

# Effect of Embedment Depth on Response of Machine Foundation on Saturated Sand

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**Abstract** In this paper, a dynamic analysis of strip machine foundation is carried out. The foundation of multiple thicknesses is placed at different depths above a saturated sand with different states (i.e., loose, medium and dense), and vertical harmonic excitation is applied with build up of the excess pore water pressure being considered. The dynamic analysis is performed numerically by using finite element software, PLAXIS 2D. The soil is assumed as elastic perfectly plastic material obeys Mohr–Coulomb yield criterion. A parametric study is carried out to evaluate the dependency of machine foundation on various parameters including the amplitude of the dynamic load, the frequency of the dynamic load and the embedment of foundation. It was concluded that increasing the embedment ratio causes a reduction in the dynamic response up to a certain embedment depth; when the depth of embedment increases higher than 1 m, the effect become less pronounced and as strength of the soil increases, the effect of embedment depth in reducing dynamic response will decrease also. The vertical displacements decrease obviously by 46, 37 and 40% for loose, medium and dense sand, respectively, when increasing the embedment of foundation from 0.5 to 1 m, while when the embedment of foundation increases from 1 to 1.5 m, the vertical displacements for loose, medium and dense sand decrease by 45, 38 and 3%, respectively. Finally, when the embedment of foundation increases from 1.5 to 2 m,

the decrements in vertical displacements are also recorded for loose, medium and dense sand by 42, 36 and 18%, respectively.

**Keywords** Machine foundation · Harmonic load · Finite elements · Elastic perfectly plastic · Foundation embedment · Saturated soil

## 1 Introduction

There is enough experimental evidence to prove that embedment greatly affects the vibration of footings. The embedment of foundation increases resonant frequencies and reduces resonant amplitudes [1]. Embedment also causes an increase in radiation damping [2]. The general consensus about the embedment of soil on the foundation from both theoretical and field observations is as follows [3]:

- The embedment effect increases the natural frequency of the foundation.
- It reduces overall amplitude of the foundation.

It is not difficult to conceive from the above statements that:

- Embedment effect increases the soil stiffness and
- Also has an incremental effect on the damping of the soil.

Embedment effects are often overestimated because soil stiffness (shear modulus) diminishes toward the soil surface due to diminishing confining pressure. This is particularly so for backfill lacking a stiff surface crust and whose effects are always much less pronounced than those of undisturbed soil.

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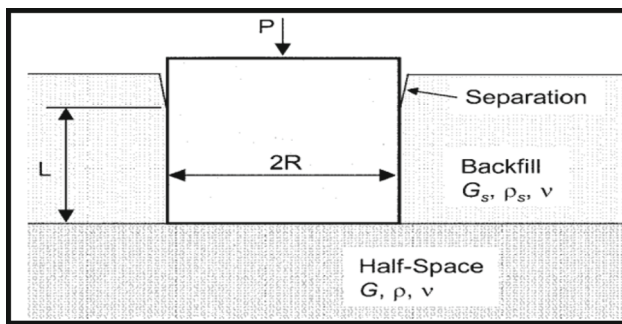


Fig. 1 Schematic of embedded foundation [4]

The lack of confining pressure at the surface often leads to the separation of the soil from the foundation and to the creation of a gap as indicated in Fig. 1 which significantly reduces the effectiveness of embedment [4]. To find an approximate correction for this effect, the engineer should consider an effective embedment depth less than the true embedment. For a given size and geometry of the foundation, and the soil properties, the stiffness and damping values for an embedded foundation are much higher than those for a surface foundation. The natural frequency of an embedded foundation will be higher, and its amplitude of vibration will be smaller compared to a foundation resting on the surface. Increasing the depth of embedment may be a very effective way in reducing the vibration amplitudes [6].

Embedment effect increases the stiffness and consequently the resonant frequency of the foundation–soil system. This property of embedment is usually included in analysis by increasing the foundation–soil impedance values [2]. Two different approaches to include the effect of embedment have been undertaken. The first method [7] assumes that the foundation rests on the surface of an elastic half-space and is embedded in an elastic layer which may have different properties from the elastic half-space. Using this approach, it is possible to include the effect of separation between foundation and the soil in an approximate fashion by reducing the shear modulus of the half-space. In this method, the added terms to the impedance functions accounting for the effect of embedment are frequency dependent. In the other method, the additional stiffness and damping terms introduced by the effect of embedment are assumed to be frequency independent. Static impedance values are merely increased by a factor which is a function of embedment depth to obtain the impedance properties of the embedded foundations. The effect of embedment is considered by using similar functions for stiffness and damping terms for all modes of vibration [2].

In practice, foundations are placed at a specified depth, below the ground surface and transmit the load to soil. Usually, increasing the depth means increasing the foundation stiffness  $K$  [5]. The other two effects, given the names

“trench” effect and “sidewall contact” effects, respectively, tend to increase the stiffness of the embedded foundation [8].

### 1.1 Trench Effect

Even in perfectly homogenous soil, a rigid footing will settle less if it is placed at the bottom of an open trench. The normal and shear stresses resulting from the overlying soil restrict the vertical movement and thus reducing the settlement of the foundation base by increasing its vertical stiffness. In addition to the analytical evidence of the trench effect presented subsequently, experimental work on a dry sand by Erden [9] and Gazetas et al. [8] showed that even with no side contact along the embedded depth, the effect of embedment to increase the vertical stiffness is not caused solely by an increase in shear modulus beneath the footing. The trench effect suggested by Gazetas et al. [8] is:

$$K_{tre}/K_{sur} = I_{tre} > 1 \quad (1)$$

where:

$K_{tre}$  is the vertical static stiffness of an embedded foundation mat with no sidewall contact,

$K_{tsur}$  is the stiffness of the same mat placed at the soil surface,

$I_{tre}$  is the dimensionless trench factor that is expected to be a function of dimensionless groups of geometric parameters and of the poisson’s ratio of the soil.

### 1.2 Sidewall Effect

Part of the applied load is transmitted to the ground through shear stresses along the vertical sides of the footing when the sides are in contact with the surrounding soil. As a result, the overall stiffness of an embedded foundation  $K_{emb}$  is larger than  $K_{tre}$  stiffness corresponding to a foundation with the same depth of embedment but without side effect [8].

$$K_{emb}/K_{tre} = I_{wall} > 1 \quad (2)$$

where  $I_{wall}$  is the dimensionless sidewall-soil contact factor, expected to be a function of the relative size of the effective sidewall–soil contact surface ( $A_s$ ).

Hushmand [2] made an investigation into the behavior of rigid foundations and structures resting on the surface or embedded in a cohesionless soil and subjected to transient active or passive excitations and forced vibrations using the centrifuge modeling technique. The investigation was aimed at studying both low and high amplitude of vibrations of foundations under machine-type loadings, earthquake- or wave-induced vibrations and other sources of dynamic loads.

Rigid model structures (aluminum towers) attached to foundations of different shapes, sizes, masses and moments of inertia were studied.

The effects of soil depth, boundary conditions and depth of foundation embedment were investigated. Mainly rocking and horizontal modes of vibration were studied. Experimental results provided information regarding the influence of different geometrical, inertial and loading conditions on the vibrational properties of the soil–structure system. In particular, the effect of foundation embedment was to increase the model resonant frequencies and to cause an appreciable change in contact pressure distribution underneath the footing. Damping ratios of the rocking-sliding vibration did not change considerably when footing size or depth of embedment changed. It is well known that increasing embedment depth of the foundation will increase the stiffness of the soil–structure system and therefore will result in an increase in natural frequency of the structure. This phenomenon was studied in a series of tests changing depth of embedment from 0 to 1.5 times the radius of the tower base. The results showed that increase in the frequency with embedment ratio is not very large. This is because of the high amplitude of the force.

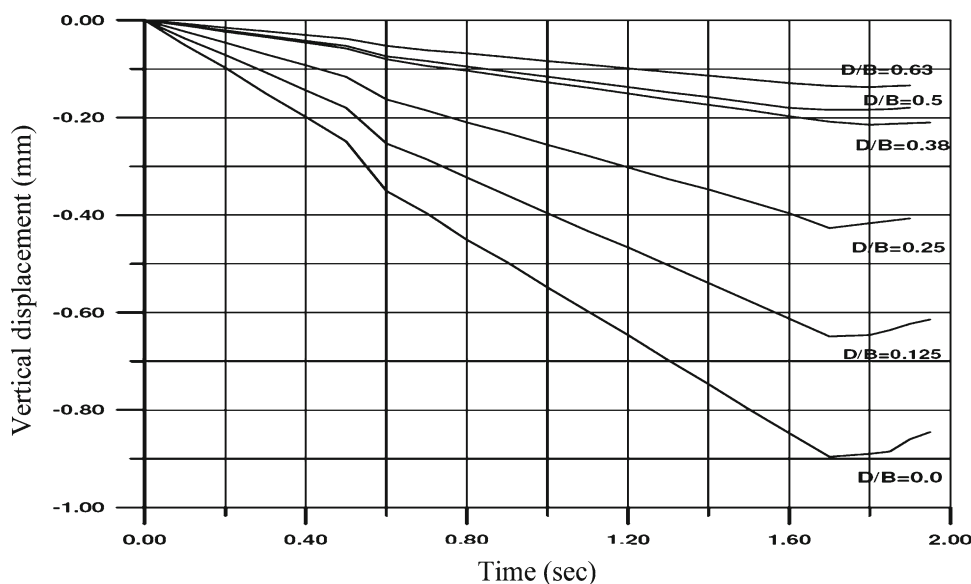
Spyrakos and Xu [10] tried to investigate the effects of embedment by making a comparison between the foundation responses for two different embedments with the response of a surface foundation. In both cases, the foundations were massless with a relative stiffness. The normalized dynamic compliances at the center of the foundation showed that the fundamental frequencies of the embedded foundation are decreased as compared to the surface foundation. This indicates that the effect of the additional inertia added from the embedment has more than counterbalanced the additional stiffness provided by the sidewalls of the foundation. Also, increasing the embedment depth of the foundation greatly

reduces the displacement of the system. Notice that for a foundation with an embedment depth of 2 m, the displacement close to resonance is only 60 % to that of a surface foundation. However, for an embedded foundation with an embedment depth of 4 m, the displacement close to resonance is only 40 % to that of a surface foundation.

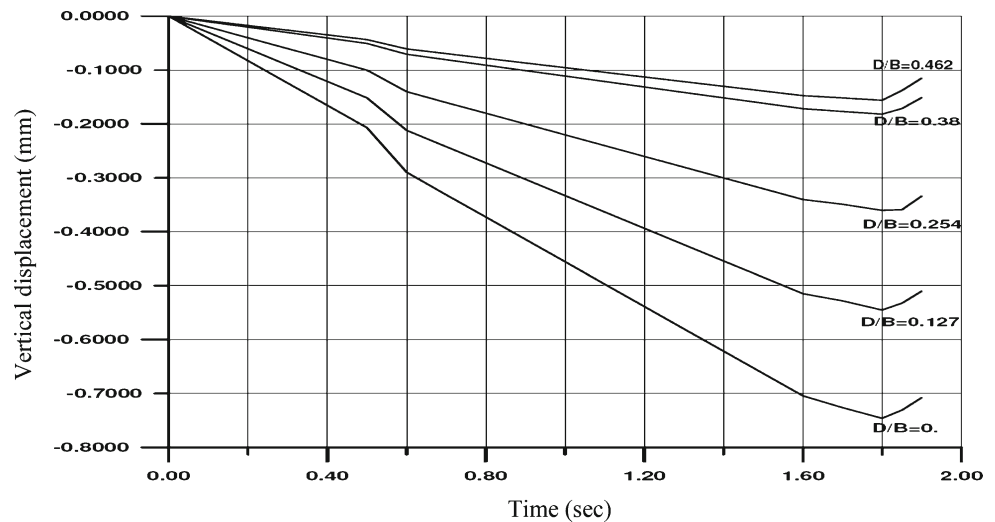
Al-Azawi et al. [5] carried out a dynamic analysis of machine foundations under vertical excitations. The effect of embedment and foundation geometry was taken into account. The stiffness and damping of soil were considered as frequency dependent. A computer program (CPESP) was coded in FORTRAN to evaluate the stiffness and damping coefficients depending on excitation frequency and embedment depth. The effect of embedment upon vertical forced vibration of a rigid footing was investigated theoretically. It was found that embedment of foundations has a significant effect on the dynamic response. It causes an increase in the dynamic stiffness and damping coefficients and leads to increase the resonant frequency and to decrease the dynamic response of foundation. A convergence in results was obvious when the depth ratio will be about 0.50. This means that the reduction in dynamic displacement will be less pronounced when the depth ratio is to be increased higher than 0.50 as shown in Fig. 2. The dynamic displacement in the vertical direction is smaller for the case of square foundations as compared to those of rectangular foundations for the same weight and contact soil pressure. The results indicated a reduction in the dynamic displacement in a range of (15–17 %) as compared to those of the rectangular foundation as shown in Fig. 3.

Livaoglu and Dogangun [11] studied the effects of foundation embedment on the seismic behavior of fluid-elevated tank–foundation–soil system with a structural frame supporting the fluid containing tank. Six different soil types defined

Fig. 2 Effect of embedment on vertical displacement [5]



**Fig. 3** Effect of embedment on vertical displacement for different depth ratios of a square foundation [5]



in the well-known seismic codes were considered. Both the sloshing effects of the fluid and soil–structure interaction of the elevated tanks located on these six different soils were included in the analyses. Fluid–elevated tank–foundation–soil systems were modeled with the finite element (FE) technique. The fluid–structure interaction was taken into account using Lagrangian fluid FE approximation implemented in the general purpose structural analysis computer program, ANSYS. FE model with viscous boundary was used to include elevated tank–foundation–soil interaction effects. The models were analyzed for the foundations with and without embedment. It was found that the tank roof displacements were affected significantly by the embedment in soft soil, however, this effect was smaller for stiff soil types. For soft soil types, embedment did not affect the other response parameters, such as sloshing displacement.

