. This empirical equation was developed based on the assumption of homogeneous and isotropic rock. However, different modifications have been proposed to improve the accuracy of Darcy equation. Each modification accounts for the effect of different observed phenomena, i.e., convective acceleration within the porous media [6], slip, desorption, and non-Darcy flow. Now you might ask are such modifications always necessary?

In order to macroscopically describe the flow through an oil reservoir, it is necessary to introduce variables that take into account the space left by the solid matrix to the fluid. One of them is the porosity represented either in percentage or in fraction, it is defined as the ratio of the pore volume to the bulk volume of the rock. Second and more important is the rock permeability (with a dimension of L^2), described as a measure of the ability of the rock to transmit a fluid. The above rock properties have been established to be dependent on grain shape and size distribution, confining pressure, temperature (due to hot water injection or steam injection), stress, and reservoir process [7–35]. Due to the unconsolidated nature of heavy oil reservoirs, thermal operations usually result in particle mobilization [35]. The subsequent particle migration leads to pore throat plugging which is one of the root cause of permeability reduction. Figure 1 presents the results observed by Amaefule et al. [24], showing the permeability reduction observed on three core samples with increasing confining pressure.

In this survey, key issues in the literature have been grouped into categories where each category addresses one aspect of the problem pointing out the strengths and weaknesses associated with them. The main goal of this exposition into non-isothermal modeling in an oil reservoir is to enlighten researchers and scientists alike on an alternative means of addressing the effect of thermal alterations in rock properties which may not be known priori in a more efficient manner. Secondly, the incorporation of the concept of anomalous diffusion into mathematical models may provide a more convenient alternative to handling reservoir heterogeneity (natural fractures). It is our opinion that current mathematical formulations or models can be improved trough the incorporation of more generalized constitutive equations due to the nature of the rock fabric i.e. equations relating the volumetric flux to pressure in oil reservoirs, and/or equations relating the conduction heat flux to the temperature.

2 Momentum Transport

There exist several momentum equations proposed to describe fluid flow in porous media, some of which were developed to match empirical observations and interestingly converge to corresponding free-fluid model (Navier–Stokes) when the porosity approaches 100 % and permeability infinity. Accurate modeling of the momentum equation is of the utmost importance as improper velocity distribution introduces error to the temperature distribution through the convective energy flux no matter what sophisticated scheme or algorithm is used to handle the corresponding heat equation. The oldest and most used momentum equation is the Darcy flow model. This model is a form of linear momentum equation, which states that the volumetrically averaged velocity is directly related to the fluid potential gradient in that direction as presented in Eq. 1.

$$\vec{u} = \frac{\mathrm{K}}{\mu}[G - \nabla \mathrm{P}] = \frac{K}{\mu}\nabla \Phi \tag{1}$$

where G is the body force term due to gravity.

$$G = \rho \mathrm{g}. \tag{2}$$

However, in the derivation of Darcy flow model, several simplifying assumptions were made concerning the porous medium and the nature or properties of the flowing fluid(s). It has been established [36], starting from the Navier–Stokes equation, that Darcy law is restricted to flows in which the viscous forces dominate over the inertia forces.

The Darcy flow model as described earlier makes the momentum equation linear, hence the resulting simplicity in solving the diffusivity equation. Amhalhel and Furmański [37] established that the Darcy equation is of one order less than the Navier–Stokes equation and that the no-slip hydrodynamic boundary condition cannot be applied. Furthermore, the maximum velocity is predicted at the impermeable surface. However, if any of the simplifying assumptions, for example, the porous medium, is heterogeneous, non-isothermal conditions prevail, or the fluid interacts either chemically with the rock surface. Darcy law in its simplest form cannot be used to model fluid flow in such systems. There have been other fluid flow models proposed in the petroleum engineering literature; this includes Brinkman-Darcy model [38], Forchheimer-Darcy model [39], Darcy-Brinkman-Forchheimer model [40], and Hsu and Cheng generalized flow model [41], which were derived using some volume average technique from the Navier-Stokes equation see references [6,36,42–45] for more description. Choi et al. [46] studied the influence of inertia and viscous terms on velocity profile. Their results show that viscous forces contribute mostly to the deviation from Darcy flow model.

Fortunately, in many oil reservoirs and aquifers, the Reynolds number based on permeability (Re_K) is $\ll 1$. In such flow conditions, Darcy equation has been established to be appropriate to describe the macroscopic fluid motion [47]. However, Darcy equation in its simplistic form does not account for evolution or variation in rock and fluid properties with time; therefore, it still has to be modified in some way. On the other hand, Darcy equation is not recommended when describing fluid flow in shale reservoirs, naturally fractured Karst reservoirs, artificially created porous media, wormhole modeling in reservoir rocks, and nanomaterials. This is because the Reynolds number in such porous media is greater than unity.

A summary of some of the widely employed constitutive equations in porous media modeling applications is listed in Table 1. Please refer to nomenclature for the definition of terms.

3 Energy Transport

The importance of proper understanding the possible heat transfer modes within any porous media is of great significance for proper prediction of temperature distribution. Two different macroscopic descriptions are available in the literature namely the heterogeneous and homogeneous models.

3.1 Heterogeneous Formulation

This formulation considers the oil reservoir by two coexisting temperature fields, i.e., the solid and fluid phases. In the presence of two temperature fields, there is an additional heat exchange between both phases, i.e., no local thermal equilibrium (NLTE). This approach is key to accurate modeling of highly transient problems and few steady-state problems as pointed out by Nield [48]. The key to the accurate formulation of heterogeneous models lies in the determination of representative heat transfer coefficient (h_{sf}) between both phases [49]. The literature is littered with experimental studies, see [50], and theoretical investigations, see [51–53], on estimating accurate and representative estimates of the heat transfer coefficient for different porous media. According

Table 1 Constitutive equations describing fluid flow in a porous medium

Flow model and year	Equation	Key facts
Darcy law [45]	$\vec{u} = \frac{\mathrm{K}}{\mu}\left[\langle G\rangle - \nabla\{\mathrm{P}\}\right]$	1. Appropriate when permeability-based Reynolds number is less than one 2. Its main limitation is that the no-slip boundary condition cannot be imposed
Forchheimer–Darcy model [39]	$\nabla P - \langle G\rangle = -\frac{\mu}{K}\vec{u} - \rho\frac{c_F\phi}{\sqrt{K}}\vec{u}\,\lvert\vec{u}\rvert$	1. Proposed a velocity square term addition to the Darcy term to account for the inertia effects in the pressure drop 2. Appropriate for very high flow velocities in porous media
Brinkman–Darcy model [38]	$\nabla P - \langle G\rangle = -\frac{\mu}{K} + \frac{\mu_e}{\phi}\nabla^2\vec{u}$	1. Derived for an assembly of spheres 2. Proposed to account for transitional flow between boundaries, i.e., boundary layer flow 3. Introduced the effective viscosity term and the Laplacian of velocity to account for the viscous effects which become significant as the porosity and permeability of the porous media becomes larger 4. Due to the method of its derivation, it has been reported that it is only applicable to porous media with porosity values greater than 0.6 5. Ambiguity in the effective viscosity term, some researchers concluded that the term depends on the porous media geometry
Darcy–Brinkman–Forchheimer [40]	$\frac{\rho}{\phi}\left[\frac{\partial\vec{u}}{\partial t} + \frac{(\vec{u}.\nabla)\vec{u}}{\phi}\right] = \langle G\rangle - \nabla P + \mu_e\nabla^2\vec{u} - \frac{\mu}{K}\vec{u} - \rho\frac{c_F}{\sqrt{K}}\vec{u}\,\lvert\vec{u}\rvert$	1. Difficult to solve numerically 2. The convective term contributes to the inertia effects experienced in a porous media 3. The presence of the convective term is important to high velocity and/or high porosity media. However, its role is not as clear as that of the Forchheimer inertia term and can be best understood as to that of the corresponding free-fluid flow
Hsu and Cheng [41]	$\frac{\rho}{\phi}\left[\frac{\partial\vec{u}}{\partial t} + \frac{(\vec{u}.\nabla)\vec{u}}{\phi}\right] = \langle G\rangle - \nabla P + \mu\nabla^2\vec{u} + B$ Where $B = -\left[\frac{\mu}{K}\vec{u} + \rho\frac{c_F}{\sqrt{K}}\vec{u}\ \lvert\vec{u}\rvert\right]$ B is the total drag force per unit volume (body force) due to the presence of the solid particles	1. Derived starting from starting the Navier–Stokes equations and utilizing the volume averaging 2. As the porosity approaches unity and the permeability of the porous media approaches infinity, the equation reduces to the classical Navier–Stokes equation 3. Difficulty to solve numerically
Generalized model (2008)	$\rho\left[\frac{\partial\vec{u}}{\partial t} + \nabla\left(\frac{\vec{u}.\vec{u}}{\phi}\right)\right] = -\nabla\phi P + \mu_e\nabla^2\vec{u} - \left[\frac{\mu\phi}{K}\vec{u} - \rho\frac{c_F\phi}{\sqrt{K}}\vec{u}\,\lvert\vec{u}\rvert\right] + \langle G\rangle$	Similar to above (Hsu and Cheng model)

to Wakao et al. [54] for the heterogeneous description, the energy balance falls into three classes as presented below.

Schumann Model This model neglects the heat conduction in both phases in the governing energy balance equations.

Continuous Solid Phase (C-S) Model This model accounts for thermal conduction in both phases. In addition, the effective thermal conductivity is introduced, which includes the thermal dispersion effect.

Dispersion–Concentric (D-C) Model This model also uses one equation based on the average fluid temperature, which is coupled to the energy equation for the heat conduction in a single particle [42]. For more information, refer to the

Table 2 Heterogeneous energy balance models

Model	Fluid phase	Solid phase
Schumann model	$\phi\left(\rho c_{\mathrm{p}}\right)_{\mathrm{f}}\frac{\partial T_{\mathrm{f}}}{\partial t}+\phi\left(\rho c_{\mathrm{p}}\right)_{\mathrm{f}}\vec{u}.\nabla T_{\mathrm{f}}=h_{\mathrm{sf}}a_{\mathrm{sf}}\left[T_{\mathrm{s}}-T_{\mathrm{f}}\right]$	$(1-\phi)\left(\rho c_{\mathrm{p}}\right)_{\mathrm{s}}\frac{\partial T_{\mathrm{s}}}{\partial t}=h_{\mathrm{sf}}a_{\mathrm{sf}}\left[T_{\mathrm{s}}-T_{\mathrm{f}}\right]$
Continuous solid phase (C-S) model	$\phi\left(\rho c_{\mathrm{p}}\right)_{\mathrm{f}}\frac{\partial T_{\mathrm{f}}}{\partial t}+\phi\left(\rho c_{\mathrm{p}}\right)_{\mathrm{f}}\vec{u}.\nabla T_{\mathrm{f}}=\nabla.\left(k_{e}.\nabla T_{\mathrm{f}}\right)+h_{\mathrm{sf}}a_{\mathrm{sf}}\left[T_{\mathrm{s}}-T_{\mathrm{f}}\right]$	$\nabla.\left(k_{es}.\nabla T_{\mathrm{s}}\right)+h_{\mathrm{sf}}a_{\mathrm{sf}}\left[T_{\mathrm{s}}-T_{\mathrm{f}}\right]=(1-\phi)\left(\rho c_{\mathrm{p}}\right)_{\mathrm{s}}\frac{\partial T_{\mathrm{s}}}{\partial t}$
Dispersion–concentric (D-C) model	$\frac{\partial T_{\mathrm{f}}}{\partial t}+\vec{u}.\nabla T_{\mathrm{f}}=\alpha'_{ax}\nabla^{2}T_{\mathrm{f}}+\frac{h_{\mathrm{sf}}a_{\mathrm{sf}}}{\phi\left(\rho c_{\mathrm{p}}\right)_{\mathrm{f}}}\left[T_{\mathrm{s}}-T_{\mathrm{f}}\right]$	$\frac{\partial T_{\mathrm{s}}}{\partial t}=\alpha'_{\mathrm{s}}\left(\frac{\partial^{2}T_{\mathrm{s}}}{\partial r^{2}}+\frac{2}{r}\frac{\partial T_{\mathrm{s}}}{\partial r}\right)$

manuscript of Wakao et al. [54]. For a more thorough review on the NLTE, readers should refer to the following references [55–58]. However, the heterogeneous energy balance models are unpopular in petroleum engineering literature and never really applied in practice. Recently, Hossain and Abu-Khamsin [59] employed the Schumann model to describe hot water injection process in an oil reservoir. Table 2 provides a summary of the above-described heterogeneous energy balance models.

3.2 Homogeneous Formulation

This formulation neglects the heat transfer between the fluid and solid phases. Hence, only a single temperature exists at any point in the porous media. A condition referred to in the literature as local thermal equilibrium (LTE), where only a single energy equation is required to describe the oil reservoir by two coexisting temperature fields, i.e., the solid and fluid phases [60].

The assumption of LTE was investigated by Wong and Dybbs [61]. They concluded that LTE holds for flow rates where the pore diameter-based Reynolds number, Re_{p} (refer to nomenclature for definition), is smaller than ten. It has also been reported in the literature that the Darcy number, *Da* (refer to nomenclature for definition), has the most influence in determining the validity of the LTE [62]. Therefore, it has been concluded that the LTE is only applicable for very small values of particle Reynolds number and the Darcy number. Vasdaz [63–65], pointed out that the LTE applies generally applied for boundary conditions which are a mixture of Dirichlet and insulation type. The complete form of the above-mentioned homogeneous energy equation is presented in Eq. 3.

$$\left[\phi+(1-\phi)\frac{\left(\rho c_{\mathrm{p}}\right)_{\mathrm{s}}}{\left(\rho c_{\mathrm{p}}\right)_{\mathrm{f}}}\right]\frac{\partial T}{\partial t}+\vec{u}.\nabla T=\nabla.\left(k_{e}.\nabla T\right) \quad (3)$$

3.3 Heat Transfer Mechanisms

The major difference between the mechanism of heat transfer in an oil reservoir and a solid body stems from the inherent nature of porous materials. From the literature [66–72], heat can be transferred in fluid-saturated porous media, by a combination of different mechanisms, namely heat convection, hydrodynamic/mechanical dispersion, radiation, thermal conduction, and forced convection.

Thermal conduction involves the transfer of heat from the porous media to the impermeable confining layers (overburden and under-burden) and also within the porous media. Its importance is dependent on the magnitude of thermal conductivities of the rock and fluid.

