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Simple Approach to Design PID Controller via Internal Model Control

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Abstract The internal model control (IMC)-based PID controller is widely used in industrial control problems. This scheme provides a good compromise among set-point tracking, disturbance attenuation, and robustness. Therefore, in this paper, we propose a simple technique to design IMC-PID controller. To illustrate the utility of the proposed technique, different types of linear and nonlinear second-order systems and approximated second-order models of higher-order systems are simulated. The proposed approach depicts quick response to set-point change, good disturbance attenuation, and optimal performances in most of the class of problems when compared to the conventional IMC-PID and other existing popular techniques. The beauty of this paper is that there is no need of highly complex mathematical approaches, and using only simple conventional IMC approach, the improved servo and regulatory results can be achieved.

Keywords Disturbance rejection · Filter · Internal model control · PID tuning · Second-order system

1 Introduction

For the past few decades, proportional integral derivative (PID) controller has a strong hold in the field of control engineering for wide area applications (such as machinery, chemical and food, mining, automobile, electrical power and apparatus, and aerospace industries) and still it is an area of intensive research [1]. PID controller designing is interesting due to simple PID structure that generally leads to numerous tuning techniques and several ways to implement it. These tuning techniques are based on either smooth and fast set-point tracking, closed-loop system stability, minimization of disturbance effect, prevention of excessive control action, actuator saturation and integrator windup, robustness to system parameters and modeling errors, or optimization of performance indices [2–9]. In closed-loop system, PID can be arranged as feed-forward or feedback control, cascade control, selectors, limiters, ratio control, etc, and may be obtained in the form of parallel or series single-loop controller, multi-loop controllers, programmable logic controllers, distributed control systems, etc [10]. The different structures of PID controller are

- Ideal structure: $K_1(s) = K_c\left(1 + \frac{1}{\tau_I s} + \tau_D s\right)$
- Series structure: $K_2(s) = K_c\left(1 + \frac{1}{\tau_I s}\right)\left(\frac{\tau_D s+1}{\alpha\tau_D s+1}\right)$
- Parallel structure: $K_3(s) = K_c\left(1 + \frac{1}{\tau_I s} + \frac{\tau_D s}{\alpha\tau_D s+1}\right)$
- Series filter: $K_4(s) = K_c\left(1 + \frac{1}{\tau_I s} + \tau_D s\right)\left(\frac{1}{\tau_f s+1}\right)$

Over the past one and a half decade, internal model control (IMC) has revolutionized the control system field. This control technique has brought a great impact on control practices in terms of performance issues like intuitive and simple control algorithm; optimal and robust design; and servo and regulatory performance [11]. Particularly, due to single user-defined tuning parameter λ, IMC-based PID control has gained widespread acceptance among all existing tuning techniques based on specifications of design, process knowledge, and computational aspects. In the literature, there exists numerous IMC-based PID tuning algorithms for delayed, oscillatory, unstable, higher-order systems, and non-

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linear systems. The main role of IMC-PID is to eliminate the problem of anti-reset windup, dead-time compensators, decouplers which are generally less associated with other PID controllers [12].

IMC-PID control scheme was first presented by Rivera et al. [13]. In this paper, it was shown that converting the IMC-based control structure into conventional feedback PID control might bring addition lag term of the form $1/(1+\varphi s)$, $\varphi > 0$ for some processes of the first- and second-order, and integrating type. Later on, Hang et al. [14] showed that IMC-PID fails for lag dominant processes with relative small dead time. Then, Horn and co-workers framed IMC-PID controller in series with a filter of second or higher order. This formulation leads to complex structure and heavy computation burden [15]. The aforementioned works drew the attention of researchers to investigate the optimum PID tuning rules based on IMC technique. As a result, Lee et al. [16] showed that PID controller tuning based on Maclaurin series brings improved dynamic performance.

Another aspect in IMC-PID design is that approximating a process to its lower-order model plays an important role in efficient design of control system. Therefore, based on model order reduction, some heuristic methods have also been explored to evaluate PID parameters [17–19]. However, Shamsuzzoha and Lee demonstrated the superiority of using higher-order IMC filter for better disturbance rejection performance [20]. Furthermore, some recent techniques of PI/PID tuning are also evaluated on the basis of user- defined percentage overshoot specification, maximum complementary sensitivity function, two-degree-of-freedom (2DOF) IMC structure, H_∞ design, and frequency-dependent uncertainty constraints [21–26]. A slight modification of robust PI/PID based on IMC has been conducted in [27], which reports the utility of mid- and high-frequency robustness for tuning parameters. Recently, 2DOF-based IMC tuning involving gain margin and maximum peak criterion is also proposed for processes having uncertain models [28]. These methods also received much attention due to optimal performance and robust tuning method. However, after going through these schemes, we realized that PID can be evolved in another fashion through basic or traditional IMC concept. The concept is based on neglecting the lag term obtained after converting IMC controller to conventional feedback controller form.

The whole paper is framed in following sections. Section 2 reviews the principle of IMC scheme. Section 3 illustrates how to evaluate PID parameters from IMC design scheme and its optimum tuning. Performance evaluation criterions are explained in Sect. 4. Simulations and efficiency of the control law are carried out for some class of second-order process and models in Sect. 5. In Sect. 6, the proposed technique is further extended to nonlinear second-order processes. Some remarks about the proposed scheme and their future prospects are given in Sect. 7. Finally, conclusions are drawn in Sect. 8.

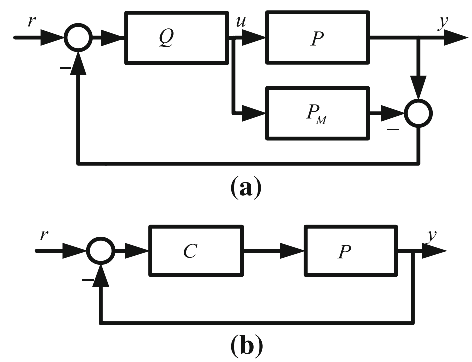


Fig. 1 **a** Internal model control and **b** conventional feedback control structures

2 Fundamentals of IMC

IMC design scheme is a type of model predictive-based control technique conceptualizing the pole–zero cancelation in which a controller constitutes plant model as an important tool to design and implement controller. The structures of IMC and conventional feedback control loop are shown in Fig. 1a, b. IMC contains the process model P_M in parallel to P. The difference between the process and its model is fed back to the controller Q. The design procedure requires two tasks: (1) determination of the inverse of the process and (2) adjustment of the IMC filter's parameter to give an *optimal* system performance [11]. The controller comprises of inverse of the process model augmented with a low-pass filter F. This filter F is generally of the form

$$F(s) = \frac{1}{(1+\lambda s)^n}, \quad n \in I, \lambda > 0 \tag{1}$$

where n is sufficiently large to make the controller proper; λ is a tuning parameter responsible for robustness, process/model mismatch, and closed-loop performance of the control system. 's' is a Laplace operator and $s = j\omega \, \forall \, \omega \in \mathbb{R}$. Thus, the IMC-based controller may be defined as

$$Q(s) = P_M^{-1}(s)F(s) \tag{2}$$

One should note that the inverse of the plant model used in (2) should be free from the predictor factor ($e^{\tau s}$) and RHP system. If the model contains delay and RHP poles and zeros, then factorize the process model into minimum phase (MP) and non-minimum phase (NMP) elements. See [11] for more details.

For a linear time invariant system, the IMC framework can be easily converted into conventional feedback controller $C(s)$ using the following conversion formula

$$C(s) = \frac{Q(s)}{1 - P_M(s)Q(s)} \tag{3}$$

On substituting (2) in (3), we get

$$C(s) = \frac{F(s)}{1 - F(s)} P_M^{-1}(s) \tag{4}$$

For a perfect model, $P_M^{-1}(s)$ can be replaced by $P^{-1}(s)$. Note that, $C(s)$ inherently provides integral action as $C(s) = \frac{1}{s}\frac{N(s)}{D(s)}$ where $N(s)$ and $D(s)$ are polynomial functions.

3 Proposed Control Law

In practice, the higher-order process control plant is approximated to either first- or second-order plant with time delay. Therefore, in this proposed approach, we stick our discussion to the design of IMC-PID controller by approximating higher-order model to second-order systems. Our proposed control strategy proceeds in the following manner.

