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Information Recommendation Between User Groups in Social Networks

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Abstract Information recommendation between different user groups has recently received a lot of attention in the information service community. However, we find that obtaining the exact optimal recommendation solution is an NP-hard problem. Based on the above finding, in this paper, we present an efficient method achieving approximate optimal recommendation solution (AAORS) to reduce this NP-hard problem to an equivalent extended Steiner tree problem and obtain the approximate optimal recommendation solution *appIRS* in polynomial time. We theoretically prove that the global trust value of *appIRS* is at least 63 % of that obtained for the exact optimal solution *optIRS*. Moreover, in real applications, based on a computed index of reputation gain, we also adjust the recommendation solution produced by the AAORS method in polynomial time and obtain the optimal recommendation solution which satisfies the global reputation constraint. The detailed theoretical analyses and extensive experiments demonstrate that our proposed methods are both efficient and effective.

Keywords Social network · Information recommendation · User group · Approximation algorithm · Steiner tree

1 Introduction

Social networks have already developed significantly in the era of Web 2.0 in facilitating information recommending and sharing between people. Compared with traditional print media, network media etc., people may join in different social groups via Web, WAP and various Apps which are based on the technology of information sharing, transmission and acquisition, where they can gain more information and more interactive experience [1]. Just by virtue of this peculiar mode of fission information transmitting and sharing generated from social relationships, the social network has quickly swept from the government, celebrities, stars, to general public, and it has been an important platform gathering specific user groups [2].

In recent years, the group model has been widely introduced in social networks. The group model is based on a close relationship through which users are combined in a community. Owing to the group model, users are introduced from a relatively closed friendship into groups thus creating a new and more open social relationship to transmit and share information more conveniently and efficiently [3,4]. Xiang et al. [5] computed the node degrees, aggregation coefficient, length of characteristic path and the inflation rate, etc., in social networks and then depicted the characteristics of social networks from different aspects. Borgatti et al. [6] designed a machine learning system ReGroup based on terminal interaction to assist users to create personalized groups in social networks. When a user requests to join in a social network group, ReGroup will learn the characteristics of group members and obtain the relevant probability model, which will be used to judge whether or not the user is suitable for this group. Through experimental evaluation of the system, the author showed that ReGroup can efficiently collaborate to create a large-scale heterogeneous social network groups. Amershi et al. [7] used two network data sets DBLP [8] and LiveJournal [9] to test and analyse the principles of growth and evolution of social network groups. Backstrom et al. [10] took two differing social networks, World of Warcraft (WoW) [11] and DBLP, as testing platforms to build models and predict the dynamic stability of

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the social network groups. The results showed that the level of diversification and social activities of network members are two key factors in maintaining stability of social network groups while the presence of a specific set of network members plays an important role in the maintenance of the stability of social network groups. Patil et al. [12] proposed a model of computing trust between social network groups. There are four main evaluation properties of the trust model, namely availability, reliability, maintainability and fault tolerance. Sagar [13] designed an effective heuristic algorithm to obtain approximate optimal active social network groups given the specific active hosts, a set of tags describing active topics, desired size of the group and a set of network members to be included.

We find that current research work on social network groups mainly focuses on the establishment and evolution of groups but little on information transmitting and sharing. While social network groups are put forward to facilitate information transmitting and sharing more efficiently [14]. Thus, based on the current popular mechanism of trust relationship between members and the evaluation models of objective reputation and subjective trust [15], we study the optimal solution problem of recommending information between groups, which is the basis of information transmitting and sharing between groups. However, we find that it is an NP-hard problem to get the exact optimal solution of recommending information.