Convective heat transfer differs from the heat transfer due to forced convection. This accounts for the heat transferred between the injected fluid, the original reservoir fluids, and the solid material [73]. However, the low velocities encountered in most oil reservoirs justify the use of the LTE model. Generally, temperature equilibrium is attained under 1 s for 1 mm diameter grains, and in 1 min for 1 cm and in 2 h for 10 cm [74].

Hydrodynamic/mechanical dispersion results from velocity variations, which arise from the velocity profile in a single pore, the velocity differences between different pores in the porous media and the tortuosity. For instance, Fig. 2 shows two fluid parcels starting near each other at locations B and C dispersed to locations farther apart B' and C' during transport in the pore space.

The overall diffusion coefficients in the longitudinal and transverse directions are defined by:

$$\kappa_{\mathrm{L}}=k+\alpha_{\mathrm{L}}\rho c_{\mathrm{p}}u_{\mathrm{L}} \quad (4)$$

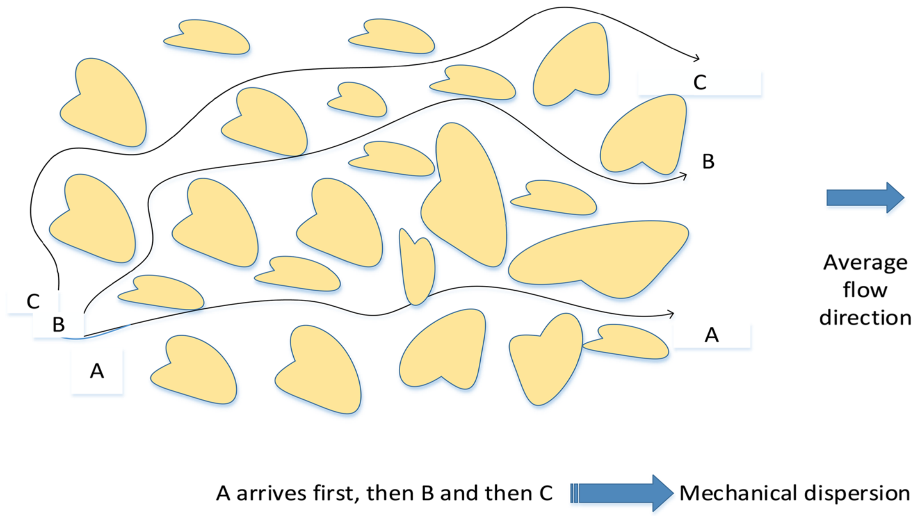
and

$$\kappa_{t}=k+\alpha_{t}\rho c_{\mathrm{p}}u_{t} \quad (5)$$

Dispersion is usually several orders of magnitude lower than heat conduction leading to its effect neglected in many heat transfer models [75]. For a detailed overview on thermal dispersion, readers should refer to the following references [44,76–82].

Radiation can be described through the electromagnetic wave theory, and it is independent of temperature and the thermodynamic properties of the medium. Its effect is usually neglected in many heat transfer models due to difficulty in its quantification at a given point in the medium [83].

Fig. 2 Mechanical dispersion in ground water transport (Redrawn from Ref. [245])



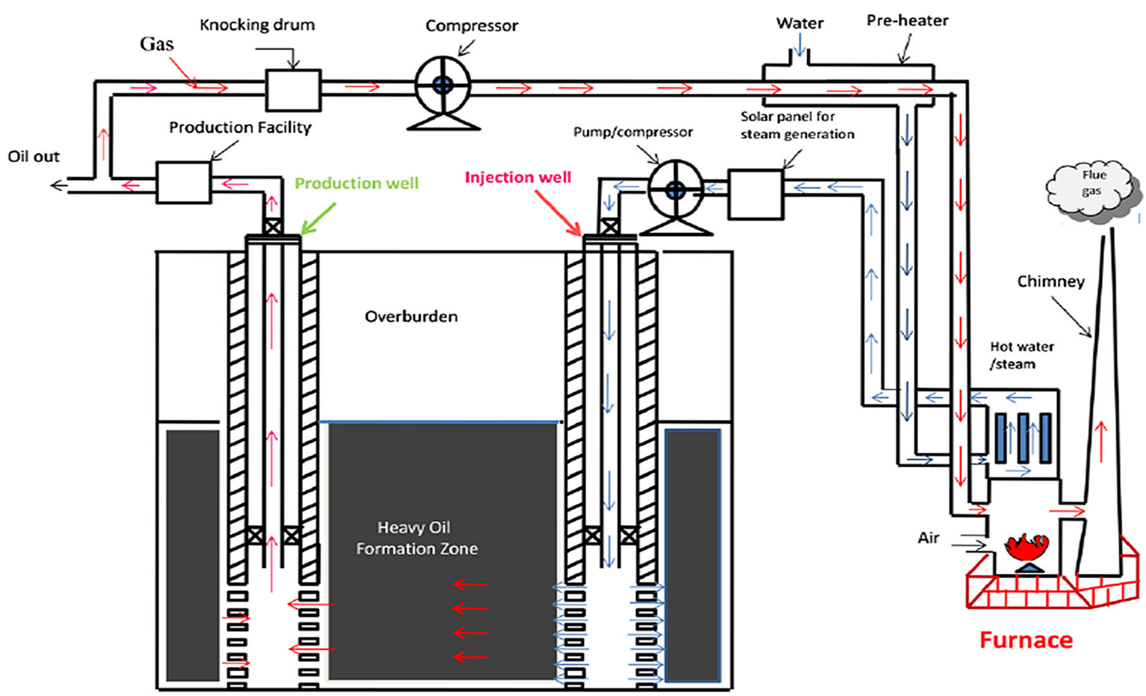


Fig. 3 Mechanism of oil recovery scheme using injection and production wells in an oil field reservoir. (*Source*: Ref. [200])

4 Steam Flooding

The observed reduction in oil viscosity μ_o during the injection of heat energy is key to effective recovery in heavy oil reservoirs. Similarly, there is an observed reduction in the viscosity of water μ_{w} but to a lesser degree. However, it has been acknowledged that the benefit of increased temperature is improving the water-to-oil mobility ratio [84–87]. The most successful and widely used process for heating a reservoir is the steam injection.

Steam injection applications in heavy oil reservoirs date back to the 1960s. The most common application of steam injection is steam flooding, also referred to as steam drive or steam displacement. The process in simple term describes the continuous injection of steam through injection well(s) with the aim of displacing original reservoir fluids toward the production wells as shown in Fig. 3. In a perfect scenario, a steam-saturated zone is formed around the injection well, with a temperature almost equal to that of the injected steam.

Predicting reservoir response during a steam flood is very important for reservoir engineering applications, and proper reservoir management and evaluation require tools or models, which accurately predict steam-flood parameters, for example the oil-to-steam ratios (OSR). A thorough review of some of the published steam-drive models indicates that the predictive capability of these models is inadequate and that improvements are needed for improved evaluation of steam-drive projects [59,88]. The available prediction techniques can be classified into three groups, namely empirical correlations, analytical models, multi-component, and multiphase numerical simulations.

Four different zones have been observed between the steam injection well and the producer, each with its own pressure, temperature, and saturation [89].

4.1 Empirical and Analytical Steam-Flood Models

In general, numerous analytic models haven been presented to predict the temperature distribution in an oil reservoir. The earlier equations were derived based on pure convective type flow in linear and radial reservoirs, for example equations presented by Lauwerier [90], Marx and Langenheim [91], Ramey [92], Malofeev and Scheinman [93], Rubinshtein [94], Mandl and Volek [95], and Avdonin [96,97]. These authors presented analytical solutions to describe the temperature distribution, thermal invasion rate, heat injection (H_o) rate required to raise the temperature to another temperature, heating efficiency (E_H), cumulative heated area as a function of time (A), theoretical economic limits for sustained hot fluid injection, to describe the injection of hot water into an oil-bearing layer. Each model is an improvement over the other by accounting for practical effects. Take for instance, radial heat conduction both within and outside the reservoir, vertical conduction within the reservoir, variable heat injection rate, no restriction on the direction of development of the heated area [92], and finite longitudinal and transverse conductivity [96,97]. Following the approach by Lauwerier [90], the temperature distribution in an oil layer can be described by Eq. 6. Refer to nomenclature for variables introduced.

$$T = T_i \operatorname{erfc}\left(\frac{\xi + |\eta| - 1}{2\sqrt{\theta(\tau - \xi)}}\right)\alpha(\tau - \xi) \quad (6)$$

where

$$\eta = \begin{cases} \frac{y}{b} & \text{for } |y| > b \\ 1 & \text{for } |y| < b \end{cases} \quad (7)$$

$$\alpha(\tau - \xi) = \begin{cases} 1 & \text{for } \tau \geq \xi \\ 0 & \text{for } \tau < \xi \end{cases} \quad (8)$$

The above quantities are defined as:

$$\xi = \frac{k_2}{b^2 c_p \rho_1 u} x, \quad \theta = \frac{\rho_1 c_{p1}}{\rho_2 c_{p2}}, \quad \tau = \frac{k_2}{b^2 \rho_1 c_{p1}} t \quad (9)$$

x = distance in flow direction, m
b = half the formation thickness, m
erfc = complementary error function

Marx and Langenheim [91] were able to predict the cumulative heated area within the oil layer when subjected to heat using Eq. 10. Refer to nomenclature for variables introduced.

$$A(t) = \left[\frac{H_o M h \alpha_2'}{4k^2 (T_{inj} - T_0)}\right]\left[e^{x^2} erf\, x + \frac{2x}{\sqrt{\pi}} - 1\right] \quad (10)$$

where

$$x = \left(\frac{2k}{Mh\alpha_2'}\right) t^{\frac{1}{2}},$$
$$M = \left[(1-\phi)\rho_r c_{pr} + S_w \phi \rho_w c_{pw} + S_o \phi \rho_o c_{po}\right] \quad (11)$$

Later, Ramey [92] extended the work of Marx and Langenheim [91] by considering the case of variable heat injection rate. He proposed that the cumulative heated area could be predicted with Eq. 12.

$$A(t) = \left(\frac{H_o(t)}{2\rho_1 c_1 b \Delta T}\right) * \left(e^{x^2} \operatorname{erfc} x\right) \quad (12)$$

The term (∗) refers to the convolution of two functions. Refer to nomenclature for variables presented.

Rubinshtein [94] derived an equation for predicting the heating efficiency (E_H) in the oil layer in between the overburden and under-burden formation using Eq. 13.

$$E_H = 1 - (1-\beta) \times \left\{\frac{\sqrt{\gamma\tau}}{\pi}\left[1 - (1-\beta)\sum_{n=1}^{m} \beta^{n-1}\left(1 + \frac{n^2}{\gamma\tau}\right)e^{-n^2/\gamma\tau}\right] + (1-\beta)\sum_{n-1}^{\infty} n\beta^{n-1}\left(1 + \frac{2n^2}{3\gamma\tau}\right)\operatorname{erfc}\frac{n}{\sqrt{\gamma\tau}}\right\} \quad (13)$$

where

$$\beta = \frac{\gamma a - 1}{\gamma a + 1}, \quad \gamma = \frac{k_1}{k_2}, \quad \text{and} \quad a^2 = \frac{k_2 \rho_1 c_1}{k_1 \rho_2 c_2} \quad (14)$$

Prats [98] investigated the thermal efficiency of thermal recovery processes in oil reservoirs using the same method as Marx and Langenheim [91]. However, he introduced far greater generality. The heat stored $Q(t)$ in the formation was said to be divided into two parts: the heat in the pay zone near the injection well and the heat in the pay zone far from the injection well. Furthermore, he presented and solved an energy balance using the Laplace transform to obtain an estimate of the heat stored in the pay zone. The presented heat balance equation is of the form:

$$Q(t) = \frac{dH(t)}{dt} + 2\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} U_{hz}(x, y, 0, t)\, dx dy \quad (15)$$

where $z = 0$ is the interface plane between the pay zone and the adjacent zone. Refer to nomenclature for other terms. Furthermore, he solved Eq.15 using the Laplace transform to get the heat stored in the pay zone as:

$$H(t) = \int_0^t Q(t') K\left(\theta_2\sqrt{t-t'}\right) dt' - F \int_0^t H_o(t')\, dt \;\; K\left(\theta_2\sqrt{t-t'}\right) dt' \quad (16)$$

The dimensionless parameters presented above can be expressed as follows:

$$K(z) = e^{z^2} \operatorname{erfc} z \quad (17)$$

$$\theta_2 = \frac{\lambda_{h2z}}{b\sqrt{\alpha_2}\,(\rho c)_1} \quad (18)$$

$$F = \frac{(\rho c)_1 - (\rho c)_2}{(\rho c)_1} \quad (19)$$

Davies and Silberberg [99] proposed a performance prediction model for five-spot steam floods based on the contributions of Marx and Langenheim [91] and Buckley and Leverett [100]. For each radial segment, they obtained the position of the steam front (through Buckley–Leveret solution to the material balance equation) and the temperature distribution ahead of the steam front by heat balance equations. They also estimated the amount of heat loss to the surrounding formations.

Ali [101] proposed a steam-flood model that incorporated the effect of relative permeability to predict the displaced oil from the moving steam chamber. In addition, he presented some post- breakthrough calculations; however, he neglected the effects of the heat lost through produced fluids.

Willman et al. [102] presented an analytical solution to the position of the steam front (steam zone advance) during a steam flood in a radial homogeneous reservoir. The solution can also be extended to include a variable rate by a superposition method. In addition, they suggested a calculative procedure using the temperature gradient to model the displacement in the hot liquid region moving ahead of the steam front.

Mandl and Volek [95] introduced for the first time the "critical time" to account for the heat transfer ahead of the steam front. They pointed out that after the critical time, it was paramount to account for convective heat transfer ahead of the steam front. The Myhill and Stegemeier [103] model combines some aspects of theories by Marx and Langenheim [91] and Ali (1982) to predict the volume of the steam zone and oil steam ratio. Using the principles of segregated flow, Van Lookeren [104] proposed a technique for predicting the geometry of the growing of steam envelope (zone) during steam injection applications into an oil reservoir (linear and radial steam-flood models). The author accounted for the effect of the steam override. In addition, the model was in agreement with results obtained from scaled laboratory experiments, steam injection projects that were available, and calculations from a numerical simulation. He observed that the steam injection rate, pressure, and effective formation permeability to steam played an important role in determining the shape of the steam envelope (zone).

