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The Use of Genetic Programming and Regression Analysis for Modeling the Modulus of Elasticity of NSC and HSC

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Abstract Artificial intelligence has recently drawn the attention of explorers to predict the physical, chemical and mechanical properties of normal-strength concrete (NSC) and high-strength concrete (HSC). This study presents gene expression programming (GEP) and regression analysis (RA) for modeling the modulus of elasticity (E_c) from the compressive strength (f_c) values of NSC and HSC. In order to create the models, experimental results of NSC and HSC are collected from the published literature. The evaluated results by training, testing and checking of the GEP and RA models are compared with the results obtained from the experimental studies, the formulations presented by some national building codes and the formulations proposed by some authors available in the literature. These comparisons and statistic results show that GEP and RA models are very effective methods for calculating the E_c from f_c of NSC and HSC.

Keywords Compressive strength · Modulus of elasticity · Genetic programming · Regression analysis

1 Introduction

In recent times, high-strength concretes (HSCs) have taken extensive attention among civil engineers and building contractors compared to normal-strength concrete (NSC) [1]. This is because durability, mechanical properties and service life of HSC are better than the properties of NSC. High strength is obtained feasible by fulfilling the conditions of

Mustafa Sarıdemir msdemir@nigde.edu.tr curing and by decreasing porosity, inhomogeneity and microcracks in the concrete and the interface zone between cement paste and aggregate. This can be accomplished by using highrange water-reducing admixtures and mineral additives such as fly ash, silica fume, metakaolin, granulated blast furnace slag, rice husk ash and some natural pozzolans [2]. They are usually utilized to generate extra strength undergoing pozzolanic reaction with Ca(OH)₂, to enhance the durability, to extend the service life and to decrease the permeability of concrete [3]. Besides, many of these materials are industrial waste products and help in decreasing the amount of cement required to obtain concrete less costly, more nature friendly and less energy intensive [2].

The cylindrical f_c values of concretes at 28 days higher or equal than 41 MPa are generally described as HSC according to ACI Committee 363 [4]. The difference between NSC and HSC is fundamentally the addition of mineral additives and chemical admixtures. The addition of chemical admixtures decreases water requirement; therefore, they reduce the porosity within the hydrated cement matrix and the transition zone [5,6]. Mineral additives, also named as cement replacement or cementitious and supplementary materials, are used as pozzolanic materials in addition to fine fillers; therefore, microstructure of hardened cement paste occurs compacter and stronger [6,7].

Usually, the f_c value of concrete is the most important mechanical property to be taken into consideration in the concrete design. Moreover, the E_c value of concrete is a very important mechanical property exhibiting the flexibility property of concrete. Besides, the E_c value of concrete supplies a bridge between stress and strain or force and deformations [8]. As the f_c value of concrete strongly affects the E_c value of concrete, different national building codes and some authors have made many attempts to formulate a relationship between the f_c and E_c values for NSC and HSC.



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Several relationships between the f_c and E_c values for NSC are given by Eqs. (1) and (2).

ACI 318 [9]:

$$E_{\rm c} = 4.73 (f_{\rm c})^{0.5} \tag{1}$$

Kim et al. [10]:

$$E_{\rm c} = 5.25 (f_{\rm c})^{0.46} \tag{2}$$

Various relationships between the f_c and E_c values for HSC are given by Eqs. (3)–(8). ACI 363 [4]:

$$E_{\rm c} = 3.32(f_{\rm c})^{0.5} + 6.9\tag{3}$$

CEB-FIP MC90 [7]:

$$E_{\rm c} = 10(f_{\rm c} + 8)^{1/3} \tag{4}$$

Euorocode 2 [11]:

$$E_{\rm c} = 9.5(f_{\rm c} + 8)^{1/3} \tag{5}$$

Rashid et al. [8]

$$E_{\rm c} = 8.9(f_{\rm c})^{0.33} \tag{6}$$

Sarıdemir [12]:

$$E_{\rm c} = 8.64(f_{\rm c})^{1/3} \tag{7}$$

Gesoğlu et al. [13]:

$$E_{\rm c} = 5.535(f_{\rm c})^{1/2} - 5.552 \tag{8}$$

where f_c (MPa) and E_c (GPa) are cylindrical f_c and E_c values of concrete, respectively.

