Arab J Sci Eng (2016) 41:3587–3604
DOI 10.1007/s13369-016-2222-3

RESEARCH ARTICLE - ELECTRICAL ENGINEERING

Bat Algorithm: Application to Adaptive Infinite Impulse Response System Identification

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Received: 27 August 2015 / Accepted: 15 May 2016 / Published online: 7 June 2016

Abstract The problem of system identification concerns with the design of adaptive infinite impulse response (IIR) system by determining the optimal system parameters of the unknown system on the minimization of error fitness function. The conventional system identification techniques have stability issues and problem of degradation in performance when modeled using a reduced-order system. Hence, a meta-heuristic optimization method is applied to overcome such drawbacks. In this paper, a new meta-heuristic optimization algorithm, called bat algorithm (BA), is utilized for the design of an adaptive IIR system in order to approximate the unknown system. Bat algorithm is inspired from the echolocation behavior of bats combining the advantages of existing optimization techniques. A proper tuning of control parameter has been performed in order to achieve a balance between intensification and diversification phases. The proposed BA method for system identification is free from the problems encountered in conventional techniques. To valuate the performance of the proposed method, mean square error, mean square deviation and computation time are measured. Simulations have been carried out considering four benchmarked IIR systems using the same-order and reduced-order systems. The results of the proposed BA method have been compared to that of the well known optimization methods such as genetic algorithm, particle swarm optimization and cat swarm optimization. The simulation results confirm that the proposed system identification method outperforms the existing system identification methods.

Keywords Bat algorithm · IIR system identification · Meta-heuristics · Mean square error · Mean square deviation · Echolocation

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1 Introduction

Adaptive infinite impulse response (IIR) filtering has maintained tremendous vitality over the past few decades, and there is a clear indication that this trend will continue. Adaptive IIR systems are widely applied in system identification and modeling problems in the field of signal processing. It is also used to describe many phenomena in almost all fields such as radar processing [1], robotics [2], parameter estimation [3], signal processing, control system and communication system [4,5]. This is due to the fact that an IIR filter models an unknown system effectively with a few number of coefficients compared to an adaptive finite impulse response (FIR) system [6].

Digital systems are classified in two categories, namely finite impulse response (FIR) system and infinite impulse response (IIR) system [6]. The output of the FIR system depends only on the input signal (present and past inputs), whereas the output of IIR system depends the not only on the input (present and past) but also on the past outputs. IIR system has two shortcomings. First, it is unstable due to the inappropriate selection of denominator coefficients. Proper selection of search space overcomes this problem. Second, it cannot provide an exact linear phase response. Apart from these shortcomings, requirement of less number of system coefficients compared to the FIR system makes it computationally efficient. Therefore, for the system identification problem, adaptive IIR system is proven to be a better option.

The implementation of an adaptive IIR system identification involves two processes. First, a suitable identification plant is chosen. Further, the optimal filter coefficients are computed using an efficient optimization algorithm. The system identification problem is articulated as an error minimization problem. The objective is to obtain an optimal set of coefficients such that the output of the adaptive IIR system exactly tracks the output of the unknown system when both the systems are subjected to the same input signal. Hence, IIR system identification is based on minimizing error objective function between the output of the adaptive filter and the output of the unknown system for the same input. In IIR filtering, the error surface (objective function) is generally non-quadratic and multimodal with respect to the filter parameters.

Traditionally, the gradient-based search algorithm such as Quasi-Newton technique, least mean square (LMS) and its variants were applied to minimize the error fitness function. Most adaptive IIR filter applications are associated with the nonlinear and multimodal error fitness function [7]. Minimization of such error fitness function using gradient-based search algorithms is difficult. This is due to the fact that the gradient-based search algorithms cannot converge to the global minima and get stuck in local minima. Moreover, the higher-order systems are associated with the stability issues as the poles of the systems are placed out of the unit circle.

To overcome these drawbacks, several practitioners rely on metaheuristic algorithms [8], which are based on natural evolution. The metaheuristic algorithms are nature inspired population-based search techniques which have the ability to provide a global optimal solution with fast convergence by incorporating random search and selection principle. Such algorithms are widely applied in solving many complex and unsolved constrained optimization problems [8].

These population-based search methods include genetic algorithm [9–11], particle swarm optimization [12], differential evolution (DE), cat swarm optimization [13], cuckoo-search algorithm [14–17], harmony search [18], gravitational search algorithm (GSA) [19,20], seeker optimization algorithm (SOA) [21] and many more. A new algorithm, BA [22], employed in this work uses the concepts of localization and tracking. The concept of localization has been explored in different applications [23–28]. The efficiency of meta-heuristic algorithms can be attributed to the fact that they imitate the best features in nature, especially the selection of the fittest in biological systems which have evolved by natural selection over millions of years.

The above-mentioned algorithms have been extensively researched for the problem of system identification and filter modeling [29–48]. Karaboga et al. [29–31] presented ant colony optimization (ACO), DE and artificial bee colony (ABC) algorithm for several benchmark IIR systems in 2004, 2005 and 2009, respectively. In 2005, Kalinli and Karaboga applied the artificial immune (AI) algorithm for the system identification problem [32]. Fang et al. [33,34] employed the quantum-behaved particle swarm optimization (PSO) and mutated quantum-behaved PSO for IIR filter design problem. The pole-zero system identification was presented by Majhi et al. [35] using PSO in 2008. Dai et al. [21] demonstrated the simulation results for modeling of standard IIR plants using SOA. Parameter value of unknown system was computed by means of PSO algorithm for the test functions in Chen et al. [36]. In [37], PSO with quantum infusion scheme for the coefficient value optimization of the unknown system model was investigated. In [38,39], PSO-based parameter value selection method has been applied for adaptive IIR system design to obtain a lesser mean square error value. Panda et al. [13] formulated the system identification task as an optimization problem using cat swarm optimization (CSO) on some standard IIR plants in 2011. Further, in 2011, a relatively new optimization technique, GSA, was utilized to model the nonlinear and linear IIR system [40]. Recently, new strategies were introduced by Saha et al. [41,42], utilizing the concepts of opposition-based bat algorithm (OBA) and HS algorithm for IIR filter design. In this work, the authors have made an attempt to apply the metaheuristic algorithms productively for IIR system modeling in signal processing applications. Another strategy inspired by the breeding behavior of cuckoo species, CSA, was studied in Patwardhan et al. [43]. Upadhyay et al. [44–47] presented a comparative study on IIR system design with different metaheuristic algorithms, DEWM, CRPSO, FFA and OHS. The latest attempt was made by Jiang et al. [48] in 2015 using a hybrid algorithm, combining the characteristics of PSO and GSA to achieve a better quality of system response by optimizing IIR filter coefficients.

In this paper, efficiency of bat algorithm is demonstrated in system identification problem. BA is inspired from the magical echolocation characteristics of the flying mammal, bats, with which they easily locate their targets, distance from the target, its speed, texture and direction. Here, four benchmark functions are tested using GA, PSO, CSO and BA for approximating the same-order and reduced-order IIR systems. The proposed BA method for system identification is testified to be the best among others on the basis of computing the mean square error (MSE), mean square deviation (MSD) and computation time. In addition, it is affirmed that the proposed BA method is superior in performance in comparison with some of the other existing system identification techniques.

The rest of the paper is organized as follows: Following a detailed literature survey in Sect. 1, mathematical formulation of adaptive IIR system identification problem is presented in Sect. 2. In Sect. 3, a brief review of the basic principles of bat algorithm and implementation steps are presented for the system identification problem. The performance of the proposed system identification method using bat algorithm is tested using four benchmark functions, and detail analysis is presented in Sect. 4. Finally, the paper is concluded in Sect. 5.

2 Problem Formulation

In this section, some basic concepts of IIR system identification are reviewed briefly. In system identification problem, the aim is to find the adaptive IIR filter coefficients such that it matches the transfer function of an unknown system. In other words, the meta-heuristic optimization algorithms are applied to alter the coefficients of an adaptive IIR system in such a way that its output matches the unknown system output, when both the systems (unknown system and adaptive IIR system) are subjected to same input signal. Figure 1 shows the schematic of an IIR system identification problem using metaheuristic optimization algorithm.

Consider IIR filter with the input–output relationship given by the following difference equation

$$y(m) + \sum_{i=1}^{M} b_i y(m-i) = \sum_{i=0}^{N} a_i x(m-i) \tag{1}$$

where a_i $(i = 0, 1, 2, \ldots, N)$ and b_i $(i = 1, 2, \ldots, M)$ are the filter coefficients, $x(m)$ and $y(m)$ are the input and output of the filter at a time instant m, respectively, N and M are the order of the numerator and denominator, respectively. Transfer function of the estimated adaptive IIR filter can be written as