Suppose the higher-order system is represented by

$$P(s) = \frac{a_0 + a_1 s + a_2 s + \cdots + a_m s^m}{b_0 + b_1 s + b_2 s^2 + \cdots + b_n s^n}, \quad m < n, a_i, b_j \in \mathbb{R}, i = 1, \ldots, m, j = 1, 2, \ldots, n \tag{5}$$

Now, we approximate the above system (5) in the generalized second-order system of the form

$$P(s) = \frac{K}{s^2 + 2\varsigma\omega s + \omega^2}(-s + z)e^{-\theta s} \tag{6}$$

In order to design IMC-based PID controller, we first separate out NMP part, i.e., $P_{NM}(s)$ and MP part, i.e., $P_M(s)$ from (6) as

$$P_{NM}(s) = (-s + z)e^{-\theta s} \tag{7}$$

$$P_M(s) = \frac{K}{s^2 + 2\varsigma\omega s + \omega^2} \tag{8}$$

The order of the plant as shown in (8) is two; therefore in order to realize IMC controller, we select the filter as

$$F(s) = (1 + \lambda s)^{-2} \tag{9}$$

Substituting (8) and (9) in (4), we get the controller:

$$C(s) = \left(\frac{(1 + \lambda s)^{-2}}{1 - (1 + \lambda s)^{-2}}\right)\left(\frac{s^2 + 2\varsigma\omega s + \omega^2}{K}\right) \tag{10}$$

After some algebraic manipulations, (10) becomes

$$C(s) = \frac{2\varsigma\omega}{K\lambda^2}\left(1 + \frac{1}{(2\varsigma/\omega)s} + \frac{1}{2\varsigma\omega}s\right)\left(\frac{1}{s + 2/\lambda}\right) \tag{11}$$

The above equation clearly represents the PID form followed by another filter function which we termed as "*lag term*". So, (11) can also be written as

$$C(s) = C_{\text{PID}}(s) \cdot C_{\text{Lag}}(s) \tag{12}$$

where

$$C_{\text{PID}}(s) = K_P\left(1 + \frac{1}{T_I s} + T_D s\right) \tag{13}$$

$$C_{\text{Lag}}(s) = \left(\frac{1}{s + 2/\lambda}\right) \tag{14}$$

and

$$K_P = \frac{2\varsigma\omega}{K\lambda^2}; \quad T_I = \frac{2\varsigma}{\omega}; \quad T_D = \frac{1}{2\varsigma\omega} \tag{15}$$

In (15), K_P is the PID gain and the integral and derivative time constants are T_I and T_D, respectively. In the proposed approach, we are neglecting $C_{\text{Lag}}(s)$, and thus, the new controller obtained will contain only PID term given by (13), i.e., $\tilde{C}(s) = C_{\text{PID}}(s)$.

Remark 1 It should be noted that the order of the controller is reduced on separating the lag term. Thus, one can state that the proposed scheme follows model order reduction in controller instead of plant-model order reduction. However, there is a difference in controller reduction and plant reduction. The controller reduction sustains closed-loop stability, closed-loop performance, and closed-loop transfer function, whereas plant reduction constitutes only open-loop consideration [29].

Remark 2 In the presented IMC design scheme, we approximate the original systems and the controller includes only the minimum phase part of the plant. Such model approximation has generally an impact in high- frequency region and $F(s)$ is a low-pass filter which governs the closed-loop bandwidth of the control system. Therefore, the controller prevents the modeling effect and brings improved results.

Now, we will discuss the effect of modeling error when the lag term in the controller is omitted. The modeling error between the PID controller with and without lag term (denoted by $C(s)$ and $\tilde{C}(s)$, respectively) can be written as

$$E(s) \triangleq \left|C(s) - \tilde{C}(s)\right| = K_P\left(1 + \frac{1}{T_I s} + T_D s\right)\left(\frac{s + (2/\lambda - 1)}{s + 2/\lambda}\right) \tag{16}$$

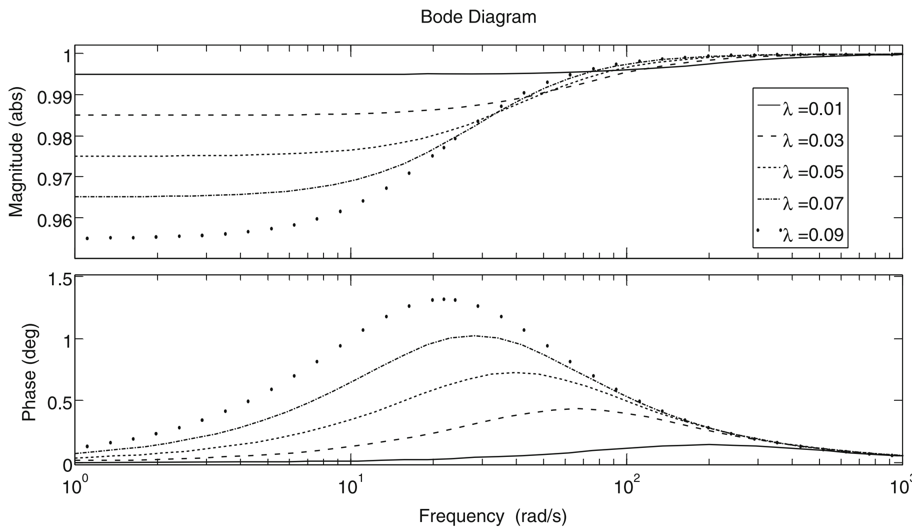


Fig. 2 Bode plot of normalized error term

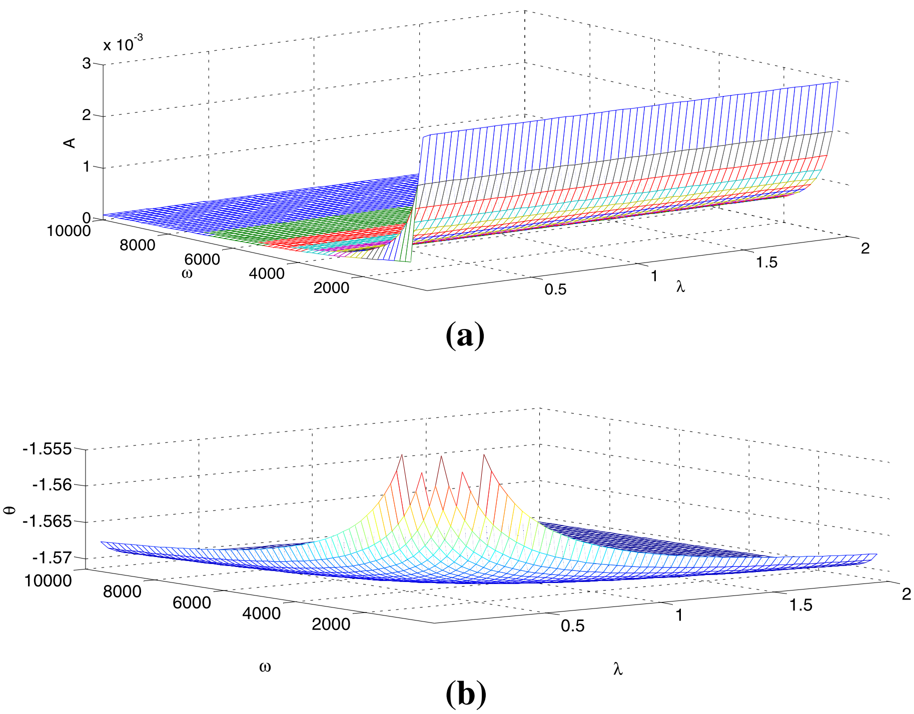


Fig. 3 **a** Magnitude and **b** phase plot of lag term for different values of λ and ω

Next, the normalized error is given as

$$\tilde{E}(s) = \left|\frac{C(s) - \tilde{C}(s)}{\tilde{C}(s)}\right| = \left(\frac{s + (2/\lambda - 1)}{s + 2/\lambda}\right) \tag{17}$$

which clearly reveals that $\tilde{E}(s)$ is a function of tuning parameter λ. So, our objective is now to optimize the value of λ to nullify the normalized error. Furthermore, a rough idea about the effect of λ on the frequency response of the normalized error term can be examined using Bode plot. Figure 2 shows the variation of $\tilde{E}$ in terms of magnitude and phase plots for different values of λ, which states that the magnitude of $\tilde{E}(s)$ become unity at high frequency for different values of λ.

