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RESEARCH ARTICLE - MECHANICAL ENGINEERING

Effects of the Difference Between the Static and the Kinetic Friction Coefficients on a Drill String Vibration Linear Approach

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Abstract Stick–slip phenomenon in drill string is a self-excited vibration that is detrimental to the drilling equipment as well as to the drilling efficiency. Although there are a number of publications on this subject, there is yet not a generally accepted interpretation of its causes. In this paper, an analytical model that differs from the classical block-on-belt one is presented. The equation of motion of the drill bit in the slip phase is obtained and its solution determined for a set of parameters usually found in actual practice, which includes a drill string length of 3000 m. By choosing different sets of friction coefficients, the influence of the difference between the static and the kinetic ones on the occurrence of stick–slip vibration and the performance of the drilling equipment is investigated.

Keywords Stick–slip vibration · Drill string · Friction coefficient · Torque · Limit cycle

1 Introduction

Wells of a depth up to 10 km are drilled onshore and offshore for the exploration of crude oil and natural gas fields [1], thereby submitting drilling equipments to conditions so severe that their components can easily fail. Besides, friction-induced vibrations occur in many engineering systems, such as grating brakes and chattering machine tools [2,3]. This type of vibration is particularly undesirable due to its detrimental effect on the performance of engineering systems, such as deep well rigs. The main components of these equipments are the rotary table, the drill string, and the drill bit.

Due to the large length–diameter ratio of the drill string–drill bit set, and the friction generated during the drilling process, the occurrence of stick–slip vibration is common, especially in large depths [4]. Due to this vibration of the drill string, not only drilling efficiency is reduced but also premature failure of the drilling equipment may be expected. In addition, stick–slip vibration adversely affects borehole quality and increases drilling costs [5].

With the development of the oil and gas industry, the field is currently moving toward deep drilling systems [6]. Since the stick–slip vibration is more likely to occur in deep wells, controlling this type of vibration is an important task. Understanding the causes of the stick–slip vibration and dynamic responses of the drill bit during the stick–slip vibration, therefore, has been the most basic work.

The study of stick–slip vibration of the drilling system can be traced back to the work of Belokobyl'skii and Prokopov in 1982 [7]. Then, Dawson et al. [8], Kyllingstad and Halsey [9], and Lin and Wang [10] studied this vibration based on a torsional pendulum model. They illustrated the stick–slip phenomenon in the drill string, but the causes for this phenomenon have not been presented. Van de Vrande [11] investigated the friction-induced stick–slip vibration to find its periodic characteristics by using block-on-belt models. In this model, the frictional torque of the drill bit is modeled as a frictional force. This model, however, fails to practically describe the real system.

Mihajlovic et al. [12,13] experimentally studied the stick–slip vibration to get improved understanding of the causes of torsional vibrations. In these works, the drill bit motion with and without stick–slip vibrations is observed. Khulief et al. [14,15] and Leine et al. [16] studied the stick–slip vibration

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by using the finite element method. Nevertheless, the effect of friction has not been discussed in these publications. Patil and Teodoriu [17] parametrically investigated the stick–slip influencing factors.

A plausible explanation for the stick–slip vibration of a drill string can be tentatively set forth by assuming that the drill string works in similar way as does a mass–damper–helical compression spring system, in which the wire is actually submitted to torsion, in the same way as the drill string is. Hence, if the axial load applied to the former system remains constant, the same occurs to the torque applied to the wire. Further, if the mass–damper–spring system receives a sudden vertical blow, or the drill bit hits a hard point in the rock formation, the flexibility and the damping of these systems do not allow them to fully transmit the applied action, thus reducing the displacement imposed to the suspended mass in one case, and possibly causing the stop of the drill bit and starting a stick phase, in the other. In this latter case, however, a difference exists, as the deformation energy stored in the drill string increases due to the continuous action or the rotary table. As a result, after a while, a large enough restitutive force is built up to set off the slip phase of the drill bit and a stable vibration takes place in drill string until the restitutive force is not enough to overcome the friction force any longer, either due to the dissipation of energy that occurs in the sliding phase of the drill bit or due to its hit to another hard point in the rock, thus causing its immediate return to the stick phase [18].