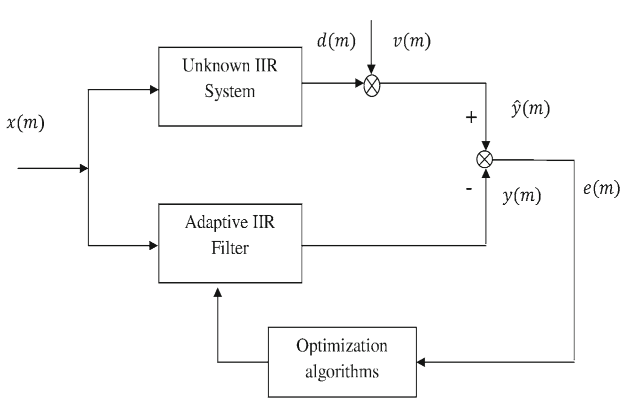

Fig. 1 Block diagram of adaptive IIR filter for system identification

$$H_E(z) = \frac{\sum_{i=0}^{N} a_i z^{-i}}{1 + \sum_{i=1}^{M} b_i z^{-i}} = \frac{N_E(z)}{D_E(z)} = \frac{\Theta^{\mathrm{T}}(i)\mathbf{a}_i}{\Phi^{\mathrm{T}}(i)\mathbf{b}_i} \tag{2}$$

where

$$\begin{aligned}
N_E(z) &= \Theta^{\mathrm{T}}(i)\mathbf{a}_i \\
D_E(z) &= \Phi^{\mathrm{T}}(i)\mathbf{b}_i \\
\Theta^{\mathrm{T}}(i) &= [z^{-1}, z^{-2}, \ldots, z^{-N}]^{\mathrm{T}}, \\
\Phi^{\mathrm{T}}(i) &= [1, z^{-1}, z^{-2}, \ldots, z^{-M}]^{\mathrm{T}}, \\
\mathbf{a}_i &= [a_0, a_1, a_2, \ldots, a_N]^T
\end{aligned}$$

and

$$\mathbf{b}_i = [1, b_1, b_2, \ldots, b_M]^{\mathrm{T}}.$$

It is assumed that the actual system is known by considering some standard IIR plants. Hence, the transfer function of the unknown system is expressed as follows:

$$H_A(z) = \frac{\sum_{i=0}^{N} \hat{a}_i z^{-i}}{1 + \sum_{i=1}^{M} \hat{b}_i z^{-i}} \tag{3}$$

where $\hat{a}_i$ $(i = 0, 1, 2, \ldots, N)$ and $\hat{b}_i$ $(i = 1, 2, \ldots, M)$ are coefficients of the actual system. Here, we considered that all the coefficients are real valued.

Now, the objective is to find the optimal filter coefficient vector $[\mathbf{a}_i, \mathbf{b}_i]$ such that the response of the estimated adaptive IIR filter $H_E(z)$ approaches the actual system response $H_A(z)$. To accomplish this objective, a fitness function which is a mean square error (MSE) between the estimated adaptive IIR filter and the actual system is articulated and the meta-heuristic optimization algorithm considered in this work is applied to determine the filter coefficient vector $[\mathbf{a}_i, \mathbf{b}_i]$. The fitness function is minimized such that the output of the estimated IIR filter closely approximates the actual system output. The mean square error objective function is defined as

$$\begin{aligned}
J(\mathbf{a}_i, \mathbf{b}_i) &= \frac{1}{L} \sum_{m=1}^{L} e^2(m) \\
&= \frac{1}{L} \sum_{m=1}^{L} \left(\hat{y}(m) - y(m)\right)^2
\end{aligned} \tag{4}$$

where $e(m)$ is the error signal, $\hat{y}(m)$ and $y(m)$ are the are response of the actual system and the adaptive IIR filter, respectively, L is the number of samples utilized to compute the fitness function. In this paper, coefficients of the IIR filter are iteratively varied by BA in a manner such that the error between adaptive IIR filter output and the actual system output is minimized.

3 Bat Algorithm

A new metaheuristic algorithm, inspired from the natural echolocation behavior of the microbats, was developed by Yang [22]. The bat algorithm is a population-based stochastic search approach for solving constrained optimization problem with multimodal fitness function. The echolocation process performed by bats investigates the presence of prey for their survival. It allows them to sense nearby movements and vibrations, even in the dark. It is similar to the principles of sonar signaling where the bat emits very high frequency sound waves and learns from the reflected echoes. They account for three parameters (i) time delay between the transmitted and detected waves, (ii) time difference between their ears and (iii) the variation in loudness, to create a three-dimensional perception of the environment. With these parameters, they inherently determine the obstacle/target size, direction and distance from the target, its speed and texture. With the efficient performance of BA, it has been applied in various applications such as image processing, feature selection, data mining, parameter estimation and many other engineering optimization problems [49–54].

The algorithm follows some idealistic rules for its successful operation.

1. The bats use their inherent magical potential to classify between an obstacle and prey, in their path.
2. In the process of searching, bats fly randomly with velocity $\mathbf{v}_i$ to achieve the position $\mathbf{x}_i$, with a fixed frequency $f_{\min}$, variable wavelength λ and a loudness parameter A_0. Based on the propinquity of the target, bats reflex toward adjusting the wavelength of emitting waves and the pulse rate $r \in [0, 1]$.
3. The loudness values can be constant or decreasing from a maximum limit.
4. There is a limit to the maximum and minimum frequency/wavelength of emitting waves.

In order to formulate BA according the adaptive system identification problem, the flowchart of bat algorithm is shown in Fig. 2 and the implementation steps are outlined below.

Step 1: Initialization. Set population size of bats, n_i, maximum number of iterations and number of parameters to be optimized (depending on the order of the system). Specify

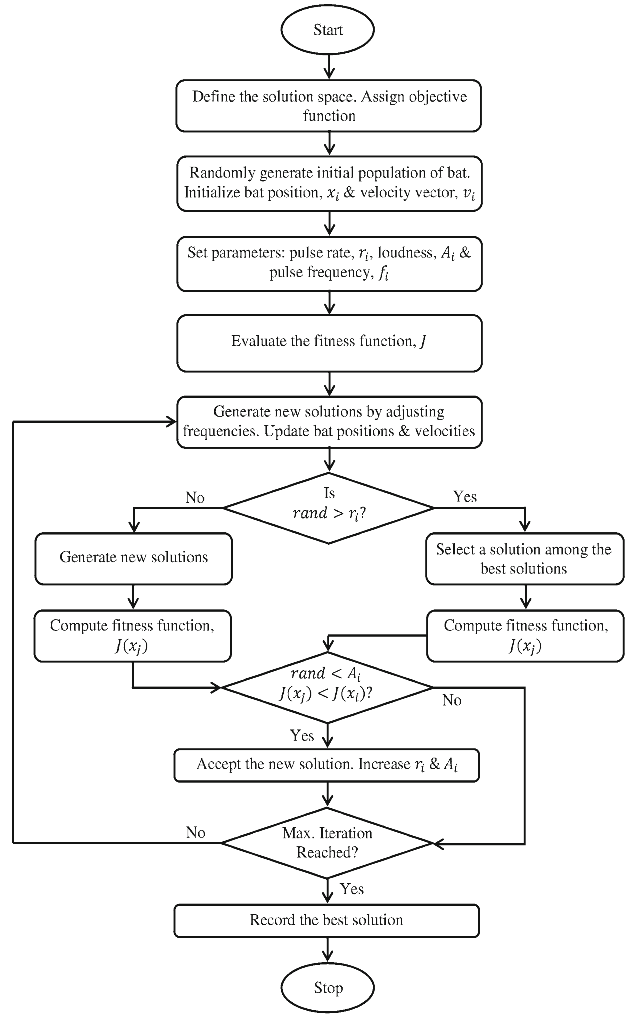

Fig. 2 Flowchart of the bat algorithm

the control parameters, A_i, r_i, $f_{\min}$, $f_{\max}$ and search space (upper and lower bound of system parameters).

Step 2: Initial Generation. Generate the initial bat population specifying their positions, x_i, $(i = 1, 2, \ldots, n)$, and velocities, v_i. Compute the fitness function, $F(x_i)$, to evaluate the effectiveness of each bat position in search of the best solution.

Step 3: Movement. After evaluating the fitness for each bat position, the best location known as the current global solution, x_j, is selected. In the next iteration, update the new solutions, x_i^{l+1}, and velocities, v_i^{l+1}, using the following equations.

$$f_i = f_{\min} + (f_{\max} - f_{\min})\,\beta \tag{5}$$

$$v_i^{l+1} = v_i^l + \left(x_i^{l+1} - x_j\right) f_i \tag{6}$$

$$x_i^{l+1} = x_i^l + v_i^{l+1} \tag{7}$$

where β is a uniformly distributed random number in [0,1].

Table 1 Control parameters of GA, PSO, CSO and BA for IIR system identification

Parameters	Symbol	GA	PSO	CSO	BA
Initial population	n_i	25	25	25	25
Maximum iterations	N_i	400	400	400	400
Tolerance	ϵ	10^{-5}	10^{-5}	10^{-5}	10^{-5}
Lower bound	$L_{\min}$	-2	-2	-2	-2
Upper bound	$L_{\max}$	2	2	2	2
Elite count	–	1	–	–	–
Creation function	–	Feasible population	–	–	–
Crossover fraction	–	0.90	–	–	–
Crossover function	–	Scattered	–	–	–
Scaling function	–	Rank	–	–	–
Selection function	–	Roulette	–	–	–
Mutation function	–	Constraint dependent	–	–	–
Cognitive constant	C_1	–	2.0	–	–
Social constant	C_2	–	2.0	–	–
Initial velocity	$v_i^{\min}$	–	0.01	–	–
Final velocity	$v_i^{\max}$	–	1.0	–	–
Lower frequency (for PSO)	$w_{\min}$	–	0.2	–	–
Higher frequency (for PSO)	$w_{\max}$	–	1.0	–	–
Seeking memory pool	SMP	–	–	5	–
Counts of dimensions to change	CDC	–	–	0.6	–
Seeking range of selected Dimension	SRD	–	–	2	–
Mixture ratio	MR	–	–	0.1	–
Inertia weight	ω	–	–	0.4	–
Acceleration constant	C	–	–	1.5	–
Initial velocity (for CSO)	$V_{\min}$	–	–	-0.1	–
Final velocity (for CSO)	$V_{\max}$	–	–	0.1	–
Loudness	A_i	–	–	–	0.5
Pulse rate	r_i	–	–	–	0.5
Lower frequency (for CSO)	$f_{\min}$	–	–	–	0
Higher frequency (for CSO)	$f_{\max}$	–	–	–	2
Stopping criteria	–	Maximum iteration/best solution	Maximum iteration	Maximum iteration	Maximum iteration

Table 2 Optimized coefficients in Example 1 for identification of second-order IIR system using the same-order system

Coefficients	Exact value	Optimized value			
		GA	PSO	CSO	BA
a_0	0.0500	-0.0877	0.0536	0.0493	0.0501
a_1	-0.4000	-0.4112	-0.4184	-0.4021	-0.4002
b_1	1.1314	1.1820	1.0876	1.1248	1.1306
b_2	-0.2500	-0.3050	-0.2077	-0.2433	-0.2497

Step 4: Local Search. If a random number rand $> r_i$, then select a solution among the best solution. If rand $< r_i$, generate new solutions for each bat using the random walk, given by

$$x_{\text{new}} = x_{old} + \epsilon A^l \tag{8}$$

where $\epsilon \in [-1, 1]$ and $A^l = \langle A_i^l \rangle$ is the average loudness for all the bats at the current iteration.