Mandal and Roychowdhury [12] presented the central response of the square raft under the step loading of 100 kN for different depth to width ratios. It was observed that the increase in the depth of embedment yields response of lesser amplitude and higher frequency.

Makhmalbaf et al. [13] considered two cases to investigate the effect of foundation embedment. First, the structure was considered as a model with surface foundation (i.e., without embedment) which was influenced by the excitation with different frequencies and then foundation was considered as completely embedded. In the second case, just the foundation was embedded in the soil. It was concluded that the foundation embedment is a positive feature. Because it can decrease the roof displacements in all frequencies of incident motion and also have a similar behavior in lower frequencies for displacements beneath the foundation. In higher excitation frequencies, embedding the foundation increases the displacements. This reveals that the filtering effect of SSI is not resulted from foundation embedment and therefore it is influenced by other factors of kinematic interaction.

Previous studies did not pay a lot of attention to the pore water pressure generated by machinery loads in saturated soils. The main aims of this paper are studying the effect of foundation embedment on the dynamic response of machine foundations on saturated sandy soil and investigating the best embedment ratio which provides the acceptable behavior for machine foundation. The excess pore water pressure generated due to vibrations will be traced.

### 1.3 Computer Program

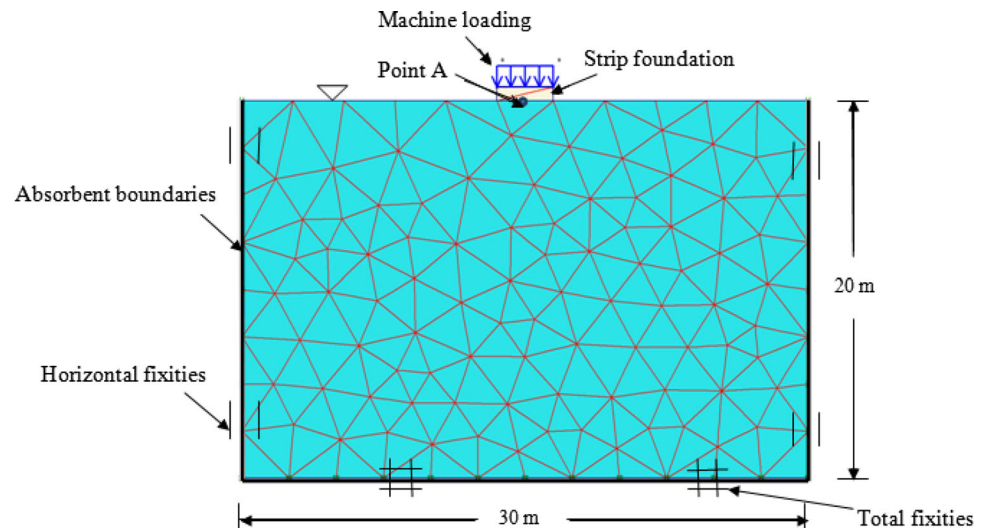
PLAXIS program will be used for numerical modeling of the problem; this program has a series of advantages [14]:

- Excess pore pressure: ability to deal with excess pore pressure phenomena. Excess pore pressures are computed during plastic calculations in undrained soil.
- Automatic load stepping: The program can run in an automatic step size and an automatic time step selection mode, providing this way robust result.
- Dynamic analysis: possibility to analyze vibrations and wave propagations in the soil and their influence on nearby structures. Excess pore pressures can be analyzed.
- Soil model: It can reproduce advanced constitutive soil models for simulation of nonlinear behavior.

## 2 Definition of the Basic Problem

Dynamic finite element analysis of strip foundation under vertical harmonic excitation is carried out in this research. A 3-m-wide strip foundation is analyzed. The general theory of bearing capacity of foundations is based on equations for strip footings. Then, this equation was modified by introducing shape factors for other shapes: rectangular, square,

**Fig. 4** Finite element mesh and boundary condition of the machine foundation problem



circular, etc. The foundation is placed basically at the top surface of sand with different states (i.e., loose, medium and dense); then, the depth of embedment is changed. The foundation is assumed to have different thicknesses. The analysis is performed numerically using the finite element software, PLAXIS 2D version 8.2. 15-noded triangular isoparametric elements are used to discretize the soil medium under the plane strain condition. The boundaries of the soil are taken as 30 m wide and 20 m deep far away from the foundation to minimize the boundary effects. To investigate the excess pore water pressure build up under machine foundation due to harmonic excitation, the soil is assumed to be saturated with water table which coincides with the ground surface. The boundary conditions and other modeling details considered for strip foundation are shown in Fig. 4. Total fixities ( $u_x = u_y = 0$ ) are applied at the base of the model, and horizontal fixities ( $u_x = 0$ ) are applied at the extreme vertical boundaries restraining the motion along the horizontal direction. Absorbent boundaries are applied along vertical and horizontal boundaries to avoid the reflection of stress waves back to the failure domain. It should be noted that in this analysis, a vertical vibration is applied and the vertical displacements and excess pore water pressure are calculated at the top central point of the foundation (node A in Fig. 4). It is important to mention here that all cases are analyzed for duration of (60 s) with time step taken as ( $\Delta t = 0.0256$  s).

### 3 Material Properties

The properties are classified into two groups:

1. Soil properties: Three states of sandy soil are used in this parametric study which are: loose, medium and dense sand. The soil deposit is assumed to obey the advanced Mohr–Coulomb yield criterion, with parameters adopted

from Bowles [15] and Murthy [16] except the dilatancy parameter. The effect of dilatancy is taken into account in the present study. The dilatancy of sand depends on both the density and the friction angle. It is suitable in PLAXIS to use the value of cohesion  $c > 0.2$  kPa for cohesionless sands and dilatancy angle  $\psi = \phi - 30$  for the soils with  $\phi > 30$ , and  $\psi = 0$  for the soils with  $\phi < 30$  [17]. Due to this, the value of cohesion is assumed equal to 1 kPa to avoid complications and the value of the angle of dilatancy is assumed as ( $\psi = \phi - 30$ ). The properties of all soil types are listed in Table 1.

2. Foundation properties: The concrete foundation is assumed as a linear elastic material with parameters shown in Table 1. The weight of the machine depends upon its type as suggested by Leonards [18]. Based on this table, the ratio between weight of foundation and weight of machine is approximately taken as 2.16 (i.e., weight of machine = 10 kN).

### 3.1 Phases Analysis of the Basic Problem

The dynamic analysis has been performed by accomplishing the following steps:

1. The model is solved under the gravity loading (geostatic stresses) only prior to placement of the foundations.
2. The model is then analyzed with equal and uniform static working load intensity of ( $10 \text{ kN/m}^2$ ) on foundation, which predicts the static behavior of the foundation under self-weight of the machine parts.
3. The dynamic analysis of the model is then performed with the application of vertical dynamic load intensity on the foundation. Any analysis, static or dynamic, in a FEM follows a standard procedure.



**Table 1** Material properties

Material	Material properties	Unit	Loose sand	Medium sand	Dense sand
Soil	Unit weight, $\gamma$	kN/m <sup>3</sup>	16**	18.5**	21**
	Young's modulus, $E$	kN/m <sup>2</sup>	18000**	35000**	65000**
	Poisson's ratio, $\nu$	–	0.3**	0.32**	0.34**
	Friction angle, $\phi$	°	32**	35**	40**
	Cohesion, $c$	kN/m <sup>2</sup>	1	1	1
	Dilatancy angle, $\psi$	°	2	5	10
	Horizontal permeability, $k_x$	m /sec	10 <sup>-2</sup> **	10 <sup>-4</sup> **	10 <sup>-5</sup> **
	Vertical permeability, $k_y$	m /sec	10 <sup>-3</sup> **	10 <sup>-4</sup> **	10 <sup>-5</sup> **
Foundation	Young's modulus of concrete, $E_{\text{concrete}}$	kN/m <sup>2</sup>	2 × 10 <sup>7</sup>		
	Unit weight of concrete, $\gamma_{\text{concrete}}$	kN/m <sup>3</sup>	24		
	Poisson's ratio of concrete, $\nu_{\text{concrete}}$	–	0.15		
Machine	Weight of machine, $W_{\text{mach.}}$	kN	10		

\* From Bowles [15]

\*\* From Murthy [16]

### 3.2 Sinusoidal Excitation

The most common problem involving dynamic loading is that of foundation for machinery. Reciprocating machines and poorly balanced rotating equipment cause periodic dynamic forces  $F(t)$  [19,20]:

$$F(t) = a \sin \omega t \quad (3)$$

where

$a$  = maximum amplitude of dynamic force,  
 $\omega = 2\pi f$  with  $f$  = operating frequency,  
 $t$  = time.

Typical operating frequencies range from 3 Hz for large reciprocating air compressors to about 200 Hz for turbines and high-speed rotary compressors [21]. The values of amplitudes range between 25 and 100 kPa, while the frequency ranges between 5 and 50 Hz.

### 4 Resonant Frequency

All machines under operation usually induce a periodic dynamic load on the foundation. Due to this induced dynamic load from the machine, the foundation including some portion of the soil underlying the foundation is subjected to vibration and it is essential that the natural frequency ( $\omega_n$ ) of this vibration should be well away from the operating frequency of the machine [3], ( $\omega_n$ ) of the system is expressed as [22]:

$$\omega_n = \sqrt{\frac{k}{m}} \quad (4)$$

where

$k$  = equivalent spring constant, and  
 $m$  = mass of machine and foundation.

If the effect of the soil ( $m_s$ ) is taken into account, the expression as shown in Eq. (4) becomes [22]:

$$\omega_n = \sqrt{\frac{k}{m + m_s}} \quad \text{and} \quad f_n = \frac{1}{2\pi} \sqrt{\frac{k}{m + m_s}} \quad (5)$$

where:

$m$  = total mass of foundation and machine (kg),  $m = \frac{w}{g}$ ,  
 $w$  = weight of foundation (kN),  $w = L \times B \times h \times \gamma_{\text{conc}}$  =  
length, width, thickness of foundation (m) and unit weight  
of the concrete foundation (kN/m<sup>3</sup>), respectively,  
 $g$  = acceleration due to gravity (m/s<sup>2</sup>),  
 $f_n$  = natural frequency in (Hz) and  
 $m_s$  = mass of soil participating in vibration (kg).

There have been several approaches suggested to determine the effective or equivalent mass of soil ( $m_s$ ), in calculating the natural frequency [22]. The choice of any one method still remains the designer's preference. Barkan [23] suggested that the mass of the vibrating soil should lie between (2/3) to (1.5) times the total mass of the vibrator and foundations; therefore, the mass of soil is considered to be

equal to the total mass of the vibrator and foundations in this study.

The equivalent spring constant can be calculated from the Lysmer and Richart’s method [24]:

$$K = \frac{4Gr_o}{1 - \nu} \tag{6}$$

where

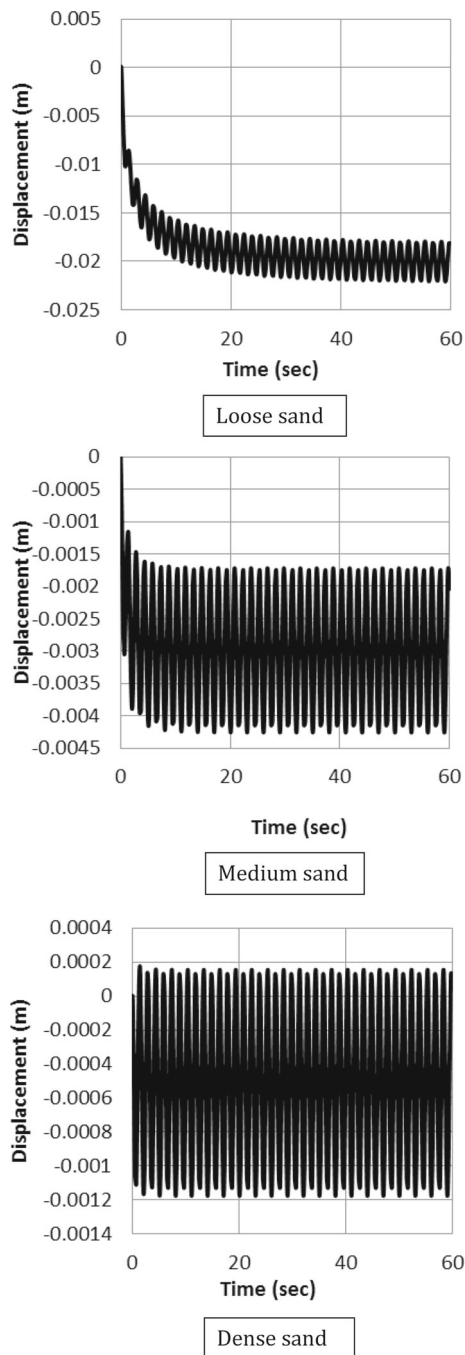
$$G = \text{shear modulus of the soil (kN/m}^2\text{)}, G = \frac{E}{2(1+\nu)}$$

$E$  = Young’s modulus of soil (kN/m<sup>2</sup>),  
 $\nu$  = Poisson’s ratio of soil and  
 $r_o$  = equivalent radius of the foundation ( $m$ ) =  $(\frac{LB}{\pi})^2$ .