Although the stick–slip phenomenon has been studied for a long time, most of the publications have been approaches for suppressing this vibration, including active and passive methods [19–24]. The mechanism of friction-induced stick–slip vibration in the drill string has been scantily studied. In addition, there is no common interpretation of the mechanism of stick–slip vibration.

Presently, the stick–slip vibration is regarded as being of a nonlinear nature [14,25,26], thus not only coupling with other modes of vibration of the drilling equipment occurs but also there may be sudden 'jumps' in the amplitude of vibration as the driving frequency increases, or decreases, respectively [27], which makes such an analysis particularly difficult.

In this paper, a linear approach to the problem is proposed as preliminary form of examination of the existing relationship between the static and the kinetic friction coefficients and the stick–slip phenomenon in a drilling equipment. By studying the difference between the two friction coefficients in this way, the conditions for the formation of stick–slip vibration and dynamic responses of the drill bit will be presented.

2 Analytical Model

2.1 Model Description

The dynamic model of the present study is shown in Fig. 1, which is a system with Coulomb friction. The system presented in Fig. 1 is equivalent to the real drilling system in which the drill bit is rotating in the rock formation and the system is driven by the rotary table. The model consists of the following parts: an object with moment of inertia J, which represents the drilling equipment, a linear spring with stiffness K_D, and a viscous damper with damping coefficient c. The object is driven by the rotary table with a clockwise velocity φ, and the bit contacts the rock formation with a normal load (weight on the drill bit) W_B. The object is forced by frictional torque T_f, which is determined as a function of the drill bit velocity [17,26]. Since the rotary table rotates at constant speed, power is transmitted to the drill bit by the drill string.

2.2 Equivalent Parameters

In this study, both the drill pipes and drill collars are assumed to be continuous shafts with constant cross section and density ρ, which allow to express the equivalent moment of inertia as:

$$J = \frac{1}{3} J_\mathrm{r} = J_\mathrm{P} + J_\mathrm{C} \qquad (1)$$

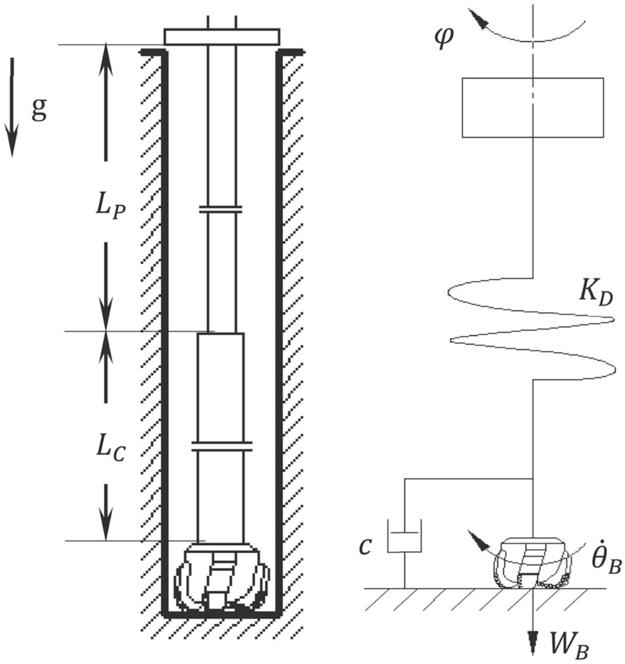

Fig. 1 Analytical model of the drilling system

with

$$J_{\mathrm{P}} = \frac{\pi\rho}{96} L_{\mathrm{P}} \left(D_{\mathrm{P}}^4 - d_{\mathrm{P}}^4\right) \tag{2}$$