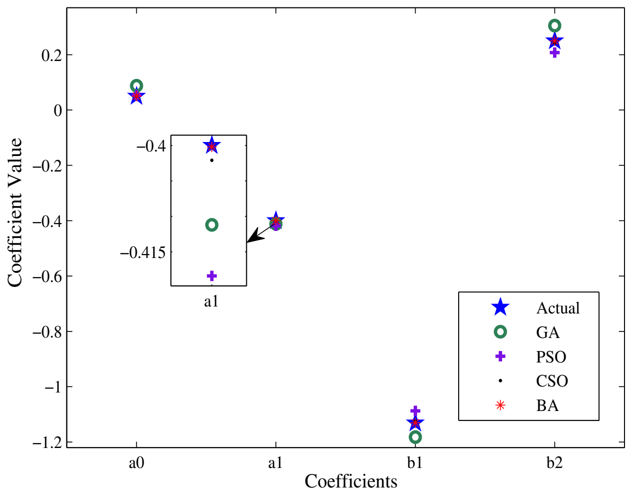

Fig. 3 Comparison of optimized coefficient values in Example 1 obtained by employing different evolutionary algorithms

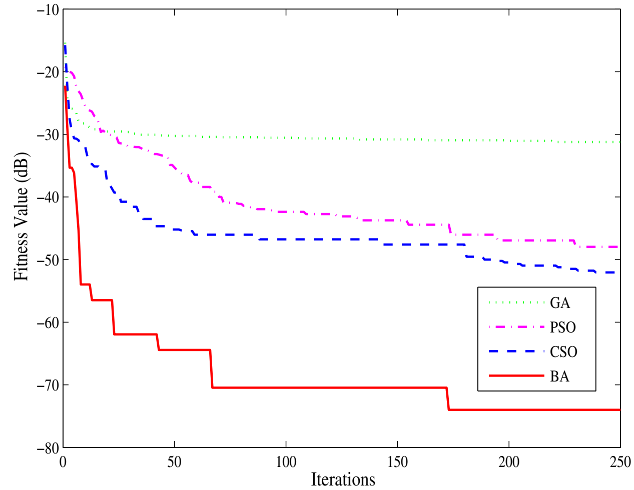

Fig. 4 Convergence profile for Example 1 modeled using same-order system obtained by employing different evolutionary algorithms

Step 5: Compare. Compute the fitness of the new solutions, $F(x_j)$. If $F(x_j) < F(x_i)$ and a random number rand $< A_i$, then retain the new solutions, x_j, as best solutions, otherwise goto step 3 if the maximum iterations has not reached.

Step 6: Update. The loudness, A_i, and pulse rate, r_i, are updated according the following equations.

$$A_i^{l+1} = \alpha A_i^l \tag{9}$$

$$r_i^{l+1} = r_i^0 \left[1 - e^{-\gamma l}\right] \tag{10}$$

where $\alpha = \gamma = 0.8$ for the problem under consideration. Approaching the prey, the loudness of the bat decreases, whereas the pulse rate increases.

Step 7: Solution. Record the best solution if the maximum number of iterations has reached. Otherwise, go to step 3. The best solution corresponds to the minimum fitness function are used to identify the unknown system.

4 Simulation Results

The system identification problem described in Sect. 2 has been implemented in MATLAB. To evaluate the performance of BA for system identification, it is examined using four established benchmark systems. In order to model an unknown system, two cases are considered: (i) using a sys-

Table 3 Statistical comparison of MSE performance matric in Example 1 for identification of second-order IIR system using the same-order system

Algorithm	Mean square error (MSE)				
	Best	Worst	Average	Median	Standard deviation
BA	2.1569×10^{-5}	2.2014×10^{-5}	2.1815×10^{-5}	2.1501×10^{-5}	2.3365×10^{-7}
CSO	6.3639×10^{-5}	6.4629×10^{-5}	6.3849×10^{-5}	6.3806×10^{-5}	2.8906×10^{-7}
PSO	1.0116×10^{-4}	2.7405×10^{-4}	1.5491×10^{-4}	1.4519×10^{-5}	5.1800×10^{-5}
GA	2.6428×10^{-4}	4.9228×10^{-3}	1.4671×10^{-3}	8.2421×10^{-4}	1.5489×10^{-3}

Table 4 Statistical comparison of MSD performance matric in Example 1 for identification of second-order IIR system using the same-order system

Algorithm	Mean square deviation (MSD)				
	Best	Worst	Average	Median	Standard deviation
BA	2.2500×10^{-8}	2.4815×10^{-8}	2.3157×10^{-8}	2.3085×10^{-8}	1.1930×10^{-9}
CSO	1.1998×10^{-6}	2.8118×10^{-5}	1.3085×10^{-5}	1.3728×10^{-5}	9.9418×10^{-6}
PSO	3.5700×10^{-5}	5.9713×10^{-3}	1.8691×10^{-3}	1.4839×10^{-3}	1.7985×10^{-3}
GA	4.9903×10^{-4}	2.8144×10^{-2}	7.9963×10^{-3}	5.1482×10^{-3}	8.9938×10^{-3}

Table 5 Statistical comparison of MSE performance matric in Example 1 for identification of second-order IIR system using the reduced-order system

Algorithm	Mean square error (MSE)				
	Best	Worst	Average	Median	Standard deviation
BA	7.9178×10^{-3}	7.9178×10^{-3}	7.9178×10^{-3}	7.9178×10^{-3}	6.9831×10^{-19}
CSO	1.7515×10^{-2}	1.7515×10^{-2}	1.7515×10^{-2}	1.7515×10^{-2}	4.9100×10^{-18}
PSO	1.7515×10^{-2}	5.5841×10^{-2}	3.8807×10^{-2}	5.5841×10^{-2}	2.0199×10^{-2}
GA	2.7122×10^{-2}	5.7830×10^{-2}	4.6893×10^{-2}	5.7285×10^{-2}	1.3236×10^{-2}

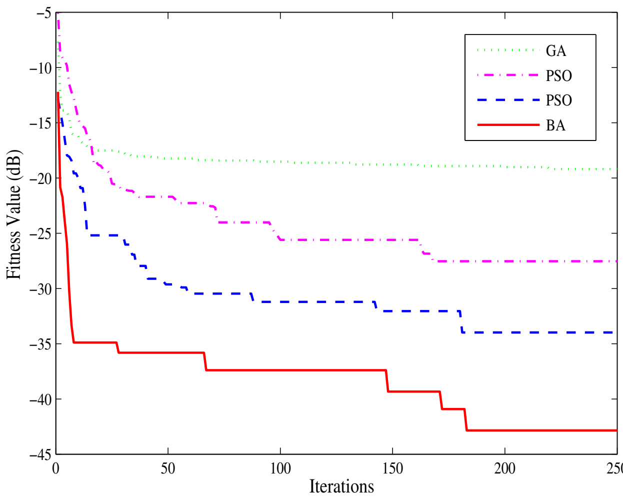

Fig. 5 Convergence profile for Example 1 modeled using reduced-order system obtained by employing different evolutionary algorithms

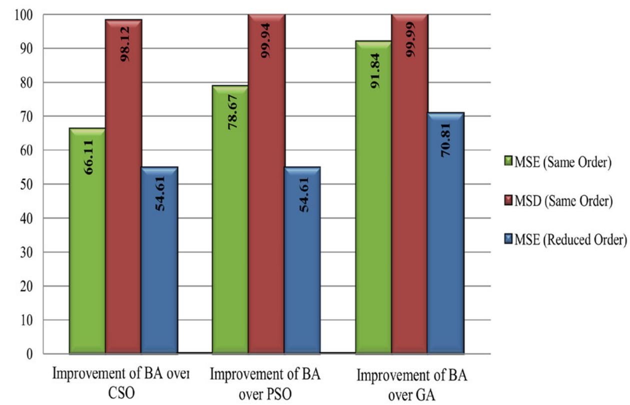

Fig. 6 Percentage improvement for Example 1 in terms of MSE and MSD as compared to other reported algorithms for same-order and reduced-order systems

tem of the same order as that of the benchmark system and (ii) using a reduced-order system. The proposed BA-based system identification method is compared with the genetic algorithm, particle swarm optimization and cat swarm optimization [13]. The control parameters selected for optimal results are summarized in Table 1. The tuning of control parameters is distinctive, and there is no explicit method available in the vast literature to obtain an optimal set of parameter values for optimal performance. Moreover, optimal parameter values can differ for different problems. Thus, after performing extensive simulations with different parameter values in the range specified by the researchers in this field, the parameter values mentioned in Table 1 are computed for IIR system identification problem. The ability to model a system using reduced-order determines the effectiveness of the algorithm. Here, the same-order system identification results are analyzed in terms of computation time, mean square error and mean square deviation, whereas the results of the reduced-order system are provided in terms of computation time and mean square error. The mean square deviation is defined as

$$\mathrm{MSD} = \frac{1}{L}\sum_{k=0}^{L-1}\left[\Psi(k) - \hat{\Psi}(k)\right]^2 \tag{11}$$

where Ψ is the vector of actual parameters, $\hat{\Psi}$ is the vector of approximated parameters and L be the total number of parameters to be optimized.