Accordingly, the frequency ratio is defined as the ratio of operating frequency ( $f$ ) to natural frequency ( $f_n$ ). The frequency ratio ( $f/f_n$ ) should be either less than 0.5 or more than 2 to avoid resonance [22]. The values of the natural frequency ( $f_n$ ) and frequency ratio ( $f/f_n$ ) are summarized in Table 2.

**Table 2** Frequency ratios for all soil types and thicknesses of foundation

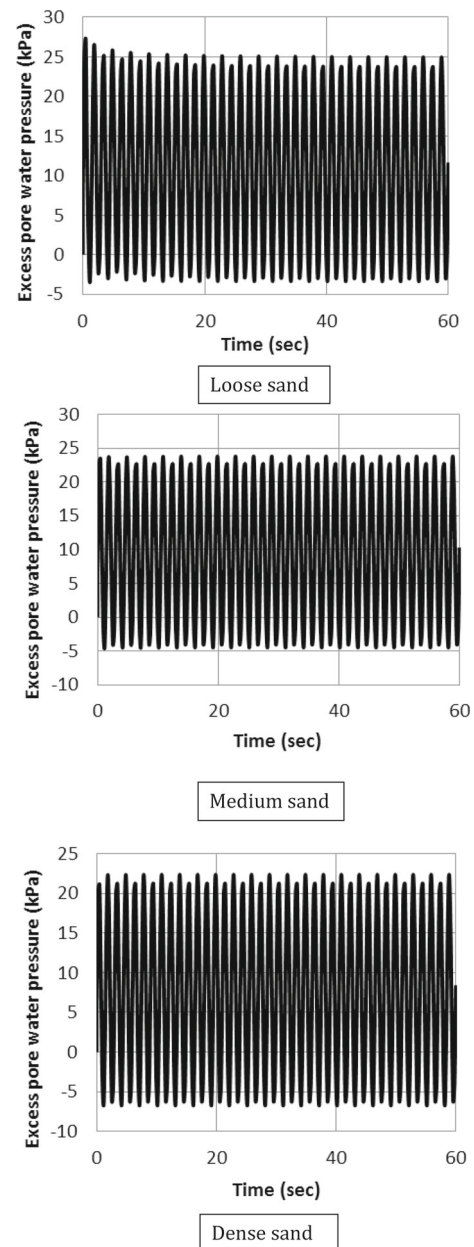
Type of soil	G (kN/m <sup>2</sup> )	K (kN/m)	h (m)	$m + m_s$ (Kg)	$f_n$ (Hz)	$f$ (Hz)	$\frac{f}{f_n}$
Loose sand	7500	36637.5	0.3	4.4	14.523	5	0.344
						25	1.721
						35	2.41
						50	3.442
			0.5	7.3	11.275	5	0.443
						25	2.217
						35	3.104
						50	4.435
			0.75	11	9.185	5	0.544
						25	2.722
						35	3.811
						50	5.443
Medium sand	13461.538	75153.844	0.3	4.4	20.8	5	0.24
						25	1.202
						35	1.682
						50	2.403
			0.5	7.3	16.149	5	0.31
						25	1.548
						35	2.167
						50	3.1
			0.75	11	13.155	5	0.38
						25	1.9
						35	0.376
						50	3.8
Dense sand	24253.731	143611.488	0.3	4.4	28.753	5	0.174
						25	0.87
						35	1.217
						50	1.74
			0.5	7.3	22.323	5	0.224
						25	1.12
						35	1.568
						50	2.24
			0.75	11	18.185	5	0.275
						25	1.375
						35	1.925
						50	2.75



**Fig. 5** Variation of displacement with time for foundation at embedment ( $D = 0.5$  m) with thickness ( $h = 0.3$  m) subjected to harmonic load with ( $a = 25$  kPa,  $f = 5$  Hz and  $\xi = 0$ ) in different types of sandy soil

## 5 Results of Analysis

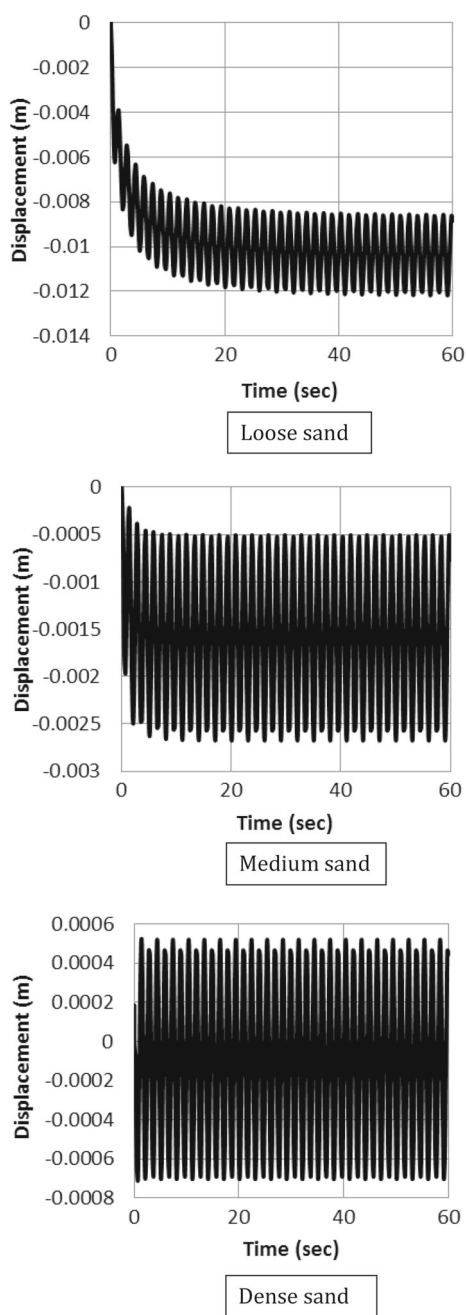
The effect of foundation embedment is investigated first, by considering four levels of embedment ( $D = 0.5, 1, 1.5$  and  $2$  m), where  $D$  is the depth of embedment, which is the depth from the surface to the horizontal soil–foundation interface [25].



**Fig. 6** Variation of excess pore water pressure with time for foundation at embedment ( $D = 0.5$  m) with thickness ( $h = 0.3$  m) subjected to harmonic load with ( $a = 25$  kPa,  $f = 5$  Hz and  $\xi = 0$ ) in different types of sandy soil

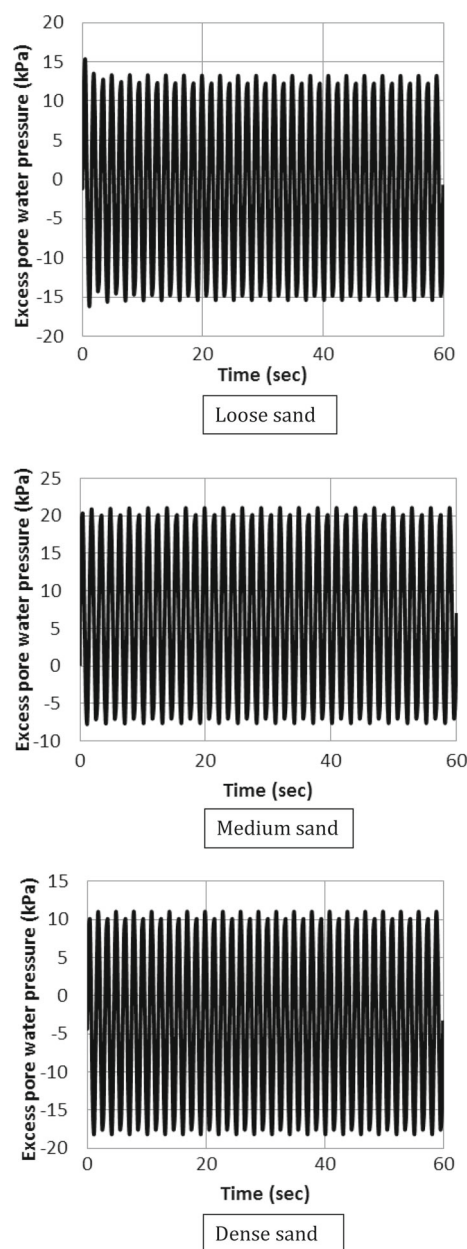
Figures 5, 7, 9 and 11 demonstrate the results of the typical displacement–time responses for loose, medium and dense sand with constant amplitude (with amount of 25 kN) and the constant frequency (with amount of 5 Hz) with constant thickness of foundation ( $h = 0.3$  m) for foundation at four embedments of (0.5, 1, 1.5 and 2 m), while Figs. 6, 8, 10 and 12 show the excess pore water pressure–time responses for loose, medium and dense sand with same frequency, amplitude of sinusoidal loading, thicknesses and embedments of foundation without damping (damping ratio  $\xi = 0$ ).





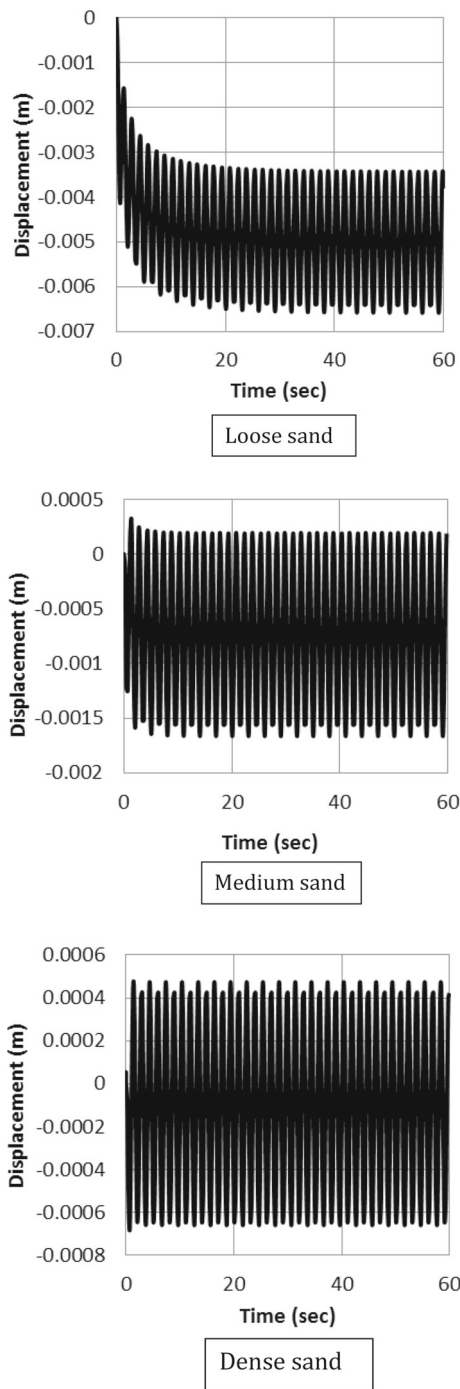
**Fig. 7** Variation of displacement with time for foundation at embedment ( $D = 1$  m) with thickness ( $h = 0.3$  m) subjected to harmonic load with ( $a = 25$  kPa,  $f = 5$  Hz and  $\xi = 0$ ) in different types of sandy soil

From the results of the displacement–time response for loose, medium and dense sand, it can be indicated that the vertical displacements decrease obviously by (46, 37 and 40 %), respectively, when increasing the embedment of foundation from 0.5 to 1 m, while when the embedment of foundation increases from 1 to 1.5 m, the vertical displacements for loose, medium and dense sand decrease by (45, 38 and 3 %), respectively, finally when the embedment of



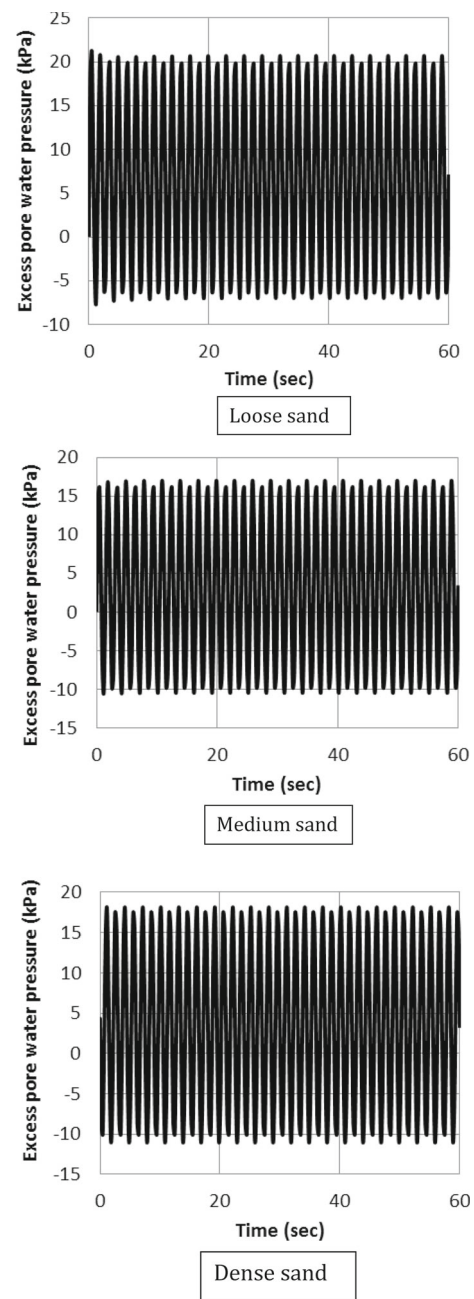
**Fig. 8** Variation of excess pore water pressure with time for foundation at embedment ( $D = 1$  m) with thickness ( $h = 0.3$  m) subjected to harmonic load with ( $a = 25$  kPa,  $f = 5$  Hz and  $\xi = 0$ ) in different types of sandy soil

foundation increases from 1.5 to 2 m, the decrements in vertical displacements are also recorded for loose, medium and dense sand by (42, 36 and 18 %), respectively. The basic reasons of the decrement in vertical displacement are the normal and shear stresses resulting from the additional inertia added from the embedment. In addition, the overlying soil restricts the vertical movement and thus reducing the settlement of the foundation base by increasing its vertical stiffness. Also, part of the applied load is transmitted to the ground through shear stresses along the vertical sides