Now, we discuss the effect of neglecting lag term. From (14), the lag term can be expressed in the magnitude-phase form, i.e., $1/(s + (2/\lambda)) = A\angle\theta$ where magnitude A and phase angle θ are, respectively,

$$|A| = \frac{1}{\sqrt{\omega^2 + \frac{4}{\lambda^2}}} > 0 \quad \forall \text{ all } \lambda, \omega \tag{18}$$

$$\angle\theta = \tan^{-1}\left(-\frac{\omega\lambda}{2}\right) < 0 \quad \forall \text{ all } \lambda, \omega \tag{19}$$

Figure 3 also states that the magnitude is always positive, while the phase angle is always negative. Now, it is clear

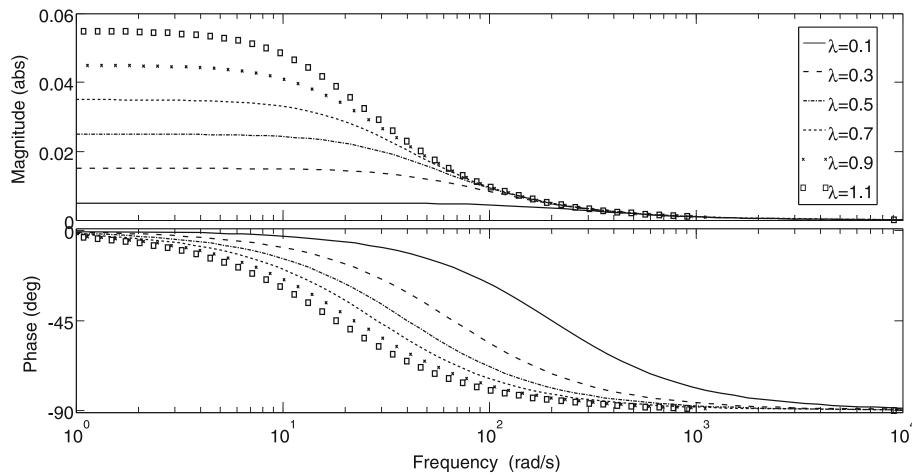


Fig. 4 Bode plot of lag term

that adding the lag term increases the gain by a small factor but phase lag significantly. Therefore, the phase margin is reduced and the closed-loop system may become oscillatory. Thus if the lag term is removed, the stability of the system improves.

Moreover, the frequency response of lag term is plotted in Fig. 4. This figure depicts the effect of magnitude and phase plot for different values of λ which shows that both the magnitude and phase vary as the λ varies. It is also observed that higher the λ, higher will be the magnitude of lag term and roll-off in phase response. Therefore, λ should be selected as small as possible.

4 Performance Assessment

In this paper, the performance assessment of the proposed controller is evaluated by the following indices.

4.1 Integral Error Criterion

Minimizing the loss or cost function proves the optimality of the controller. Normally, the typical criterion to minimize the error function of the form

$$I = \int_0^\infty t^n \left|e(t)\right|^m \mathrm{d}t \tag{20}$$

is utilized to evaluate the performance indices such as integrated error (IE), integral of the squared error (ISE), integral of the absolute error (IAE), and integral of the time weighted absolute error (ITAE). These performance indices can be defined as

$$IE = \int_0^\infty e(t)dt;\quad ISE = \int_0^\infty e(t)^2 dt;\ IAE = \int_0^\infty |e(t)|\,dt;\quad ITAE = \int_0^\infty t\,|e(t)|\,dt; \tag{21}$$

where $e(t) = r(t) - y(t)$ is an error signal. IE simply accumulates the net error and describes the performance of monotonic response. ISE denotes indirectly several characteristics like settling time, overshoot, speed of response, and all other important features of the transient response [30]. IAE is a measure of disturbance rejection for integral controller [31]. ITAE accounts for long duration error.

4.2 Maximum Sensitivity to Modeling Error

The sensitivity and complementary sensitivity functions denoted by M_s and M_t, respectively, form the basis of robustness to process variations and stability of a control system. These specifications are based on Nyquist stability criterion of a loop transfer function and are defined by

$$M_s = \max_\omega \left|\frac{1}{1 + P(i\omega)C(i\omega)}\right|,\quad M_t = \max_\omega \left|\frac{P(i\omega)C(i\omega)}{1 + P(i\omega)C(i\omega)}\right| \tag{22}$$

It is suggested that the maximum sensitivities should be in range of 1 to 2 for suitable control. Moreover, the small value of peak of sensitivity denotes large distance of Nyquist plot from critical point [1].

5 Numerical Studies

In this section, we consider seven different classes of systems. For each class of system, controller has been designed using the existing methods and the proposed method. All the computation and simulations have been carried out in MATLAB and SIMULINK environment.

Table 1 PID parameters by the existing methods

Methods	K_P	T_I	T_D	Augmented filter
Proposed	0.0734	0.01178	1×10^{-3}	–
Rivera et al.	7.34210×10^{-4}	0.01181	1×10^{-3}	–
Lee et al.	2.11500×10^{-4}	0.00680	0.0067	–
SIMC	7.00105×10^{-4}	0.01125	–	–
Honeywell	18.66949×10^{-4}	0.0118	9.9746×10^{-4}	$\frac{9.9746\times10^{-4}s+1}{9.9746\times10^{-5}s+1}$

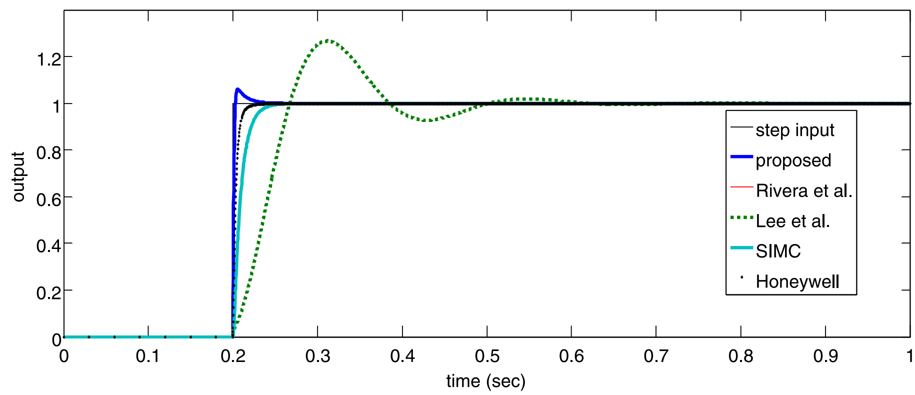


Fig. 5 Simulation results of closed-loop system with PID controllers for unit step input

5.1 Second-Order Over Damped System

To illustrate the procedure of the proposed PID design, consider an example of DC motor speed control in which plant $P(s)$ is a second-order given by [32] as

$$P(s) = \frac{1.362 \times 10^8}{s^2 + 1000s + 8.476 \times 10^4} \tag{23}$$

For IMC-based design, treat the model of plant P_M same as P. Now, applying the technique mentioned in Sect. 3 (using (8) and (9)), the conventional feedback controller can be evaluated as

$$C(s) = \left(0.0734 + \frac{6.233}{s} + 7.34 \times 10^{-5}s\right)\left(\frac{1}{s+200}\right) \tag{24}$$

In (24), $\lambda = 0.01$ is taken as a suitable value and the lag term to be neglected is $C_{\text{Lag}}(s) = 1/(s + 200)$.

To justify the efficacy and efficiency of the proposed technique, PID parameters are evaluated using some existing methods described by researchers [13,16,19,33]. The PID tuning formulas of these methods are presented in Appendix, and their calculated values are given in Table 1. The reasons behind selecting these particular methods for comparison are as follows:

- IMC-PID developed by Rivera and co-workers [13] is the first and foremost scheme,
- the second [16] and the third tuning [19] techniques have been selected because they are the state-of-the art and popular schemes, respectively, in the area of IMC-PID,
- the last technique (Honeywell tuning rule) developed by Aström et al. [33] is employed in industries.

Table 2 Performance comparison for set-point change

Method	IE	ISE	IAE	ITAE	M_S	M_t
Proposed	0.0001	0.0018	0.0006	0.0019	1.005	1.001
Rivera et al.	0.01	0.01	0.0053	0.0101	1.001	1.001
Lee et al.	0.02	0.061	0.0285	0.0660	1.001	1.001
SIMC	0.01	0.01	0.0054	0.0101	1.08	1.001
Honeywell	0.00393	0.0039	0.0019	0.0040	1.00	1.001

A closed-loop response of the process with PID controller for unit step input is shown in Fig. 5. The comparison of responses of the proposed method and other methods clearly states that the speed of response using the proposed PID controller is faster than that of other methods with a very low overshoot. The performance indices of the proposed method and other methods described are listed in the first four columns of the Table 2. The proposed technique has least value for various integral error indices and hence shows optimal and better performance in comparison with all other mentioned techniques. Moreover, the proposed technique has same level of robustness ($M_s = 1.005$) except for SIMC method [19] as mentioned in the second last column of the Table 2.