and

$$J_{\mathrm{C}} = \frac{\pi\rho}{96} L_{\mathrm{C}} \left(D_{\mathrm{C}}^4 - d_{\mathrm{C}}^4\right) \tag{3}$$

where J_{r} represents the actual moment of inertia of the drill string, J_{P} and J_{C} represent the moment of inertia of the drill pipes and drill collars, D_{P} and D_{C} are the external diameters of the drill pipes and drill collars, d_{P} and d_{C} are their internal diameters, while L_{P} and L_{C} represent the lengths of the drill pipes and drill collars, respectively. Accordingly, the equivalent drill string stiffness can be given as:

$$K_{\mathrm{D}} = \frac{K_{\mathrm{P}} K_{\mathrm{C}}}{K_{\mathrm{P}} + K_{\mathrm{C}}} \tag{4}$$

with

$$K_{\mathrm{P}} = \frac{\pi G}{32 L_{\mathrm{P}}} \left(D_{\mathrm{P}}^4 - d_{\mathrm{P}}^4\right) \tag{5}$$

and

$$K_{\mathrm{C}} = \frac{\pi G}{32 L_{\mathrm{C}}} \left(D_{\mathrm{C}}^4 - d_{\mathrm{C}}^4\right) \tag{6}$$

where K_{P} and K_{C} represent the equivalent stiffness of the drill pipes and drill collars, respectively, while G is the shear modulus of the drill string material.

The contact points on the drill bit have different arms (distance from a contact point to the bit center), thus requiring an integral method to their proper evaluation. In the present work, this can be expressed as:

$$\bar{R}_{\mathrm{B}} = \int_0^{R_{\mathrm{B}}} \frac{2\pi r}{\pi R_{\mathrm{B}}^2} \cdot r \cdot \mathrm{d}r = \frac{2}{3} R_{\mathrm{B}} \tag{7}$$

where r denotes the moment arm for a certain point and R_{B} represents the actual diameter of the drill bit. In recent studies, the frictional torque on the drill bit was considered to be proportional to the R_{B} [28] or even to the diameter of the drill bit [17]. By comparing these frictional torque models to the one in this paper, it is believed that the moment arm represented by Eq. (7) improves the results.

2.3 States of the Stick–Slip Bit

Stick phase ($\dot{\theta}_{\mathrm{B}} = 0$):

$$T_{\mathrm{f}} = T_{\mathrm{S}} \tag{8}$$

when

$$-\mu_{\mathrm{S}} W_{\mathrm{B}} \bar{R}_{\mathrm{B}} \le T_{\mathrm{S}} \le \mu_{\mathrm{S}} W_{\mathrm{B}} \bar{R}_{\mathrm{B}} \tag{9}$$

Slip phase ($\dot{\theta}_{\mathrm{B}} \neq 0$):

$$T_{\mathrm{f}} = T_{\mathrm{K}} \tag{10}$$

when

$$T_{\mathrm{K}} = \mu_{\mathrm{K}} W_{\mathrm{B}} \bar{R}_{\mathrm{B}} \tag{11}$$

where $\dot{\theta}_{\mathrm{B}}$ denotes the drill bit velocity, T_{S} and T_{K} represent the frictional torque on the drill bit in the stick phase and the slip phase, W_{B} is the friction force, and μ_{S} and μ_{K} denote the static friction coefficient and kinetic friction coefficient, ($\mu_{\mathrm{S}} > \mu_{\mathrm{K}}$), respectively.

2.4 Equation of Motion of the Drill Bit

For convenience of analysis, the drill bit is assumed to be in a critical state of stick phase transiting to slip phase. The differential equation of motion of the drill bit in the slip phase can be expressed by Newton's law of motion as:

$$J\ddot{\theta}_{\mathrm{B}} + c\dot{\theta}_{\mathrm{B}} + K_{\mathrm{D}} (\theta_{\mathrm{B}} - \varphi t) + \mu_{\mathrm{K}} W_{\mathrm{B}} \bar{R}_{\mathrm{B}} sgn\left(\dot{\theta}_{\mathrm{B}}\right) = 0 \tag{12}$$

where θ_{B} is the angular displacement of the drill bit in the clockwise direction, $(\dot{\cdot})$ denotes the derivative with respect to time, and $sgn(\cdot)$ is the sign function.