Jones [105] proposed model is an extension of the model proposed by Myhill and Stegemeier (1987) through the introduction of the capture efficiency. The model converts the oil displacement rate obtained from the results of Myhill and Stegemeier (1987) steam flood to the corresponding actual oil production rate using his correlation obtained from the results of 14 different steam-flood projects. He assumed that any steam flood consists of three production stages: The first stage is controlled by initial oil viscosity, the second stage is controlled by hot oil mobility and reservoir permeability, and the final stage is dominated by the remaining mobile fraction of original oil in place.

Jensen et al. [106] presented an improved steam-drive model over models proposed by Myhill and Stegemeier [103], van Lookeren [104]. The proposed model is based on reservoir information and operating conditions from various field-scale steam-drive projects. The new model employed the use of correlations to predict steam-flood process parameters both before and after steam breakthrough. Most importantly, the proposed model showed greater accuracy over existing models when compared with some 15 field-scale steam-drive projects.

Interestingly, Bödvarsson [107] was the first researcher to investigate the propagation of the thermal front in a porous media fully saturated with a single fluid (single-phase fluid flow). He assumed the heat transferred through thermal conduction was very small compared to that by thermal convection. This lead to a simplified energy balance, from which he derived analytically an equation for the position of the thermal front. Furthermore, he showed that the temperature front always lagged the fluid front by a constant related to the ratio of the heat capacity of rock and fluid. Some years later, Bödvarsson and Tsang [68] proposed an analytical model that predicted the rate at which the thermal front advanced during the injection of cooler water into a fractured geothermal reservoir. They assumed the geothermal system was comprised of equal-spaced, horizontal fractures, each intersecting the injection well. Furthermore, it has been established that for many practical scenarios, the effect of thermal conductivity was negligible for heat transport in homogeneous geothermal porous media [108].

An analytical solution was derived by Ziagos and Blackwell [109] to predict the temperature in an underground thin aquifer. The authors in the proposed equation took into account the heat transferred through conduction into both overburden and under-burden layers following the injection of a hot fluid using Fourier transform technique. They assumed that the aquifer was of infinite size in the horizontal direction.

Chen and Sylvester [110] presented an evaluation of three existing analytical steam-flood models: Jones [105], Ali [101], and Miller and Leung [111]. For each model presented, they considered the oil recovery mechanism(s), the associated predictive capability and comparison with field data. In addition, each of the presented models was improved to help its ability to predict production rate and/or history match for typical field production data.

Chandra and Mamora [112] modified Jones' model due to its inadequacy in properly predicting the peak oil production. The predicted peak oil production was always smaller than that observed in the field. This modification was achieved through results obtained from the simulated performance of a 5-spot steam-flood pattern. Furthermore, the improved model was found to give satisfactory production performance up to 20 years for the simulated steam flood.

Shook [113] investigated the effect of thermal conductivity for flow in heterogeneous media and concluded that neglecting the effect of thermal conductivity was applicable for heterogeneous non-fractured media. Similarly, Stopa and Wojnarowski [114] derived an expression that can help predict the position of the velocity front, during cold water injection applications in geothermal reservoirs. They noticed the speed of the thermal front was not temperature-dependent. Furthermore, in the developed model, they accounted for temperature-dependent rock and water system.

Ziagos and Blackwell [115] proposed a method that predicts the temperature profile within an unconfined aquifer of semi-infinite thickness. Likewise, they deduced a technique that can predict the extent of the zone of influence as well as its magnitude for any combination of thermal and hydrological parameters. Further improvements to the temperature distribution models accounting for various scenarios have been proposed. Consider for example when we have finite confining layers separated by multiple fractures [116–119], or modeling thermal injection backflow tests [117,120–125], or applications to naturally fractured geothermal reservoirs [126] and development of analogies to tracer transport [127–132].

Lawal and Vesovic [133] developed an analytical model to describe the possible buoyancy-induced natural convection in a one-dimensional, vertical, and semi-infinite reservoir column. The reservoir was assumed to be fully saturated with undersaturated heavy oil and was subjected to a constant temperature from the bottom. For analysis, they assumed that the density and viscosity were temperature-dependent using typical Athabasca bitumen correlation. They showed that the vertical distributions of in situ oil density, velocity, and Nusselt number were consistent with the induced temperature gradient. They concluded that at any time, the oil density increases vertically away from the heat source, a gravitationally unstable condition, which can trigger the onset of convection. The temperature distribution was obtained by Eq. 20:

$$T\,(z,t) = T_0 + (T_1 - T_0)\,\mathrm{erfc}\left(\frac{z}{2\sqrt{\alpha t}}\right) \tag{20}$$

Barends [134] derived an analytical solution that predicts the temperature distribution in porous rocks while considering the effect of convection, conduction, dispersion, and thermal bleeding. He derived equations describing the temperature distribution considering both linear and radial flow situations using the Boltzmann and Laplace transformation methods. These analytical solutions were validated with COMSOL software giving an excellent match. In addition, a sensitivity study was performed with MAPLE for the assessment of specific effects. The solutions were all derived under the assumption of LTE between fluid and rock grains. Accordingly, the temperature profile in the oil layer can be described by Eq. 21.

$$T - T_0 = 2\frac{(T_1 - T_0)}{\sqrt{\pi}}\mathrm{e}^{\frac{xv}{2\alpha}}\int_{\frac{x}{2\sqrt{\alpha t}}}^{\infty}\mathrm{e}^{-\sigma^2-\left(\frac{xv}{4D\sigma}\right)^2}\,\mathrm{erfc}\left[\left(\frac{x^2 h'\sqrt{\alpha'}}{8DH\sigma^2}+\frac{z}{2\sqrt{\alpha'}}\right)\left(t-\frac{x^2}{4\alpha\sigma^2}\right)\right]\mathrm{d}\sigma \tag{21}$$

Miura and Wang [135,136] proposed a modification to the Edmunds and Peterson model [137] used for to predict the cumulative steam-to-oil ratio (CSOR). The analytical model was derived by a combination of the material/energy balance and the gravity drainage theory. In the proposed model, the CSOR was allowed to be a function of the average reservoir properties as well as the time-dependent variables. The time-dependent variables include the injection temperature, temperature of the produced fluids, and the rising chamber height. They proved that the new model was able to predict the CSOR of a well more accurately than the Edmunds and Peterson model and was also verified with field data. In addition, the proposed model could be used to predict the instantaneous steam-to-oil ratio (iSOR).

Recently, Wei et al. [138] derived an analytical solution for the development of the steam chamber during steam-assisted gravity drainage (SAGD) applications in heavy oil reservoirs. According to the authors, the steam chamber shape is affected by the steam injection rate with a convex-like parabola for small injection rates and an inverted triangle shape with increasing steam injection.

All the above analytical models were developed based on flow in the transverse direction only. These one-dimensional models fall into two categories namely.

Boundary conditions dominated models: generally applicable to relatively thin productive layers. Examples of such models include but not limited to models proposed by [90,93,96,97]. This category of models is adequate for thin productive layers.

The second model assumes that the porous rock is so thick that the heat losses to the impermeable surrounding layers can be neglected. Conceptually, the reservoir is assumed to be made up of a highly permeable fracture network and a relatively small permeability matrix blocks superimposed together [73]. Most geothermal models fall into this category.

Kocabas [73] developed for the first time an analytical solution for predicting the transient temperature profile in two-dimensional laterally/vertically confined layer. He accounted for the heat loss to the surrounding strata and considered the effect of both the longitudinal and transverse heat dispersion. An advantage of this model is that it allows for the understanding of the role on the temperature profile the boundary conditions and fluid mechanics controls play at the same time. Due to much larger transition zones observed in reality, the concept of hydrodynamic heat dispersion was incorporated in the model. He assumed an incompressible fluid, with constant linear flow velocities in both directions. The dimensionless temperature profile (T_{D}) was obtained by the application of Laplace transform; see Eq. 22. Interested readers are recommended to review original work by the author for definitions of dimensionless variables.

$$T_{\mathrm{D}} = \frac{1}{2}\left\{\operatorname{erfc}\left(\frac{x_{\mathrm{D}} - \tau}{2\sqrt{\tau}}\right) + \exp\left(x_{\mathrm{D}}\right) \operatorname{erfc}\left(\frac{x_{\mathrm{D}} + \tau}{2\sqrt{\tau}}\right)\right\} - T_{\mathrm{D}\,\sin\upsilon 2} \tag{22}$$

where

$$T_{\mathrm{D}\,\sin\upsilon 2} = \sum_{n=0}^{\infty}\int_0^{t_{\mathrm{D}}} \frac{\exp\left[-\frac{\{(2n+1)z_{Db} - z_{\mathrm{D}}\}^2}{4\tau}\right] + \exp\left[-\frac{\{(2n+1)z_{Db} + z_{\mathrm{D}}\}^2}{4\tau}\right]}{\pi\sqrt{\tau\left(t_{\mathrm{D}} - \tau\right)}} F_{\sin\upsilon}\,\mathrm{d}\tau \tag{23}$$

and

$$F_{\sin\upsilon} = \int_0^{\infty} \frac{1}{\left(\omega^2 + a\right)^{1/2}} \exp\left[-\left\{\frac{\omega^2 + a}{2}\left(t_{\mathrm{D}} - \tau\right)\right\}\right] I_{n+\frac{1}{2}}\left[\frac{\omega^2 + a}{2}\left(t_{\mathrm{D}} - \tau\right)\right] \sin x\omega\,\mathrm{d}\omega \tag{24}$$

In Eqs. (22) to (24), a is a constant equal to 0.25, $I_{n+\frac{1}{2}}$ is the modified Bessel function of order $n + \frac{1}{2}$, ω is the Fourier sine transform variable, $Sinv$ is the inverse Fourier sine, and τ is the time convolution variable. Please refer to the original manuscript for the definition of other dimensionless variables/terms.

Lastly, Li et al. [139] derived a mathematical model to predict the transient temperature distribution in a confined aquifer, bounded in both longitudinal directions by rocks each having different properties. They obtained the semi-analytic solution for the dimensionless temperature distribution using Laplace transform techniques. They considered in their model the effect of advection in the aquifer and the conduction in the porous media. Table 3 provides a summary of some of the various analytical solutions existing in the literature as regards the temperature distribution in a reservoir. In some cases, expressions were derived for the thermal efficiency or temperature profile in the reservoir. In addition, the assumptions made by the authors in terms of the thermal conductivity in different directions and overburden and under-burden rocks are presented.

In summary, the analytical solutions proposed in the literature eliminate a majority of the non-isothermal physical processes due to the simplifying assumptions usually invoked to arrive at them.

4.2 Experimental Studies

A plethora of experimental studies has been conducted over the years, each focusing on different issues some of which include improving oil recovery in heavy oil reservoirs through the addition of pure steam, steam mixed with surfactants, steam with hydrocarbons, and even hot water. Other studies include the effect of temperature on fines migration and subsequent pore throat plugging. In fact, experiments related to water re-injection into subsurface rocks, natural convection, and permeability impairment, etc., are rampant in the literature. Due to space restriction only, literature related to temperature distribution and EOR with hot water, and steam (or its variants) is discussed.

The first study devoted to hot fluid injection was presented by Cheppelear and Volek [140]. These authors considered experimentally and theoretically the heat transfer process by injecting a hotter fluid into an initially cool porous rock saturated with the same fluid and surrounded by heat conducting cap and base rocks. The viscosity dependence was accounted for in their mathematical model, but the specific heat and density of various materials were independent of temperature. The mathematical model was developed assuming a two-dimensional case using a finite difference numerical scheme. Both the theoretical and experimental results indicated that centerline temperatures were significantly higher than boundary temperatures. Likewise, comparisons of experimental and theoretical results with a cold-to-hot viscosity ratio of 19:1 were in reasonable agreement. Their theoretical calculations showed that the effect of the temperature dependence of viscosity was very significant at ratios of

Table 3 Features of some available analytical models related to temperature distribution in an oil reservoir

Author	Flow geometry in reservoir layer	Rock thermal conductivity				Solutions derived for	
		In the reservoir layer		In the surrounding strata		Temp distribution	Thermal efficiency
		Horizontal direction	Vertical direction	Horizontal direction	Vertical direction		
Avdonin [96]	Linear, radial	Finite	Infinite	Zero	Finite	Yes	No
Avdonin [96]	Linear, radial	Zero	Finite	Zero	Finite	Yes	No
Lauwerier [90]	Linear	Zero	Infinite	Zero	Finite	Yes	Yes
Malofeev and Scheinman [93]	Radial	Zero	Infinite	Zero	Finite	Yes	Yes
Rubinshtein [94]	Radial	Finite	Finite	Finite	Finite	No	Yes
Marx and Langenheim [91]	Radial	Zero	Finite	Zero	Finite	No	Yes
Rubinshtein [94]	Radial	Finite	Infinite	Finite	Finite	Yes	No
Willman et al. [102]	Radial	Zero	Finite	Zero	Finite	No	Yes
Kocabas [73]	Linear	Finite	Infinite	Zero	Finite	Yes	No
Barends [134]	Linear, radial	Zero	Finite	Zero	Finite	Yes	No
Lawal and Vesovic [133]	Linear	Zero	Finite	Zero	Infinite	Yes	No
Li et al. [139]	Radial	Finite	Zero	Zero	Finite	Yes	No
Miura and Wang [136]	Linear	Zero	Infinite	Zero	Finite	No	No
Wei et al. [138]	Linear	Finite	Finite	Zero	Infinite	No	No

100:1 to 1000:1, which are typical of those that occur when injecting hot water to flood heavy oil reservoirs.

Similarly, Baker [141–143] conducted experiments to understand the heat transfer mechanisms in a reservoir using steam to displace a water-saturated porous rock assuming radial fluid flow model. In other words, he assumed the steam front to be a right circular cylinder. He was able to measure the temperature profile in the reservoir and the confining layers through some set of fixed thermocouples. This way he could quantify the fraction of the heat subsequently lost to the overburden and substratum. However, all flooding experiments were performed under low pressure (15 psig). Furthermore, the obtained radial temperature distributions matched perfectly the theoretical results of [90,92,144]. Additionally, he calculated the thermal efficiency both numerically and with the experimentally measured temperature values. He realized that higher thermal efficiencies were obtained at higher rates of heat injection. For the case of steam injection, the author found that thermal efficiency decreased with cumulative injected heat and that the heating process was more efficient at higher heat injection rates.

Ferguson [145] carried out further investigations based on the conclusions of Goite and his colleagues [146,147]. He was interested in determining the optimum propane-to-steam mass ratio to achieve the best recovery. He observed a rapid increase in oil production with steam-to-propane experiments as opposed to using pure steam. He concluded that the optimum mass ratio of propane to steam was approximately 5:100.