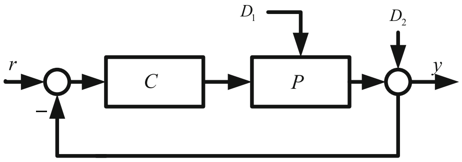


Fig. 6 Control system with input and output disturbances

To measure the disturbance rejection performance, the step input of magnitude 0.5 is applied at the input and output (See D_1 and D_2 signal in Fig. 6). The disturbance response of the proposed method is very sharp, and its magnitude is very low when the disturbance acts at input of the plant as shown in Fig. 7a. In this figure, the peak magnitude of the disturbance response is 0.3, whereas for other methods it is quite large. Likewise, when the disturbance acts at output of the plant, the speed of disturbance rejection is very fast as compared to other methods; see Fig. 7b. However, the magnitudes of the responses are almost same for all other methods. Thus, it can be said that the proposed technique speeds up the response of the control system for both set-point change and disturbance rejection attributes. Also, it minimizes the effect (overshoot) of the disturbance.

The optimality of the proposed controller for disturbance rejection is evaluated which is shown in Table 3. Since all the indices are minimum for the proposed method for both input and output disturbances, therefore, it can be concluded that the proposed controller shows the optimal performance.

5.2 Second-Order Under Damped System

To incorporate other type of commonly used second-order processes, we consider an under damped system described by

$$P(s) = \frac{1}{s^2 + 8s + 25} \tag{25}$$

where $\varsigma = 0.8$. Using the perfect model and the proposed method, the PID controller obtained after neglecting the lag term is

Fig. 7 **a** Input and **b** output disturbance responses of second-order overdamped system

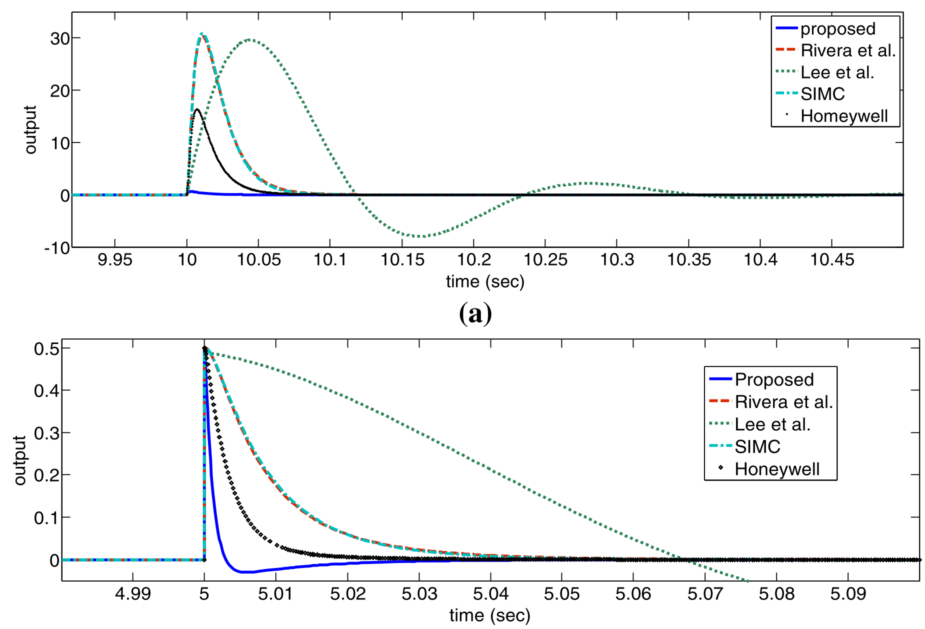
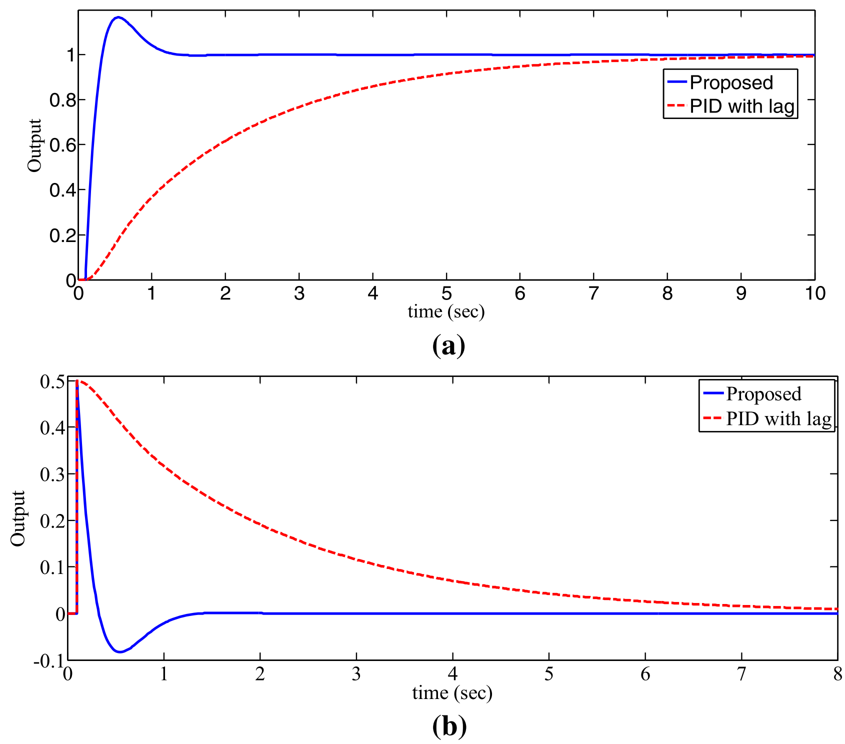


Table 3 Performance comparison for disturbance

Method	Input disturbance				Output disturbance			
	IE	ISE	IAE	ITAE	IE	ISE	IAE	ITAE
Proposed	0.00837	0.008444	0.002956	0.0847	0.00005	0.0009051	0.000143	0.004553
Rivera et al.	0.8369	0.8369	16.84	8.387	0.005	0.005	0.001322	0.02505
Lee et al.	1.674	2.909	52.52	29.36	0.01	0.0308	0.007268	0.1565
SIMC	0.8369	0.8369	17.3	8.387	0.005	0.005	0.001353	0.02505
Honeywell	0.3287	0.3287	3.412	3.292	0.00196	0.001963	0.000479	0.009827

Fig. 8 Responses of second-order underdamped system for **a** unit step input and **b** disturbance



$$C(s) = \frac{1}{\lambda^2}\left(8 + \frac{25}{s} + s\right) \tag{26}$$

Taking $\lambda = 0.3162$, we obtain the output of the system for step-type input as shown in Fig. 8. Further, a step-type disturbance of magnitude of 0.5 is applied at the output. It is observed that set-point tracking and disturbance rejection are faster than that of PID having lag term.

5.3 Second-Order Under Damped System Having Delay and RHP Zero

We consider plant having delay term as well as RHP zeros from [18] as:

$$P(s) = \frac{-0.5s + 1}{(s+1)(2s+1)}e^{-s} \tag{27}$$

where the MP part is

$$P_M(s) = \frac{1}{(s+1)(2s+1)} \tag{28}$$

The frequency responses of (27) and (28) are shown in Fig. 9a. The response depicts that there is uncertainty due to ignorance of RHP zeros and delay term. As shown in Fig. 9a, modeling errors are visualized after 5 rad/s approximately. Now we design controller using proposed approach. The proposed method utilizes only MP part to design PID controller which yields

$$C(s) = \frac{1}{2\lambda^2}\left(1.5 + \frac{0.5}{s} + s\right) \tag{29}$$

where λ is set to 0.3162. From Fig. 9b, c, it is seen that an improved performance is achieved with respect to unit step input and disturbance rejection. However, the PID tuning parameters for (27) evaluated by Wang et al. [18] are $K_P = 1.1194$, $K_I = 0.3569$, $K_D = 0.9765$, which gives a little bit faster response but significant overshoot and almost same setting time in comparison with that of proposed one. And, the response of the PID with lag term is poor among all.