It is very well known that the elastic properties of concrete are affected by elastic properties of the component materials and structure of the interface region between aggregate particles and cement matrix [14, 15]. Because of the intrinsic inflexibility and large space, which is engaged in the concrete, the aggregate particles exert the effect on the E_c of NSC and HSC. Not only the aggregate particles inflexibility, but also the aggregate particles type influences the E_c of concrete [15]. In addition, properties, amounts and kinds of mineral additives and chemical admixtures used in the concrete mixtures, and the shapes, sizes, vibrating, curing and examining methods of concrete samples affect the E_c of concrete [16].

The main objective of the present study is proposing new equations to predict E_c from cylindrical f_c of NSC and

HSC by GEP and RA methods. For this purpose, the concrete with cylindrical f_c lower than 41 MPa are defined as NSC, while the concrete with cylindrical f_c higher or equal than 41 MPa are defined as HSC. The proposed GEP and RA models for NSC and HSC were trained and tested with the results collected from various experimental studies. Therefore, the explicit equations were also obtained from GEP and RA. These equations were checked with the results collected from different experimental studies independent from the experimental studies used in training and testing. For the calculation of $E_{\rm c}$ from the corresponding $f_{\rm c}$ of NSC and HSC, the number of experimental data employed for training is 88 and 262, testing is 44 and 88, and checking is 67 and 74, respectively. The evaluated results by proposed GEP and RA models were compared with the results of experimental studies, national building codes and various equations proposed by some authors. The comparisons show a good agreement between the results of experimental studies and the results of GEP and RA models according to the other results.

2 Gene Expression Programming

GEP has five basic parts. These are the sets of function and terminal, suitability function, control variables and stop situation, which must be explained when employing GEP to solve a problem. A mathematical equation is developed by GEP using a data set in this problem. GEP method employs a fixed length of character strings named as "expression tree" (ET) to develop this equation [17,18]. As shown in Fig. 1, crossover randomly constituted pieces of two trees to assemble good knowledge from the parents and to build the fitness of the next generation. As shown in Fig. 2, mutation defends the model against premature assembly and improves the out of local properties of search. Sometimes, a randomly constituted knot is modified by another one from the same set [19].

2.1 Gene Expression Programming Models

In the present study, for modeling the E_c values depending on the f_c values of NSC, among 132 experimental data collected from the different literature [20–23], approximately 70% of the entire data (88 sets) were randomly separated as training set and the remaining of the entire data (44 sets) were randomly taken as testing set. In addition, 67 experimental data collected from the literature [10,24–26] were only distinguished as checking set. Similarly, for modeling the E_c values depending on the f_c values of HSC, among 350 experimental data collected from the literature [2, 13, 20, 22, 24, 27–32], approximately 75% of the entire data (262 sets) were randomly separated as training set and the remaining of the entire data (88 sets) were randomly taken as testing set. Besides,

Fig. 1 Example of crossover



Fig. 2 Example of mutation

Table 1	Ranges of
experime	ental data employed to
training,	testing and checking

nput-output variables	NSC		HSC		
	Minimum	Maximum	Minimum	Maximum	
f _c (MPa)	7.98	40.90	41.10	119.90	
E _c (GPa)	7.77	34.10	22.57	54.43	

74 experimental data collected from the literature [25,33–35] were only distinguished as checking set. The limit values of input and output variable for training, testing and checking sets of NSC and HSC are shown in Table 1. As stated above, the further part of modeling makes up of five base steps for solving a problem by employing GEP. The first step was selecting the set of suitability functions. The second step was making up the selection of terminal sets for E_c values and the set of functions for f_c values to constitute the chromosomes. In this step, the terminal sets composed clearly of the independent variable, $E_c = f(f_c)$. To determine the relationship between the E_c and f_c values, the mathematical expressions