Tinss [148] conducted experiments on heavy oil samples from Kulin oil field in Indonesia using steam and propane combination. He observed rapid oil production in his experiments for a 5:100 propane-to-steam mass ratio. Furthermore, he noticed an increase in the API gravity of the produced oil, as well as a reduction in oil viscosity. In addition, better injectivity was achieved when propane was combined with steam leading to a reduction in the maximum injection pressure of 85–78 psig.

Rivero [149] conducted experiments using heavy oil samples from Hamaca field. His goal was to understand the benefits of steam additives in the recovery of Hamaca oil. He noticed a similar trend (as above studies) of accelerated oil production. He concluded that the optimum recovery was achieved with a propane-to-steam mass ratio of 2.5:100. Furthermore, Simangunsong [150] carried out both experimental investigation to understand how additives such as propane and petroleum distillate improve the recovery of heavy oil during steam injection. For his experiments, heavy oil samples from San Ardo field under the then current reservoir conditions were studied. He observed a rapid jump in oil production when the injected steam was mixed with the additives. In fact, the oil production increased by about 30 % for 5:100 propane-to-steam injection and 38 % for 5:100 petroleum distillate-to-steam injection. In addition, he

Table 4 Typical data required for thermal reservoir simulation

Group	Property	Requirements
Reservoir	Principal values of the anisotropic absolute permeability and thermal conductivity, assigned to the directions x, y, and z	Three values of permeability and conductivity, respectively, for each block
	Porosity and heat capacity of reservoir rocks	Two values, respectively, for each block
	Relative permeability for each phase	One relation for each phase at each grid block; each relation is a function of saturations and temperature
	Capillary pressure	Two relations as functions of saturations; several pairs allowed
	Reservoir geometry	Specify coordinate system to be used and locations of wells and boundaries
	Rock matrix compressibility	One value for each block
Overburden and under-burden formations	Thermal conductivity and heat capacity	At least one of each for both Caprock and base rock
	Rock density	
Initialization values	Saturations, pressure, temperature, and composition	One value for each variable at each grid block
Fluid property	Density and viscosity of each phase; compressibility of the fluids	One relation for each phase; each relation should depend on temperature, pressure, and possibly composition
	Component properties and K values (for compositional simulation)	Should be a function of pressures and temperature
	Latent heat of vaporization and saturation pressure	Latent heat of vaporization and pressure/temperature relation at saturation for each component that undergoes a phase change
	Enthalpy and internal energy of each phase	A relation for each quantity for each phase as a function of temperature, pressure, and possibly composition
Well and boundary conditions	Specify well type, and inner boundary conditions, rates, pressures, and temperatures	Maximum and minimum values, constraints and penalties

derived a simplified analytical model capable of predicting the steam front position and the cumulative oil recovery for a one-dimensional steam flood. He concluded that the rapid increase in oil production observed was due to the reduction in oil viscosity as a result of mixing steam with propane and petroleum distillate. More recent literature related to experimental studies on thermal EOR applications can be found in references listed here [151–177].

4.3 Numerical Simulation

The use of numerical reservoir simulation for steam flood or hot water injection performance prediction has been reported in the literature with applications dating to over 20 years. With rapid increasing numerical, simulation, and computational capabilities, almost all important reservoir phenomena can be modeled adequately. Non-isothermal numerical models are similar to the conventional black-oil simulation with the additional modeling of the energy balance. That is, thermal effects are considered.

Numerical models have the advantage of encompassing all important physics in terms of accurate modeling of the temperature transients in a reservoir. However, the implementation of numerical models requires proper understanding of the issues that are relevant and important. Table 4 describes the amount of data required for the development of any numerical thermal model.

The earliest numerical models were developed for varying applications encompassing a large spectrum some of which are one-dimensional/two-phase flow and heat transfer neglecting the effect of heat losses [178], estimation of the recharge rate and the time of evolution for a fault charged hydrothermal system [179], economic analysis for comparing costs associated with different thermal recovery schemes [180], compositional steam flooding numerical models [181, 182], equation of state thermal simulation [183], investigating multidimensional heat transfer problems associated with hot water or steam injection into an oil reservoir [144, 184–186], heat flow in fractured carbonate reservoir [187–191], natural convection [192, 193], understanding the effect of temperature-dependent rock properties on three-phase fluid flow during a steam flood [186,194–196], oil recovery correlation applicable to typical heavy oil reservoirs [197,198], studies devoted to investigating the effect of steam distillation and solution gas during steam flooding [199] and application of steam injection for removal of non-aqueous phase liquids from subsurface [70,71]. However, a major drawback of the above studies was the simplistic assumptions incorporated into their numerical models. Take for instance the injected fluid was considered to be non-condensable, temperature-independent rock properties, convection only in one direction, etc.

Hossain et al. [16,200] developed a one-dimensional numerical model to investigate the effects of the reservoir fluid and injection steam velocities on the temperature distribution in a one-dimensional reservoir. For the first case, they assumed that the reservoir rock and fluids had different temperatures. They observed little or no difference between the fluid and rock temperatures. Secondly, they considered when the reservoir rock and fluid temperature were equal. The authors solved the governing energy balance equations using an explicit finite difference scheme. The convective term and diffusive term were discretized using central differencing. Results showed that fluid velocity, initial steam injection rate, and time have strong effects on the temperature profile. However, in both cases, the fluid velocity was assumed to be a linear function of time and was a function of rock and fluid properties.

Cicek [201] considered the steam displacement of oil in a naturally fractured reservoir by developing a three-dimensional, three-phase, compositional, dual-porosity/dual-permeability model. The effects of capillary pressure and gravity were all incorporated into the simulation model. Cicek [202] again presented a detailed study on the effects of the reservoir and operational properties on the performance of steam displacement considering an inverted nine-spot pattern in a naturally fractured reservoir. In both studies, a fully implicit numerical scheme was developed. Subsequently, the Newton–Raphson method was employed to linearize the resulting sets of equations.

Wu et al. [203] developed a model for predicting the breakthrough time for steam during steam injection into heavy oil reservoirs based on the production performance data. However, the authors concluded that due to some features of the model, the model is best applied during the early time period of steam-drive applications and numerical simulations during the latter stages. Recent investigations have shown that temperature variations can lead to continuous alterations in rock and fluid properties [204–206]. This continuous alteration of fluid and pore space can be captured or modeled by fluid memory models especially in geothermal areas [19].

Again, Hossain et al. [15] developed a finite difference numerical model to investigate the permeability, porosity, and pore volume changes that occur during steam flooding process in a reservoir. The following assumptions were during their analysis; instantaneous thermal equilibrium between rock and fluid, the Boussinesq approximation was applicable. Their results showed the reduction in permeability, increase in porosity, and increase in pore volume during the steam injection process. They concluded that higher cumulative oil recovery would be predicted when the alterations of rock properties are included in recovery calculations. However, the authors assumed a constant fluid velocity for the energy balance.

Recently, new mathematical models have been proposed to describe the temperature evolution in a reservoir during steam injection process [59,207]. They included the effects of fluid memory through a modified Darcy law. The model was derived assuming a one-dimensional linear reservoir for both the case of instantaneous thermal equilibrium and unequal fluid and rock temperatures. Their study produced new dimensionless numbers that are specific to and influence the performance of a thermal process in an oil reservoir.

Civan [8] proposed an empirical model to describe the permeability impairment in porous rocks incorporating the contributions from fines deposition and non-isothermal conditions such as steam flooding or hot water injection. He developed a one-dimensional, finite difference, numerical scheme to predict the temperature distribution in a reservoir during non-isothermal conditions assuming thermal equilibrium between the flowing fluid system and the porous matrix. From the numerical results, it became evident that temperature variation had a significant effect on permeability impairment, with a higher degree of permeability impairment observed during non-isothermal conditions than isothermal conditions. The proposed model could easily be extended to two- or three-dimensional cases to account for the dispersion in various directions.

Yoshida et al. [208,209] developed a mathematical model capable of predicting the flow and temperature profile for a system comprising of horizontal wells intersected by transverse fractures. They assumed only single-phase gas flow conditions. Sensitivity studies were conducted to understand the influence of fracture conductivity and the fracture half-length on the temperature behavior of the system. They observed that the wellbore temperature was strongly affected by the fracture half-length and the fracture conductivity. The proposed model is very useful for evaluating created fracture parameters with real-time post-fracture temperature measurements.

Mozaffari et al. [69] developed a three-dimensional, three-phase simulation model to investigate the steam injection process in heavy oil reservoir using a finite difference scheme. Although the proposed numerical approach was rigorous, in that it accounted for three-phase relative permeability, capillary pressure, pressure- and temperature-dependent fluid properties, and interphase mass transfer between water and steam. Yet, the effect of temperature on rock properties (porosity, absolute permeability, and relative permeability) was not included, the oil was assumed to be nonvolatile, and the hydrocarbon gas was considered insoluble in the liquid phases. The authors pointed out that steam injection could result in an overall recovery improvement of almost 60 % from nothing for a fixed period of time.

Very recently, Irawan and Bathaee [210] developed a three-phase mathematical model for the prediction of flow and temperature distribution for water-alternating-gas (WAG) process in a heterogeneous porous media. They included the effects of gravity, turbulence, relative permeability, and capillary pressure. The flow model was developed in the cylindrical coordinate, with the flow in the tangential direction neglected. The governing equations were solved using implicit finite difference scheme. However, the model did not consider temperature-dependent relative permeability and changing rock properties.

Tables 5, 6, and 7 present a summary of the comparison of the treatment of some rock and fluid properties, distribution of components in fluid phases, and features, respectively, in randomly selected non-isothermal simulation models in the literature.

Under certain scenarios, steam injection is not the best possible option for the production of heavy oil reservoirs, for example some shallow reservoirs, or a very deep reservoir due to heat losses either within the reservoir or across the wellbore [199]. Recently, Lasgari [200,201] developed an electrical joule heating simulation model applicable for heavy oil reservoir production applications for the prediction of temperature distribution. They were able to study the effect of water vaporization near the wellbore as well as the effect of water- saturated fractures during the heating process. They concluded that the vaporization of water reduces the generation of heat within the reservoir and that the water saturation and the electrical conductivity of the water within the fractures are critical to the success of the heating process. Other recent technologies include electromagnetic heating [211–220] and downhole heaters [221–229]. It is worth reiterating that although numerical methods incorporate most of the physical processes, they usually need to be validated against benchmark solution to ascertain their accuracy, suffer from reliability problems, large memory requirements, and sometimes excessive computation time.

5 Summary

The literature review reveals a collection of mathematical formulations that vary in their assumptions and approaches to model non-isothermal flow in porous media and, consequently, their accuracies. Unfortunately, most of all the above-formulated lack in a fundamental aspect that is pertinent to thermal EOR operations. That is, accounting for continuous thermal alteration of the characteristics of reservoir rock and fluid.

Additionally, the disordered structure of naturally fractured reservoir rocks (see Fig. 4) has been pointed to be more in line with the anomalous diffusion models, characterized by the mean displacement of particles proportional to the fractional power of time [230,231]. In fact, the transport pathways created by the natural and induced fractures have been shown to be fractals [230]. The petroleum engineering literature is littered with models based on fractal derivatives; for example, refer to references [232–234]. This approach has successfully been applied to capture the stochastic nature of heterogeneity, i.e., natural fractures in reservoirs.

Furthermore, classical diffusion model(s) assume that the random motion of diffusing particles follows a Gaussian probability density characterized with a variance proportional to the first power of time, i.e., the mean square displacement of a particle is a linear function of time. Thus, one might ask what happens when the mean square displacement (variance) grows faster or perhaps slower than the Gaussian diffusion process? A general relationship between the mean square variance and time was presented by [230] as follows:

$$\sigma_r^2 \sim Dt^{\gamma} \text{ where } \begin{cases} \gamma = 1 & \text{Normal diffusion} \\ \gamma \neq 1 & \text{Anomalous diffusion} \\ \gamma > 1 & \text{Super diffusion} \\ \gamma < 1 & \text{Sub diffusion} \end{cases} \tag{25}$$

A comprehensive mathematical model that can incorporate all the factors discussed thus far will be a huge step up to more realistic, robust, and accurate simulation for non-isothermal fluid flow. One of such approaches is the memory-based models (fractional diffusion models), in that they capture the hereditary nature of the porous media. Applications of such models in petroleum engineering are few and can be found in references [19,21,59,235,236].