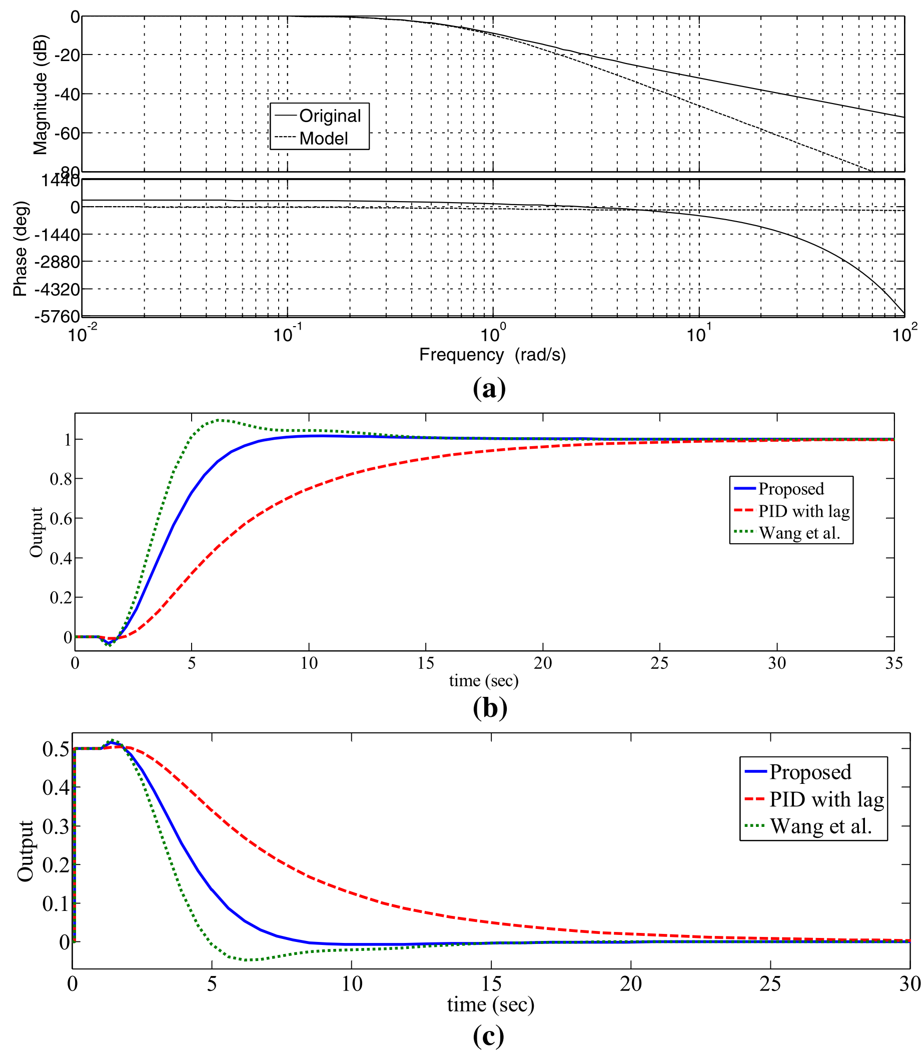
5.4 Second-Order System Depicting Pole–Zero cancelation

A peculiar type of system having pole–zero cancelation property is described by the transfer function as:

$$P(s) = \frac{2s + 4}{s^2 + 4s + 4} \tag{30}$$

where the pole–zero cancelation occurs at $s = -2$. Normally, the pole–zero cancelation is not frequent in real-time conditions since real-time environment produce uncertainties and the poles and zeros do not overlap each other

Fig. 9 **a** Frequency, **b** output for unit step input and **c** disturbance responses of second-order having time delay and RHP zero



at same location. However, we include precise overlapping of pole–zero and the obtained model after cancelation is

$$P_M(s) = \frac{2}{s+2} \tag{31}$$

The proposed IMC-PID controller for this plant-model mismatched case is

$$C(s) = \frac{1}{4\lambda^2}\left(1 + \frac{2}{s}\right) \tag{32}$$

where we select $\lambda = 0.1$. Here, as usual for a first-order plant model, PI form is obtained. Figure 10 demonstrates the closed-loop response of the proposed method and the conventional PID with lag term. The result shows that in comparison with PID with lag term, the proposed method depicts the faster response for unit step input (Fig. 10a) and faster disturbance attenuation (Fig. 10b).

5.5 Fourth-Order Over Damped System

Consider a higher-order system from [19]

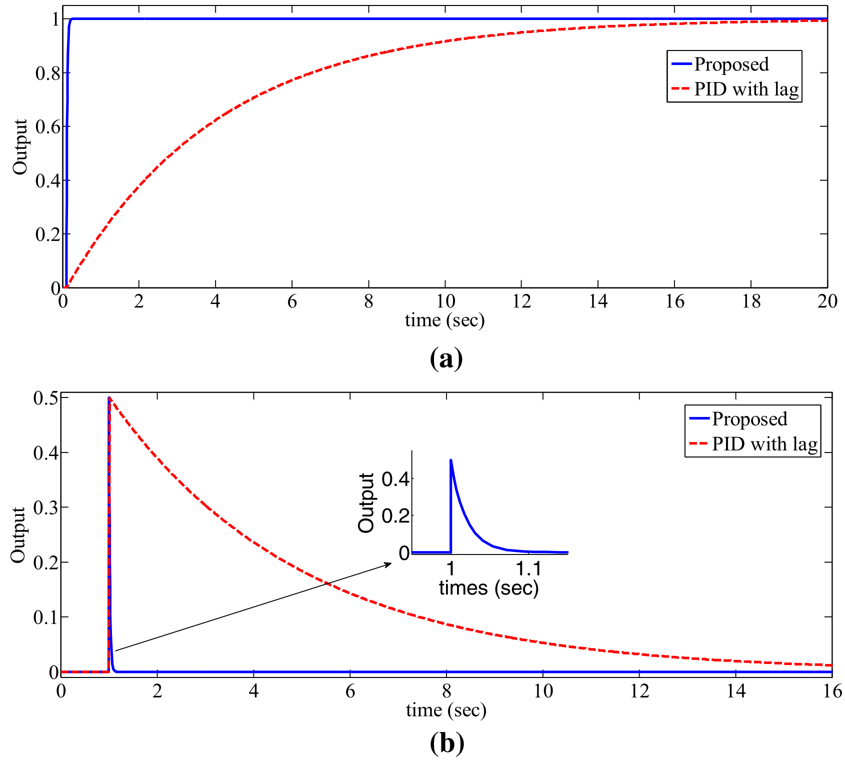
$$P(s) = \frac{1}{(s+1)(0.2s+1)(0.04s+1)(0.0008s+1)} \tag{33}$$

for which Skogestad's procedure produces the approximated second-order plus dead-time (SOPDT) model as

$$P_M(s) = \frac{1}{(s+1)(0.22s+1)} e^{-0.028s} \tag{34}$$

Here, the fourth-order process is reduced to SOPDT model, thereby creating the plant-model mismatching. The modeling inaccuracies can be easily depicted in Fig. 11a where the magnitude and phase responses vary drastically after 1000 rad/s. Using (34), the proposed IMC-PID method yields

Fig. 10 **a** Output for unit step input and **b** disturbance responses of second-order system having pole–zero cancelation



$$C(s) = \frac{1}{4.5455\lambda^2}\left(5.5455 + \frac{4.5455}{s} + s\right) \quad (35)$$

where we set $\lambda = 1$. The PID settings for (33) using Skogestad's method of SIMC are $K_P = 17.9$, $T_I = 0.224$, $T_D = 0.22$ [19]. The comparison of proposed method, SIMC method, and PID with lag term is carried out for unit step input and output load disturbance, as shown in Fig. 11b, c, respectively. Although the setting time is almost same for all responses, the proposed method produces nice and smooth behavior, whereas SIMC gives significant overshoot and oscillations of approximately 40% and PID with lag term gives sluggish response.

5.6 Fourth-Order System Having Zero on LHP

To account for the unstructured modeling, we take another process of [19] as:

$$P(s) = \frac{2(15s + 1)}{(20s + 1)(s + 1)(0.1s + 1)^2} \quad (36)$$

The approximated model for this fourth-order model suggested by Skogestad is

$$P_M(s) = \frac{1.5}{(s + 1)(0.15s + 1)}e^{-0.05s} \quad (37)$$

The uncertainty of the original system and its reduced model can be observed easily in frequency response plot as shown in Fig. 12a, where the behavior matching exists for original system and its reduced model up to 50 rad/s only and vary rapidly afterward. The controller design for this approximated model using proposed scheme gives

$$C(s) = \frac{1}{10\lambda^2}\left(7.6667 + \frac{6.6667}{s} + s\right) \quad (38)$$

where $\lambda = 0.3162$. The SIMC settings for the same model are $K_P = 6.67$, $T_I = 0.4$, $T_D = 0.15$ [19]. The comparisons of the responses for unit step input and output disturbance rejection are shown in Fig 12b, c. The proposed controller provides superior response in comparison with SIMC method with reference to overshoot. As usual due to the presence of lag term, the response of PID with lag term is poor among all.