 $(+, -, \times, x^{1/3})$ and $(+, \times, x^{1/3})$ were used in the models of NSC and HSC, respectively. In the third step, the number of chromosomes, the gene number and the length of the head size were selected. In this step, one gene and two head size lengths were firstly utilized, and then, the gene numbers and the head size lengths were raised during each successive study, and the training and testing performances of each model were observed. In the present study, after many trials, the number of genes and length of head sizes were defined as one and four for the both models, respectively. The linking function was chosen in the fourth step. The multiplication was employed as linking function for the GEP models of



Table 2 GEP parameters used

for proposed models

174
.,.
+, ×, 3Rt
20
4
1
Multiplication
0.044
0.1
0.3
0.3
0.1
0.1



Fig. 3 Expression tree of GEP model proposed for NSC



NSC and HSC. The set of genetic operators, which brought about change, and their rates were chosen in the final step. The combinations of sets of all genetic operators are given in Table 2. The best of generation individuals was observed at the number of chromosome as 20 for calculating E_c values from the corresponding f_c values for the both models.

The explicit formulations based on GEP models for predicting the E_c values of NSC and HSC are obtained by Eq. (9). For the GEP models of NSC and HSC, the expression tree of formulations which are actually $E_c = ((c2) \times 3Rt(d(0) - c1)))$ and $E_c = ((c0) \times 3Rt(d(0)))$ is shown in Figs. 3 and 4, respectively. The real parameter is $d0 = f_c$, and the constants are c0=9.84, c1=6.00 and c2=8.66 in the formulation for the GEP models of NSC and HSC. The final formulations based on GEP models for the E_c of NSC and HSC are given by Eqs. (10) and (11), respectively.



$E_{\rm c} =$	$f(f_{\rm c})$	(9)
C	5 (50)	(-)

NSC:
$$E_{\rm c} = 8.66(f_{\rm c} - 6)^{1/3}$$
 $f_{\rm c} < 41 {\rm MPa}$ (10)

HSC: $E_c = 9.84(f_c)^{1/3}$ $f_c \ge 41$ MPa (11)

3 Regression Analysis

A regression analysis (RA) model includes independent input variable and corresponding output variable for common examples, and afterward preparing a predefined mathematical relationship to the input and output variables. On the basis of the predefined mathematical expressions, regression analyses are divided as linear and/or nonlinear regression. As mentioned above, various regression formulations based on nonlinear correlation have been proposed by different national building codes and some authors to model the E_c values of NSC and HSC. These formulations mostly express E_c as function of the f_c or the f_c together with the specific gravity of the aggregate used in the concrete mixture. In this study, a RA was made by using the computer program for the calculation of E_c values from the corresponding f_c values of NSC and HSC.

3.1 Regression Analysis Models

The RA models were separately performed with the curve fitting technique called as shifted powder function for NSC and HSC. For the comparison of the results obtained from the GEP and RA models, the same data were used in the training, testing and checking sets of the GEP and RA models. The relationship between the ages, E_c and f_c values of NSC and HSC are given in Fig. 5a, b as three dimensional, respectively. The relationship between the E_c and f_c values of NSC and HSC are evaluated by using formulations obtained from shifted power regression model as seen in Fig. 6a, b. The



Fig. 5 a Effect of age on E_c and f_c of NSC, **b** effect of age on E_c and f_c of HSC

general form of the shifted powder regression model is given in Eq. (12).

$$E_{ci} = a(f_{ci} - b)^{c} \tag{12}$$

where E_{ci} and f_{ci} are the modulus of elasticity (GPa) and compressive strengths (MPa) at the same day of NSC and HSC. The *a*, *b* and *c* represent the values of the constants obtained from RA. The constant, standard error (SE), linear correlation coefficient (*R*) and R-square (R^2) of shifted power regression models are given in Table 3. Equations (13) and (14) have been derived for describing a relationship between the E_c and f_c values of NSC and HSC at all ages according to constants given in Table 3.

NSC:
$$E_c = 6.95(f_c - 6.87)^{0.4}$$
 $f_c < 41 \text{ MPa}$ (13)

HSC:
$$E_{\rm c} = 6.35(f_{\rm c} - 13.45)^{0.46}$$
 $f_{\rm c} \ge 41 \,{\rm MPa}$ (14)



Fig. 6 a Relationship between E_c with f_c of NSC, **b** the relationship between E_c with f_c of HSC

4 Results and Discussion

In this study, three statistical parameters were utilized to compare the performance of GEP and RA models. These parameters are mean absolute percentage error (MAPE), root-mean-squared error (RMSE) and R-square (R^2) presented in Eqs. (13), (14) and (15), respectively. These parameters are evaluated from the relationship of experimental results with GEP and RA models, the formulations presented by some national building codes and the formulations proposed by some authors available in the literature.