4.1 Design Examples and Modeled System

Example 1 A second-order system is considered and its transfer function is given by

$$H_t(z) = \frac{0.05 - 0.4z^{-1}}{1 - 1.314z^{-1} + 0.25z^{-2}} \tag{12}$$

$H_t(z)$ is modeled in this example to testify the superiority of BA using the same-order system as described in case 1 and reduced-order system in case 2.

Case 1: Same Order

In this case, the second-order system is modeled using a second-order unknown system with the transfer function given by

Table 6 Percentage improvement for different performance measures in Example 1 in comparison with other reported algorithms

Parameters	Percentage improvement in Example 1		
	BA compared to CSO	BA compared to PSO	BA compared to GA
MSE (same order)	66.11	78.67	91.84
MSD (same order)	98.12	99.94	99.99
MSE (reduced order)	54.61	54.61	70.81

Table 7 Optimized coefficients in Example 2 for identification of third-order IIR system using the same-order system

Coefficients	Exact value	Optimized value			
		GA	PSO	CSO	BA
a_0	−0.2000	−0.2258	−0.2105	−0.2050	−0.2066
a_1	−0.4000	−0.2717	−0.3778	−0.3927	−0.3996
a_2	0.5000	0.4643	0.4670	0.5038	0.4994
b_1	0.6000	0.7742	0.6123	0.6077	0.5983
b_2	−0.2500	−0.4379	−0.3134	−0.2519	−0.2497
b_3	0.2000	0.3206	0.2249	0.2031	0.1991

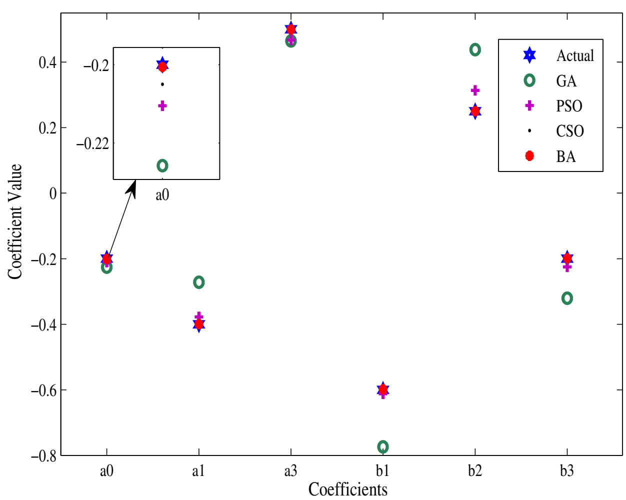

Fig. 7 Comparison of optimized coefficient values in Example 2 obtained by employing different evolutionary algorithms

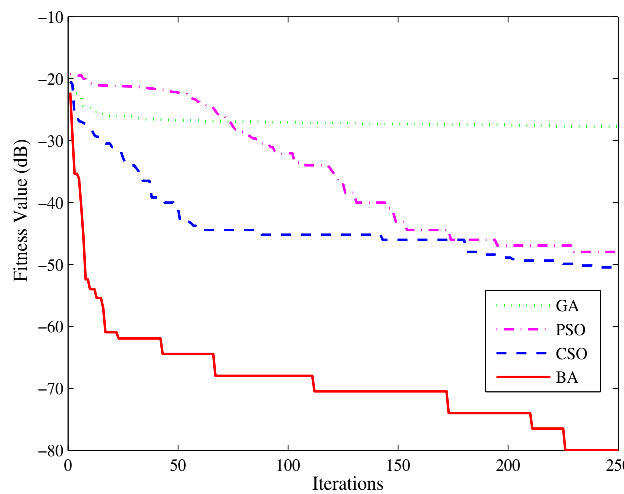

Fig. 8 Convergence profile for Example 2 modeled using same-order system obtained by employing different evolutionary algorithms

$$H_{SO}(z) = \frac{a_0 + a_1 z^{-1}}{1 - b_1 z^{-1} - b_2 z^{-2}} \tag{13}$$

The problem of system identification is reduced to optimize the numerator and denominator coefficients a_0, a_1 and b_1, b_2, respectively. The coefficients obtained are reported in Table 2 which leads to the best approximation to the unknown system using the evolutionary algorithms. From Table 2 and Fig. 3, it can be observed that BA contributes to the best approximation of the actual value of system coefficients compared to the reported algorithms. Statistical analysis of the performance of all algorithms is measured in terms of best, worst, mean, median and standard deviation of computed MSE and MSD of the identified system. The MSE and MSD values are summarized in Tables 3 and 4, respectively. The best MSE values obtained are 2.1569×10^{-5}, 6.3639×10^{-5}, 1.0116×10^{-4}, and 2.6428×10^{-4} for BA, CSO, PSO and GA, respectively. The best MSD values noticed for BA, CSO, PSO and GA are 2.25×10^{-8}, 1.1998×10^{-6}, 3.57×10^{-5}, and 4.9903×10^{-4}, respectively. It is evident from the above observations that the proposed BA-based system identification method yields the best results in terms of MSE and MSD as compared to GA, PSO and CSO. Convergence profiles for the least MSE values using different algorithms are demonstrated in Fig. 4. It is apparent from Fig. 4 that BA required 170 iterations to converge to the minimum fitness value of about −73 dB. Furthermore, based on the precipitousness it is concluded that the convergence speed of BA is much higher than that of the CSO, PSO and GA.

Table 8 Statistical comparison of MSE performance matric in Example 2 for identification of third-order IIR system using the same-order system

Algorithm	Mean square error (MSE)				
	Best	Worst	Average	Median	Standard deviation
BA	2.3037×10^{-5}	2.3045×10^{-5}	2.3039×10^{-5}	2.3482×10^{-5}	2.9143×10^{-12}
CSO	6.35201×10^{-5}	6.35201×10^{-5}	6.35201×10^{-5}	6.35201×10^{-5}	1.68717×10^{-18}
PSO	6.35202×10^{-5}	6.35206×10^{-5}	6.35203×10^{-5}	6.35202×10^{-5}	1.47673×10^{-10}
GA	7.32034×10^{-4}	6.15287×10^{-3}	2.51099×10^{-3}	2.19245×10^{-3}	1.48508×10^{-3}

Table 9 Statistical comparison of MSD performance matric in Example 2 for identification of third-order IIR system using the same-order system

Algorithm	Mean square deviation (MSD)				
	Best	Worst	Average	Median	Standard deviation
BA	1.3501×10^{-7}	1.3755×10^{-7}	1.3592×10^{-7}	1.6810×10^{-7}	2.2795×10^{-12}
CSO	1.22363×10^{-5}	1.22363×10^{-5}	1.22363×10^{-5}	1.22363×10^{-5}	8.24073×10^{-12}
PSO	1.21551×10^{-5}	1.23941×10^{-5}	1.22585×10^{-5}	1.22488×10^{-5}	7.30148×10^{-8}
GA	4.25648×10^{-3}	2.60326×10^{-2}	1.28355×10^{-2}	1.03561×10^{-2}	8.08953×10^{-3}

Table 10 Statistical comparison of MSE performance matric in Example 2 for identification of third-order IIR system using the reduced-order system

Algorithm	Mean square error (MSE)				
	Best	Worst	Average	Median	Standard deviation
BA	8.3264×10^{-4}	8.3264×10^{-4}	8.3264×10^{-4}	8.3264×10^{-4}	4.2975×10^{-20}
CSO	1.3938×10^{-3}	1.3938×10^{-3}	1.3938×10^{-3}	1.3938×10^{-3}	1.0842×10^{-19}
PSO	1.3938×10^{-3}	1.3938×10^{-3}	1.3938×10^{-3}	1.3938×10^{-3}	2.9692×10^{-19}
GA	1.6505×10^{-2}	6.6687×10^{-2}	3.2599×10^{-2}	2.4585×10^{-2}	1.6105×10^{-2}

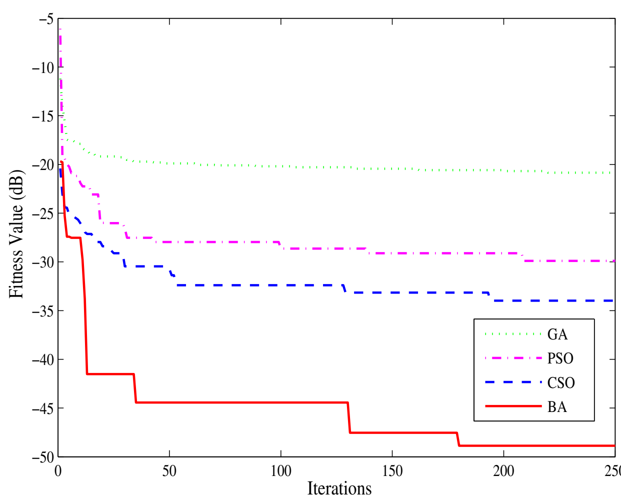

Fig. 9 Convergence profile for Example 2 modeled using reduced-order system obtained by employing different evolutionary algorithms

Case 2: Reduced Order

In this case, the second-order system is modeled using a first order unknown system with the transfer function given by

$$H_{RO}(z) = \frac{a_0}{1 - b_1 z^{-1}} \quad (14)$$

Here, MSE and convergence profile are the performance metric that are considered to valuate the performance of reduced-order system identification problem. Statistical results are taken into consideration to analyze the comparative performance of the BA, CSO, PSO and GA. Table 5 presents the MSE values. The best MSE values observed for BA, CSO, PSO and GA are 7.9178×10^{-3}, 1.7515×10^{-2}, 1.7515×10^{-2}, and 2.7122×10^{-2}, respectively. From the above observation, it is inferred that BA algorithm gives the best results for system identification problem compared to CSO, PSO and GA. Figure 5 depicts the convergence behavior of MSE values. It is observed from Fig. 5 that BA reaches the minimum fitness value of about −43 dB in 180 iterations.