5.7 Second-Order Undamped System

Consider a second-order oscillatory system

$$P(s) = P_M(s) = \frac{1}{s^2 + 10} \quad (39)$$

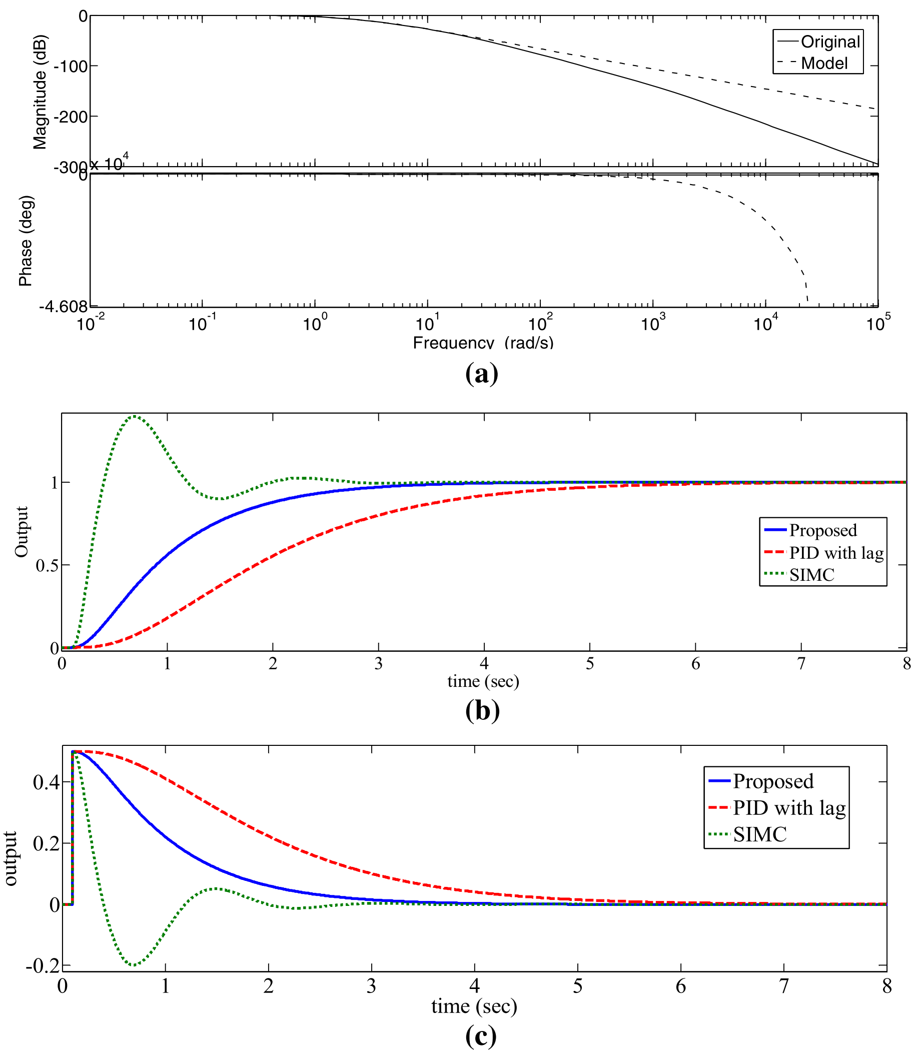


Fig. 11 **a** Frequency, **b** output for unit step input and **c** disturbance responses of fourth-order overdamped system

having zero damping factor. The PID controller using proposed scheme gives

$$C(s) = \frac{1}{\lambda^2}\left(\frac{10}{s} + s\right) \tag{40}$$

We set $\lambda = 1$ and the response for unit step input and disturbance are shown in Fig. (13). Since the nature of system is oscillatory, still the proposed controller is able to attenuate oscillations and disturbance, whereas the conventional controller with lag term gives unstable response.

To illustrate the optimality of the proposed technique, the performance indices in terms of integral error are also evaluated in Table 4 for processes described in Sect. 5.2 to 5.7. The proposed technique gives minimum value of integral errors in comparison with conventional PID with lag term. However, for the case described in Sect. 5.3, tuning done by Wang et al. method shows least value due to faster speed of response, but it introduces overshoot also. Likewise, for the processes described in Sects. 5.5 and 5.6, SIMC depicts least error performance due to oscillatory behavior around set point. Lastly, for plant of Sect. 5.7, the response is optimal for the proposed method, but the controller with lag term produces unstable output.

6 Application to Nonlinear Systems

Nonlinear systems have much richer and complex behavior than linear systems [34–36]. We extended our proposed work to the class of nonlinear systems so-called separable systems, which comprise a linear part defined by its transfer function, and a nonlinear part defined by a time-independent relationship Γ between its input u and output u_Γ (see Fig. 14). In this paper, we assume that the nonlinear part consists of discontinuous nonlinearity, commonly known as hard nonlinearity (such as saturation, backlash, and coulomb friction). For such systems, the proposed scheme involves only the linear part

Fig. 12 **a** Frequency, **b** output for unit step input and **c** disturbance responses of fourth-order having zero on LHP

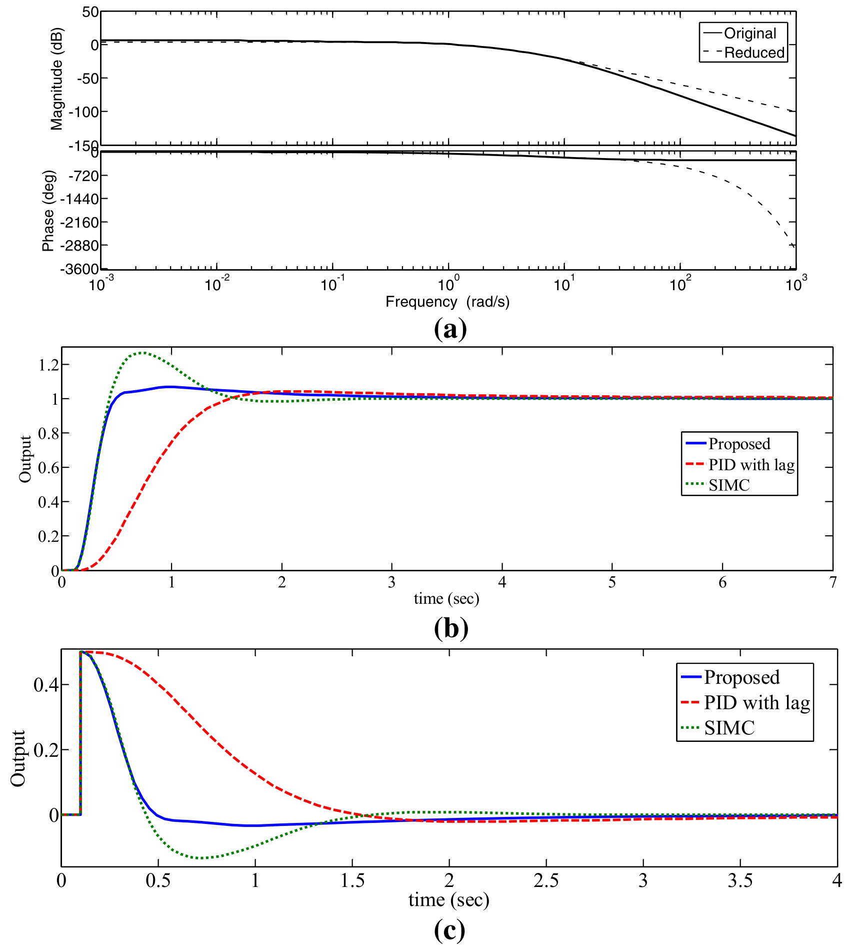


Fig. 13 **a** Output for unit step input and **b** disturbance responses of second-order oscillatory system