MAPE =
$$\frac{1}{n} \left[\frac{\sum_{i=1}^{n} |t_i - o_i|}{\sum_{i=1}^{n} t_i} \times 100 \right]$$
 (15)

RMSE =
$$\sqrt{\frac{1}{n} \sum_{i=1}^{n} (t_i - o_i)^2}$$
 (16)



Table 3 Results of shiftedpower regression models

	Equation	Parameters					R	\mathbb{R}^2
		Va	lue	SE	Range (95% confidence)			
NSC	$E_{\rm c} = a(f_{\rm c} - b)^{\rm c}$	а	6.95	1.274	4.413–9.481	2.394	0.849	0.721
		b	6.87	1.252	4.379-9.359			
		c	0.40	0.052	0.293-0.499			
HSC	$E_{\rm c} = a(f_{\rm c} - b)^{\rm c}$	а	6.35	3.609	-0.760 to 13.450	3.62	0.850	0.722
		b	13.45	12.398	-10.960 to 37.860			
		с	0.46	0.115	0.228-0.681			

$$R^{2} = \frac{(n \sum t_{i} o_{i} - \sum t_{i} \sum o_{i})^{2}}{(n \sum t_{i}^{2} - (\sum t_{i})^{2})(n \sum o_{i}^{2} - (\sum o_{i})^{2})}$$
(17)

where *t* is the target value, *o* is the output value, and *n* is total number of data.

4.1 The Results of GEP and RA Models of NSC

In order to create empirical models and to indicate the generalization ability of models obtained from the GEP and RA, the database obtained from the experimental study for NSC is subdivided into three sets, namely training, testing and checking sets. None of the experimental results used for checking set was utilized in the training and testing sets. Figures 7, 8 and 9 show the results evaluated by the proposed formulation in the GEP and RA models, ACI 318 [4], ACI 363 [4] and Kim et al. [10] versus the experimental results of NSC for training, testing and checking sets, respectively. Also, the linear least square fit line and the R^2 values are given in these figures for the training, testing and checking data. Figures 7 and 8 exhibit how well the nonlinear



Fig. 7 Comparison of experimental with calculated training results of NSC



Fig. 8 Comparison of experimental with calculated testing results of NSC



Fig. 9 Comparison of experimental with calculated checking results of NSC

relation between parameters is obtained from training and testing results of GEP and RA models for NSC, respectively. Figure 9 shows the accuracy in generalization of the check-

Table 4Statistical parametersof experimental results with the

predicted results for NSC

Statistical parameters	NSC						
	GEP	RA	ACI 318	ACI363	Kim et al.		
Training							
MAPE	10.163	8.637	12.151	12.171	10.882		
RMSE	2.582	2.372	3.125	2.962	2.726		
R^2	0.722	0.721	0.709	0.709	0.710		
Testing							
MAPE	10.438	9.049	12.426	12.475	10.920		
RMSE	2.796	2.568	3.199	3.135	2.865		
R^2	0.703	0.704	0.710	0.710	0.710		
Checking							
MAPE	3.961	6.923	4.222	5.340	4.104		
RMSE	2.058	2.283	2.131	2.154	2.020		
R^2	0.719	0.719	0.717	0.717	0.718		



Fig. 10 Comparison of experimental with calculated training results of HSC



Fig. 11 Comparison of experimental with calculated testing results of HSC



Fig. 12 Comparison of experimental with calculated checking results of HSC

ing results obtained from proposed GEP and RA models for NSC. The whole of the results demonstrate a prosperous performance of GEP and RA models for calculating E_c from the corresponding f_c of NSC for the each of training, testing and checking sets.