6 Suggested Future Trends

We propose a generalized constitutive equation of the form presented in Eq. 26 to relate the volumetric flux to the fluid potential in an oil reservoir.

$$\vec{u} = -\eta D_t^{1-\gamma} \nabla \Phi \tag{26}$$

Equation 26 implies a fluid velocity proportional to the time fractional derivative of the gradient of fluid potential in the

Table 5 Comparison of treatment of rock and fluid properties in some available steam-flood simulation models in the literature

Author	Rel perm	Oil/water viscosity	Gas viscosity	Oil/water density	K values (Kv)	Porosity	Permeability
Spillete [144]	$k_r\,(S)$	$\mu_o\,(T)$	$\mu_g\,(P, T)$	$\rho_o\,(T)$	Not applicable	Const.	Const.
Shutler [195]	$k_r\,(S)$	$\mu_o\,(P, T)$	$\mu_g\,(P, T)$	$\rho_o\,(P, T)$	Not applicable	Const.	Const.
Shutler (1970)	$k_r\,(S)$	$\mu_o\,(P, T)$	$\mu_g\,(P, T)$	$\rho_o\,(P, T)$	Not applicable	Const.	Const.
Abdalla and Coats [198]	$k_r\,(S)$	$\mu_o\,(T)$	$\mu_g\,(T)$	$\rho_o\,(T)$	Not applicable	$\phi\,(P)$	Const.
Vinsome [185]	$k_r\,(S)$	$\mu_o\,(T)$	$\mu_g\,(T)$	$\rho_o\,(P, T, C)$	Not applicable	Const.	Const.
Coats et al. [247]	$k_r\,(S, T)$	$\mu_o\,(T)$	$\mu_g\,(T)$	$\rho_o\,(P, T, C)$	Not applicable	$\phi\,(P)$	Const.
Weinstein et al. [246]	$k_r\,(S)$	$\mu_o\,(T, C)$	$\mu_g\,(T)$	$\rho_o\,(P, T, C)$	Not applicable	Const.	Const.
Coats [248]	$k_r\,(S, T)$	$\mu_o\,(T, C)$	Not clear	$\rho_o\,(P, T, C)$	$Kv\,(P, T)$	$\phi\,(P)$	Const.
Abou-Kassem [182]	$k_r\,(S, T)$	$\mu_o\,(T, C)$	$\mu_g\,(T, P)$	$\rho_o\,(T)$	$Kv\,(P, T)$	Const.	Const.
Rubin and Buchanan [249]	$k_r\,(S, T)$	$\mu_o\,(T)$	$\mu_g\,(T)$	$\rho_o\,(T)$	$Kv\,(P, T, C)$	$\phi\,(C_c, \rho_c)$	Const.
Ishimoto et al. [183]	$k_r\,(S, T)$	$\mu_o\,(T, C, P)$	$\mu_g\,(T, P, C)$	$\rho_o\,(T, P, C)$	$Kv\,(P, T, C)$	Const.	Const.
Sarathi [194]	$k_r\,(S, T)$	$\mu_o\,(T, C)$	$\mu_g\,(T, P, C)$	$\rho_o\,(T, P, C)$	$Kv\,(P, T)$	$\phi\,(P)$	Const.
Jensen et al. [187]	$k_r\,(S, T)$	$\mu_o\,(T)$	$\mu_g\,(T, C)$	$\rho_O\,(T, C)$	$Kv\,(P, T, C)$	$\phi\,(P)$	Const.
Class et al. [186]	$k_r\,(S)$	$\mu_o\,(T)$	$\mu_g\,(T, C)$	$\rho_O\,(T)$	Not applicable	Const.	Const.
Cicek [201]	$k_r\,(S)$	$\mu_o\,(T)$	$\mu_g\,(T)$	$\rho\,(T)$	Yes	Const.	Const.
Hossain et al. [204]	Not applicable	Const.	Not applicable	Const.	Not applicable	$\phi\,(T)$	$K\,(T)$
Hossain et al. [235]	Not applicable	Const.	Not applicable	Const.	Not applicable	Const.	Const.
Agarwal et al. [240]	$k_r\,(S)$	$\mu_o\,(T, P)$	$\mu_g\,(P, T)$	$\rho\,(P, T)$	$Kv\,(P, T)$	Const.	Const.
Rousset [250]	Not applicable	$\mu_o\,(T)$	Not applicable	$\rho\,(P, T)$	Not applicable	Const.	Const.
App [251]	Not applicable	$\mu_o\,(T, P)$	Not applicable	$\rho\,(P, T)$	Not applicable	$\phi\,(P, T)$	Const.
Hossain et al. [59]	Not applicable	$\mu_o\,(T)$	Not applicable	Const.	Not applicable	$\phi\,(T)$	$K\,(T)$
Civan [8]	Not applicable	$\mu_{\mathrm{w}}\,(T)$	Not applicable	Const.	Not applicable	$\phi\,(T)$	$K\,(T)$
Mozaffari et al. [69]	$k_r\,(S)$	$\mu_o\,(T)$	$\mu_g\,(T)$	$\rho\,(P, T)$	Not applicable	$\phi\,(P)$	Const.
Yoshida et al. [209]	$k_r\,(S)$	$\mu_o\,(T)$	Not applicable	$\rho\,(P, T)$	Not applicable	Const.	Const.
Irawan and Bathaee [210]	$k_r\,(S)$	Not clear	Not clear	$\rho\,(P, T)$	Not applicable	$\phi\,(P)$	Const.

Table 6 Distribution of components in fluid phases typical of non-isothermal simulation

No	Component	Phases		
		Aqueous	Oleic	Vapor
1	Water	X	–	X
2	Light oil	–	X	X
3	Intermediate oil	–	X	X
4	Heavy oil	–	X	X

reservoir. Subsequently, the fractional derivative operator $D_t^{1-\gamma}$ must be interpreted using a suitable definition; Caputo [237,238], or Reimann–Liouville [239,240]. Such a generalized constitutive equation has the inherent ability to capture both the classic physics and hereditary nature (long memory) of subsurface reservoir rocks. In fact, Eq. 26 simplifies to the classic Darcy equation for certain value of the fractional-order derivative γ.

Incorporating the proposed constitutive equation into the fluid mass balance results in a nonlinear fractional diffusion

Table 7 Major features of some numerical simulation models devoted to modeling steam-flood process in the literature

Researcher	Steam distillation effect	Dimension of reservoir geometry	No of phases	Gravity override effect	No of components in phases		Memory effect	Capillary pressure effect
					Oil	Gas		
Spillete [144]	No	2	2	Yes	1	0	No	Yes
Shutler [195]	No	1	3	Yes	1	2	No	Yes
Shutler (1970)	No	2	3	Yes	1	2	No	Yes
Abdalla and Coats [198]	No	2	3	No	1	1	No	Yes
Vinsome [185]	No	3	3	Yes	1	2	No	Yes
Coats et al. [247]	Yes	3	3	Yes	3	3	No	Yes
Coats [199]	Yes	3	3	Yes	2	2	No	Yes
Weinstein et al. [246]	No	1	3	No	2	2	No	No
Coats [248]	Yes	3	3	Yes	2	2	No	Yes
Abou-Kassem [182]	Yes	2	3	Yes	3	4	No	Yes
Rubin and Buchanan [249]	Yes	2	4	Yes	2	4	No	Yes
Ishimoto et al. [183]	Yes	1	3	Yes	3	3	No	Yes
Sarathi [194]	Yes	2	3	Yes	3	4	No	Yes
Jensen et al. [187]	No	2	3	Yes	1	1	No	Yes
Cicek [201]	No	3	3	Yes	Not applicable	Not applicable	No	Yes
Hossain et al. [204]	No	1	1	No	1	Not applicable	No	No
Hossain et al. [235]	No	1	3	No	1	No	No	No
Civan [8]	Not applicable	1	3	No	Not applicable	No	No	No
App [251]	No	1	2	No	1	No	No	No
Mozaffari et al. [69]	No	3	3	Yes	1	1	No	Yes
Hossain et al. [72]	No	1	3	No	1	1	Yes	No
Lashgari et al. (2015)	No	3	3	Yes	Not applicable		No	Yes
Irawan and Bathaee [210]	Not applicable	2	3	Yes	Not applicable		No	Yes

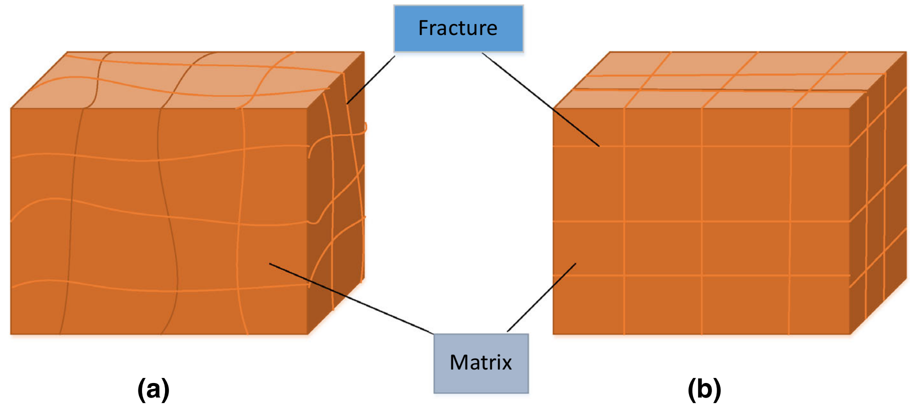


Fig. 4 **a** Actual, and **b** idealized naturally fractured dual-porosity reservoir model (Redrawn from Ref. [252])

model. Numerical schemes for handling fractional diffusion equations are well established in the literature [241–244]. Lastly, calibrating the accuracy of such fractional diffusion models (memory-based models) with thermal flooding experimental data is recommended.

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References

1. Muradov, K.M.; Davies, D.R.: Prediction of temperature distribution in intelligent wells. In: SPE Russian Oil and Gas Technical Conference and Exhibition. Society of Petroleum Engineers (2008)
2. Muskat, M.: The flow of homogeneous fluids through porous media. Soil. Sci. **46**(2), 169 (1938)
3. Nield, D.A.; Bejan, A.: Convection in Porous Media. Springer, New York (2006)
4. Ertekin, T.; Abou-Kassem, J.H.; King, G.R.: Basic Applied Reservoir Simulation. Society of Petroleum Engineers Richardson, TX (2001)
5. Civan, F.: Porous Media Transport Phenomena. Wiley, Hoboken (2011)
6. Yamamoto, K.; Iwamura, N.: Flow with convective acceleration through a porous medium. J. Eng. Math. **10**, 41–54 (1976)
7. Bhat, S.K.; Kovscek, A.R.: Permeability modification of diatomite during hot-fluid injection. J. Pet. Technol. **50**, 98–100 (1998)
8. Civan, F.: Non-isothermal permeability impairment by fines migration and deposition in porous media including dispersive transport. Transp. Porous Media. **85**, 233–258 (2010)
9. Civan, F.: Scale effect on porosity and permeability: kinetics, model, and correlation. AIChE J. **47**, 271–287 (2001)
10. Kolodzie S. Jr.: Analysis of pore throat size and use of the Waxman–Smits equation to determine OOIP in Spindle Field, Colorado: SPE-9382-MS. In: SPE Annual Technical Conference and Exhibition. Society of Petroleum Engineers (1980)
11. Ochi, J.; Vernoux, J.-F.: Permeability decrease in sandstone reservoirs by fluid injection. J. Hydrol. **208**, 237–248 (1998)
12. Pape, H.; Clauser, C.; Iffland, J.: Permeability prediction based on fractal pore-space geometry. Geophysics **64**, 1447–1460 (1999)
13. Ross, C.; Ikeda, M.; Tang, G.-Q.; Kovscek, A.R.: Alteration of reservoir diatomites by hot water injection. In: SPE Western Regional and Pacific Section AAPG Joint Meeting. Society of Petroleum Engineers (2008)
14. Vaidya, R.N.; Fogler, H.S.: Formation damage due to colloidally induced fines migration. Colloids Surf. **50**, 215–229 (1990)
15. Hossain, M.E.; Mousavizadegan, S.H.; Islam, M.R.: The effects of thermal alterations on formation permeability and porosity. Pet. Sci. Technol. **26**, 1282–1302 (2008)
16. Hossain, M.E.; Mousavizadegan, S.H.; Islam, M.R.: Variation of rock and fluid temperature during thermal operation in porous media. Pet. Sci. Technol. **27**, 597–611 (2009)
17. İşcan, A.G.; Kök, M.V.; Bagcı, A.S.: Estimation of permeability and rock mechanical properties of limestone reservoir rocks under stress conditions by strain gauge. J. Pet. Sci. Eng. **53**, 13–24 (2006)
18. Nooruddin, H.A.; Hossain, M.E.: Modified Kozeny-Carmen correlation for enhanced hydraulic flow unit characterization. J. Pet. Sci. Eng. **80**, 107–115 (2011)
19. Caputo, M.: Diffusion of fluids in porous media with memory. Geothermics **28**, 2–19 (1998)
20. Iaffaldano, G.; Caputo, M.; Martino, S.: Experimental and theoretical memory diffusion of water in sand. Hydrol. Earth Syst. Sci. Discuss. **2**, 1329–1357 (2005)
21. Martino, S.; Caputo, M.; Iaffaldano, G.: Experimental and theoretical memory diffusion of water in sand. Hydrol. Earth Syst. Sci. Discuss. **10**, 93–100 (2006)
22. Civan, F.: Temperature effect on advection–diffusion transport involving fines migration and deposition in geological porous media. Article No. 452. In: 2008 IAHR International Groundwater Symposium, Istanbul Turkey (2008)
23. Civan, F.: Predictability of porosity and permeability alterations by geochemical and geomechanical rock and fluid interactions. In: SPE International Symposium on Formation Damage Control. Society of Petroleum Engineers (2000)
24. Amaefule, J.O.; Kersey, D.G.; Marshall, D.M.; Powell, J.D.; Valencia, L.E.; Keelan, D.K.: Reservoir description: a practical synergistic engineering and geological approach based on analysis of core data. In: SPE Annual Technical Conference and Exhibition. Society of Petroleum Engineers (1988)
25. Krumbein, W.C.; Monk, G.D.: Permeability as a function of the size parameters of unconsolidated sand. Trans. AIME. **151**, 153–163 (1943)
26. Pang, Z.; Liu, H.: The study on permeability reduction during steam injection in unconsolidated porous media. J. Pet. Sci. Eng. **106**, 77–84 (2013)