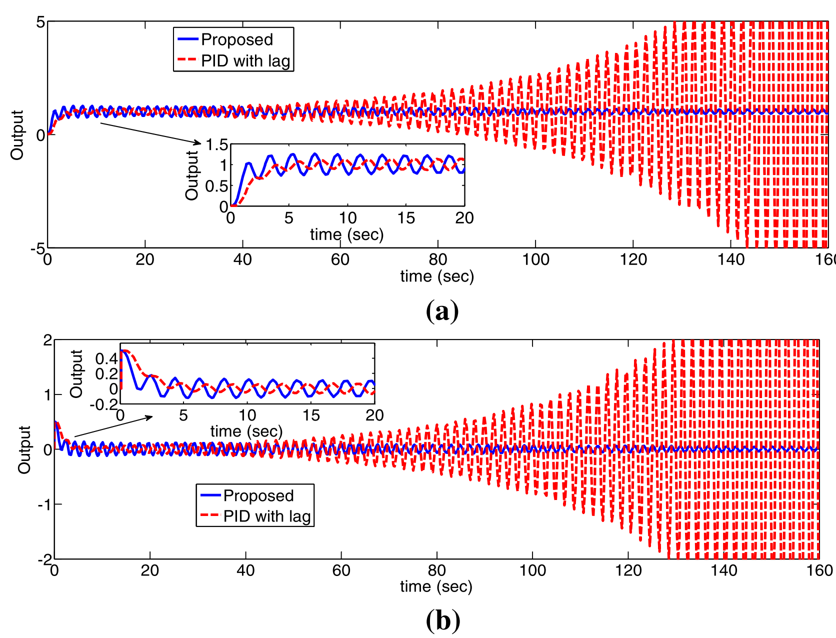


Table 4 Performance comparison for different processes

Process	Method	Set-point change				Output disturbance			
		IE	ISE	IAE	ITAE	IE	ISE	IAE	ITAE
Second-order over damped system	Proposed	0.01	0.0638	0.172	0.07121	0.005	0.01595	0.0858	0.03561
	PID with lag	1.986	1.019	1.986	3.987	0.9931	0.2547	0.9931	1.993
Second-order under damped system having delay and RHP zero	Proposed	4	3.311	4.195	11.02	2	0.828	2.096	5.497
	PID with lag	7.959	7.999	5.361	46.86	4	1.34	4	23.42
	Wang et al.	2.802	2.883	3.752	10.17	1.401	0.7211	1.875	5.064
Second-order system depicting pole–zero cancelation	Proposed	0.02	0.01	0.02	0.0024	1.972	0.505	1.972	9.366
	PID with lag	3.942	2.002	3.942	15.16	0.01	0.0025	0.01	0.0102
Fourth-order over damped system	Proposed	1	0.6033	1.004	0.9032	0.5	0.1508	0.502	0.4516
	PID with lag	2	1.337	2	2.956	1	0.3343	1	1.478
	SIMC	0.01251	0.1855	0.421	0.2928	0.006257	0.04637	0.2105	0.1464
Fourth-order system having zero on LHP	Proposed	4	3.311	4.195	11.02	2	0.828	2.096	5.497
	PID with lag	7.99	5.361	7.999	46.86	4	1.34	4	23.42
	SIMC	2.802	2.883	3.752	10.17	1.404	0.7211	1.875	5.064
Second-order undamped system	Proposed	1.013	2.933	17.19	1087	0.5067	0.734	8.607	545.2
	PID with lag	3.956	809	211.1	26730	1.974	202.5	105.6	13370

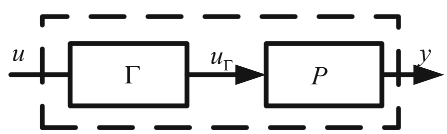


Fig. 14 Block diagram of a nonlinear system with separable nonlinearity

to obtain PID controller. To demonstrate the effectiveness of the proposed method, we present the following examples.

6.1 Second-Order Integrating Type System Having Saturation Nonlinearity

Consider a second-order integrating type system

$$P(s) = \frac{4}{s(0.5s+1)} \tag{41}$$

having input saturation nonlinearities defined as

$$\Gamma : \begin{cases} u_\Gamma = u, & |u| \le \delta \\ u_\Gamma = \delta sgn(u), & |u| > \delta \end{cases}$$

where $\delta = 0.05$. To design PID controller, only the linear part is selected and using the proposed scheme, we get controller as

$$C(s) = \frac{1}{\lambda^2}\left(\frac{1}{4} + 0.125s\right) \tag{42}$$

where $\lambda = 0.4$. The responses of system with proposed controller and PID with lag term with reference to a unit step input and disturbance are shown in Fig. 15. The results show the sluggishness in performance of the conventional approach of PID with lag term in comparison with the proposed one.

6.2 Second-Order System Having Backlash Nonlinearity

We simulate the forced-actuated mass-damper-spring system given by

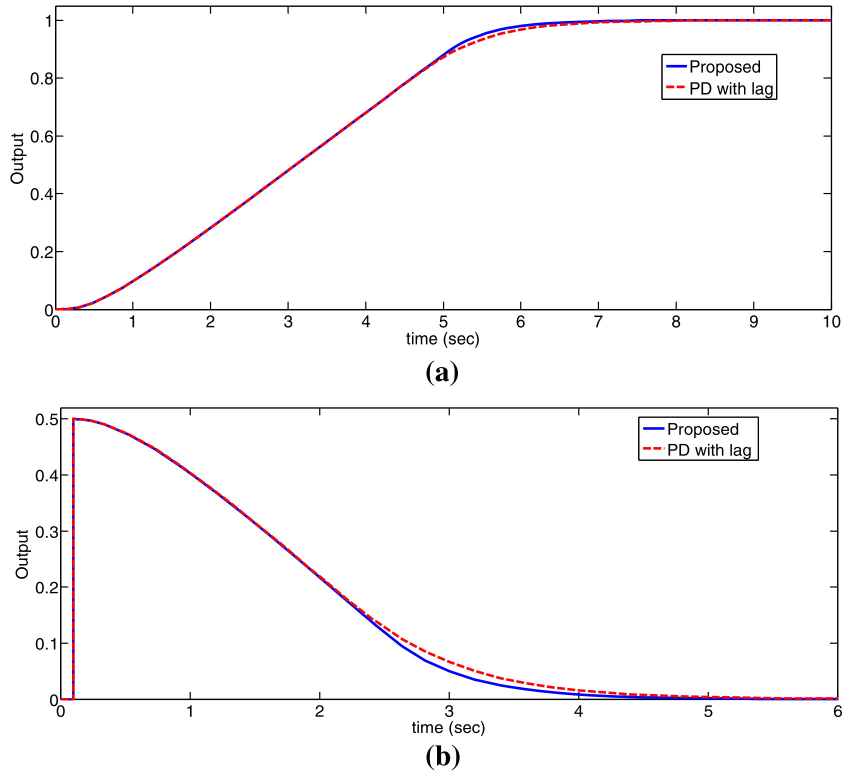
$$P(s) = \frac{1}{ms^2 + bs + k} \tag{43}$$

Here, we set $m = 1$, $k = 1$ and $b = 2$ in (43) which makes the plant model to have double pole at -1. The friction force is modeled by the backlash model as shown in Fig. 16, where dead zone is $\delta = 1.5$. As suggested, the proposed IMC-PID method utilizes only the linear part to design PID controller which yields

$$C(s) = \frac{1}{\lambda^2}\left(2 + \frac{1}{s} + s\right) \tag{44}$$

where $\lambda = 0.58$. Figure 17a shows the response for a unit step input which depicts that the proposed scheme produces quick response with lesser overshoot than that of PID with

Fig. 15 **a** Output for unit step input and **b** disturbance responses of second-order system having saturation-type nonlinearity



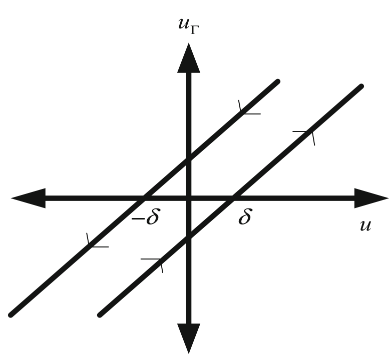


Fig. 16 Input–output plot for backlash nonlinearity

lag term. Similarly, for step input disturbance of magnitude 0.5, the disturbance rejection response is sluggish for system having PID with lag term (See Fig. 17b).

The optimality of the proposed scheme for aforementioned second-order systems with nonlinear actuators is also illustrated in Table 5. The performance is minimum for all error indices when compared to that of the system having PID with lag term.