The percentage improvement in the performance of BA over CSO, PSO and GA is graphically presented in Fig. 6 and listed in Table 6 for both same-order and reduced-order system identification. The percentage improvement observed in MSE for same-order system is 66.11 %, 78.67 % and 91.84 % for BA compared to CSO, PSO and GA, respec-

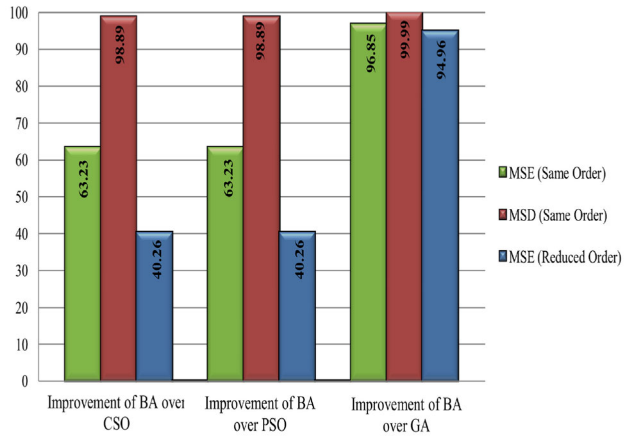

Fig. 10 Percentage improvement for Example 2 in terms of MSE and MSD as compared to other reported algorithms for same-order and reduced-order systems

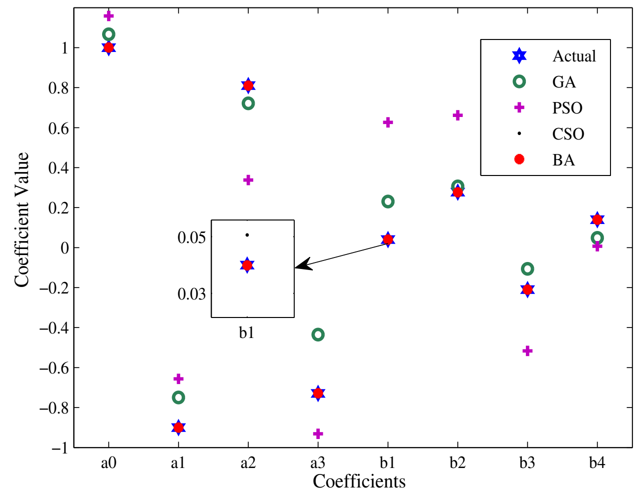

Fig. 11 Comparison of optimized coefficient values in Example 3 obtained by employing different evolutionary algorithms

tively. The percentage improvement obtained in MSD for BA compared to CSO, PSO and GA is 98.12 %, 99.94 % and 99.99 %, respectively, for same-order system. The percentage improvement noticed in MSE for the reduced-order system is 54.61 %, 54.61 % and 70.81 % for BA compared to CSO, PSO and GA, respectively. The above-mentioned results are verified from Table 6.

Example 2 Transfer function of the third-order IIR system, which is modeled below using the same-order and reduced-second-order systems, is given by

$$H_t(z) = \frac{-0.2 - 0.4z^{-1} + 0.5z^{-2}}{1 - 0.6z^{-1} + 0.25z^{-2} - 0.2z^{-3}} \tag{15}$$

Case 1: Same Order

The third-order system is approximated using a third-order unknown system with the transfer function given by

$$H_{SO}(z) = \frac{a_0 + a_1z^{-1} + a_2z^{-2}}{1 - b_1z^{-1} - b_2z^{-2} - b_3z^{-3}} \tag{16}$$

In this case, the optimization of system parameters, $a_0, a_1, a_2, b_1, b_2, b_3$, is executed using four algorithms. The estimated coefficients are enlisted in Table 7. Observations are made from Fig. 7 that the coefficient values obtained using BA approaches to the actual parameter values as compared to other methods. Figure 8 depicts the comparison

Table 11 Percentage improvement for different performance measures in Example 2 in comparison with other reported algorithms

Parameters	Percentage improvement in Example 2		
	BA compared to CSO	BA compared to PSO	BA compared to GA
MSE (same order)	63.23	63.23	96.85
MSD (same order)	98.89	98.89	99.99
MSE (reduced order)	40.26	40.26	94.96

Table 12 Optimized coefficients in Example 3 for identification of fourth-order IIR system using the same-order system

Coefficients	Exact value	Optimized value			
		GA	PSO	CSO	BA
a_0	1.0000	1.0670	1.1587	0.9951	1.0004
a_1	−0.9000	−0.7493	−0.6562	−0.8839	−0.9002
a_2	0.8100	0.7214	0.3380	0.8206	0.8099
a_3	−0.7290	−0.4350	−0.9309	−0.7253	−0.7286
b_1	−0.0400	−0.2308	−0.6264	−0.0506	−0.0399
b_2	−0.2775	−0.3064	−0.6618	−0.2930	−0.2768
b_3	0.2101	0.1065	0.5165	0.1962	0.2102
b_4	−0.1400	−0.0489	−0.0067	−0.1461	−0.1396

Table 13 Statistical comparison of MSE performance matric in Example 3 for identification of fourth-order IIR system using the same-order system

Algorithm	Mean square error (MSE)				
	Best	Worst	Average	Median	Standard deviation
BA	1.7315×10^{-5}	1.8029×10^{-5}	1.7615×10^{-5}	1.7660×10^{-5}	3.4863×10^{-9}
CSO	5.9421×10^{-5}	5.9444×10^{-5}	5.9428×10^{-5}	5.9425×10^{-5}	8.3021×10^{-9}
PSO	6.1146×10^{-5}	1.4251×10^{-4}	8.7325×10^{-5}	7.7249×10^{-5}	2.6268×10^{-5}
GA	7.1586×10^{-3}	4.4913×10^{-2}	1.7415×10^{-2}	1.2474×10^{-2}	1.2255×10^{-2}

Table 14 Statistical comparison of MSD performance matric in Example 3 for identification of fourth-order IIR system using the same-order system

Algorithm	Mean square deviation (MSD)				
	Best	Worst	Average	Median	Standard deviation
BA	2.0015×10^{-8}	2.3761×10^{-8}	2.1386×10^{-8}	2.1397×10^{-8}	5.3168×10^{-9}
CSO	1.3908×10^{-5}	1.4134×10^{-5}	1.44037×10^{-5}	1.4052×10^{-5}	9.6703×10^{-8}
PSO	1.1272×10^{-4}	1.3378×10^{-3}	4.7619×10^{-4}	2.4282×10^{-4}	4.7374×10^{-4}
GA	2.5197×10^{-2}	1.6039×10^{-1}	8.4828×10^{-2}	7.6787×10^{-2}	3.8506×10^{-2}

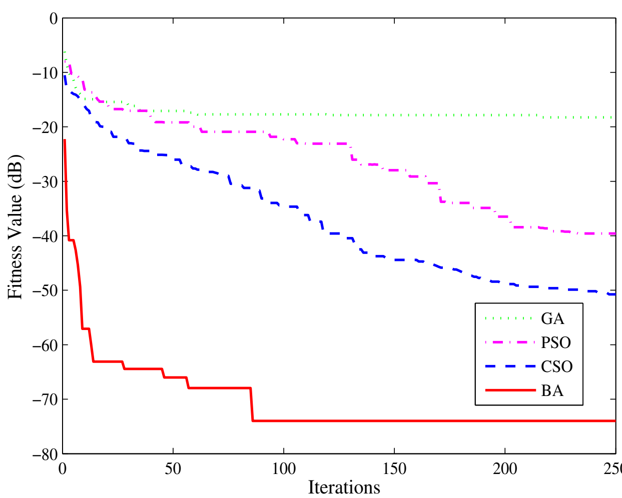

Fig. 12 Convergence profile for Example 3 modeled using same-order system obtained by employing different evolutionary algorithms

of the convergence profiles of BA, CSO, PSO and GA. It is seen that the least fitness value nears to −80 dB is prevailed by BA at 225th iteration. Moreover, the values of MSE and MSD ascertained are given in Tables 8 and 9, respectively. The best observed MSE for BA, CSO PSO and GA are 2.3037×10^{-5}, 6.3520×10^{-5}, 6.3520×10^{-5} and 7.3203×10^{-4}, respectively, and the best MSD values are 1.3501×10^{-7}, 1.2236×10^{-5}, 1.2155×10^{-5} and 4.2564×10^{-3}, respectively. It can be concluded from these measurements that the system identified using the BA approach possesses least MSE and MSD and an optimal approximation is delivered by BA.

Case 2: Reduced Order

Here, the system identification is based on modeling the third-order system using a second-order unknown system whose transfer function is given by

$$H_{RO}(z) = \frac{a_0 + a_1 z^{-1}}{1 - b_1 z^{-1} - b_2 z^{-2}} \tag{17}$$

The optimized coefficients yield a system with the MSE values given in Table 10. The convergence plot for the reduced-order approximation is shown in Fig. 9, and a minimum error of around −48 dB is obtained at 180th iteration by BA.