For a drilling equipment operating at a constant speed of rotation, below the resonance frequency, in the clockwise direction, the drill bit complies with the applied torque, also rotating in the same direction after starting from sliding in the slip phase. In such conditions, the differential equation of motion of the drill bit can be written as:

$$J\ddot{\theta}_{\mathrm{B}} + c\dot{\theta}_{\mathrm{B}} + K_{\mathrm{D}} (\theta_{\mathrm{B}} - \varphi t) + \mu_{\mathrm{K}} W_{\mathrm{B}} \bar{R}_{\mathrm{B}} = 0 \tag{13}$$

Equation (13) is a second-order inhomogeneous differential equation whose solution comprises a solution of the associated homogeneous equation and a particular one. The solution of this equation can then be written as:

$$\theta_{\mathrm{B}} = A e^{-\xi\omega_{\mathrm{n}} t} \sin\left(\sqrt{1-\xi^2}\,\omega_{\mathrm{n}} t + \psi\right) + \varphi t - \frac{\mu_{\mathrm{K}} W_{\mathrm{B}} \bar{R}_{\mathrm{B}}}{K_{\mathrm{D}}} \tag{14}$$

where

$$\omega_{\mathrm{n}} = \sqrt{\frac{K_{\mathrm{D}}}{J}} \tag{15}$$

is the natural frequency of the system;

$$\xi = \frac{c}{2J} \Big/ \sqrt{\frac{K_D}{J}} \tag{16}$$

is the damping ratio; A and ψ are the amplitude and the initial phase angle, which are both determined by the initial conditions of the drill bit, and expressed, respectively, as:

$$A = \sqrt{\left(\theta_{B0} + \frac{\mu_K W_B \bar{R}_B}{K_D}\right)^2 + \left[\frac{\varphi - \dot{\theta}_{B0} + \xi\omega_n\left(\theta_{B0} + \frac{\mu_K W_B \bar{R}_B}{K_D}\right)}{\sqrt{1-\xi^2}\omega_n}\right]^2} \tag{17}$$

and

$$\psi = \arctan \frac{\sqrt{1-\xi^2}\omega_n\left(\theta_{B0} + \frac{\mu_K W_B \bar{R}_B}{K_D}\right)}{\varphi - \dot{\theta}_{B0} + \xi\omega_n\left(\theta_{B0} + \frac{\mu_K W_B \bar{R}_B}{K_D}\right)} \tag{18}$$

where θ_{B0} is the initial angular displacement and $\dot{\theta}_{B0}$ is the initial angular velocity in the slip phase.

3 Analysis of the Stick–Slip Vibration

3.1 Drilling System Investigated

To determine the dynamic response of the drill bit, a drill string 3000 m long is studied. In this example, $L_P = 2800$ m; $L_C = 200$ m; $D_P = 127$ mm; $d_P = 108.6$ mm; $D_C = 165.1$ mm; $d_C = 57.2$ mm; $R_B = 108$ mm; $W_B = 160$ kN; $\varphi = 100$ rpm; $\rho = 7850\,\text{kg/m}^3$; $G = 80$ GPa; $\xi = 0.1$. The system parameters are $K_D = 335.6\,\text{N}\cdot\text{m/rad}$, $J = 124.7\,\text{kg}\cdot\text{m}^2$, and $\omega_n = 1.64$ rad/s. These parameters were chosen to simulate actual drilling systems [28]. Without loss of generality, the kinetic friction is fixed at $\mu_K = 0.5$, while distinct values of μ_S are assumed ($\mu_S = 0.85, 0.8, 0.75, 0.7$, and 0.65).