**Fig. 9** Variation of displacement with time for foundation at embedment ( $D = 1.5$  m) with thickness ( $h = 0.3$  m) subjected to harmonic load with ( $a = 25$  kPa,  $f = 5$  Hz and  $\xi = 0$ ) in different types of sandy soil

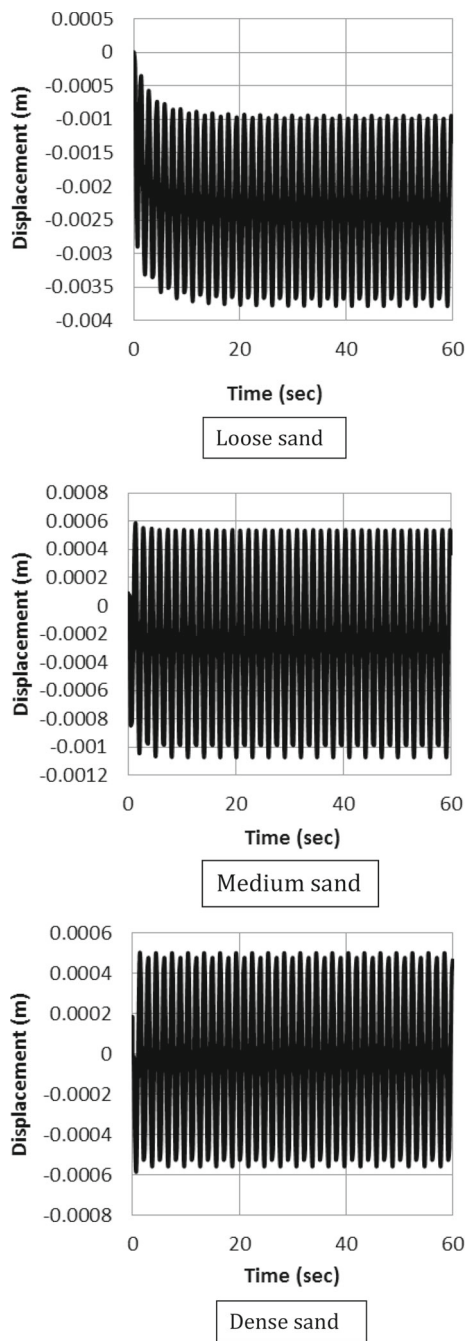
of the foundation when the sides are in contact with the surrounding soil. It is noticed that the decay in displacement decreases with embedment depth. This is because the weight of the foundation overrides the diminishing confining pressure caused by the removed soil, so that the settlement caused by foundation weight increases. This can



**Fig. 10** Variation of excess pore water pressure with time for foundation at embedment ( $D = 1.5$  m) with thickness ( $h = 0.3$  m) subjected to harmonic load with ( $a = 25$  kPa,  $f = 5$  Hz and  $\xi = 0$ ) in different types of sandy soil

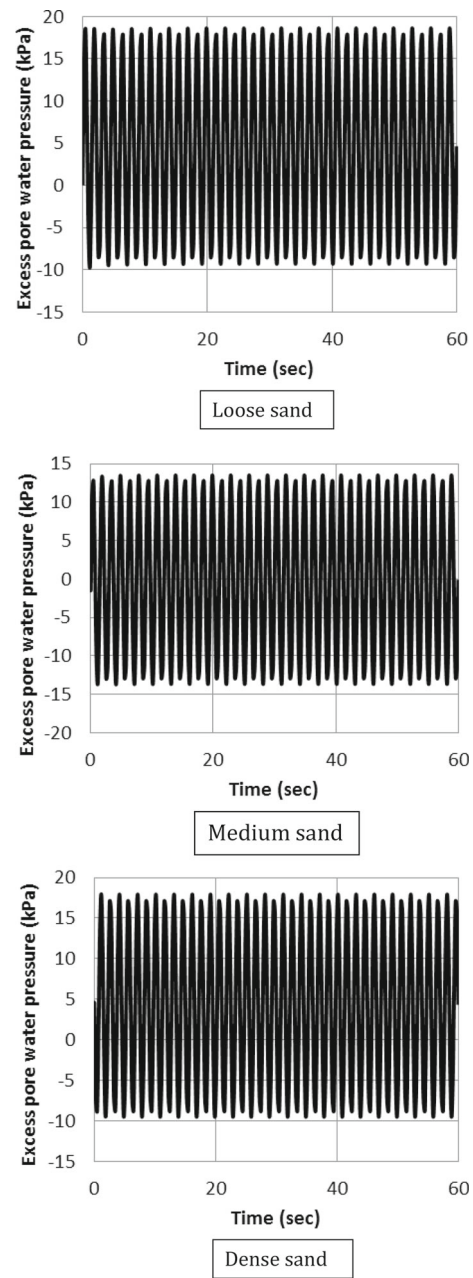
be caused by the effect of the additional inertia added from the embedment which has more than counterbalanced the additional stiffness provided by the sidewalls of the foundation.

From these results, it is also apparent that the embedment generally leads to a beneficial reduction in dynamic response for all soil types but with different percentages accompanied by an increase in soil strength, it seems that



**Fig. 11** Variation of displacement with time for foundation at embedment ( $D = 2$  m) with thickness ( $h = 0.3$  m) subjected to harmonic load with ( $a = 25$  kPa,  $f = 5$  Hz and  $\xi = 0$ ) in different types of sandy soil

the effect of embedment on the dynamic response of foundation becomes less pronounced by increasing the modulus of elasticity (and consequently improvement in soil strength from loose to dense sand). This conclusion agrees well with Livaoglu and Dogangun [11] who stated that the amount of embedment appears to cause a decrease in displacements especially for softer soils and the effect of embedment on

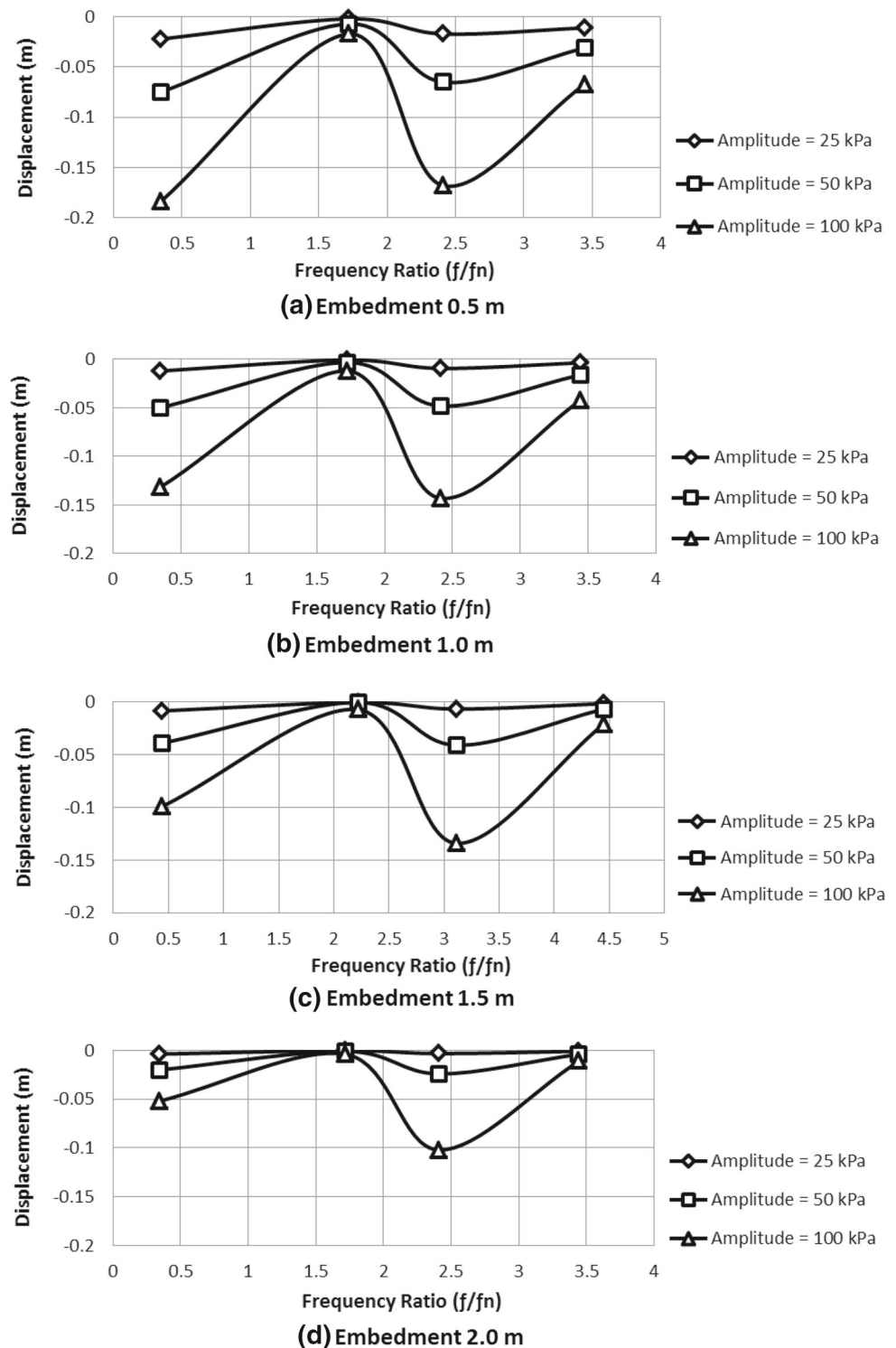


**Fig. 12** Variation of excess pore water pressure with time for foundation at embedment ( $D = 2$  m) with thickness ( $h = 0.3$  m) subjected to harmonic load with ( $a = 25$  kPa,  $f = 5$  Hz and  $\xi = 0$ ) in different types of sandy soil

the dynamic behavior on stiffer soils is not as significant, for very stiff soils, the embedment effect disappears completely. It is also noticeable that the percentage decrease in maximum vertical displacement does not follow a particular trend due to the nonlinear restoration characteristics of soil medium.

It is significant mentioning here that the sensitivity of the dynamic response decreases as the increasing of the embedment higher than (1 m), this is because confining pressure in

**Fig. 13** Variation of the maximum displacement with frequency ratio for foundation at different embedments with thickness ( $h = 0.3$  m) and without damping ( $\xi = 0$ ) in loose sandy soil

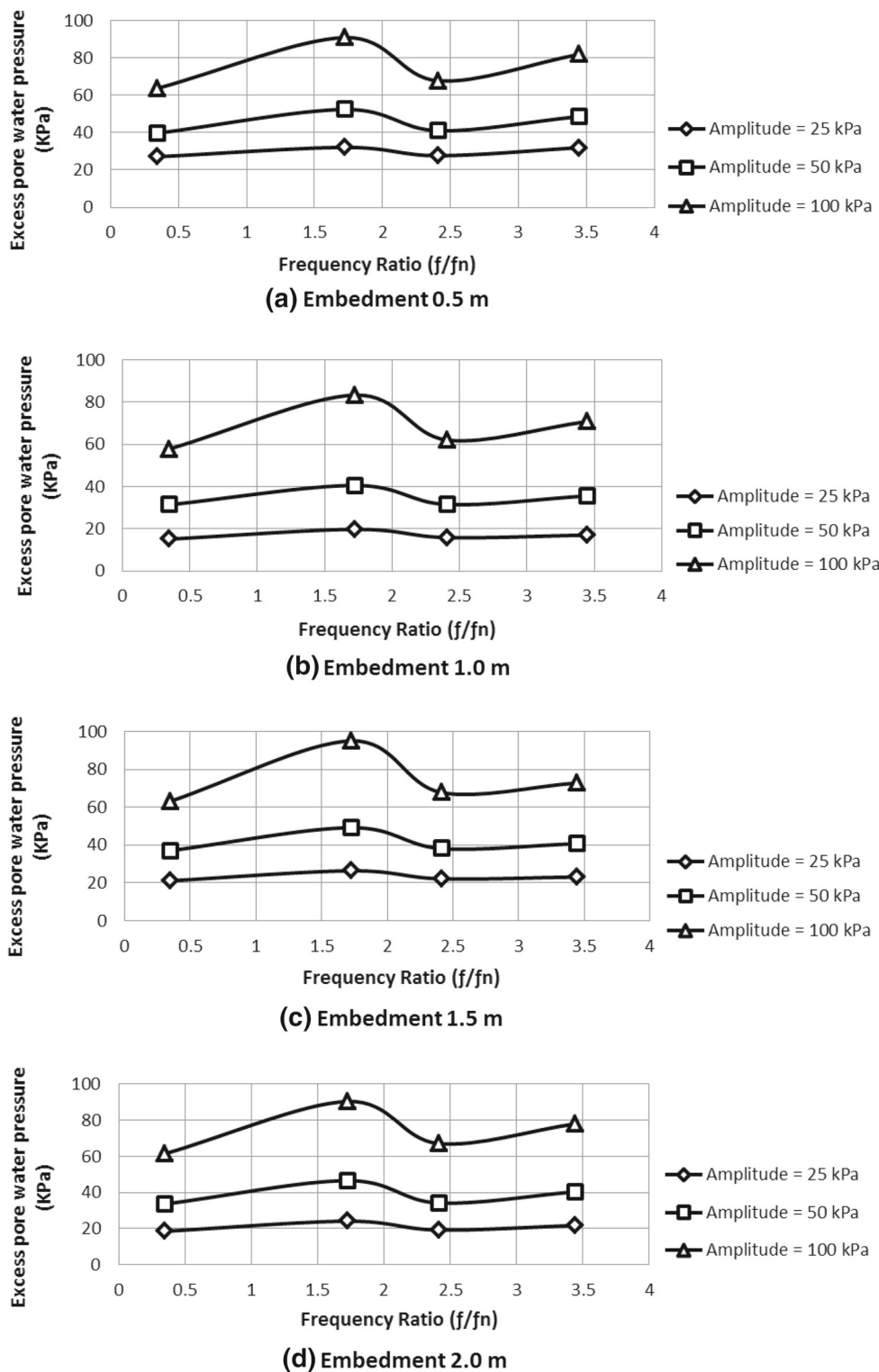


the soil at the foundation base changes from zero to a finite value from no embedment to an initial depth of embedment. These results are consistent in trend with the experimental studies of Hushmand [2] who found that the major increase in stiffness of the embedded foundation occurs from the zero embedment to the first embedment depth of 0.5 times

foundation width, and additional increase in embedment depth has a minor effect on stiffness of the soil–foundation system.