Furthermore, the improvement in the performance of the BA in the system identification problem over CSO, PSO and GA is evaluated and demonstrated in Fig. 10. It indicates that the results obtained in terms of MSE and MSD values for the same-order and reduced-order system modeling using BA are improved tremendously over GA. In addition, improvements are also observed over PSO and CSO to a large extent. The percentage values are reflected in Table 11.

Example 3 A fourth-order system is considered, and its transfer function is given by

$$H_t(z) = \frac{1 - 0.9z^{-1} + 0.81z^{-2} - 0.729z^{-3}}{1 + 0.04z^{-1} + 0.277z^{-2} - 0.2101z^{-3} + 0.14z^{-4}} \tag{18}$$

Case 1: Same Order

In this case, fourth-order system is modeled using a fourth-order unknown system with the transfer function given by

Table 15 Statistical comparison of MSE performance matric in Example 3 for identification of fourth-order IIR system using the reduced-order system

Algorithm	Mean square error (MSE)				
	Best	Worst	Average	Median	Standard deviation
BA	7.6325×10^{-4}	7.6325×10^{-4}	7.6325×10^{-4}	7.6325×10^{-4}	9.6156×10^{-12}
CSO	6.7051×10^{-3}	6.7051×10^{-3}	6.7051×10^{-3}	6.7051×10^{-3}	2.9990×10^{-11}
PSO	6.7051×10^{-3}	1.5666×10^{-2}	8.5485×10^{-3}	6.8008×10^{-3}	3.7473×10^{-3}
GA	1.9375×10^{-2}	9.2520×10^{-2}	4.6595×10^{-2}	4.4555×10^{-2}	2.3287×10^{-2}

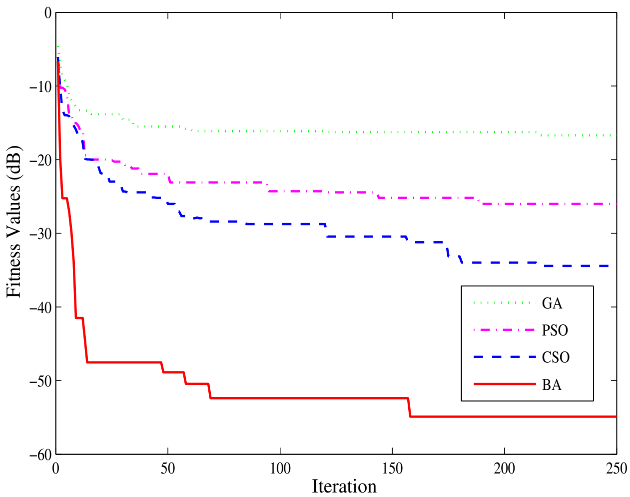

Fig. 13 Convergence profile for Example 3 modeled using reduced-order system obtained by employing different evolutionary algorithms

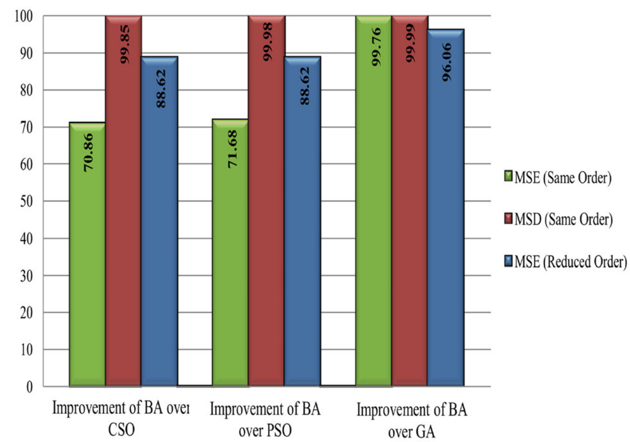

Fig. 14 Percentage improvement for Example 3 in terms of MSE and MSD as compared to other reported algorithms for same-order and reduced-order systems

$$H_{SO}(z) = \frac{a_0 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3}}{1 - b_1 z^{-1} - b_2 z^{-2} - b_3 z^{-3} - b_4 z^{-4}} \tag{19}$$

The problem of system identification is reduced to optimize the numerator and denominator coefficients a_0, a_1, a_2, a_3 and b_1, b_2, b_3, b_4, respectively. The coefficients obtained are listed in Table 12. From Table 12 and Fig. 11, it can be noticed that BA gives the best approximation of the actual value of system coefficients compared to the reported algorithms. Tables 13 and 14 summarize the MSE and MSD values, respectively. The best MSE values observed are $1.7315 \times 10^{-5}, 5.9421 \times 10^{-5}, 6.1146 \times 10^{-5}$, and 7.1586×10^{-3} for BA, CSO, PSO and GA, respectively. The best MSD values noted for BA, CSO, PSO and GA are $2.0015 \times 10^{-8}, 1.3908 \times 10^{-5}, 1.1272 \times 10^{-4}$, and 2.5197×10^{-2}, respectively. From the above observations, it is apparent that the proposed BA-based system identification method gives the best results in terms of MSE and MSD as compared with GA, PSO and CSO. Figure 12 shows the convergence profile for MSE values. It is evident from Fig. 12 that BA requires 85 iterations to converge to the minimum fitness value of about −75 dB. Furthermore, based on the steepness it is inferred that the convergence speed of BA is much higher than that of CSO, PSO and GA.

Case 2: Reduced Order
In this case, fourth-order system is modeled using a third-order unknown system with the transfer function given by

$$H_{RO}(z) = \frac{a_0 + a_1 z^{-1} + a_2 z^{-2}}{1 - b_1 z^{-1} - b_2 z^{-2} - b_3 z^{-3}} \tag{20}$$

Here, MSE and convergence profile are the performance metrics that are considered to valuate performance of the reduced-order system identification problem. The MSE values obtained with BA, CSO, PSO and GA are reported in Table 15. The MSE values observed for BA, CSO, PSO and GA are $7.6325 \times 10^{-4}, 6.7051 \times 10^{-3}, 6.7051 \times 10^{-3}$ and 1.9375×10^{-2}, respectively. From the above fact, it is inferred that BA yields best results compared to CSO, PSO and GA. The convergence behavior of MSE values using BA, CSO, PSO and GA is shown in Fig. 13. Figure 13 shows that BA converges to the minimum fitness value of about −55 dB in 155 iterations. Moreover, BA has very fast convergence rate compared to CSO, PSO and GA.

The percentage improvement in the performance of BA over CSO, PSO and GA for both same-order and reduced-

Table 16 Percentage improvement for different performance measures in Example 3 in comparison with other reported algorithms

Parameters	Percentage improvement in Example 3		
	BA compared to CSO	BA compared to PSO	BA compared to GA
MSE (same order)	70.86	71.68	99.76
MSD (same order)	99.85	99.98	99.99
MSE (reduced order)	88.62	88.62	96.06

Table 17 Optimized coefficients in Example 4 for identification of fifth-order IIR system using the same-order system

Coefficients	Exact value	Optimized value			
		GA	PSO	CSO	BA
a_0	0.1084	0.5083	0.2484	0.1038	0.1064
a_1	0.5419	0.7449	0.3789	0.5403	0.5326
a_2	1.0837	1.0303	1.6960	1.0813	1.0774
a_3	1.0837	1.0714	1.4109	1.0803	1.0925
a_4	0.5419	0.7067	0.8467	0.5447	0.5513
a_5	0.1084	0.3578	0.2684	0.1145	0.1091
b_1	−0.9853	−0.6080	−1.0628	−0.9768	−0.9890
b_2	−0.9738	−0.9316	−0.7275	−0.9632	−0.9709
b_3	−0.3864	−0.3451	−0.4842	−0.3827	−0.3878
b_4	−0.1112	−0.3382	−0.3291	−0.1137	−0.1093
b_5	−0.0113	−0.1848	−0.2238	−0.0167	−0.0121

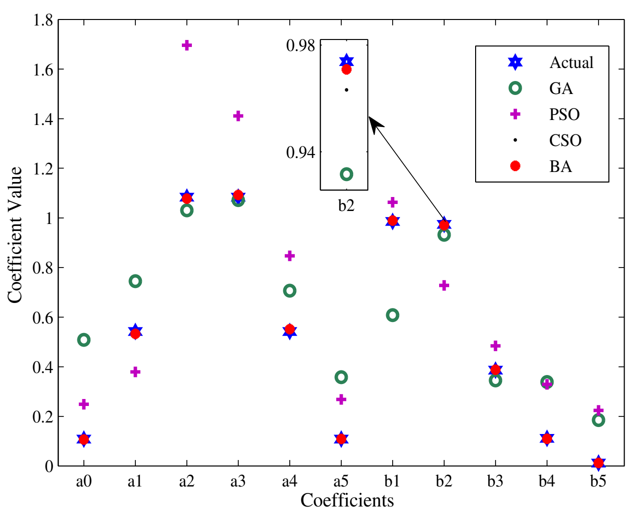

Fig. 15 Comparison of optimized coefficient values in Example 4 obtained by employing different evolutionary algorithms

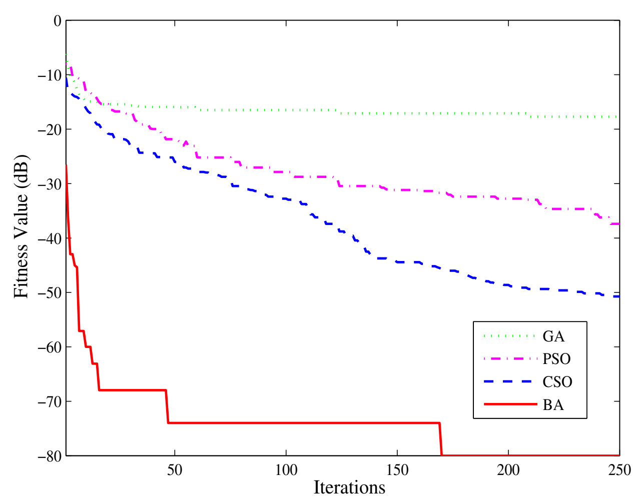

Fig. 16 Convergence profile for Example 4 modeled using same-order system obtained by employing different evolutionary algorithms

order systems are graphically presented in Fig. 14 and listed in Table 16. To compute the percentage improvement of BA over CSO, PSO and GA, MSE and MSD are the performance measures that are considered for same-order system identification, whereas in reduced-order system identification, MSE is the only performance measure that is considered. The percentage improvement obtained in MSE for same-order system is 70.86 %, 71.68 % and 99.76 % for BA compared to CSO, PSO and GA, respectively. The percentage improvement noticed in MSD for BA compared to CSO, PSO and GA is 99.85 %, 99.98 % and 99.99 %, respectively, for the same-order system. The percentage improvement observed in MSE for the reduced-order system is 88.62 %, 88.62 % and 96.06 % for BA compared to CSO, PSO and GA, respectively.