3.2 Dynamic Responses of the Drill Bit

Since different combinations of the two friction coefficients, herein denominated cases, are selected, responses of the drill bit slip phase change. The different cases of Eq. (12) depend on whether the drill bit rotates clockwise or anticlockwise, the latter of which does not occur. The drill bit keeps rotating clockwise during the slip process. Also, after stick phase is broken, the drill bit gets an abrupt acceleration and rotates forward. The frictional torque and viscous force, however, block the motion of the bit. The bit is accelerated and then decelerated gradually and finally gets stuck at a place where the drive torque from the drill string cannot overcome the frictional torque.

The relative motion between the rotary table and the drill bit while the latter is driven by the rotary table is investigated. The relative responses ($\theta_{Br} = \theta_B - \varphi t$ and $\dot{\theta}_{Br} = \dot{\theta}_B - \varphi$) between the drill bit and the rotary table are obtained for different static friction coefficient cases.

Case 1: $\mu_S = 0.85$

$$\theta_{Br} = -14.21e^{-0.164t}\sin(1.63t + 1) - 17.16 \tag{19}$$

$$\dot{\theta}_{Br} = -23.3e^{-0.164t}\,[0.995\cos(1.63t + 1) - 0.1\sin(1.632t + 1)] \tag{20}$$

Case 2: $\mu_S = 0.8$

$$\theta_{Br} = -12.73e^{-0.164t}\sin(1.63t + 0.94) - 17.16 \tag{21}$$

$$\dot{\theta}_{Br} = -20.87e^{-0.164t}\,[0.995\cos(1.63t + 0.94) - 0.1\sin(1.63t + 0.94)] \tag{22}$$

Case 3: $\mu_S = 0.75$

$$\theta_{Br} = -11.23e^{-0.164t}\sin(1.63t + 0.86) - 17.16 \tag{23}$$

$$\dot{\theta}_{Br} = -18.42e^{-0.164t}\,[0.995\cos(1.63t + 0.86) - 0.1\sin(1.63t + 0.86)] \tag{24}$$

Case 4: $\mu_S = 0.7$

$$\theta_{Br} = -9.87e^{-0.164t}\sin(1.63t + 0.76) - 17.16 \tag{25}$$

$$\dot{\theta}_{Br} = -16.19e^{-0.164t}\,[0.995\cos(1.63t + 0.76) - 0.1\sin(1.63t + 0.76)] \tag{26}$$

Case 5: $\mu_S = 0.65$

$$\theta_{Br} = -8.62e^{-0.164t}\sin(1.63t + 0.63) - 17.16 \tag{27}$$

$$\dot{\theta}_{Br} = -14.14e^{-0.164t}\,[0.995\cos(1.63t + 0.63) - 0.1\sin(1.63t + 0.63)] \tag{28}$$

For these five cases, the period characteristics are shown in Table 1. As can be seen from this table, stick–slip vibrations occur for only three out of the five cases analyzed. For those

Table 1 Period characteristics of the drill bit

	Case 1	Case 2	Case 3	Case 4	Case 5
Time interval of the slip phase (s)	2.69	2.81	3.04	–	–
Time interval of the stick phase (s)	1.82	1.51	1.1	–	–
Period of the stick–slip motion (s)	4.51	4.32	4.14	–	–

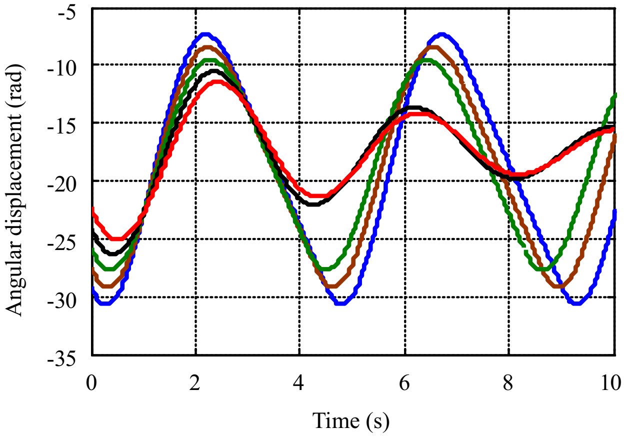

Fig. 2 Angular displacement of the drill bit relative to the rotary table: *blue*, $\mu_S = 0.85$; *brown*, $\mu_S = 0.8$; *green*, $\mu_S = 0.75$; *black*, $\mu_S = 0.7$; *red*, $\mu_S = 0.65$