27. Heller, R.; Vermylen, J.; Zoback, M.: Experimental investigation of matrix permeability of gas shales. Am. Assoc. Pet. Geol. Bull. **98**, 975–995 (2014)
28. Liu, S.; Harpalani, S.; Pillalamarry, M.: Laboratory measurement and modeling of coal permeability with continued methane production: part 2-Modeling results. Fuel **94**, 117–124 (2012)
29. Latham, J.-P.; Xiang, J.; Belayneh, M.; Nick, H.M.; Tsang, C.-F.; Blunt, M.J.: Modelling stress-dependent permeability in fractured rock including effects of propagating and bending fractures. Int. J. Rock Mech. Min. Sci. **57**, 100–112 (2013)
30. Manga, M.; Beresnev, I.; Brodsky, E.E.; Elkhoury, J.E.; Elsworth, D.; Ingebritsen, S.E.; Mays, D.C.; Wang, C.: Changes in permeability caused by transient stresses: field observations, experiments, and mechanisms. Rev. Geophys. **50**, 1–24 (2012)
31. Bagci, S.; Kok, M.V.; Turksoy, U.: Determination of formation damage in limestone reservoirs and its effect on production. J. Pet. Sci. Eng. **28**, 1–12 (2000)
32. Schembre, J.M.; Tang, G.; Kovscek, A.R.: Effect of temperature on relative permeability for heavy-oil diatomite reservoirs. In: SPE Western Regional Meeting. Society of Petroleum Engineers (2005)
33. Pang, Z.-X.; Liu, H.-Q.; Liu, X.-L.: Characteristics of formation damage and variations of reservoir properties during steam injection in heavy oil reservoir. Pet. Sci. Technol. **28**, 477–493 (2010)
34. Sola, B.S.; Rashidi, F.: Experimental study of hot water injection into low-permeability carbonate rocks. Energy Fuels **22**, 2353–2361 (2008)
35. Schembre, J.M.; Kovscek, A.R.: Thermally induced fines mobilization: its relationship to wettability and formation damage. In: SPE International Thermal Operations and Heavy Oil Symposium and Western Regional Meeting. Society of Petroleum Engineers (2004)
36. Kaviany, M.: Principles of Heat Transfer in Porous Media. Springer, New York (1995)
37. Amhalhel, G.; Furmanski, P.: Problems of modeling flow and heat transfer in porous media. J. Power Technol. **85**, 55 (1997)
38. Brinkman, H.C.: On the permeability of media consisting of closely packed porous particles. Appl. Sci. Res. **1**, 81–86 (1949)
39. Forchheimer, P.H.: Wasserbewegung Durch Boden. Zeitschrift des Vereines Dtsch. Ingenieure **45**(50), 1781–1788 (1901)
40. Lauriat, G.; Prasad, V.: Non-Darcian effects on natural convection in a vertical porous enclosure. Int. J. Heat Mass Transf. **32**, 2135–2148 (1989)
41. Hsu, C.T.; Cheng, P.: Thermal dispersion in a porous medium. Int. J. Heat Mass Transf. 33, 1587–1597 (1990)
42. Bayram, A.: Non-isothermal flow models with mass diffusion for a stationary porous media by employing representative elementary volume. J. Inst. Sci. Technol. Dumlupinar Univ. Üniversitesi Fen Bilim. Enstitüsü Derg. 23, 49–58 (2010)
43. Batchelor, G.K.; Young, A.D.: An Introduction to Fluid Dynamics. Cambridge University Press, Cambridge (2000)
44. Bear, J.: Dynamics of Fluids in Porous Media. Courier Corporation, Chelmsford (2013)
45. Darcy, H.: Les fontaines publiques de la ville de Dijon. Exposition et application á suivre et des formules á employer dans les questions de duistribution d'eau (1856)
46. Choi, C.Y.; Kulacki, F.A.: Non-Darcian effects on mixed convection in a vertical porous annulus. In: Proceedings of 9th IHTC-Heat Transfer (1993)
47. Barak, A.Z.: Comments on 'High velocity flow in porous media' by Hassanizadeh and Gray. Transp. Porous Media **2**, 533–535 (1987)
48. Nield, D.A.: Effects of local thermal nonequilibrium in steady convective processes in a saturated porous medium: forced convection in a channel. J. Porous Media **1**, 181–186 (1998)
49. Nield, D.A.; Bejan, A.: Heat transfer through a porous medium. In: Convection in Porous Media, pp. 31–46. Springer, Berlin (2013)
50. Yang, J.; Wang, J.; Bu, S.; Zeng, M.; Wang, Q.; Nakayama, A.: Experimental analysis of forced convective heat transfer in novel structured packed beds of particles. Chem. Eng. Sci. **71**, 126–137 (2012)
51. Alazmi, B.; Vafai, K.: Analysis of variants within the porous media transport models. J. Heat Transf. **122**, 303–326 (2000)
52. Carrillo, L.P.: Convective heat transfer for viscous fluid flow through a metallic packed bed. Interciencia **30**, 81 (2005)
53. Quintard, M.; Whitaker, S.: Theoretical Analysis of Transport in Porous Media. Marcel Dekker, New York (2000)
54. Wakao, N.; Kaguei, S.; Funazkri, T.: Effect of fluid dispersion coefficients on particle-to-fluid heat transfer coefficients in packed beds: correlation of Nusselt numbers. Chem. Eng. Sci. **34**, 325–336 (1979)
55. Nield, D.A.: A note on local thermal non-equilibrium in porous media near boundaries and interfaces. Transp. porous media. **95**, 581–584 (2012)
56. Yang, K.; Vafai, K.: Restrictions on the validity of the thermal conditions at the porous-fluid interface-an exact solution. J. Heat Transf. **133**, 112601 (2011)
57. Yang, K.; Vafai, K.: Analysis of temperature gradient bifurcation in porous media-an exact solution. Int. J. Heat Mass Transf. **53**, 4316–4325 (2010)
58. Yang, K.; Vafai, K.: Analysis of heat flux bifurcation inside porous media incorporating inertial and dispersion effects-an exact solution. Int. J. Heat Mass Transf. **54**, 5286–5297 (2011)
59. Hossain, M.E.; Abu-khamsin, S.A.: Development of dimensionless numbers for heat transfer in porous media using a memory concept. J. Porous Media **15**, 18 (2012)
60. Levec, J.; Carbonell, R.G.: Longitudinal and lateral thermal dispersion in packed beds. Part I: theory. AICHE J. **31**, 581–590 (1985)
61. Wong, K.F.; Dybbs, A.: An experimental study of thermal equilibrium in liquid saturated porous media. Int. J. Heat Mass Transf. **19**, 234–235 (1976)
62. Amiri, A.; Vafai, K.: Analysis of dispersion effects and non-thermal equilibrium, non-Darcian, variable porosity incompressible flow through porous media. Int. J. Heat Mass Transf. **37**, 939–954 (1994)
63. Vadasz, P.: Explicit conditions for local thermal equilibrium in porous media heat conduction. Transp. porous media. **59**, 341–355 (2005)
64. Vadasz, P.: Absence of oscillations and resonance in porous media dual-phase-lagging Fourier heat conduction. J. Heat Transf. **127**, 307–314 (2005)
65. Vadasz, P.: On the paradox of heat conduction in porous media subject to lack of local thermal equilibrium. Int. J. Heat Mass Transf. **50**, 4131–4140 (2007)
66. Spillette, A.G.: Heat transfer during hot fluid injection into an oil reservoir. J. Can. Pet. Technol. **4**(04), 213–218 (1965)
67. Sauty, J.P.; Gringarten, A.C.; Landel, P.A.: The effect of thermal dispersion on injection of hot water in aquifers. In: Proceedings of the Second Invitational Well Testing Symposium, Berkeley, California, pp. 122–131 (1978)
68. Bödvarsson, G.S.; Tsang, C.F.: Injection and thermal breakthrough in fractured geothermal reservoirs. J. Geophys. Res. Solid Earth. **87**, 1031–1048 (1982)
69. Mozaffari, S.; Nikookar, M.; Ehsani, M.R.; Sahranavard, L.; Roayaie, E.; Mohammadi, A.H.: Numerical modeling of steam injection in heavy oil reservoirs. Fuel **112**, 185–192 (2013)
70. Falta, R.W.; Pruess, K.; Javandel, I.; Witherspoon, P.A.: Numerical modeling of steam injection for the removal of nonaqueous phase liquids from the subsurface: 1. Numerical formulation. Water Resour. Res. **28**, 433–449 (1992)

71. Falta, R.W.; Pruess, K.; Javandel, I.; Witherspoon, P.A.: Numerical modeling of steam injection for the removal of nonaqueous phase liquids from the subsurface: 2. Code validation and application. Water Resour. Res. **28**, 451–465 (1992)
72. Hossain, M.E.; Abu-Khamsin, S.A.; Al-Helali, A.-A.: A mathematical model for thermal flooding with equal rock and fluid temperatures. J. Porous Media. **18**, 731–744 (2015)
73. Kocabas, I.: Thermal transients during nonisothermal fluid injection into oil reservoirs. J. Pet. Sci. Eng. **42**, 133–144 (2004)
74. Sauty, J.-P.; Gringarten, A.C.; Fabris, H.; Thiéry, D.; Menjoz, A.; Landel, P.A.: Sensible energy storage in aquifers: 2. field experiments and comparison with theoretical results. Water Resour. Res. **18**, 253–265 (1982)
75. Graf, T.: Simulation of Geothermal Flow in Deep Sedimentary Basins in Alberta. Alberta Energy Resources Conservation Board, Alberta (2009)
76. Vafai, K.: Handbook of Porous Media. CRC Press, Boca Raton (2015)
77. Hunt, A.; Ewing, R.; Ghanbarian, B.: Percolation Theory for Flow in Porous Media. Springer, Berlin (2014)
78. Mahdi, R.A.; Mohammed, H.A.; Munisamy, K.M.; Saeid, N.H.: Review of convection heat transfer and fluid flow in porous media with nanofluid. Renew. Sustain. Energy Rev. **41**, 715–734 (2015)
79. Dullien, F.A.L.: Porous Media: Fluid Transport and Pore Structure. Academic Press, London (2012)
80. Rubin, H.: Heat dispersion effect on thermal convection in a porous medium layer. J. Hydrol. **21**, 173–185 (1974)
81. Ripple, C.D.; James, R.V.; Rubin, J.: Packing-induced radial particle-size segregation: influence on hydrodynamic dispersion and water transfer measurements. Soil Sci. Soc. Am. J. **38**, 219–222 (1974)
82. Tyvand, P.A.: Heat dispersion effect on thermal convection in anisotropic porous media. J. Hydrol. **34**, 335–342 (1977)
83. Planck, M.: The Theory of Heat Radiation. Blakiston, Philadelphia (1906)
84. Shah, D.O.: Surface Phenomena in Enhanced Oil Recovery. Springer, Berlin (1981)
85. Lashgari, H.; Lotfollahi, M.; Delshad, M.; Sepehrnoori, K.; De Rouffignac, E.P.: Steam-surfactant-foam modeling in heavy oil reservoirs. In: SPE Heavy Oil Conference-Canada. Society of Petroleum Engineers (2014)
86. Zhao, D.W.; Wang, J.; Gates, I.D.: Thermal recovery strategies for thin heavy oil reservoirs. Fuel **117**, 431–441 (2014)
87. Zhao, D.W.; Gates, I.D.: On hot water flooding strategies for thin heavy oil reservoirs. Fuel **153**, 559–568 (2015)
88. Jensen, T.B.; Sharma, M.P.; Harris, H.G.: A critical evaluation of preliminary design techniques for steam drive projects. J. Pet. Sci. Eng. **5**, 67–85 (1990)
89. Lumley, D.E.; Bee, M.; Jenkins, S.; Wang, Z.: 4-D seismic monitoring of an active steamflood. doi:10.1190/1.1887497 (1995)
90. Lauwerier, H.A.: The transport of heat in an oil layer caused by the injection of hot fluid. Appl. Sci. Res. Sect. A **5**, 145–150 (1955)
91. Marx, J.W.; Langenheim, R.H.: Reservoir heating by hot fluid injection. Pet. Trans. AIME. **216**, 312–315 (1959)
92. Ramey Jr., H.J.: Discussion of reservoir heating by hot fluid injection. Trans. AIME **216**, 364–365 (1959)
93. Malofeev, G.E.; Scheinman, A.B.: The calculation of oil recovery from a stratum upon injecting hot water into it. Neft. Khoz. **41**, 31–35 (1963)
94. Rubinshtein, L.I.: The total heat losses in injection of a hot liquid into a stratum. Neft. Gaz. **2**, 41 (1959)
95. Mandl, G.; Volek, C.W.: Heat and mass transport in steam-drive processes. Soc. Pet. Eng. J. **9**, 59–79 (1969)
96. Avdonin, N.A.: Some formulas for calculating the temperature field of a stratum subject to thermal injection. Neft. Gaz. **3**, 37–41 (1964)
97. Avdonin, N.A.: On some formulae for the calculation of the temperature field of a layer under heat injection. Izv. Vuzov. Neft. Gaz. **3**, 37–41 (1964)
98. Prats, M.: The heat efficiency of thermal recovery processes. J. Pet. Technol. **21**, 323–332 (1969)
99. Davies, L.G.; Silberberg, I.H.: A method of predicting oil recovery in a five-spot steamflood. J. Pet. Technol. **20**, 1050–1058 (1968)
100. Buckley, S.E.: Leverett, Mc: Mechanism of fluid displacement in sands. Trans. AIME **146**, 107–116 (1942)
101. Ali, S.M.F.: Steam injection theories—a unified approach. In: SPE California Regional Meeting. Society of Petroleum Engineers (1982)
102. Willman, B.T.; Valleroy, V. V.; Runberc, C.W.: Laboratory studies of oil recovery by steam injection. SPE 2515 (1960)
103. Myhill, N.A.; Stegemeier, G.L.: Steam-drive correlation and prediction. J. Pet. Technol. **30**, 173–182 (1978)
104. van Lookeren, J.: Calculation methods for linear and radial steam flow in oil reservoirs. Soc. Pet. Eng. J. **23**, 427–439 (1983)
105. Jones, J.: Steam drive model for hand-held programmable calculators. J. Pet. Technol. **33**, 1583–1598 (1981)
106. Jensen, T.B.; Sharma, M.P.; Harris, H.G.: An improved evaluation model for steam-drive projects. J. Pet. Sci. Eng. **5**, 309–322 (1991)
107. Bodvarsson, G.: Thermal problems in the siting of reinjection wells. Geothermics **1**, 63–66 (1972)
108. Woods, A.W.; Fitzgerald, S.D.: The vaporization of a liquid front moving through a hot porous rock. J. Fluid Mech. **251**, 563–579 (1993)
109. Ziagos, J.P.; Blackwell, D.D.: A model for the transient temperature effects of horizontal fluid flow in geothermal systems. J. Volcanol. Geotherm. Res. **27**, 371–397 (1986)
110. Chen, H.-L.; Sylvester, N.D.: Appraisal of analytical steamflood models. In: SPE California Regional Meeting. Society of Petroleum Engineers (1990)
111. Miller, M.A.; Leung, W.K.: A simple gravity override model of steamdrive. In: SPE Annual Technical Conference and Exhibition. Society of Petroleum Engineers (1985)
112. Chandra, S.; Mamora, D.D.: Improved steamflood analytical model. In: SPE International Thermal Operations and Heavy Oil Symposium. Society of Petroleum Engineers (2005)
113. Shook, G.M.: Predicting thermal breakthrough in heterogeneous media from tracer tests. Geothermics **30**, 573–589 (2001)
114. Stopa, J.; Wojnarowski, P.: Analytical model of cold water front movement in a geothermal reservoir. Geothermics **35**, 59–69 (2006)
115. Mongelli, F.; Pagliarulo, P.: Influence of water recharge on heat transfer in a semi-infinite aquifer. Geothermics **26**, 365–378 (1997)
116. Satman, A.: Solutions of heat, and fluid-flow problems in naturally fractured reservoirs: part 1-heat-flow problems. SPE Prod. Eng. **3**, 463–466 (1988)
117. Ghassemi, A.; Tarasovs, S.; Cheng, A.: Integral equation solution of heat extraction-induced thermal stress in enhanced geothermal reservoirs. Int. J. Numer. Anal. Methods Geomech. **29**, 829–844 (2005)
118. Gringarten, A.C.; Witherspoon, P.A.; Ohnishi, Y.: Theory of heat extraction from fractured hot dry rock. J. Geophys. Res. **80**, 1120–1124 (1975)
119. Cheng, A.H.-D.; Ghassemi, A.; Detournay, E.: Integral equation solution of heat extraction from a fracture in hot dry rock. Int. J. Numer. Anal. Methods Geomech. **25**, 1327–1338 (2001)
120. Kocabas, I.; Horne, R.N.: A new method of forecasting the thermal breakthrough time during reinjection in geothermal reservoirs. In: Proceedings, 15th Workshop on Geothermal Reservoir Engineering (1990)
121. Tor, B.: Nonisothermal effects in water-injection well tests. SPE Form. Eval. **4**, 281–286 (1989)