Example 4 The transfer function of the fifth-order IIR system, which is modeled below using the same-order and reduced-second-order system, is given by

$$H_t(z) = \frac{0.1084 + 0.5419z^{-1} + 1.0837z^{-2} + 1.0837z^{-3} + 0.5419z^{-4} + 0.1084z^{-5}}{1 + 0.9853z^{-1} + 0.9738z^{-2} + 0.3864z^{-3} + 0.1112z^{-4} + 0.0113z^{-5}} \quad (21)$$

Case 1: Same Order

The fifth-order system is approximated using a fifth-order unknown system with the transfer function given by

$$H_{SO}(z) = \frac{a_0 + a_1z^{-1} + a_2z^{-2} + a_3z^{-3} + a_4z^{-4} + a_5z^{-5}}{1 - b_1z^{-1} - b_2z^{-2} - b_3z^{-3} - b_4z^{-4} - b_5z^{-5}} \quad (22)$$

In this case, optimization of system parameters, $a_0, a_1, a_2, a_3, a_4, a_5, b_1, b_2, b_3, b_4, b_5$ is performed using BA, CSO, PSO and GA. The optimized coefficients are summarized in Table 17. Figure 15 shows that the estimated coefficient values obtained using BA approximately match the actual parameter values. Comparison of the convergence profiles for BA, CSO, PSO and GA is demonstrated in Fig. 16. It is seen that the minimum fitness value of about −80 dB is achieved by BA at 170th iteration. Furthermore, the values of MSE and MSD noted are summarized in Tables 18 and 19, respectively. The observed MSE values for BA, CSO, PSO and GA are 5.4017×10^{-5}, 6.3551×10^{-5}, 7.2739×10^{-5} and 1.3336×10^{-2}, respectively, and the best MSD values are 5.8182×10^{-6}, 1.0109×10^{-4}, 7.7019×10^{-4} and 2.9050×10^{-2}, respectively.

Case 2: Reduced Order

Here, the system identification is based on modeling the fifth-order system using a fourth-order unknown system whose transfer function is given by

$$H_{RO}(z) = \frac{a_0 + a_1z^{-1} + a_2z^{-2} + a_3z^{-3} + a_4z^{-4}}{1 - b_1z^{-1} - b_2z^{-2} - b_3z^{-3} - b_4z^{-4}} \quad (23)$$

The MSE values corresponding to the optimized coefficients are given in Table 20. The convergence plot for the reduced-

Table 18 Statistical comparison of MSE performance matric in Example 4 for identification of fifth-order IIR system using the same-order system

Algorithm	Mean square error (MSE)				
	Best	Worst	Average	Median	Standard deviation
BA	5.4017×10^{-5}	5.5102×10^{-5}	5.4957×10^{-5}	5.4899×10^{-5}	1.2658×10^{-7}
CSO	6.3551×10^{-5}	6.4493×10^{-5}	6.3937×10^{-5}	6.3972×10^{-5}	2.9027×10^{-7}
PSO	7.2739×10^{-5}	9.1496×10^{-5}	7.7614×10^{-5}	7.6137×10^{-5}	5.7365×10^{-6}
GA	1.3336×10^{-2}	6.4171×10^{-2}	3.3988×10^{-2}	3.0856×10^{-2}	1.4807×10^{-2}

Table 19 Statistical comparison of MSD performance matric in Example 4 for identification of fifth-order IIR system using the same-order system

Algorithm	Mean square deviation (MSD)				
	Best	Worst	Average	Median	Standard deviation
BA	5.8182×10^{-6}	6.0214×10^{-6}	5.8989×10^{-6}	5.8981×10^{-6}	8.6598×10^{-6}
CSO	1.0109×10^{-4}	2.3621×10^{-3}	4.5339×10^{-4}	2.3241×10^{-4}	6.8032×10^{-4}
PSO	7.7019×10^{-4}	7.8223×10^{-3}	4.6080×10^{-3}	4.6871×10^{-3}	2.7763×10^{-3}
GA	2.9050×10^{-2}	1.5968×10^{-1}	6.8121×10^{-2}	6.6429×10^{-2}	3.6147×10^{-2}

Table 20 Statistical comparison of MSE performance matric in Example 4 for identification of second-order IIR system using the reduced-order system

Algorithm	Mean square error (MSE)				
	Best	Worst	Average	Median	Standard deviation
BA	4.3986×10^{-5}	2.6589×10^{-5}	3.7621×10^{-5}	3.2549×10^{-5}	1.0268×10^{-5}
CSO	6.9475×10^{-5}	1.4516×10^{-4}	7.8534×10^{-5}	7.0347×10^{-5}	2.4988×10^{-5}
PSO	6.9373×10^{-5}	3.8379×10^{-3}	5.1656×10^{-4}	9.7148×10^{-5}	1.2458×10^{-3}
GA	8.4596×10^{-2}	2.9049×10^{5}	3.2386×10^{4}	5.8357×10^{-1}	9.6788×10^{4}

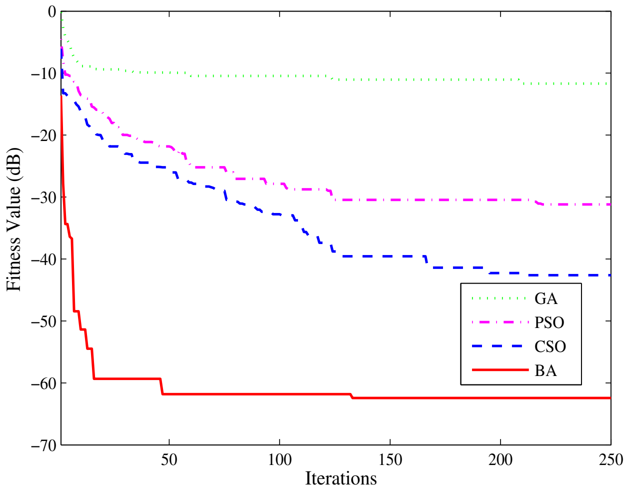

Fig. 17 Convergence profile for Example 4 modeled using reduced-order system obtained by employing different evolutionary algorithms

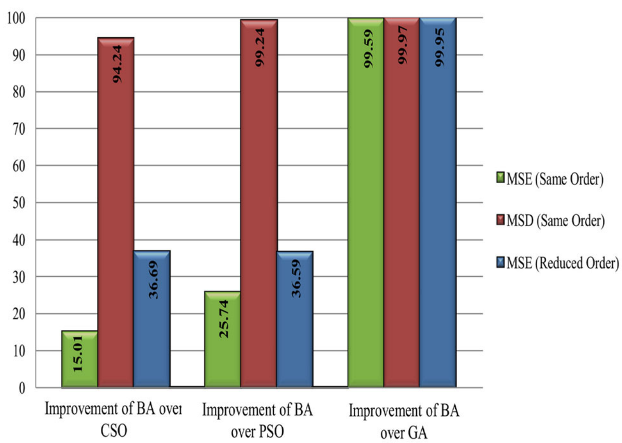

Fig. 18 Percentage improvement for Example 4 in terms of MSE and MSD as compared to other reported algorithms for same-order and reduced-order systems

order approximation is shown in Fig. 17, and a minimum error of around −63 dB is obtained at 135th iteration by BA. From Fig. 17, it is noticed that the bat algorithm converges faster to the minimum error fitness value compared to other reported algorithms.

Furthermore, the improvement in the performance of the BA in the system identification problem over CSO, PSO and GA is shown in Fig. 18 and its numerical values are reported in Table 21. A tremendous improvement in the performance of BA in terms of MSE and MSD values for the same-order and reduced-order system modeling is reported over GA, PSO and CSO. From Table 21 and Fig. 18, it can be concluded that the BA gives superior performance when compared to the other reported algorithms.

Table 21 Percentage improvement for different performance measures in Example 4 in comparison with other reported algorithms

Parameters	Percentage improvement in Example 4		
	BA compared to CSO	BA compared to PSO	BA compared to GA
MSE (same order)	15.01	25.74	99.59
MSD (same order)	94.24	99.24	99.97
MSE (reduced order)	36.69	36.59	99.95

Table 22 Computational time (in seconds) reported for different examples modeled using same-order system with different algorithms

Algorithm	Example 1	Example 2	Example 3	Example 4
BA	3.9016	4.2091	4.5626	4.6935
CSO	29.3125	66.0000	99.4218	166.9218
PSO	9.5468	21.3125	32.2656	54.4531
GA	65.7013	171.2500	297.3437	598.0468

4.2 Comparative Analysis of Computation Time for the Reported Algorithms

Table 22 summarizes the computation time observed with BA, CSO, PSO and GA for all the four examples. The computation time of BA for adaptive IIR system identification is 3.9016, 4.2091, 4.5626 and 4.6935 to model second-, third-, fourth- and fifth-order unknown system, respectively. From Table 22, it can be observed that the BA converges very fast to the optimal solution as compared to other reported algorithms. In addition, the percentage improvement in the computation time of BA over CSA, PSO and GA is calculated for all examples and given in Table 23 and demonstrated in Fig. 19. As can be observed from Table 23 and Fig. 19, bat algorithm exhibits a considerable improvement in the computation time compared to the other reported algorithms.