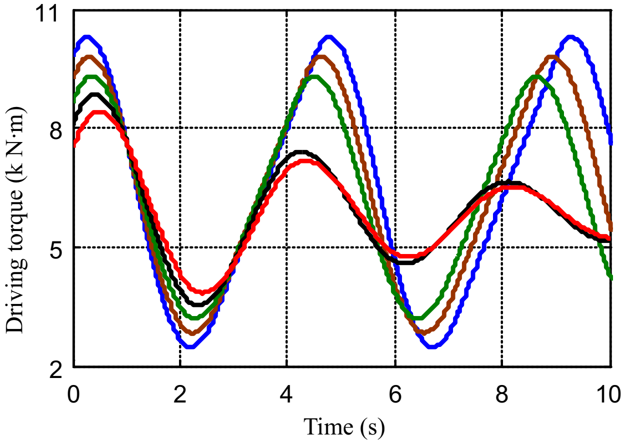

Fig. 3 Driving torque from the drill string to the drill bit: *blue*, $\mu_S = 0.85$; *brown*, $\mu_S = 0.8$; *green*, $\mu_S = 0.75$; *black*, $\mu_S = 0.7$; *red*, $\mu_S = 0.65$

in which stick–slip vibrations occur, a decrease in the static friction coefficient leads to an increase in the period of the slip phase as well as a decrease in the period of the stick phase.

The law for different drill bit responses will be discussed referring to Figs. 2, 3, 4, 5, and 6. The time history of the angular displacement of the drill bit relative to the rotary table for different static friction coefficients is presented in Fig. 2. As can be observed in this figure, stick–slip vibrations of the drill bit occur only for the static friction coefficient cases where $\mu_S = 0.85$, 0.8, and 0.75. During the slip phase, the relative angular displacement increases in the initial small period of time, then it decreases and increases again in the final period of time. In this figure, the minus sign on the relative angular displacement means that the drill bit lags behind the rotary table.

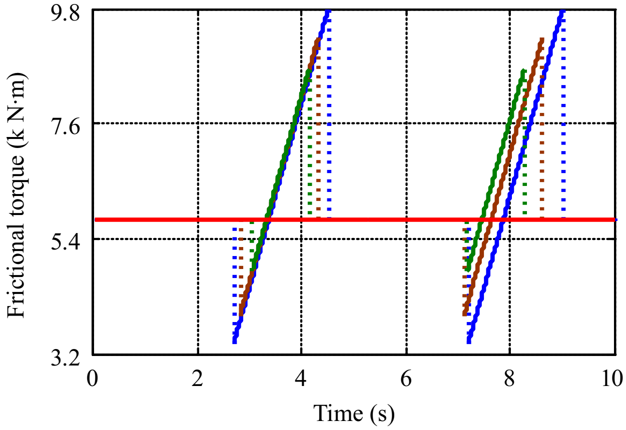

Fig. 4 Frictional torque on the drill bit: *blue*, $\mu_S = 0.85$; *brown*, $\mu_S = 0.8$; *green*, $\mu_S = 0.75$; *black*, $\mu_S = 0.7$; *red*, $\mu_S = 0.65$