On the other hand, from the results of the excess pore water pressure responses, it can be noticed that the excess pore water pressure decreases by (44, 12 and 19%), respec-

**Fig. 14** Variation of the maximum excess pore water pressure with frequency ratio for foundation at different embedments with thickness ( $h = 0.3$  m) and without damping ( $\xi = 0$ ) in loose sandy soil

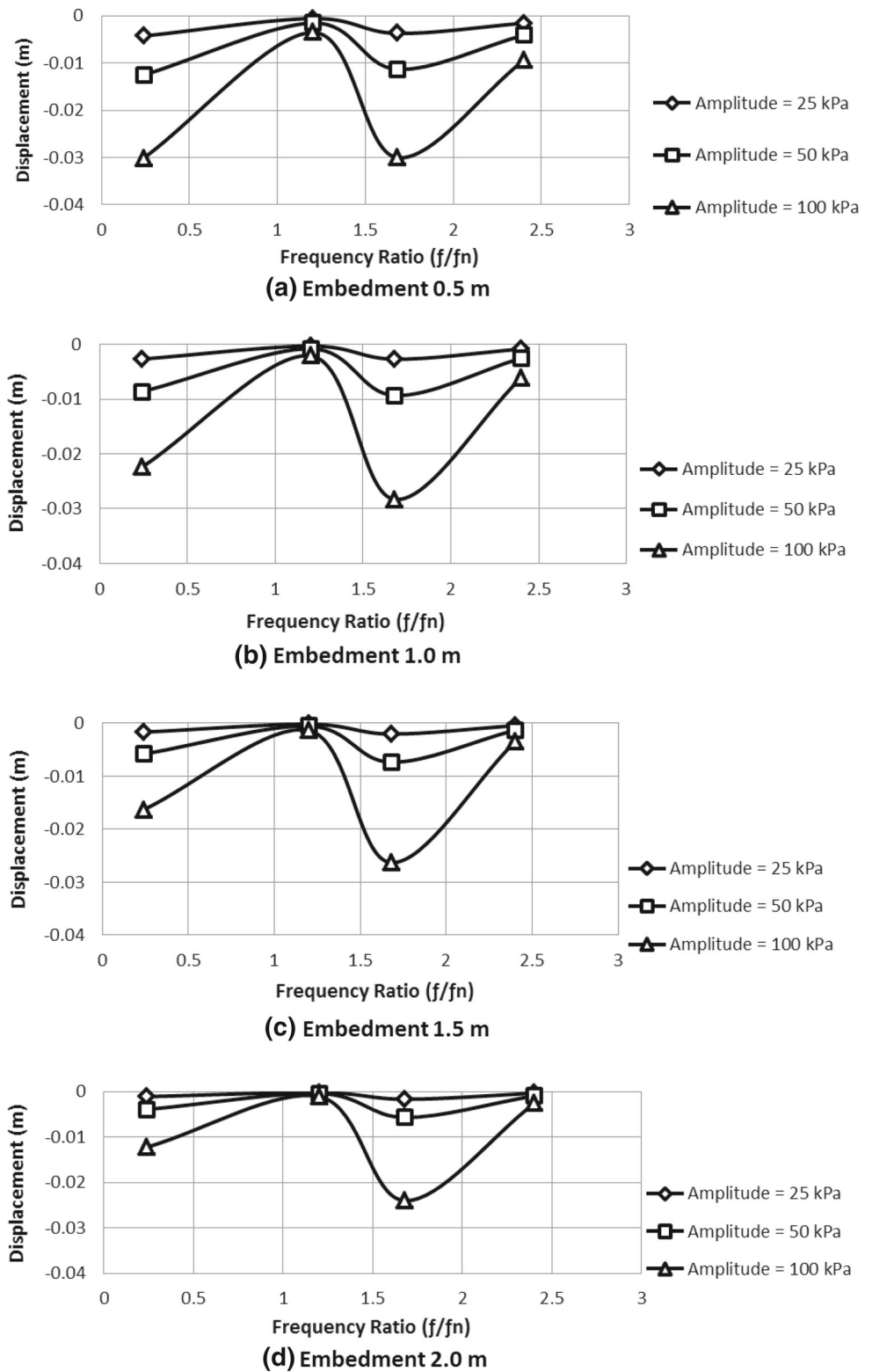


tively, for loose, medium and dense sand by increasing the embedment of foundation from 0.5 to 1 m, while when the embedment of foundation increases from (1 to 1.5 m), the excess pore water pressure increases by 39% for loose and decrease by 19, and 22% for medium and dense

sand, respectively. Finally, when the embedment of foundation increases from 1.5 to 2 m, the amounts of excess pore water pressure decrease for loose and medium sand by 12, 21%, respectively, and increase by 27% for dense sand.



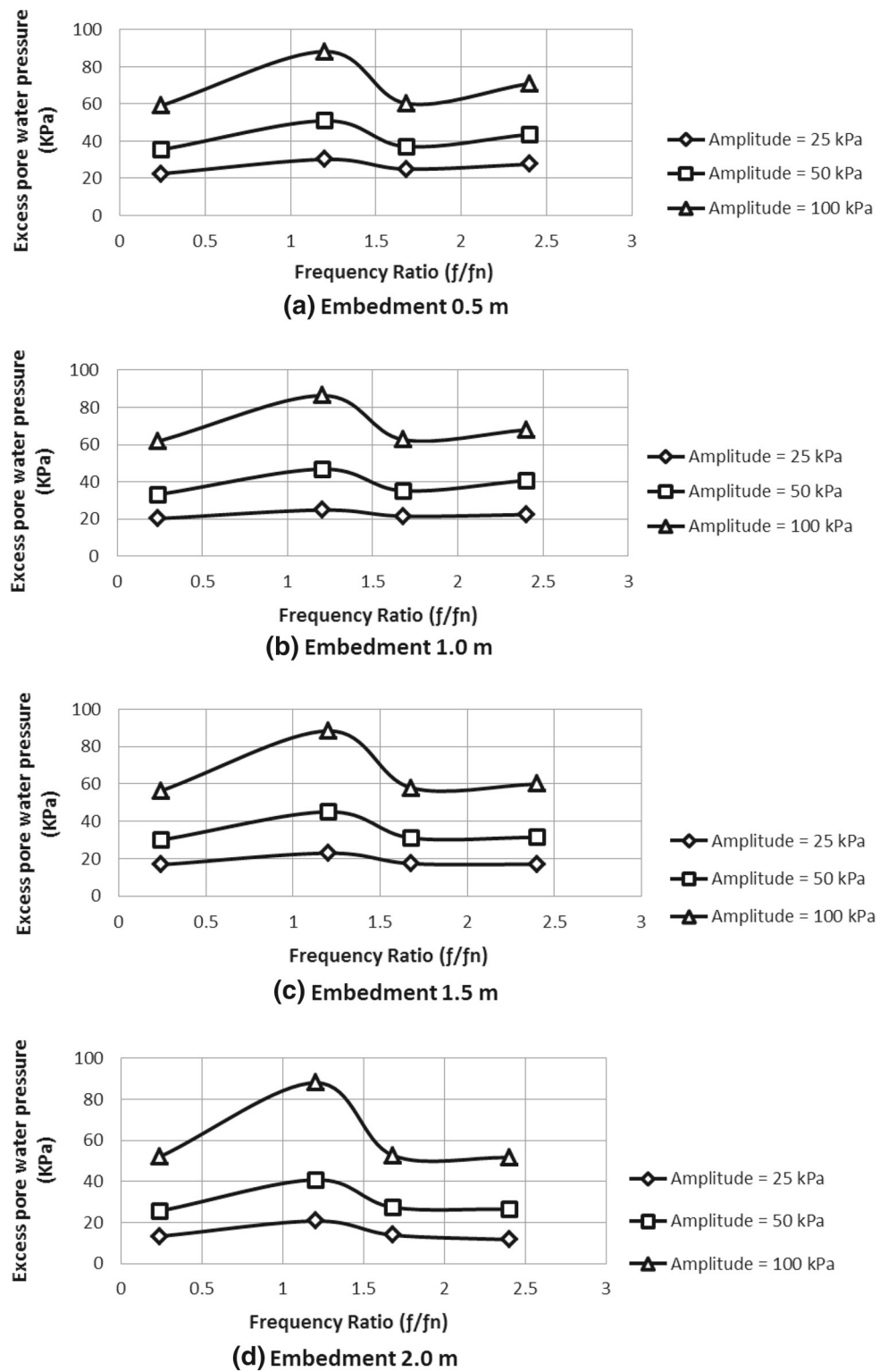
**Fig. 15** Variation of the maximum displacement with frequency ratio for foundation at different embedments with thickness ( $h = 0.3$  m) and without damping ( $\xi = 0$ ) in medium sandy soil



Figures 13, 15 and 17 portray the relationships between the frequency ratio ( $f/f_n$ ) and the maximum vertical displacements for three amplitudes of vertical harmonic load for machine foundation constructed over loose, medium and

dense sand, for the thickness of foundation 0.3 m and for different embedments of the foundation ( $D = 0.5, 1, 1.5$  and 2 m).

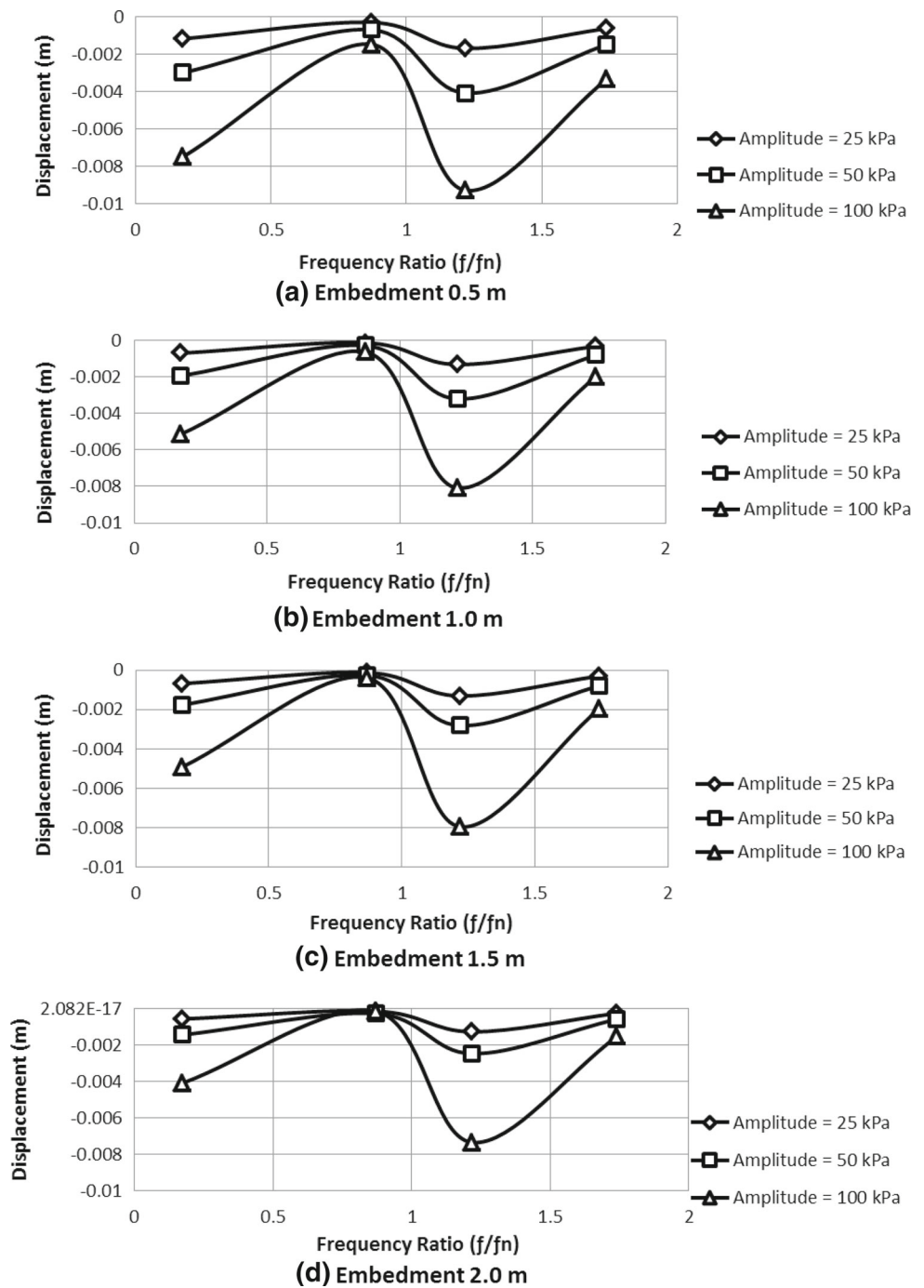
**Fig. 16** Variation of the maximum excess pore water pressure with frequency ratio for foundation at different embedments with thickness ( $h = 0.3$  m) and without damping ( $\xi = 0$ ) in medium sandy soil