Based on the above, in order to obtain the approximate optimal recommendation solution, we propose an efficient method achieving approximate optimal recommendation solution (AAORS) which has a time complexity of

$$O\left(n_1 \times n_2 + (n_1 + n_2)^{e/(e-1)} + (n_1 + n_2)log(n_1 + n_2)\right)$$
$$\approx O\left(n_1 \times n_2 + (n_1 + n_2)^{1.58} + (n_1 + n_2)log(n_1 + n_2)\right)$$

where n_1 and n_2 denote the numbers of members in two social network groups G_1 and G_2, respectively. The AAORS method first evaluates individual objective reputation and subjective trust of members in groups G_1 and G_2 and then expresses G_1 and G_2 as a directed weighted graph $\wp$ so that the optimal solution of information recommendation is equivalent to the problem of finding the extended Steiner tree (EST) [16] on the graph $\wp$. We theoretically prove that the global trust value of *appIRS* obtained for AAORS is at least 63 % of that obtained for the exact optimal solution *optIRS*. Furthermore, in real applications, we observe that the global objective reputation should not be lower than a certain threshold ∂. In order to obtain the optimal recommendation solution satisfying the global objective threshold ∂, we calculate the index of reputation gain and utilize it to quickly adjust the recommendation solution produced by the AAORS method in polynomial time complexity of $O(n_1^3 + n_2 \times n_1^2)$. The detailed theoretical analyses and extensive experiments demonstrate that our proposed methods are both efficient and effective.

The rest of this paper is organized as follows. Section 2 presents the problem description. Section 3 describes our AAORS method which can be used to efficiently obtain the approximate optimal recommendation solution. Given the global constraint to the problem due to objective reputation, Sect. 4 presents a novel strategy to extend the recommendation solution produced by the AAORS method. Both the AAORS method and the extended X-AAORS methods are implemented by computationally efficient algorithms exhibiting polynomial time complexity. We present the experimental study in Sect. 5. Finally, Sect. 6 concludes the paper with directions for the future work.

2 Problem Description

In this section, we describe in detail the problem of obtaining the optimal solution of information recommendation between groups.

Assume that there are two groups G_1 and G_2 in social networks, respectively, with member sets of $G_1 = \{v_1^1, \ldots, v_{n_1}^2\}$ and $G_2 = \{v_1^2, \ldots, v_{n_2}^2\}$. Without loss of generality, we just analyse the process of information recommendation from G_1 to G_2. According to the definition in the [17], each member $v_i^1 (1 \leq i \leq n_1)$ in G_1 is associated with a value of objective reputation that is the common opinion from all members in the group including v_i^1, denoted as $obj(v_i^1)$. Meanwhile, according to the definition in [18], each member $v_j^2 (1 \leq j \leq n_2)$ in G_2 has subjective trust towards c_j members $v_1^1, \ldots, v_{c_j}^1$ in G_1 where $c_j \leq n_1$, denoted as $sbj(v_j^2, v_1^1)$, $\ldots$, $sbj(v_j^2, v_{c_j}^1)$ respectively. Figure 1 shows the process of information recommendation from G_1 which has six members to G_2 which has three members.

We can see from Fig. 1 that six members in G_1 are all associated with the values of objective reputation. For each member $v_i^1 (1 \leq i \leq n_1)$, the value of its objective reputation is greater, which means v_i^1 is more reliable. All three members of group $G_2 (v_1^2, v_2^2$ and $v_3^2)$ receive information recommended from some members in $G_1 : v_1^2$ receives information from v_1^1, v_4^1 and v_6^1; v_2^2 receives information from v_1^1, v_2^1, v_3^1 and v_5^1; v_3^2 receives information from v_2^1 and v_3^1. And a value of subjective trust is associated with each recommendation path. The greater the value of subjective trust is, the more the members in G_2 will subjectively trust information from ones in G_1.

It is not difficult to see that the process of information recommendation from G_1 to G_2 forms a weighted directed acyclic graph $G = (N, E, W)$:

(1) the set of vertices $N = G_1 \cup G_2 = \{v_1^1, \ldots, v_{n_1}^1, v_1^2, \ldots, v_{n_2}^2\}$;