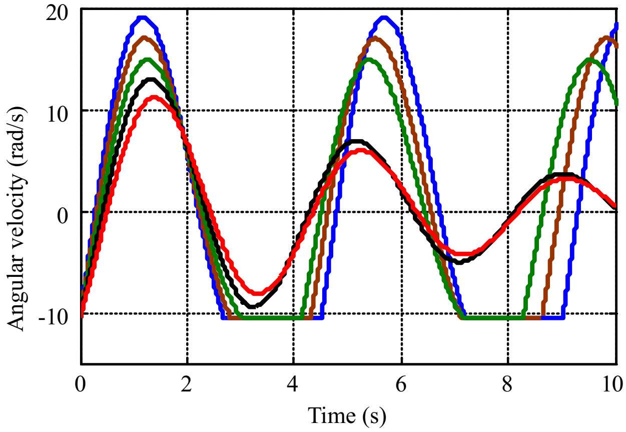

Fig. 5 Relative angular velocity of the drill bit: *blue*, $\mu_S = 0.85$; *brown*, $\mu_S = 0.8$; *green*, $\mu_S = 0.75$; *black*, $\mu_S = 0.7$; *red*, $\mu_S = 0.65$

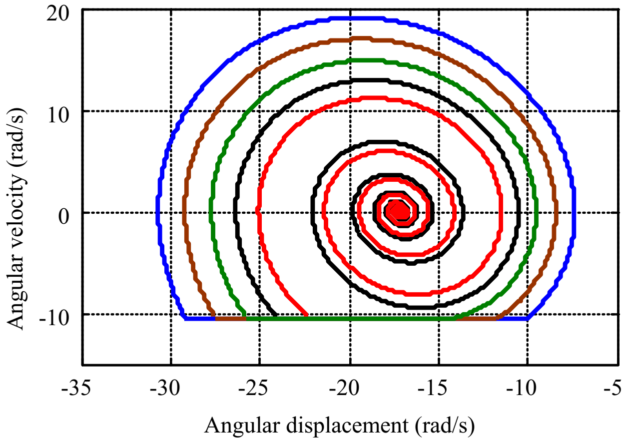

Fig. 6 Phase trajectory of the relative motion of the drill bit: *blue*, $\mu_S = 0.85$; *brown*, $\mu_S = 0.8$; *green*, $\mu_S = 0.75$; *black*, $\mu_S = 0.7$; *red*, $\mu_S = 0.65$

In the stick phase, the drill bit keeps static and the relative angular displacement increases uniformly due to constant rotation of the rotary table. Once the relative angular displacement reaches the value for which the torque on the drill string overcomes the frictional torque, the stick phase ends and the slip phase starts. With the decrease in the static friction coefficient, the fluctuation of angular displacement decreases. For the cases where stick–slip vibration does not occur, the relative angular displacement decreases and tends to stabilize at a constant value.

The time history of the driving torque transferred from the drill string to the drill bit for different static friction coefficient cases is presented in Fig. 3. The driving torque is the product of the relative angular displacement and the drill string stiffness. For a given drill string, stiffness is constant. As a result, the shapes of the driving torque curves are reversals of the relative angular displacement curves. With an increase in the static friction coefficient, the torque fluctuation decreases. The drill bit keeps still during the stick phase, when the driving torque balances the frictional one.

The time history of the frictional torque on the drill bit for different static friction coefficient cases is shown in Fig. 4. In this figure, the period characteristics for both stick and slip phases can be clearly observed. Because the kinetic friction coefficient is the same for different friction groups, the kinetic frictional torque remains unchanged. For $\mu_S = 0.85, 0.8$, and 0.75, the frictional torque presents a segmented characteristic. During the stick phase, the frictional torques increase uniformly and leap at the critical states. With the decreases in the static friction coefficient, the frictional torque fluctuations decrease. For the cases where stick–slip vibrations do not occur, the frictional torque is constant. In this figure, parts of the blue lines, the brown lines, and the green ones are covered by the red line, while the black line coincides with the red one.