For the loose sand, the maximum displacement for three amplitudes applied occurs at frequency ratio of 0.344 for embedment of the foundation ( $D = 0.5$  m). For embedment of the foundation ( $D = 1$  m), the maximum displacement for amplitudes applied ( $a = 25$  and  $50$  kN/m<sup>2</sup>) occurs at

frequency ratio of 0.344, but it is recorded at frequency ratio of 2.41 for amplitudes applied ( $a = 100$  kN/m<sup>2</sup>), while for embedments of the foundation ( $D = 1.5$  and  $2$  m) the maximum displacement for amplitudes applied ( $a = 25$  kN/m<sup>2</sup>) occurs at frequency ratio of 0.344, but it focuses at frequency

**Fig. 17** Variation of the maximum displacement with frequency ratio for foundation at different embedments with thickness ( $h = 0.3$  m) and without damping ( $\xi = 0$ ) in dense sandy soil

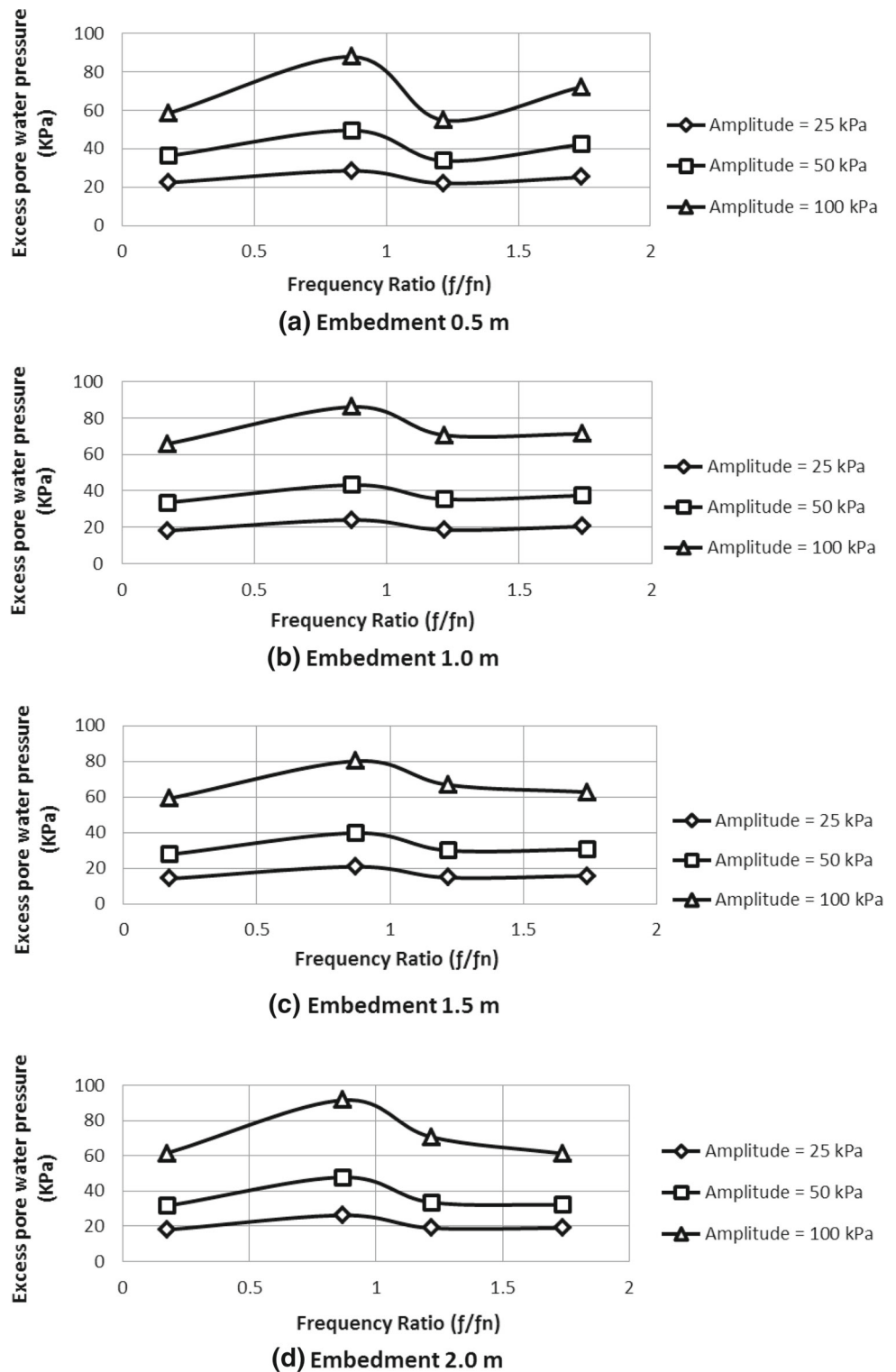


ratio of 2.41 for amplitudes applied ( $a = 50$  and  $100$  kN/m<sup>2</sup>). This observation agrees well with Chopra and Dargush [26] who stated that with increasing depths of embedment, the dynamic amplification begins to shift to the right to higher-frequency ratios. This phenomenon is associated with the propagation of generalized Rayleigh waves within the elastic soil layer. For medium and dense sand, the maximum displacement for the three amplitudes applied is found to be at frequency ratio 2.41 for different embedments of the

foundation ( $D = 0.5, 1, 1.5$  and  $2$  m). It is also apparent that the curves of the relations for the three amplitudes for loose, medium and dense sand coincide with each other especially at frequency ratio 1.721.

From Figs. 14, 16 and 18 which exhibit the relationships between the frequency ratio ( $f/f_n$ ) and the maximum excess pore water pressure for three amplitudes of vertical harmonic load for machine foundation with thickness 0.3 m constructed over loose, medium and dense sand, and

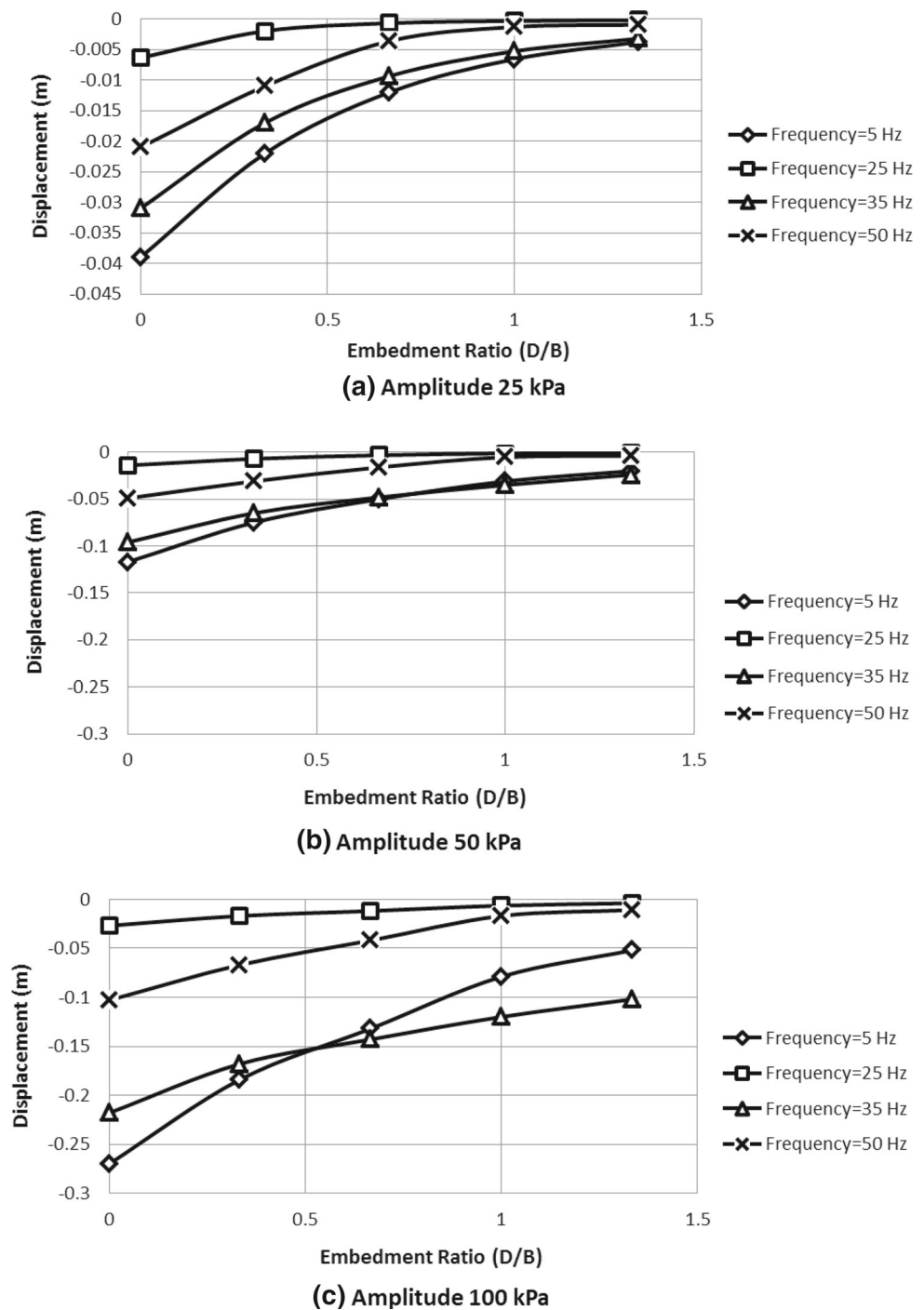
**Fig. 18** Variation of the maximum excess pore water pressure with frequency ratio for foundation at different embedments with thickness ( $h = 0.3$  m) and without damping ( $\xi = 0$ ) in dense sandy soil



for different values of embedment of the foundation ( $D = 0.5, 1, 1.5$  and  $2$  m), it can be noted that the maximum excess pore water pressure is recorded at frequency ratio 1.721 for all types of soil and magnitudes of the foundation embedment.

Other cases for different values of load amplitude and frequency for different embedments are also taken into account in this parametric study; the results are not shown here elaborately due to the lack of space, but these results are summarized in normalized form for embedment effect cor-

**Fig. 19** Variation of the maximum displacement with embedment ratio for foundation at different amplitudes and frequencies with thickness ( $h = 0.3$  m) and without damping ( $\xi = 0$ ) in loose sandy soil