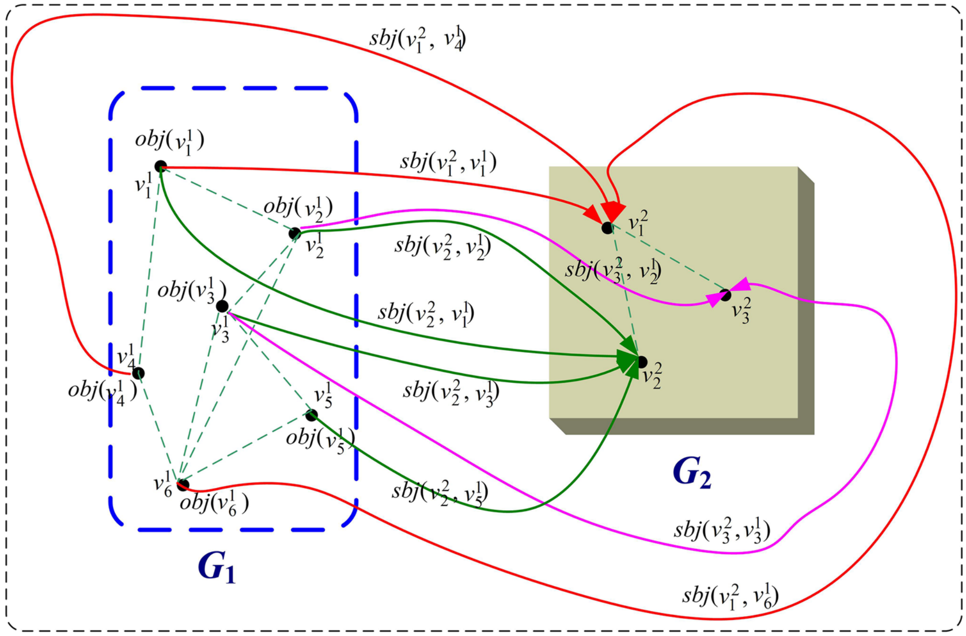


Fig. 1 The example of the process of information recommendation from G_1 to G_2

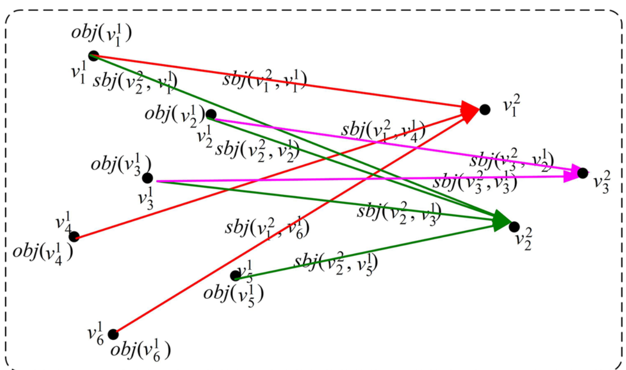


Fig. 2 The weighted directed acyclic graph of information recommendation from G_1 to G_2

(2) the set of edges $E = \{v_i^1 \rightarrow v_j^2 |$ there is a recommendation path from v_i^1 to $v_j^2\}$;

(3) the weight function W on the sets of N and E as W: $N \cup E \rightarrow Z^+$, such that for each v_i^1 in G_1, the value of W equals $obj(v_i^1)$, while for each edge $v_i^1 \rightarrow v_j^2$, the value of W equals $sbj(v_j^2, v_i^1)$.

Figure 2 shows the weighted directed acyclic graph $G = (N, E, W)$ of information recommendation from G_1 to G_2 with $N = \{v_1^1, v_2^1, v_3^1, v_4^1, v_5^1, v_6^1, v_1^2, v_2^2, v_3^2\}$, $E = \{v_1^1 \rightarrow v_1^2, v_4^1 \rightarrow v_1^2, v_6^1 \rightarrow v_1^2, v_1^1 \rightarrow v_2^2, v_2^1 \rightarrow v_2^2, v_3^1 \rightarrow v_2^2, v_5^1 \rightarrow v_2^2, v_2^1 \rightarrow v_3^2, v_3^1 \rightarrow v_3^2\}$ and $W = \{obj\,(v_1^1), obj\,(v_2^1), obj(v_3^1), obj\,(v_4^1), obj(v_5^1), obj(v_6^1), sbj(v_1^1, v_1^2), sbj(v_4^1, v_1^2), sbj(v_6^1, v_1^2), sbj(v_1^1, v_2^2), sbj\,(v_2^1, v_2^2), sbj(v_3^1, v_2^2), sbj(v_5^1, v_2^2), sbj(v_2^1, v_3^2), sbj(v_3^1, v_3^2)\}$.