7 Discussions

Here, we make few observations regarding the numerical studies conducted in the earlier section. In IMC design scheme, the only tuning parameter in the controller is the filter parameter λ. A larger λ provides a slower response and is less sensitive to model mismatches, while smaller λ provides the faster closed-loop response, but the controller action may be aggressive and produce tighter response. Thus, a trade-off is required for handling robustness/performance and servo/regulator. Morari and co-workers showed that the closed-loop transfer function $T(s)$ from r to y in Fig. 1a is given by $T(s) = P_M^+(s)F(s)$ where NMP part $P_M^+(s)$ contains all time delays and RHP zeros of $P_M(s)$. Therefore, we can say that the closed-loop response is directly affected by $F(s)$, i.e., for faster response, λ should be small. Researchers present different argument regarding selection criterion for λ, but there is no unique justification for evaluation of λ. For example, Wang et al. [18] illustrated the dependency of λ on the open-loop process bandwidth. Isaksson and Grabe [17] and Shamsuzzoha [37] formulated tuning rule on the basis of maximum sensitivity function. Vilanova [26] proposes the limiting value of λ based on frequency-dependent weight that defines the modeling error. Liu and Gao [25] explored the admissible tuning range based on upper bound of multiplicative uncertainty, and likewise. Basically, tuning of λ actually depends on how a PID form is evolved from the IMC design.

Moreover, we have already discussed in Sect. 3 that λ should be as small as possible for the proposed scheme. However, it is, in fact, very easy to observe the influence of λ on stability and performance as we set λ equal to maximum time constant and then gradually reduce its value. Therefore,

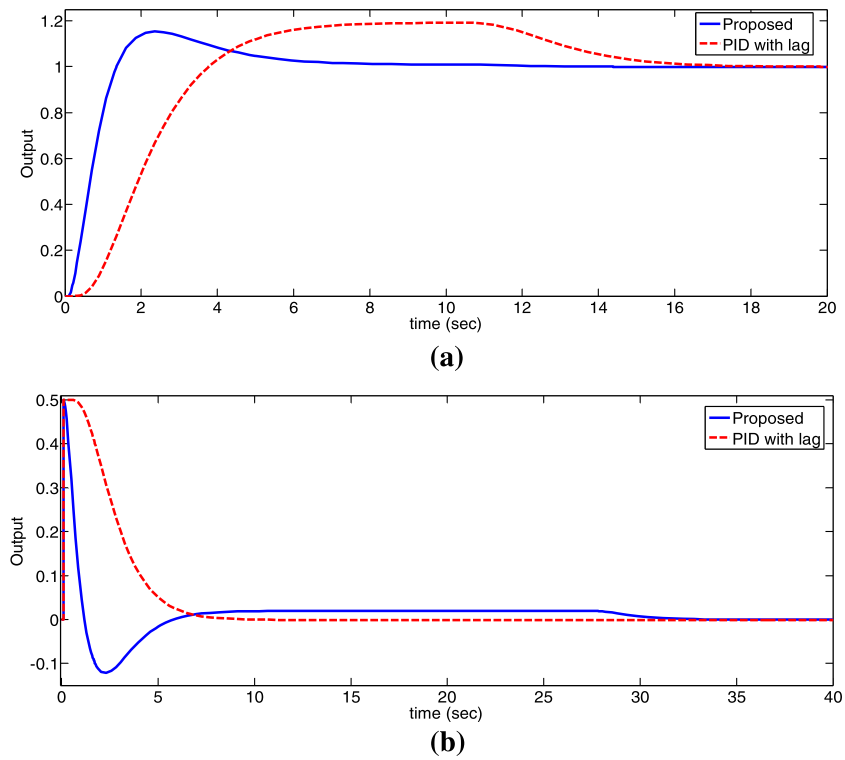


Fig. 17 **a** Output for unit step input and **b** disturbance responses of second-order system having backlash-type nonlinearity

Table 5 Performance comparison for processes having nonlinearity

Second-order Process	Method	Set-point change				Output disturbance			
		IE	ISE	IAE	ITAE	IE	ISE	IAE	ITAE
With saturation-type nonlinearity	Proposed	3.007	2.122	3.007	6.009	0.8829	0.3137	0.8829	1.074
	PID with lag	3.003	2.124	3.003	6.165	0.9123	0.3179	0.9123	1.172
With backlash-type nonlinearity	Proposed	0.1018	0.4459	1.091	2.623	0.1261	0.9976	9.103	0.4166
	PID with lag	0.2952	1.666	3.5	17.28	0.5208	1.47	3.464	1.416

after extensive simulation studies, we found that λ should be less than the maximum time constant of the plant. This gives a reasonably fast response with moderate input usage and good robustness margins. Hence, manual tuning is a viable option, and in many cases it is the preferred choice [38–41].

There are some other research gaps and issues which may create interest among control researchers in days to come. The observations which we marked out for future work in this area are mentioned below.

Remark 3 We observed that when the proposed scheme is applied to the processes like, (1) second-order system having delay and RHP zeros, (2) reduced second-order models of fourth-order systems as illustrated in Sects. 5.3, 5.5 and 5.6, respectively, then the proposed approach provides smooth response for set-point tracking and disturbance rejection, but the speed of response is slower than that obtained from Wang et al. and SIMC design.

Remark 4 The proposed IMC-based PID deals with some interesting examples like systems having attributes of pole–zero cancelation, oscillatory nature and discontinuous-type nonlinearities. To solve such examples, complex algorithms are required and probably simple IMC- based schemes are not yet explored. However, the examples considered in this paper are not covering all aspects of different dynamics of second-order processes like unstable systems, double integrators systems, and systems having highly nonlinear dynamics. Also, the concrete formulation of λ tuning law is yet to explore.

8 Conclusion

This paper establishes a simple analytical technique to derive the tuning parameters of PID controller based on IMC scheme. The proposed approach is verified for different types of second-order system and approximated second-order models of higher-order systems. The results are verified through simulations which demonstrate the superiority of the proposed method for set-point change, disturbance rejection attributes, and robustness due to modeling inaccuracies as compared to other existing popular methods. The results show that for all types of system, response is smooth with minimum overshoot in comparison with other methods; and also in most of the examples, the performance indices are better except in few cases, where existing methodology is better. Here, it is not claimed that the proposed method is the best among the other existing methods for all class of problems, but it is observed that for the most of the class of systems, the proposed approach produces improved performance, and more interestingly, this approach is based on conventional concept of internal model control.

9 Appendix

PID tuning using Rivera et al. [13]

To obtain PID parameters, we first convert (23) in the form

$$P(s) = \frac{k}{(\tau_1 s + 1)(\tau_2 s + 1)}, \tau_1 > \tau_2 \tag{A1}$$

And then using formula given in Table I (B) in [13], we get

$$K_P = \frac{\tau_1 + \tau_2}{k\lambda}, T_I = \tau_1 + \tau_2, T_D = \frac{\tau_1 \tau_2}{\tau_1 + \tau_2} \tag{A2}$$

PID tuning using Lee [16]

In this design scheme, we rearrange the plant format described by (23) into

$$P(s) = \frac{k}{(\tau^2 s^2 + 2\varepsilon\tau s + 1)} \tag{A3}$$

To obtain the PID settings, we substitute the plant parameters of (A3) in (24) of [16] which give

$$K_P = \frac{T}{2k\lambda}, T_I = 2\varepsilon\tau - \frac{\lambda}{2}, T_D = T_I - 2\varepsilon\tau + \frac{\tau^2}{\lambda T_I} \tag{A4}$$

PID tuning using Skogestad [19]

In this method, we first convert the plant expressed in (23) in to the form as mentioned in (A1). Next, approximate it into first-order time delayed system as $P_M(s) = \frac{ke^{-\theta s}}{\tilde{\tau}s+1}$ where $\theta = \frac{\tau_2}{2}, \tilde{\tau} = \tau_1 + \frac{\tau_2}{2}$. Now using SIMC PI setting given in [19], we get

$$K_P = \frac{\tilde{\tau}}{k(\lambda + \theta)}, T_I = \min\{\tau_1, 4(\lambda + \theta)\} \tag{A5}$$

PID tuning using Honeywell [33]

In this method, we have consider the structure of PID of the form $C(s) = K_P\left(1 + \frac{1}{T_I s}\right)\left(\frac{T_D s+1}{\alpha T_D s+1}\right)$. To evaluate the tuning, we consider the method expressed in [33] which gives

$$K_P = \frac{3}{k}, T_I = \tau_1 + \tau_2, T_D = \frac{\tau_1 \tau_2}{\tau_1 + \tau_2}, \alpha = 0.1 \tag{A6}$$

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