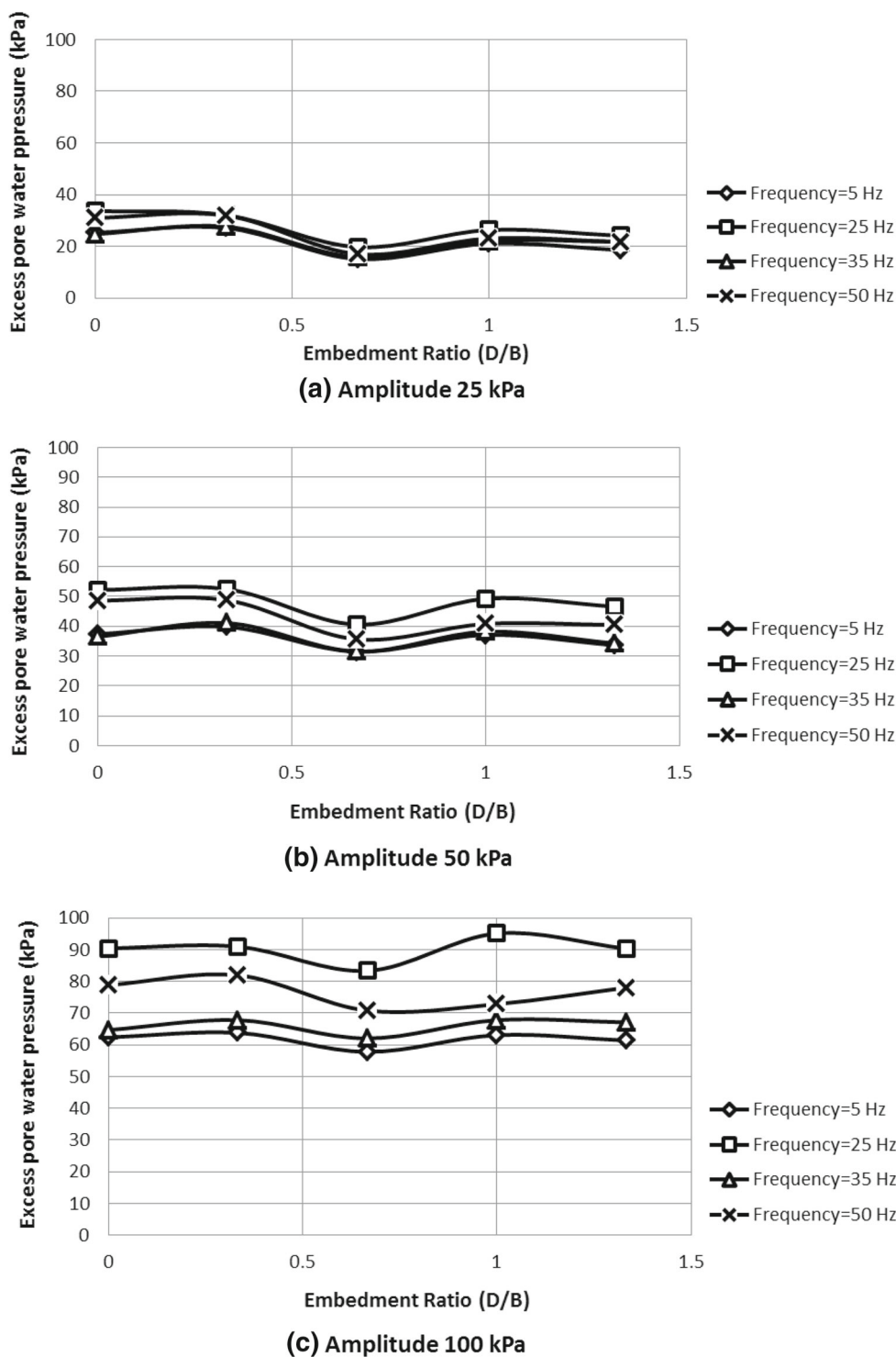
responding to embedment ratio ( $D/B = 0.333, 0.666, 1$  and  $1.333$ ) (where  $D$  the depth of embedment and  $B$  half the width of strip foundation) [25]. For rigorous investigation on embedment effect, embedment ratio ( $D/B$ ) relations with displacement and excess pore water pressure are portrayed.

Figure 19 shows the relationships between the embedment ratio and the maximum vertical displacement in loose sand at different values of load amplitude and the load frequency. It is evident that when the embedment ratio increases, the

vertical displacement decreases. A convergence in results is obvious when the embedment ratio is about (1). This finding is corroborate in trend with Al-Azawi et al. [5], who concluded that the reduction in dynamic displacement will be less pronounced when the depth ratio increased higher than 0.5. Figure 20 demonstrates the relationships between the embedment ratio and the maximum excess pore water pressures for loose sand at different values of load amplitude and frequency. It can be indicated that when the embedment



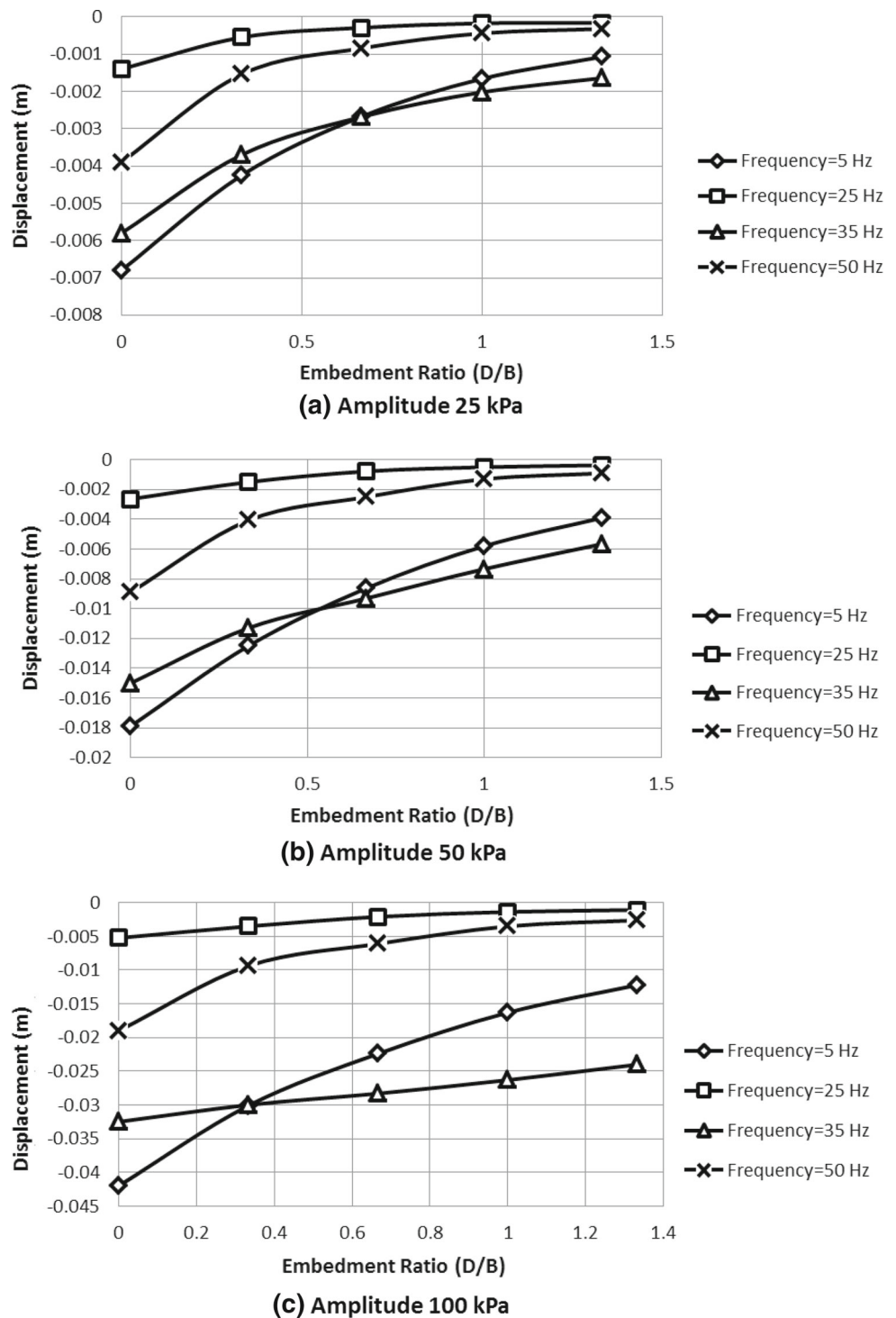
**Fig. 20** Variation of the maximum excess pore water pressure with embedment ratio for foundation at different amplitudes and frequencies with thickness ( $h = 0.3$  m) and without damping ( $\xi = 0$ ) in loose sandy soil



ratio increases, the maximum excess pore water pressures decreases until the embedment ratio equals to the 0.666, after that the excess pore water pressure increases by increasing the embedment ratio but the rate of decreasing still higher than the rate of increasing for excess pore water pressure.

For medium sand, Fig. 21 depicts the relationships between the embedment ratio and the maximum vertical displacements at different values of load amplitude and frequency. It can be seen that a convergence in results is slight when the embedment ratio is about (1) and the divergence in the results becomes obvious when the amplitude of loading

**Fig. 21** Variation of the maximum displacement with embedment ratio for foundation at different amplitudes and frequencies with thickness ( $h = 0.3$  m) and without damping ( $\xi = 0$ ) in medium sandy soil



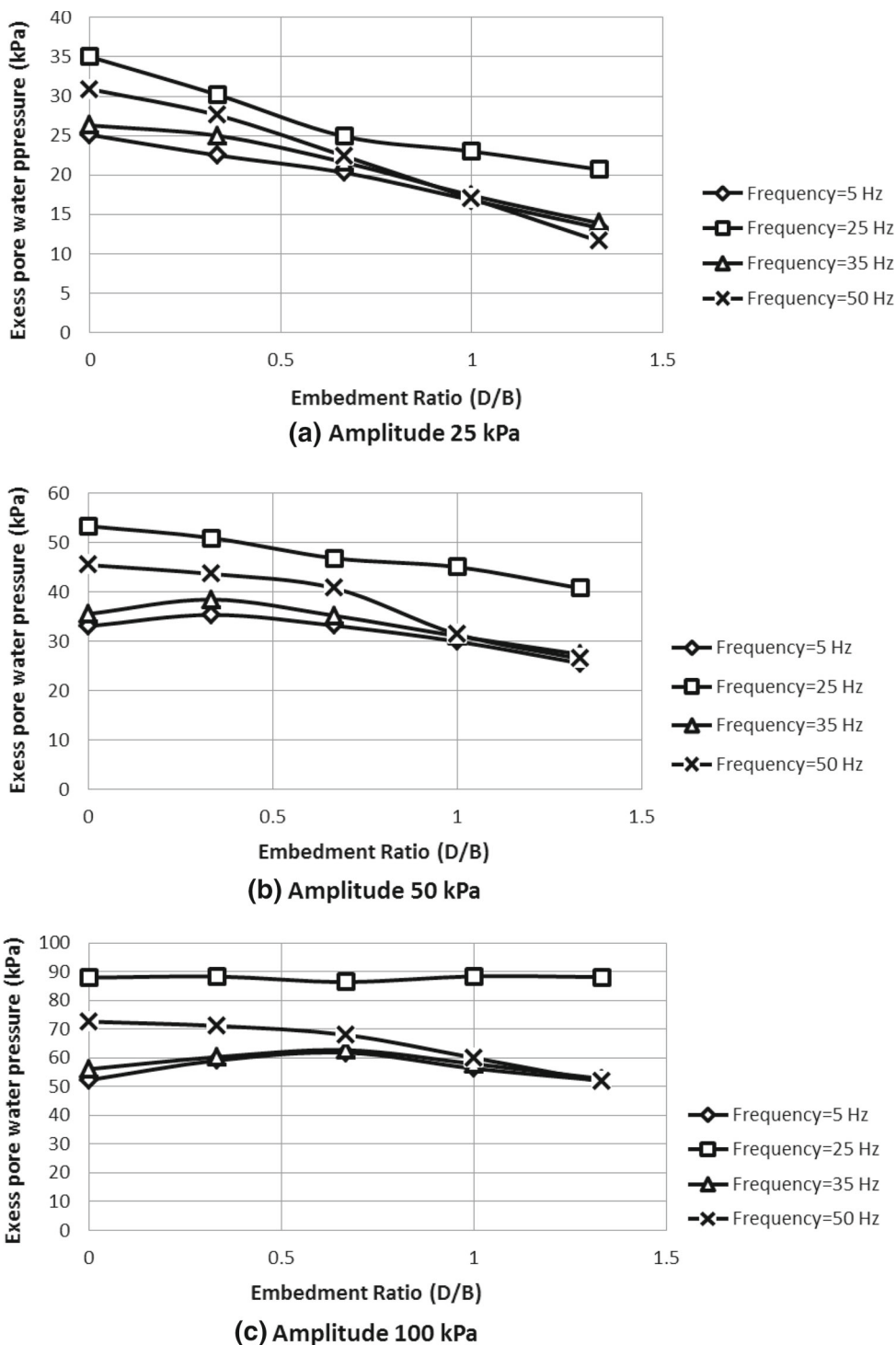
increases. On the other hand, Fig. 22 manifests the relationships between the embedment ratio and the maximum excess pore water pressure at different values of load amplitude and frequency.

It can be shown that the maximum excess pore water pressure overall decreases by increasing the embedment ratio and a convergence in the results occurs at embedment ratio equals

to (1) for frequencies (5, 35, 50 Hz) and divergence in the results is recorded for frequency (25 Hz).

For dense sand, Fig. 23 depicts the relationships between the embedment ratio and the maximum vertical displacement at different values of load amplitude and frequency. It can be indicated that no convergence in results is recorded at any embedment ratio. On the other hand, Fig. 24 portrays the

**Fig. 22** Variation of the maximum excess pore water pressure with embedment ratio for foundation at different amplitudes and frequencies with thickness ( $h = 0.3$  m) and without damping ( $\xi = 0$ ) in medium sandy soil



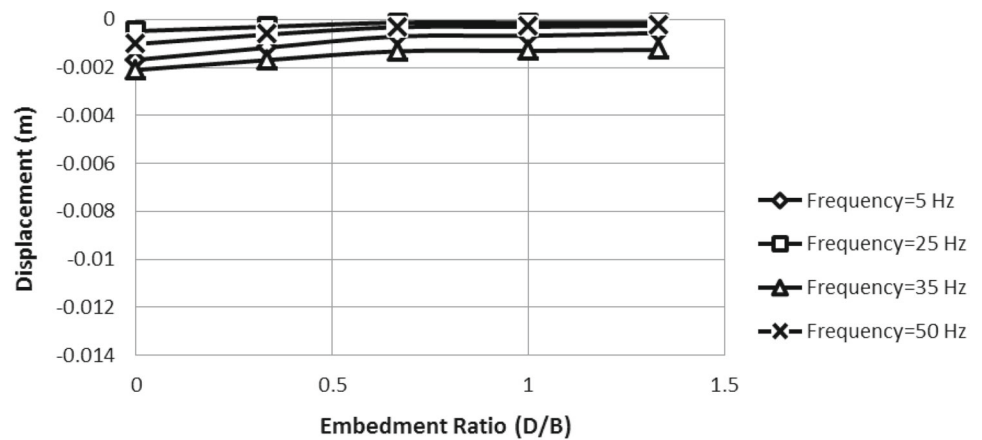
relationships between the embedment ratio and the maximum excess pore water pressures at different values of load amplitude and the load frequency. It can be seen that the excess pore water pressure decreases by increasing the embedment ratio until the embedment ratio equals to 1 after that, the magnitudes of the excess pore water pressure increases. It is also noted here that a convergence in results occurs at embed-

ment ratio equal to (1) for frequencies (5, 35, 50Hz) and divergence in the results is noticed for frequency (25 Hz).