122. Jahanbani Ghahfarokhi, A.; Jelmert, T.A.; Kleppe, J.; Ashrafi, M.; Souraki, Y.; Torsaeter, O.: Investigation of the applicability of thermal well test analysis in steam injection wells for Athabasca heavy oil. In: SPE Europec/EAGE Annual Conference. Society of Petroleum Engineers (2012)
123. Benson, S.M.; Bodvarsson, G.S.: Nonisothermal effects during injection and falloff tests. SPE Form. Eval. **1**, 53–63 (1986)
124. Ambastha, A.K.; Ramey, H.J.: Thermal recovery well test design and interpretation. SPE Form. Eval. **4**, 173–180 (1989)
125. Aeschliman, D.P.; Noble, N.J.; Meldau, R.F.: Thermal efficiency of steam injection test well with insulated tubing. In: Annual Technical Meeting. Petroleum Society of Canada (1983)
126. Cendejas, F.A.; Rodriguez, J.R.: Heat transfer processes during low or high enthalpy fluid injection into naturally fractured reservoirs. Mich, MX, Gerencia de Proyectos Geotermoelectricos, Morelia (1994)
127. Ramirez, J.; Samaniego, F.V.; Rivera, J.R.; Rodriguez, F.: Tracer flow in naturally fractured reservoirs. SPE Pap. **25900**, 12–14 (1993)
128. Kocabas, I.; Islam, M.R.: Concentration and temperature transients in heterogeneous porous media: part II: radial transport. J. Pet. Sci. Eng. **26**, 221–233 (2000)
129. Kocabas, I.; Islam, M.R.: Concentration and temperature transients in heterogeneous porous media: part I: linear transport. J. Pet. Sci. Eng. **26**, 211–220 (2000)
130. Dindoruk, D.M.; Dindoruk, B.: Analytical solution of nonisothermal Buckley-Leverett flow including tracers. SPE Reserv. Eval. Eng. **11**, 555–564 (2008)
131. Dindoruk, B.; Dindoruk, D.M.: Analytical and numerical solution of nonisothermal Buckley–Leverett flow including tracers. In: SPE Annual Technical Conference and Exhibition. Society of Petroleum Engineers (2006)
132. Bai, M.; Roegiers, J.-C.: Modeling of heat flow and solute transport in fractured rock masses. In: 8th ISRM Congress. International Society for Rock Mechanics. International Society for Rock Mechanics (1995)
133. Lawal, K.A.; Vesovic, V.: Analytic investigation of convection during conduction heating of a heavy-oil reservoir. In: SPE Annual Technical Conference and Exhibition. Society of Petroleum Engineers (2010)
134. Barends, F.: Complete solution for transient heat transport in porous media, following Lauwerier's concept. In: SPE Annual Technical Conference and Exhibition. Society of Petroleum Engineers (2010)
135. Miura, K.; Wang, J.: An analytical model to predict cumulative steam/oil ratio (CSOR) in thermal-recovery SAGD process. J. Can. Pet. Technol. **51**, 268–275 (2012)
136. Miura, K.; Wang, J.: An analytical model to predict cumulative steam oil ratio (CSOR) in thermal recovery SAGD process. In: Canadian Unconventional Resources and International Petroleum Conference. Society of Petroleum Engineers (2013)
137. Edmunds, N.; Peterson, J.: A unified model for prediction of CSOR in steam-based bitumen recovery. In: Canadian International Petroleum Conference. Petroleum Society of Canada (2007)
138. Wei, S.; Lin-Song, C.; Huang, S.; Huang, W.: Steam chamber development and production performance prediction of steam assisted gravity drainage. In: SPE Heavy Oil Conference-Canada. Society of Petroleum Engineers (2014)
139. Li, K.-Y.; Yang, S.-Y.; Yeh, H.-D.: An analytical solution for describing the transient temperature distribution in an aquifer thermal energy storage system. Hydrol. Process. **24**, 3676–3688 (2010)
140. Cheppelear, J.E.; Volek, C.W.: The injection of a hot liquid into a porous medium. Soc. Pet. Eng. J. **9**, 100–114 (1969)
141. Baker, P.E.: An experimental study of heat flow in steam flooding. Soc. Pet. Eng. J. **9**, 89–99 (1969)
142. Baker, P.E.: Effect of pressure and rate on steam zone development in steamflooding. Soc. Pet. Eng. J. **13**, 274–284 (1973)
143. Baker, P.E.: Heat wave propagation and losses in thermal oil recovery processes. In: 7th World Petroleum Congress. World Petroleum Congress. Chevron Research Co (1967)
144. Spillette, A.G.; Nielsen, R.L.: Two-dimensional method for predicting hot waterflood recovery behavior. J. Pet. Technol. **20**, 627–638 (1967)
145. Ferguson, M.A.: Further experimental studies of steam-propane injection to enhance recovery of Morichal oil. http://hdl.handle.net/1969.1/ETD-TAMU-2000-THESIS-F465 (2000)
146. Goite Marcano, J.G.: Experimental study of Morichal heavy oil recovery using combined steam and propane injection. http://hdl.handle.net/1969.1/ETD-TAMU-1999-THESIS-G66 (1999)
147. Goite, J.G.; Mamora, D.D.; Ferguson, M.A.: Experimental study of Morichal heavy oil recovery using combined steam and propane injection. In: SPE Latin American and Caribbean Petroleum Engineering Conference. Society of Petroleum Engineers (2001)
148. Tinss, J.C.: Experimental studies of steam-propane injection to enhance recovery of an intermediate crude oil. http://hdl.handle.net/1969.1/ETD-TAMU-2001-THESIS-T568 (2001)
149. Rivero, J.A.: Experimental studies of enhancement of injectivity and in-situ oil upgrading by steam propane injection for the Hamaca oil field. http://hdl.handle.net/1969.1/ETD-TAMU-2002-THESIS-R58 (2003)
150. Simangunsong, R.: Experimental and analytical modeling studies of steam injection with hydrocarbon additives to enhance recovery of San Ardo heavy oil. http://hdl.handle.net/1969.1/4308 (2006)
151. Naderi, K.; Babadagli, T.; Coskuner, G.: Bitumen recovery by the steam-over-solvent injection in fractured reservoirs (SOS-FR) method: an experimental study on Grosmont carbonates. Energy Fuels. **27**, 6501–6517 (2013)
152. Hashemi-Kiasari, H.; Hemmati-Sarapardeh, A.; Mighani, S.; Mohammadi, A.H.; Sedaee-Sola, B.: Effect of operational parameters on SAGD performance in a dip heterogeneous fractured reservoir. Fuel **122**, 82–93 (2014)
153. Mohsenzadeh, A.; Escrochi, M.; Afraz, M.V.; Al-wahaibi, Y.M.; Ayatollahi, S.: Experimental investigation of heavy oil recovery from fractured reservoirs by secondary steam-gas assisted gravity drainage. In: SPE Heavy Oil Conference Canada. Society of Petroleum Engineers (2012)
154. Shafiei, A.; Dusseault, M.B.; Zendehboudi, S.; Chatzis, I.: A new screening tool for evaluation of steamflooding performance in naturally fractured carbonate reservoirs. Fuel **108**, 502–514 (2013)
155. Farzaneh, S.A.; Sohrabi, M.: A review of the status of foam application in enhanced oil recovery. In: EAGE Annual Conference and Exhibition incorporating SPE Europec. Society of Petroleum Engineers (2013)
156. Ardali, M.; Barrufet, M.; Mamora, D.D.; Qiu, F.: A critical review of hybrid steam/solvent processes for the recovery of heavy oil and bitumen. In: SPE Annual Technical Conference and Exhibition. Society of Petroleum Engineers (2012)
157. Naderi, K.; Babadagli, T.: Field scale application of the SOS-FR (Steam-Over-Solvent Injection in Fractured Reservoirs) method: optimal operating conditions. In: SPE Improved Oil Recovery Symposium. Society of Petroleum Engineers (2012)
158. Moreno-Arciniegas, L.; Babadagli, T.: Optimal application conditions of solvent injection into oil sands to minimize the effect of asphaltene deposition: an experimental investigation. SPE Reserv. Eval. Eng. **17**, 530–546 (2014)
159. Zendehboudi, S.; Rajabzadeh, A.R.; Bahadori, A.; Chatzis, I.; Dusseault, M.B.; Elkamel, A.; Lohi, A.; Fowler, M.: Connec-

tionist model to estimate performance of steam-assisted gravity drainage in fractured and unfractured petroleum reservoirs: enhanced oil recovery implications. Ind. Eng. Chem. Res. **53**, 1645–1662 (2014)
160. Saaid, Md: I.; Mahat, S.Q.A.; Lal, B.; Mutalib, M.I.A.; M. Sabil, K.: Experimental investigation on the effectiveness of 1-butyl-3-methylimidazolium perchlorate ionic liquid as a reducing agent for heavy oil upgrading. Ind. Eng. Chem. Res. **53**, 8279–8284 (2014)
161. Pathak, V.; Babadagli, T.; Edmunds, N.: Experimental investigation of bitumen recovery from fractured carbonates using hot solvents. J. Can. Pet. Technol. **52**, 289–295 (2013)
162. Mohammadpoor, M.; Torabi, F.: An extensive review on the effective sequence of heavy oil recovery. In: SPE heavy oil conference Canada. Society of Petroleum Engineers (2012)
163. Liu, H.Q.; Wang, J.; Hou, P.C.; Wang, B.K.: Experimental investigation of enhanced oil recovery by thermal foam flooding. In: Advanced Materials Research, pp. 814–819. Trans Tech Publ (2011)
164. Naderi, K.; Babadagli, T.: Experimental analysis of heavy oil recovery and CO_2 storage by alternate injection of steam and CO_2 in deep naturally fractured reservoir. In: SPE Heavy Oil Conference Canada. Society of Petroleum Engineers (2012)
165. Pathak, V.; Babadagli, T., Edmunds, N.R.: Hot solvent injection for heavy oil/bitumen recovery: an experimental investigation. In: Canadian Unconventional Resources and International Petroleum Conference. Society of Petroleum Engineers (2010)
166. Tang, G.-Q.; Inouye, A.; Lowry, D.; Lee, V.: Recovery mechanism of steam injection in heavy oil carbonate reservoir. In: SPE Western North American Region Meeting. Society of Petroleum Engineers (2011)
167. Jia, H.; Yuan, C.; Zhang, Y.; Peng, H.; Zhong, D.; Zhao, J.: Recent progress of high pressure air injection process (HPAI) in light oil reservoir: laboratory investigation and field application. In: SPE Heavy Oil Conference Canada. Society of Petroleum Engineers (2012)
168. Mukhametshina, A.; Hascakir, B.: Bitumen extraction by expanding solvent-steam assisted gravity drainage (ES-SAGD) with asphaltene solvents and non-solvents. In: SPE Heavy Oil Conference-Canada. Society of Petroleum Engineers (2014)
169. Souraki, Y.; Ashrafi, M.; Karimaie, H.; Torsaeter, O.: Experimental investigation and numerical simulation of steam flooding in heavy oil fractured reservoir. In: SPE Western North American Region Meeting. Society of Petroleum Engineers (2011)
170. Popov, Y.; Spasennykh, M.; Miklashevskiy, D.; Parshin, A.V.; Stenin, V.; Chertenkov, M.; Novikov, S.; Tarelko, N.: Thermal properties of formations from core analysis: evolution in measurement methods, equipment, and experimental data in relation to thermal EOR. In: Canadian Unconventional Resources and International Petroleum Conference. Society of Petroleum Engineers (2010)
171. Hirsch, T.; Feldhoff, J.F.; Hennecke, K.; Pitz-Paal, R.: Advancements in the field of direct steam generation in linear solar concentrators-a review. Heat Transf. Eng. **35**, 258–271 (2014)
172. Mohammed, M.; Babadagli, T.: Efficiency of solvent retrieval during steam-over-solvent injection in fractured reservoirs (SOS-FR) method: core scale experimentation. In: SPE Heavy Oil Conference-Canada. Society of Petroleum Engineers (2013)
173. Coskuner, G.; Naderi, K.; Babadagli, T.: An enhanced oil recovery technology as a follow up to cold heavy oil production with sand. J. Pet. Sci. Eng. **133**, 475–482 (2015)
174. Zhong, L.; Li, Y.; Sun, Y.; Lu, W.; Qin, F.; Hou, J.; Dong, Z.; Zhao, L.: Investigation on principles of enhanced offshore heavy oil recovery by coinjection of steam with flue gas. In: SPE Enhanced Oil Recovery Conference. Society of Petroleum Engineers (2013)
175. Ochs, S.O.; Class, H.; Färber, A.; Helmig, R.: Methods for predicting the spreading of steam below the water table during subsurface remediation. Water Resour. Res. **46**, 1–16 (2010)
176. Lamoureux-Var, V.; Kowalewski, I.; Kohler, E.: Forecasting H_2S generated from steamed oil sands insights into H_2S generation through experimental investigation. In: AAPG Hedberg Conference. Vail, Colorado (2010)
177. Ahmadi, M.A.; Zendehboudi, S.; Bahadori, A.; James, L.; Lohi, A.; Elkamel, A.; Chatzis, I.: Recovery rate of vapor extraction in heavy oil reservoirs? Experimental, statistical, and modeling studies. Ind. Eng. Chem. Res. **53**, 16091–16106 (2014)
178. Fayers, F.J.: Some theoretical results concerning the displacement of a viscous oil by a hot fluid in a porous medium. J. Fluid Mech. **13**, 65–76 (1962)
179. Bodvarsson, G.S.; Benson, S.M.; Witherspoon, P.A.: Theory of the development of geothermal systems charged by vertical faults. J. Geophys. Res. Solid Earth. **87**, 9317–9328 (1982)
180. Wilson, L.A.; Root, P.J.: Cost comparison of reservoir heating using steam or air: SPE-1116-PA. J. Pet. Technol. **18**, 233–239 (1966)
181. Ferrer, J.: A three-phase, two-dimensional compositional thermal simulator for steam injection processes. J. Can. Pet. Technol. **16**, 78–90 (1977)
182. Abou-Kassem, J.H.: Investigation of grid orientation in a two-dimensional, compositional, three-phase steam model. http://hdl.handle.net/1880/22121 (1981)
183. Ishimoto, K.; Pope, G.A.; Sepchrnoori, K.: An equation-of-state steam simulator. In Situ (United States) **11**, 1–37 (1987)
184. Jordon, J.K.; Rayne, J.R.; Marshall, S.W.: A calculation procedure for estimating the production history during hot water injection in linear reservoirs. In: 20th Technical Conference on Petroleum Production., The Pennsylvania State U., University Park, PA, 9–10 May 1957
185. Vinsome, P.K.W.: A numerical description of hot-water and steam drives by the finite-difference method. In: Fall Meeting of the Society of Petroleum Engineers of AIME. Society of Petroleum Engineers (1974)
186. Class, H.; Helmig, R.; Bastian, P.: Numerical simulation of non-isothermal multiphase multicomponent processes in porous media: 1. An efficient solution technique. Adv. Water Resour. **25**, 533–550 (2002)
187. Jensen, T.B.; Sharma, M.P.; Harris, H.G.; Whitman, D.L.: Numerical investigations of steam and hot-water flooding in fractured porous media. In: SPE/DOE Enhanced Oil Recovery Symposium. Society of Petroleum Engineers (1992)
188. Dreher, K.D.; Kenyon, D.E.; Iwere, F.O.: Heat flow during steam injection into a fractured carbonate reservoir. Paper SPE 14902 presented at the 1986 SPE. In: DOE fifth Symposium on Enhanced Oil Recovery, Tulsa, Oklahoma, April pp. 20–23 (1986)
189. Sahuquet, B.C.; Ferrier, J.J.: Steam-drive pilot in a fractured carbonated reservoir: Lacq Superieur field. J. Pet. Technol. **34**, 873–880 (1982)
190. Reis, J.C.: Oil recovery mechanisms in fractured reservoirs during steam injection. In: SPE/DOE Enhanced Oil Recovery Symposium. Society of Petroleum Engineers (1990)
191. Sumnu, M.D.; Brigham, W.E.; Aziz, K.; Castanier, L.M.: An experimental and numerical study on steam injection in fractured systems. In: SPE/DOE Improved Oil Recovery Symposium. Society of Petroleum Engineers (1996)
192. Rayleigh, L.: LIX. On convection currents in a horizontal layer of fluid, when the higher temperature is on the under side. Philos. Mag. Ser. 6 **32**, 529–546 (1916)
193. Karra, P.S.; Aziz, K.: A numerical study of transient natural convection in porous media. In: Proceedings of 17th Annual Conference of Canadian Society of Chemical Engineeres, Ontario (1967)