Based on the weighted directed acyclic graph G, we give the problem definition of obtaining the optimal solution of information recommendation from G_1 to G_2.

2.1 Problem Definition

Assume there are two groups G_1 and G_2 in social networks whose member sets are, respectively, $G_1 = \{v_1^1, \ldots, v_{n_1}^1\}$ and $G_2 = \{v_1^2, \ldots, v_{n2}^2\}$. Each member $v_i^1 (1 \leq i \leq n_1)$ in G_1 is associated with a value of objective reputation, denoted as $obj(v_i^1)$, while each member $v_j^2 (1 \leq j \leq n_2)$ in G_2 has subjective trust towards c_j members $v_1^1, \ldots, v_{c_j}^1$ in G_1 where $c_j \leq n_1$, denoted as $sbj(v_j^2, v_1^1), \ldots, sbj(v_j^2, v_{c_j}^1)$ respectively. Then the optimal solution *optIRS* of information recommendation from G_1 to G_2 can be defined as follows: Obtaining $subG_1 \subseteq G_1$ such that for $\forall v_j^2 (1 \leq j \leq n_2) \in G_2$, v_j^2 will receive information from some members in G_1, and ascertaining the recommendation path from members in $subG_1$ to members in G_2 such that $optIRS(subG_1, G_2) = \sum_{v_i^1 \in subG_1} obj(v_i^1) + \sum_{\forall v_j^2 \in G_2} sbj(v_j^2, v_i^1)$ is greatest.

From the above problem definition, we can see that for two information recommendation solutions IRS_A and IRS_B, if $IRS_A(subG_1, G_2) > IRS_B(subG_2, G_2)$, then IRS_A is better than IRS_B, which means that information recommendation from $subG_1$ to G_2 is more reliable than that from $subG_2$ to G_2.

3 AAORS: Obtain the Approximate Recommendation Solution

It is an NP-hard problem to obtain the exact optimal solution of information recommendation between groups which is shown in Theorem 1.

Theorem 1 *Assume there are two groups G_1 and G_2 in social networks whose member sets are, respectively, $G_1 =$*

$\{v_1^1, \ldots, v_{n_1}^1\}$ and $G_2 = \{v_1^2, \ldots, v_{n_2}^2\}$. Each member $v_i^1 (1 \le i \le n_1)$ in G_1 is associated with a value of objective reputation, denoted as $obj(v_i^1)$, while each member $v_j^2 (1 \le j \le n_2)$ in G_2 has subjective trust towards c_j members $v_1^1, \ldots, v_{c_j}^1$ in G_1 where $c_j \le n_1$, denoted as $sbj(v_j^2, v_1^1), \ldots, sbj(v_j^2, v_{c_j}^1)$, respectively. Then it is an NP-hard problem to obtain the exact optimal solution optIRS of information recommendation from G_1 to G_2.

Proof The time complexity of obtaining the optimal recommendation solution is mainly due to searching the space of recommendation solutions. It is self-evident that all instances of recommendation solutions must cover all n_2 members in G_2 but not all members in G_1. Then we will get the exact time complexity of obtaining *optIRS* by analysing the number of instances for the space of recommendation solutions.

- The number of instances covering one member in G_1 is $nINS^{(1)} = C_{n_1}^1 \cdot C_{n_2}^{n_2} = n_1$.
- The number of instances covering two members in G_1 is

$$\begin{aligned} nINS^{(2)} &= C_{n_1}^2 \cdot \left(C_{n_2}^1 + \cdots + C_{n_2}^i + \cdots + C_{n_2}^{n_2-1}\right) \\ &= C_{n_1}^2 \cdot (2^{n_2} - 2). \\ &\ldots \end{aligned}$$