4.3 Comparison with the Existing Methods

The superiority in the performance of the adaptive IIR system identification using BA is proved by comparing its MSE with the existing techniques for system identification problem using DE, ABC, SOA, QPSO, MuQPSO, PSO, PSO–QI, CSA and GSA for both the same-order and reduced-order unknown systems [30–40]. The MSE values are summarized in Table 24.

For Example 1: Karaboga applied DE algorithm for the design of adaptive IIR filter whose response matches the second-order unknown system and reported the mean square error of 6.8500×10^{-2} with reduced-order system [30]. Fang et al. [33] presented the design of adaptive IIR filter using QPSO, and mean square error of 1.7300×10^{-1} is

Table 23 Percentage improvement in computation time for all examples in comparison with other reported algorithms

Example	Percentage improvement in computation time		
	BA compared to CSO	BA compared to PSO	BA compared to GA
Example 1	86.69	59.13	94.06
Example 2	93.62	80.25	97.54
Example 3	95.41	85.86	98.47
Example 4	95.61	91.38	99.22

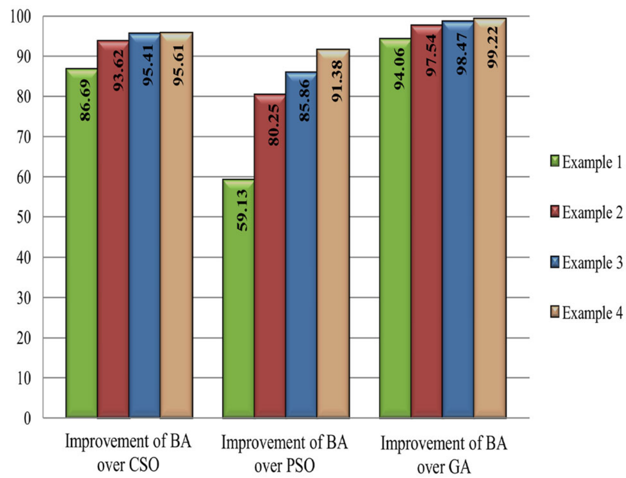

Fig. 19 Percentage improvement in computational time for different examples as compared to other reported algorithms

achieved with reduced-order system. Fang et al. [34] introduced the design of second-order adaptive IIR filter using MuQPSO, and MSE of 2.0600×10^{-1} is obtained for a reduced-order system. Majhi et al. [35] reported the design of second-order adaptive IIR filter using PSO and MSE of 1.5849×10^{-4} has been archived. Karaboga [31] implemented ABC algorithm for reduced-order system modeling, and MSE of 7.0600×10^{-2} is attained. Dai et al. [21] investigated the design of adaptive IIR filter using SOA and yields MSE value of 8.2773×10^{-2} for reduced-order system identification. Chen et al. [36] employed PSO for the reduced-order system modeling and reported the MSE value of 2.7500×10^{-1}. PSO algorithm is utilized for the reduced-order system identification by Durmus and Gun [39], and MSE value of 1.5000×10^{-2} is attained. CSO algorithm is applied for the same-order and reduced-order system identification by Panda et al. [13] which gives the MSE value of 6.3639×10^{-5} and 1.7515×10^{-2}, respectively. The MSE of 1.7200×10^{-1} was reported by Rashedi et al. [40] for reduced-order system modeling which utilizes GSA. In this work, we employed BA which yields MSE value of 2.1569×10^{-5} and 7.9178×10^{-3} for same- and reduced-order system modeling, respectively.

For Example 2: Fang et al. [34] presented the design of the third-order IIR filter using mutated quantum-behaved PSO with MSE value of 2.0410×10^{-3}. Luitel et al. introduced PSO–QI technique for modeling of same- and reduced-order systems which leads to the MSE value of 7.7910×10^{-4} and 4.0000×10^{-3}, respectively [37]. Panda et al. [13] applied CSO algorithm for same- and reduced-order system modeling which gives MSE of 6.3520×10^{-5} and 1.3938×10^{-3}, respectively. In this work, we employed BA which yields MSE value of 2.3037×10^{-5} and 8.3264×10^{-4} for same- and reduced-order system modeling, respectively.

For Example 3: Majhi et al. [35] introduced PSO algorithm for same-order system identification and best MSE value of 1.5849×10^{-4} is obtained. The PSO–QI technique was presented by Luitel et al. [37] for same- and reduced-order system modeling which gives the MSE value of 7.2450×10^{-4} and 5.0000×10^{-3}, respectively. The best MSE vales of 5.9421×10^{-5} and 6.7051×10^{-3} are archived by Panda et al. [13] with same- and reduced-order system, respectively. In this work, we employed BA which yields MSE value of 1.7315×10^{-5} and 7.6325×10^{-4} for same- and reduced-order system modeling, respectively.

For Example 4: Krusinski et al. [38] applied PSO algorithm for the design of the fifth-order IIR filter which yields a MSE value of 3.1623×10^{-4}. Panda et al. [13] applied CSO algorithm for same- and reduced-order system modeling which gives MSE of 6.3551×10^{-5} and 6.9475×10^{-5}, respectively. In this work, we employed BA which yields MSE value of 5.8182×10^{-6} and 4.3986×10^{-5} for same- and reduced-order system modeling, respectively.

5 Conclusions

In this paper, we have articulated IIR system identification as an optimization-based IIR system approximation problem using a new evolutionary optimization tool, bat algorithm. Next, BA is introduced for modeling the unknown system and is tested on four benchmarked systems to evaluate its performance. To measure the sustainability of the proposed method, extensive simulations have been carried out for modeling the unknown system using the same-order and reduced-order models. Comparison of simulation results is assessed in terms of mean square error, mean square deviation and computation time. It is observed that the BA-based IIR system identifica-

Table 24 Comparison in the performance of different reported algorithms for all examples in terms of MSE using same-order and reduced-order system

Example	Reference	Year	Algorithm	MSE	
				Same order	Reduced order
Example 1	Karaboga [30]	2005	DE	NR*	6.8500×10^{-2} (−11.6431 dB)
	Fang et al. [33]	2006	QPSO	NR*	1.7300×10^{-1} (−7.6196 dB)
	Majhi et al. [35]	2008	PSO	1.5849×10^{-4} (−38 dB)	NR*
	Karaboga [31]	2009	ABC	NR*	7.0600×10^{-2} (−11.5120 dB)
	Fang et al. [34]	2009	MuQPSO	NR*	2.0600×10^{-1} (−6.8613 dB)
	Dai et al. [21]	2010	SOA	NR*	8.2773×10^{-2} (−10.8211 dB)
	Chen et al. [36]	2010	PSO	NR*	2.7500×10^{-1} (−5.6067 dB)
	Durmus et al. [39]	2011	PSO	NR*	1.5000×10^{-2} (−18.2391 dB)
	Panda et al. [13]	2011	CSO	6.3639×10^{-5} (−41.9628 dB)	1.7515×10^{-2} (−17.5659 dB)
	Rashedi et al. [40]	2011	GSA	NR*	1.7200×10^{-1} (−7.6447 dB)
	Present study	–	BA	2.1569×10^{-5} (−46.6617 dB)	7.9178×10^{-3} (−21.0140 dB)
Example 2	Fang et al. [34]	2009	MuQPSO	2.0410×10^{-3} (−26.9016 dB)	NR*
	Luitel et al. [37]	2010	PSO–QI	7.7910×10^{-4} (−31.0841 dB)	4.0000×10^{-3} (−23.9794 dB)
	Panda et al. [13]	2011	CSO	6.3520×10^{-5} (−41.9709 dB)	1.3938×10^{-3} (−28.5580 dB)
	Present study	–	BA	2.3037×10^{-5} (−46.3757 dB)	8.3264×10^{-4} (−30.7954 dB)
Example 3	Majhi et al. [35]	2008	PSO	1.5849×10^{-4} (−38 dB)	NR*
	Luitel et al. [37]	2010	PSO–QI	7.2450×10^{-4} (−31.3996 dB)	5.0000×10^{-3} (−23.0103 dB)
	Panda et al. [13]	2011	CSO	5.9421×10^{-5} (−42.2606 dB)	6.7051×10^{-3} (−21.7359 dB)
	Present study	–	BA	1.7315×10^{-5} (−47.6158 dB)	7.6325×10^{-4} (−31.1733 dB)
Example 4	Krusinski et al. [38]	2004	PSO	3.1623×10^{-4} (−35 dB)	NR*
	Panda et al. [13]	2011	CSO	6.3551×10^{-5} (−41.9688 dB)	6.9475×10^{-5} (−41.5817 dB)
	Present study	–	BA	5.8182×10^{-6} (−52.3521 dB)	4.3986×10^{-5} (−43.5669 dB)

* *NR* not reported

tion exhibits superior performance compared to the existing evolutionary algorithms, GA, PSO and CSO. Furthermore, the incorporation of BA requires less number of control parameter tuning, which increases the flexibility of IIR system identification and reduces its computational time.

Further, this work can be extended for the identification of complex fractional systems. In addition, the proposed method needs to be explored as a future scope, for Volterra-based nonlinear system identification problem.

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