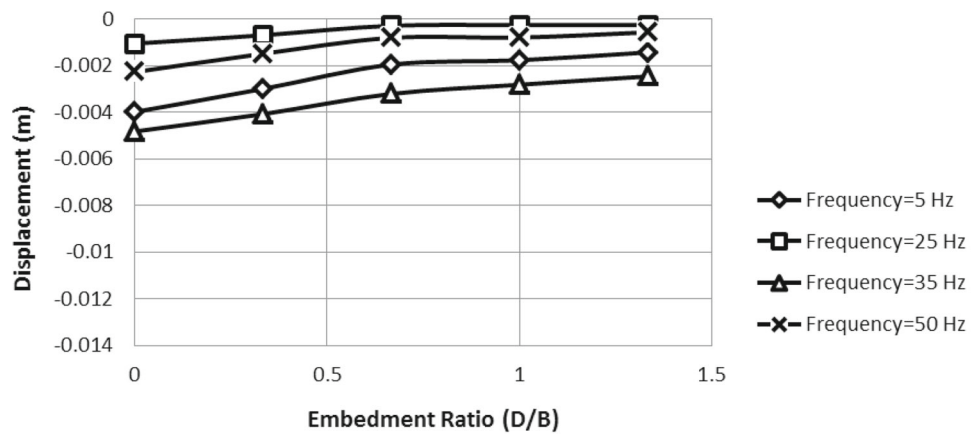
### 6 Conclusions

From the extended parametric study carried out in this study by utilizing the finite element program PLAXIS 2D V8.2 for

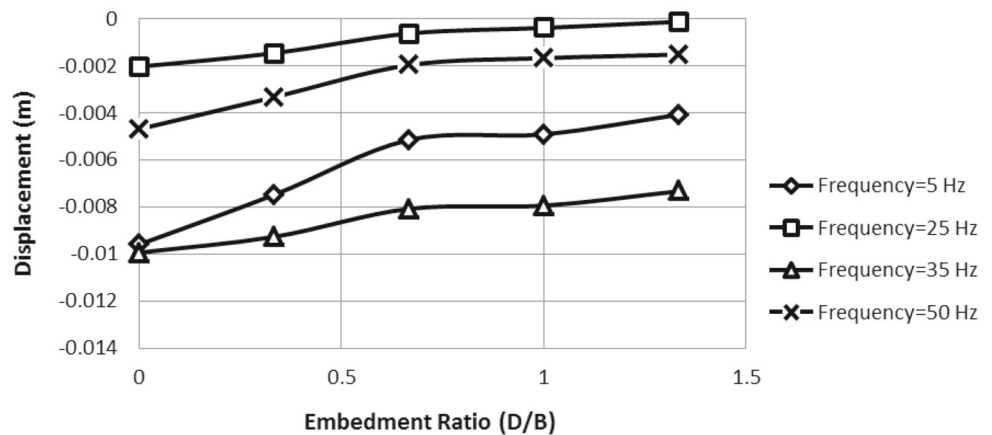
**Fig. 23** Variation of the maximum displacement with embedment ratio for foundation at different amplitudes and frequencies with thickness ( $h = 0.3$  m) and without damping ( $\xi = 0$ ) in dense sandy soil



(a) Amplitude 25 kPa



(b) Amplitude 50 kPa



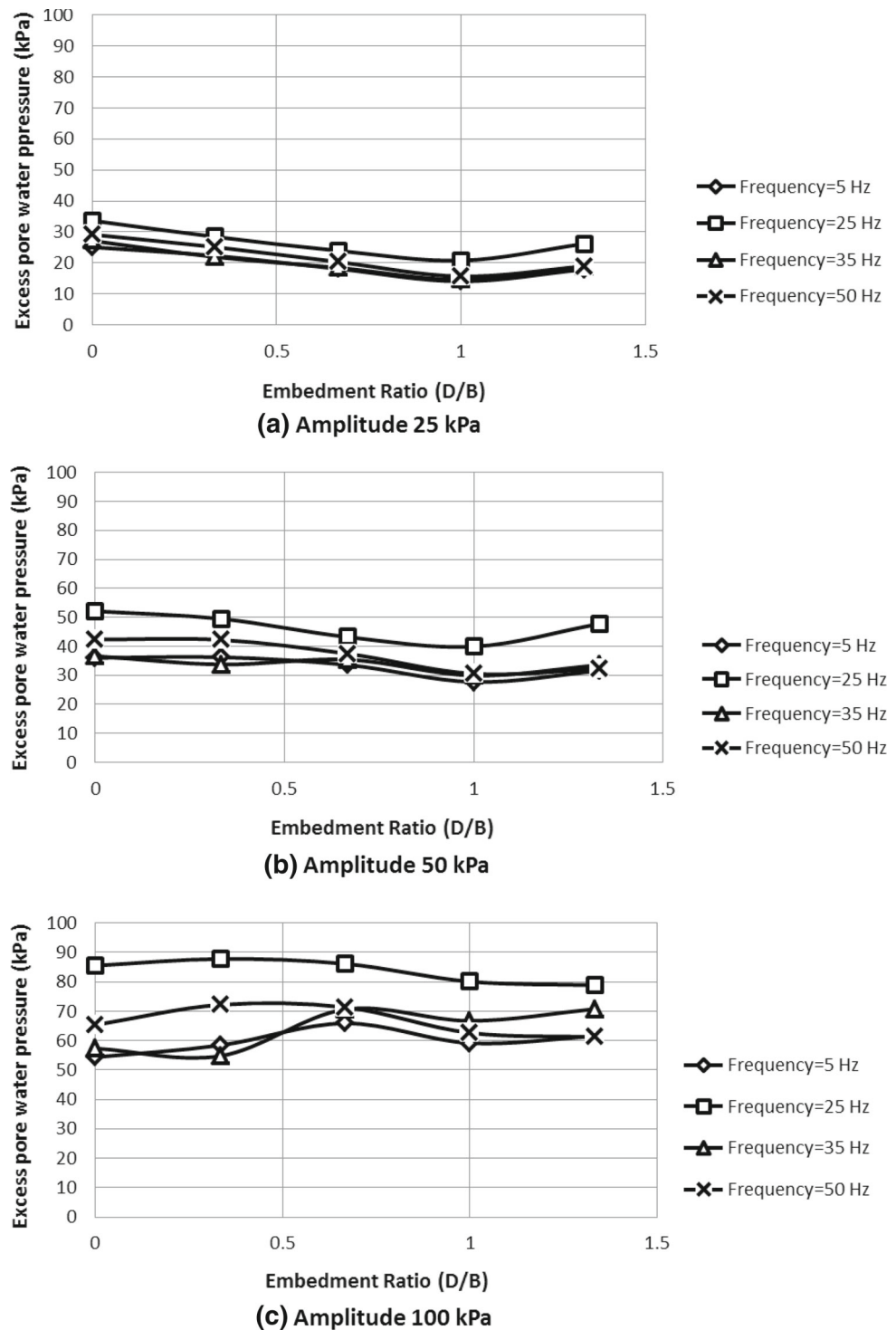
(c) Amplitude 100 kPa

the analysis of machine foundation rested on sandy soil with different densities, the following conclusions can be drawn:

1. For foundation at surface resting on sand of different relative densities, relations between frequency with displacement and excess pore water pressure are not smooth and exhibit undulations (peaks and valleys), but for all
2. Embedment generally leads to a beneficial reduction in dynamic response (displacement and excess pore water pressure) for all soil types but with different percentages



**Fig. 24** Variation of the maximum excess pore water pressure with embedment ratio for foundation at different amplitudes and frequencies with thickness ( $h = 0.3$  m) and without damping ( $\xi = 0$ ) in dense sandy soil



accompanied by an increase in soil strength, and this effect is less pronounced as the embedment increases greater than 1 m and as the soil modulus of elasticity increases (as changing the soil state from loose to dense sand).

3. The vertical displacements for loose, medium and dense sand decrease obviously by about 46, 37 and 40%,

respectively, when increasing the embedment of foundation from 0.5 to 1 m, while when the embedment of foundation increases from 1 to 1.5 m, the vertical displacements decrease by 45, 38 and 3 %, respectively.

4. The sensitivity of the dynamic response decreases as the increasing of the embedment higher than 1 m, and this is because confining pressure in the soil at the foundation



base changes from zero to a finite value from no embedment to an initial depth of embedment.

- The maximum excess pore water pressure is recorded at frequency ratio 1.721 for all types of soil and magnitudes of the foundation embedment.

## References

- Beredugo, Y.O.; Novak, M.: Coupled horizontal and rocking vibrations of embedded footings. *Can. Geotech. J.* **9**(4), 477–497 (1972)
- Hushmand, B.: Experimental Studies of Dynamic Response of Foundation. Ph.D. thesis, California Institute of Technology (1983)
- Chowdhury, I.; Dasgupta, S.P.: Dynamics of Structures and Foundations—A Unified Approach. CRC Press-Balkema, London (2009)
- American Concrete Institute: Foundation for Dynamic Equipment, Reported by ACI Committee 351 (2004)
- Al-Azawi, T.K.; Al-Azawi, R.K.; Al-Jaberi, Z.K.: Stiffness and damping properties of embedded machine foundation. *J. Eng. Univ. Baghdad* **12**(2), 429–443 (2006)
- Prakash, S.; Puri, K.V.: Foundation for vibrating machines. *J. Struct. Eng.*, pp. 1–38 (special issue). SERC, Madras, India (2006)
- Baranov, V.A.: On the calculation of excited vibrations of an embedded foundation (in Russian) *Vopr. Dyn. Prochn.* **14**, 195–209 (Cited by Prakash and Puri, 2006) (1967)
- Gazetas, G.; Dobry, R.; John, L.: Vertical response of arbitrarily shaped embedded foundations. *J. Geotech. Eng. ASCE* **111**(6), 750–771 (1985)
- Erden, S.M.: Influence of Shape and Embedment of Dynamic Foundation Response. Ph.D. thesis, University of Massachusetts, Amherst, Mass. (1974)
- Spyrakos, C.C.; Xu, C.: Dynamic analysis of flexible massive strip foundations embedded in layered soil by hybrid BEM–FEM. *Soil Dyn. Earthq. Eng.* **82**, 2541–2550 (2004)
- Livaoglu, R.; Dogangun, A.: Effect of foundation embedment on seismic behavior of elevated tanks considering fluid–structure–soil interaction. *Soil Dyn. Earthq. Eng.* **27**, 855–863 (2007)
- Mandal, J.J.; Roychowdhury, S.: Response of rectangular raft foundations under transient loading. In: Proceedings, 12th International Conference of International Association for Computer Methods and Advanced in Geomechanis (IACMAG), India, pp. 524–530 (2008)
- Makhmalbaf, M.O.; Alitalesh, M.; Samani, M.Z.; Tutunchian, M.A.: Parametric Study on Displacements of Buildings Considering the Dynamic Soil-Structure Interaction Effects. *Electron. J. Geotech. Eng.* **16**, 1617–1628 (2011)
- Catala, M.A.: Excess Pore Pressure Generation due to Pseudostatic Tests in Saturated Sand. Ph.D. thesis, Delft University of Technology, Netherland (2005)
- Bowles, J.E.: Foundation Analysis and Design, 5<sup>th</sup> edn. pp. 1175 McGraw-Hill Companies, Inc., (1996)
- Murthy, V.N.S.: Geotechnical Engineering, Principles and Practices of Soil Mechanics and Foundation Engineering. Marcel Dekker, Inc, New York (2006)
- Brinkgreve, R.B.J.: PLAXIS-2D Version 8 User Manuals. Delft University of Technology and PLAXIS, b. v., the Netherlands. (2002)
- Leonards, G.A.: Foundation Engineering. McGraw-Hill Book Co., Inc., New York (1962)
- Das, B.M.; Ramana, G.V.: Principles of Soil Dynamics, 2nd edn. Cengage Learning, Inc, Boston (2010)
- Fattah, M.Y.; Al-Neami, M.A.; Jajjawi, N.H.: Prediction of liquefaction potential and pore water pressure beneath machine foundations. *Cent. Eur. J. Eng.* **4**(3), 226–249 (2013). doi:[10.2478/s13531-013-0165-y](https://doi.org/10.2478/s13531-013-0165-y)
- Al-Sherefi, M.H.J.: Non-Linear Dynamic Response of Soils. MSc. thesis, Civil Engineering Department, University of Baghdad (2000)
- Rao, N.S.V.: Foundation Design: Theory and Practice. Wiley, Hoboken (2011)
- Barkan, D.D.: Dynamics of Bases and Foundations. McGraw-Hill Book Co., New York (1962)
- Richart, F.E.; Hall, J.R.; Woods, R.D.: Vibrations of Soils and Foundations, p. 414. Prentice-Hall, Englewood Cliffs, NJ (1970)
- Gazetas, G.: Analysis of machine foundations: state of the art. *Soil Dyn. Earthq. Eng.* **2**(1), 2–42 (1983)
- Chopra, M.B.; Dargush, G.F.: Dynamic response of embedded circular foundations using a poroelastic BEM. *Comput. Methods Adv. Geomech.* **29**(4), 2207–2210 (1997)