The statistical analysis results of the training, testing and checking sets of GEP and RA models, ACI 318 [4], ACI 363 [4] and Kim et al. [10] formulations for calculating E_c of NSC are given in Table 4. All of the MAPE, RMSE and R^2 values given in Table 4 exhibit that the recommended GEP and RA models are appropriate for calculating the E_c values of NSC similar to the experimental results at different curing days. Statistical parameters evaluated from the results of RA model are also compared with those obtained from the GEP model. The best value of R^2 is observed in the GEP model for NSC.



Statistical parameters	HSC								
	GEP	RA	ACI363	CEB-FIP MC90	Eurocode 2	Rashid et al.	Sarıdemir	Gesoğlu et al.	
Training									
MAPE	11.4226	8.2306	11.9338	14.8223	12.2191	10.9570	11.2362	8.5763	
RMSE	4.6299	3.6894	5.8839	5.9808	4.8341	5.1416	5.4334	3.7632	
R^2	0.7219	0.7216	0.7211	0.7218	0.7218	0.7219	0.7219	0.7211	
Testing									
MAPE	12.2021	8.8545	12.6896	15.5184	13.0044	12.0626	12.3481	9.2063	
RMSE	4.9545	3.7461	6.2639	6.1990	5.2519	5.5980	5.8846	3.8436	
R^2	0.7584	0.7585	0.7563	0.7575	0.7575	0.7584	0.7584	0.7563	
Checking									
MAPE	8.6668	11.2580	21.8716	6.4464	8.4173	18.0921	19.3405	10.6325	
RMSE	4.8338	5.6191	10.0022	3.5323	4.7707	8.5305	9.0393	5.3735	
R^2	0.6901	0.6907	0.6879	0.6890	0.6890	0.6901	0.6901	0.6879	

Table 5 Statistical parameters of experimental results with the predicted results for HSC

4.2 The Results of GEP and RA Models of HSC

Similar to the experimental results of NSC, the database obtained from the experimental results for E_c and f_c of HSC is separated into three sets as training, testing and checking sets. The evaluated results of the training, testing and checking sets of GEP and RA models, and the results calculated by using ACI 363 [4], CEB-FIP MC90 [7], Euorocode 2 [11], Rashid et al. [8], Sarıdemir [12] and Gesoğlu et al. [13] formulations for calculating E_c values of HSC were compared with experimental results as seen in Figures 10, 11 and 12, respectively. These figures show how well the nonlinear relation between parameters is obtained from training, testing and checking results of GEP and RA models for HSC, respectively. All of the findings demonstrate a prosperous performance of the GEP and RA models for calculating E_c from the corresponding f_c of HSC for the each of training, testing and checking sets.

The statistical analysis results of the training, testing and checking sets of the GEP and RA models, ACI 363 [4], CEB-FIP MC90 [7], Euorocode 2 [11], Rashid et al. [8], Sarıdemir [12] and Gesoğlu et al. [13] formulations for calculating E_c of HSC are given in Table 5. All of the MAPE, RMSE and R^2 values given in Table 5 demonstrate that the proposed GEP and RA models are suitable for calculating E_c values of HSC, which are compatible with the experimental results at the different curing days. The statistical values of the training, testing and checking sets of GEP and RA models indicate a good correlation between the E_c and fc values of HSC. The best value of R^2 is observed in the RA model for HSC among the R^2 values.

5 Conclusions

In the present study, two soft computing methods, namely GEP and RA, are applied for predicting the E_c values from the corresponding f_c values of NSC and HSC. These methods are efficient approaches for predicting the $E_{\rm c}$ values from the f_c values of NSC and HSC. Therefore, the GEP and RA models are recommended in order to predict the E_c values of NSC and HSC. The recommended models are empirical and based on the experimental studies. Furthermore, the formulations are obtained from the developed models in the GEP and RA. All of the statistical results of MAPE, RMS and R^2 show that these models have successful performances for predicting the $E_{\rm c}$ values from the $f_{\rm c}$ values of NSC and HSC for each of the training, testing and checking sets. In addition, the comparisons of MAPE, RMS and R^2 values reveal that these models are found to be more accurate according to some national building codes and other formulations. Finally, the formulations obtained from these models are so easy that they can be used by anyone not certainly familiar with GEP and RA.

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