194. Sarathi, P.: Thermal numerical simulator for laboratory evaluation of steamflood oil recovery. National Institute for Petroleum and Energy Research, Bartlesville, OK (USA) (1991)
195. Shutler, N.D.: Numerical, three-phase simulation of the linear steamflood process: SPE-2233-PA. Soc. Pet. Eng. J. **9**, 232–246 (1969)
196. Abdus, S.; Parrish, D.R.: A two-dimensional analysis of reservoir heating by steam injection. Soc. Pet. Eng. J. **11**, 185–197 (1971)
197. Gomaa, E.E.: Correlations for predicting oil recovery by steamflood. J. Pet. Technol. **32**, 325–332 (1980)
198. Abdalla, A.; Coats, K.H.: A three-phase, experimental and numerical simulation study of the steam flood process. In: Fall Meeting of the Society of Petroleum Engineers of AIME. Society of Petroleum Engineers (1971)
199. Coats, K.H.: Simulation of steamflooding with distillation and solution gas. Soc. Pet. Eng. J. **16**, 235–247 (1976)
200. Hossain, M.E.; Mousavizadegan, S.H.; Islam, M.R.: Rock and fluid temperature changes during thermal operation in EOR process. J. Nat. Sci. Sustain. Technol. **2**, 347–378 (2007)
201. Cicek, O.: Numerical simulation of steam displacement of oil in naturally fractured reservoirs using fully implicit compositional formulation: a comparative analysis of the effects of capillary and gravitational forces in matrix/fracture exchange term. In: SPE Annual Technical Conference and Exhibition. Society of Petroleum Engineers (2005)
202. Cicek, O.: A parametric study of the effects of reservoir and operational properties on the performance of steam displacement of heavy oil in naturally fractured reservoirs. In: SPE International Thermal Operations and Heavy Oil Symposium. Society of Petroleum Engineers (2005)
203. Wu, K.; Li, X.; Zhai, Y.: The model for predicting stream breakthrough timing during steam drive development of heavy oil reservoirs: SPE-150504-MS. In: SPE Heavy Oil Conference and Exhibition. Society of Petroleum Engineers (2011)
204. Hossain, M.E.; Mousavizadegan, S.H.; Islam, M.R.: A new porous media diffusivity equation with the inclusion of rock and fluid memories: SPE-114287-MS. Society of Petroleum Engineers (2008)
205. Hossain, M.E.: Comprehensive modelling of complex petroleum phenomena with an engineering approach. J. Porous Media. **15**, 173–186 (2012)
206. Hossain, M.E.; Abu-Khamsin, S.A.; Al-Helali, A.-A.: Use of the memory concept to investigate the temperature profile during a thermal EOR process: In: SPE/DGS Saudi Arabia Section Technical Symposium and Exhibition. Society of Petroleum Engineers (2011)
207. Hossain, M.E.; Abu-khamsin, S.A.: Utilization of memory concept to develop heat transfer dimensionless numbers for porous media undergoing thermal flooding with equal rock and fluid temperatures. J. Porous Media **15**, 18 (2011)
208. Yoshida, N.; Zhu, D.; Hill, A.D.: Temperature prediction model for a horizontal well with multiple fractures in a shale reservoir. In: SPE Annual Technical Conference and Exhibition. Society of Petroleum Engineers (2013)
209. Yoshida, N.; Zhu, D.; Hill, A.D.: Temperature-prediction model for a horizontal well with multiple fractures in a shale reservoir. SPE Prod. Oper. **29**, 261–273 (2014)
210. Irawan, S.; Bathaee, M.: Numerical modeling of flow and temperature distribution in heterogeneous wellbore medium?: SPE-172568-MS. In: SPE Middle East Oil & Gas Show and Conference. Society of Petroleum Engineers (2015)
211. Peraser, V.; Patil, S.L.; Khataniar, S.; Dandekar, A.Y.; Sonwalkar, V.S.: Evaluation of electromagnetic heating for heavy oil recovery from Alaskan Reservoirs. In: SPE Western Regional Meeting. Society of Petroleum Engineers (2012)
212. Kovaleva, L.A.; Minnigalimov, R.Z.; Zinnatullin, R.R.: Destruction of water-in-oil emulsions in radio-frequency and microwave electromagnetic fields. Energy Fuels. **25**, 3731–3738 (2011)
213. Kasevich, R.S.: Method and apparatus for in-situ radiofrequency heating. https://www.google.com/patents/US7891421 (2011)
214. Hakala, J.A.; Stanchina, W.; Soong, Y.; Hedges, S.: Influence of frequency, grade, moisture and temperature on Green River oil shale dielectric properties and electromagnetic heating processes. Fuel Process. Technol. **92**, 1–12 (2011)
215. Rassenfoss, S.: Oil sands get wired-seeking more oil. Fewer emissions. J. Pet. Technol. **64**, 34–45 (2012)
216. Banerjee, D.K.; Stalder, J.L.: Process for enhanced production of heavy oil using microwaves. https://www.google.com/patents/US7975763 (2011)
217. Bogdanov, I.; Torres, J.; Corre, B.: Numerical simulation of electromagnetic driven heavy oil recovery. In: SPE Improved Oil Recovery Symposium. Society of Petroleum Engineers (2012)
218. Bogdanov, I.; Torres, J.; Kamp, A.M.; Corre, B.: Comparative analysis of electromagnetic methods for heavy oil recovery. In: SPE Heavy Oil Conference and Exhibition. Society of Petroleum Engineers (2011)
219. Hossan, M.R.; Dutta, P.: Effects of temperature dependent properties in electromagnetic heating. Int. J. Heat Mass Transf. **55**, 3412–3422 (2012)
220. Bientinesi, M.; Petarca, L.; Cerutti, A.; Bandinelli, M.; De Simoni, M.; Manotti, M.; Maddinelli, G.: A radiofrequency/microwave heating method for thermal heavy oil recovery based on a novel tight-shell conceptual design. J. Pet. Sci. Eng. **107**, 18–30 (2013)
221. Farmayan, W.F.; Giles, S.P.; Brignac, J.P.; Munshi, A.W.; Abbasi, F.; Clomburg, L.A.; Anderson, K.G.; Tsai, K.; Siddoway, M.A.: Downhole burner systems and methods for heating subsurface formations. US Patent 7,950,453. 31 May 2011
222. Kovscek, A.R.: Emerging challenges and potential futures for thermally enhanced oil recovery. J. Pet. Sci. Eng. **98**, 130–143 (2012)
223. Aouizerate, G.; Durlofsky, L.J.; Samier, P.: New models for heater wells in subsurface simulations, with application to the in situ upgrading of oil shale. Comput. Geosci. **16**, 519–533 (2012)
224. Nguyen, S.V.; Vinegar, H.J.: Heating systems for heating subsurface formations. https://www.google.com/patents/US7931086 (2011)
225. Harris, C.K.; Karanikas, J.M.; Nguyen, S.V.: Parallel heater system for subsurface formations. https://www.google.com/patents/US8042610 (2011)
226. Hart, A.; Leeke, G.; Greaves, M.; Wood, J.: Down-hole heavy crude oil upgrading by CAPRI: Effect of hydrogen and methane gases upon upgrading and coke formation. Fuel **119**, 226–235 (2014)
227. Remey, E.E. de S.; Giuliani, V.; Harris, C.K.: Insulated conductor heaters with semiconductor layers. https://www.google.com/patents/US8939207 (2015)
228. Nguyen, S.V.; Bass, R.M.: Induction heaters used to heat subsurface formations. https://www.google.com/patents/US8162059 (2012)
229. Bottazzi, F.; Repetto, C.; Tita, E.; Maugeri, G.: Downhole electrical heating for heavy oil enhanced recovery: a successful application in offshore Congo. In: IPTC 2013: International Petroleum Technology Conference (2013)
230. Ozcan, O.: Fractional diffusion in naturally fractured unconventional reservoirs. http://hdl.handle.net/11124/10641 (2014)
231. Fomin, S.; Chugunov, V.; Hashida, T.: Mathematical modeling of anomalous diffusion in porous media. Fract. Differ. Calc. **1**, 1–28 (2011)
232. Chang, J.; Yortsos, Y.C.: Pressure transient analysis of fractal reservoirs. SPE Form. Eval. **5**, 31–38 (1990)

233. Dassas, Y.; Duby, P.: Diffusion toward fractal interfaces potentiostatic, galvanostatic, and linear sweep voltammetric techniques. J. Electrochem. Soc. **142**, 4175–4180 (1995)
234. O'Shaughnessy, B.; Procaccia, I.: Analytical solutions for diffusion on fractal objects. Phys. Rev. Lett. **54**, 455 (1985)
235. Hossain, M.E.; Islam, M.R.: A comprehensive material balance equation with the inclusion of memory during rock-fluid deformation. Adv. Sustain. Pet. Eng. Sci. **1**, 141–162 (2009)
236. Caputo, M.: Diffusion with space memory modelled with distributed order space fractional differential equations. Ann. Geophys. **46**, (2003)
237. Kilbas, A.A.; Marzan, S.A.: Nonlinear differential equations with the Caputo fractional derivative in the space of continuously differentiable functions. Differ. Equ. **41**, 84–89 (2005)
238. Daftardar-Gejji, V.; Jafari, H.: Analysis of a system of nonautonomous fractional differential equations involving Caputo derivatives. J. Math. Anal. Appl. **328**, 1026–1033 (2007)
239. Heymans, N.; Podlubny, I.: Physical interpretation of initial conditions for fractional differential equations with Riemann-Liouville fractional derivatives. Rheol. Acta **45**, 765–771 (2006)
240. Agarwal, R.; Belmekki, M.; Benchohra, M.: A survey on semilinear differential equations and inclusions involving Riemann-Liouville fractional derivative. Adv. Differ. Equ. **2009**, 1–47 (2009)
241. Yuste, S.B.; Acedo, L.: An explicit finite difference method and a new von Neumann-type stability analysis for fractional diffusion equations. SIAM J. Numer. Anal. **42**, 1862–1874 (2005)
242. Yuste, S.B.: Weighted average finite difference methods for fractional diffusion equations. J. Comput. Phys. **216**, 264–274 (2006)
243. Murillo, J.Q.; Yuste, S.B.: On an explicit difference method for fractional diffusion and diffusion-wave equations. In: ASME 2009 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference, pp. 1031–1036. American Society of Mechanical Engineers (2009)
244. Murio, D.A.: Implicit finite difference approximation for time fractional diffusion equations. Comput. Math. Appl. **56**, 1138–1145 (2008)
245. Hemond, H.F.; Fechner, E.J.: Chemical fate and transport in the environment. Elsevier, Amsterdam (2014)
246. Weinstein, H.G.; Wheeler, J.A.; Wood, E.G.: Numerical model for thermal processes. Soc. Pet. Eng. J. **17**, 65–78 (1977)
247. Coats, K.H.; George, W.D.; Chu, C.; Marcum, B.E.: Three-dimensional simulation of steamflooding. Soc. Pet. Eng. J. 14, 573–592 (1974)
248. Coats, K.H.: A highly implicit steamflood model. Soc. Pet. Eng. J. 18, 369–383 (1978)
249. Rubin, B.; Buchanan, W.L.: A general purpose thermal model. Soc. Pet. Eng. J. **25**, 202–214 (1985)
250. Rousset, M.: Reduced-order modeling for thermal simulation. Dissertation, Stanford University (2010)
251. App, J.F.: Field cases: Nonisothermal behavior due to Joule-Thomson and transient fluid expansion/compression effects. In: SPE Annual Technical Conference and Exhibition, New Orleans, Louisiana, 4–7 Oct (2009)
252. Warren, J.E.; Root, P.J.: The behavior of naturally fractured reservoirs. Soc. Pet. Eng. J. **3**, 245–255 (1963)