Similarly, the time history of the relative angular velocity of the drill bit for distinct static friction coefficients is depicted in Fig. 5. As can be observed from this figure, the relative angular velocity of the drill bit firstly increases and then decreases for $\mu_S = 0.85$, 0.8, and 0.75. Then, in the stick phase, the relative angular velocity keeps at a constant value which is determined by the rotary table velocity. With an increase in the static friction coefficient, fluctuations of the relative angular velocity decrease. For the cases where stick–slip vibrations occur, the relative angular velocities change periodically, both in the stick and in the slip states.

Finally, in Fig. 6, the phase trajectories of the relative motion between the drill bit and rotary table are shown for distinct static friction coefficients. As can be observed from this figure, after the bit starting at the initial phase point, two types of shape appear for the phase trajectories. For the cases where $\mu_S = 0.85, 0.8$, and 0.75, stable closed-phase trajectories are formed. Actually, the stick–slip vibration of the drilling system is a kind of self-excited vibration. With the increase in the difference between the two friction coefficients, the limit cycle becomes larger. For a determined drilling system in which stick–slip vibration occurs, the limit cycle is unique. In contrast to this, for the cases where $\mu_S = 0.7$ and 0.65, the phase trajectories shrink around the same focal point, which means a stable state will be formed for the drilling system.

4 Discussion

Stick–slip vibration is characterized by the alternation of the stick and the slip phases. While in the stick phase, the drill bit keeps still and the torque on the drill string mounts. In contrast to that, in the slip phase, the drill bit keeps sliding, redistributing the load and releasing some of the deformation energy stored in the drilling equipment during the stick phase, which is the bane of all deep wells drilling equipment, while the driving torque on the drilling tool fluctuates. Stick–slip vibration of the drill string is induced by the frictional torque on the drill bit, which means to say that the presence of frictional torque is a requirement for the occurrence of the stick–slip vibration. Consequently, on which conditions stick–slip vibration in oil and gas drill strings occurs is a question which invites examination. In this respect, how the difference between static and kinetic coefficients affects the drill bit motion? These questions may be tentatively answered from the results set forth in the previous section.

Assuming the limiting case where the static and the kinetic friction coefficients are equal, that is $\mu_S = \mu_K = 0.5$, as a starting point, the response of the drill bit can be easily obtained from the solution of its equation of motion, which indicates, in this case, a state of uniform motion. Likewise, as can be seen from the dynamic responses presented in Figs. 2, 3, 4, 5, and 6, the presence of a small difference between the two coefficients is still not enough to trigger stick–slip vibrations.

However, for the selected friction groups, the stick–slip vibration occurs in three out of the five cases presented. From Fig. 6, which depicts the phase trajectory of the relative motion of the drill bit, it is found that stick–slip vibration occurs only if the curve that represents the phase trajectory intersects the straight line that represents the stick state, which is to say the stick–slip vibration occurs whenever the difference between the two friction coefficients is big enough to cause this effect. Accordingly, for a drilling system with parameters presented in Sect. 3.1, the critical value of the difference between the two friction coefficients is found to be 0.236.

5 Conclusion

In this paper, an increase in the difference between a static friction coefficient and the kinetic one also means that the former is larger. Based on the dynamic responses of different friction groups, their respectively law of motion can be found. With an increase in the static friction coefficient, the motion of the drill bit changes from uniform to stick–slip motion. For the cases where stick–slip do not occur, responses of the drill bit fluctuates in the initial period of time and then remains stable, whereas, for the cases where stick–slip vibration occur, the time interval of the stick phase increases and the slip phase decreases, leading to a decrease in the period of the stick–slip vibration. The amplitudes of the angular displacements and the angular velocities increase and the limit cycle of the drill bit becomes larger with the increase in the static friction coefficient.

Since the existence of a difference between the static and kinetic friction coefficients is a fact, one might think that the occurrence of the stick–slip vibration would only depend on the drilling equipment chosen for the task. However, when looked into more detail, many factors seem to influence this phenomenon, such as drill bit type, rock formation type, and lubrication conditions, so that it may be possible to suppress the stick–slip vibration by properly matching these parameters.

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