- The number of instances covering $G_1 t (1 \le t \le n_1)$ members in G_1 is

$$\begin{aligned} nINS^{(t)} = C_{n_1}^t \cdot \Big\{ &\Big(C_{n_2}^1 \cdot C_{n_2-1}^1 \ldots C_{n_2-t+1}^1 \\ &+ \cdots + C_{n_2}^1 \cdot C_{n_2-1}^1 \ldots C_{n_2-t+1}^{n_2-t} \\ &+ \cdots + C_{n_2}^1 \cdot C_{n_2-1}^{n_2-t+2} \cdot C_{t-2}^1 \cdot C_{t-3}^1 \ldots C_1^1\Big) \\ &\ldots \\ &+ \Big(C_{n_2}^i \cdot C_{n_2-i}^1 \ldots C_{n_2-t+1}^1 \\ &+ \cdots + C_{n_2}^i \cdot C_{n_2-i}^1 \ldots C_{n_2-t+1}^{n_2-t} \\ &+ C_{n_2}^i \cdot C_{n_2-i}^{n_2-i-t+2} \cdot C_{t-2}^1 \cdot C_{t-3}^1 \ldots C_1^1\Big) \\ &+ \cdots + C_{n_2}^{n_2-t+1} \cdot C_{t-1}^1 \cdot C_{t-2}^1 \cdot C_{t-3}^1 \\ &\ldots C_1^1 \Big\} \\ = C_{n_1}^t \cdot (t^{n_2} - t)&. \\ &\ldots \end{aligned}$$

Then the time complexity of obtaining *optIRS* is

$$\begin{aligned} O(n_1, n_2) &= nINS^{(1)} + nINS^{(2)} \cdots + nINS^{(n_1)} \\ &= \sum_{u=1}^{n_1} C_{n_1}^u \cdot \left(u^{n_2} - u\right). \end{aligned}$$

From $O(n_1, n_2)$, we can see that obtaining *optIRS* requires exponential time complexity that characterizes NP problems. Furthermore, for a given instance of recommendation solution space *IRS* which covers $G_1 t (1 \le t \le n_1)$ members $\{v_1^1, \ldots, v_t^1\}$, determining whether it is the optimal solution (i.e. *IRS* = *optIRS*) can be reduced to the minimum coverage problem [19] of the weighted directed bipartite graph $G(\{v_1^1, \ldots, v_t^1\}, G_2, W)$, where W is the values of objective reputation and subjective trust associated with *IRS*. According to the graph theory [20], the minimum coverage problem of a bipartite graph is NP-hard. On the other hand, it is easy to see that obtaining *optIRS* can be reduced to the minimum coverage problem of the bipartite graph by time complexity of $O(n_1)$. And hence obtaining *optIRS* is also NP-hard.

From Theorem 1, we can see that obtaining the exact optimal solution *optIRS* of information recommendation needs much CPU time, and hence, in this section, we propose an effective method achieving approximate optimal recommendation solution (AAORS) to quickly obtain the approximate optimal solution *appIRS*. Its basic idea can be described as follows:

We first form the process of information recommendation from G_1 to G_2 as a weighted direct acyclic graph $G = (N, E, W)$ with the set of vertices $N = G_1 \cup G_2 = \{v_1^1, \ldots, v_{n_1}^1, v_1^2, \ldots, v_{n_2}^2\}$ and the set of edges $E = \{v_i^1 \to v_j^2 |$ there is a recommendation path from v_i^1 to $v_j^2\}$. We define the weight function W on the sets of N and E as W: $N \cup E \to Z^+$, such that for each v_i^1 in G_1, the value of W equals $obj(v_i^1)$, while for each edge $v_i^1 \to v_j^2$, the value of W equals $sbj(v_j^2, v_i^1)$. Since there no connected edges exist inside the vertices set $G_1 (G_2)$ although weighted directed edges do exist between G_1 and G_2, it is easy to see that G is a bipartite graph. Next, we add a control node Θ in G. For each node ς in G_1, we add a directed edge $e\langle\Theta, \varsigma\rangle$ from Θ to ς with the weight of $obj(\varsigma)$. And then we delete the previous weight of the node ς. Here we get a new weighted directed acyclic graph $G' = (N', E', W')$ with the set of vertices $N' = G_1 \cup G_2 \cup \{\Theta\} = \{v_1^1, \ldots, v_{n_1}^1, v_1^2, \ldots, v_{n_2}^2, \Theta\}$, the set of edges $E' = E \cup \{\Theta \to v_i^1 | 1 \le i \le n_1\}$ and the set of weight $W' = W \cup \{\forall v_i^1 \in G_1, W(\Theta \to v_i^1) = obj(v_i^1)\} - \{\forall v_i^1 \in G_1, W(v_i^1) = obj(v_i^1)\}$. Finally, we use the EST-A algorithm proposed in the literature [16] to produce the extended Steiner tree $\overline{ESTree}(\hat{N}, \hat{E}, \hat{W})$ in G' and obtain the approximate optimal solution appIRS. The set of members $parG_1$ in appIRS consist of the members taking part in information recommendation in G_1, that is $parG_1 = \hat{N} - \{v_1^2, \ldots, v_{n_2}^2, \Theta\} \subseteq G_1$. And if there exists an edge $v^{(p)} \to v_j^2$ in $\hat{E}$ such that $v^{(p)} \in parG_1$ and $v_j^2 \in G_2$, then this edge is the recommendation path from $v^{(p)}$ to v_j^2 and the weight $sbj(v^{(p)}, v_j^2)$ of this edge equals the value of subjective trust of v_j^2 towards $v^{(p)}$.

Based on the above analysis, the AAORS method can be implemented below.

Algorithm 1: AAORS
Input: For groups $G_1 = \{v_1^1, \ldots, v_{n_1}^1\}$ and $G_2 = \{v_1^2, \ldots, v_{n_2}^2\}$, each member $v_i^1 (1 \le i \le n_1)$ in G_1 is associated with a value of objective reputation, denoted as $obj(v_i^1)$, and each member $v_j^2 (1 \le j \le n_2)$ in G_2 has the subjective trust towards c_j members $v_1^1, \ldots, v_{c_j}^1$ in G_1 where $c_j \le n_1$, denoted as $sbj(v_j^2, v_1^1), \ldots, sbj(v_j^2, v_{c_j}^1)$, respectively.
Output: The approximate optimal solution appIRS of information recommendation from G_1 to G_2.
Begin

1. Construct the weighted directed acyclic graph $G = (N, E, W)$:
   (a) N:- $G_1 \cup G_2 = \{v_1^1, \ldots, v_{n_1}^1, v_1^2, \ldots, v_{n_2}^2\}$;
   (b) E:- $\{v_i^1 \to v_j^2 |$ there exists a recommendation path from v_i^1 to $v_j^2\}$;
   (c) W:- $\{\forall v_i^1 \in G_1, W(v_i^1) = obj(v_i^1)\} \cup \{\forall v_i^1 \to v_j^2 \in E, W(v_i^1 \to v_j^2) = sbj(v_j^2, v_i^1)\}$;
2. Construct a new weighted directed acyclic graph $G' = (N', E', W')$ based on G;
   (a) N':- $G_1 \cup G_2 \cup \{\Theta\} = \{v_1^1, \ldots, v_{n_1}^1, v_1^2, \ldots, v_{n_2}^2, \Theta\}$;
   (b) E':- $E \cup \{\Theta \to v_i^1 | 1 \le i \le n_1\}$;
   (c) W':- $W \cup \{\forall v_i^1 \in G_1, W(\Theta \to v_i^1) = obj(v_i^1)\} - \{\forall v_i^1 \in G_1, W(v_i^1) = obj(v_i^1)\}$;
3. Produce the extend Steiner tree $\overline{ESTree}(\hat{N}, \hat{E}, \hat{W})$ on G' using the EST-A algorithm [16];
4. $parG_1$:- $\hat{N} - \{v_1^2, \ldots, v_{n_2}^2, \Theta\}$;
5. *rePATH*:- $\hat{E}$;
6. $appIRS(parG_1, G_2)$:- $\sum_{v_i^1 \in parG_1} \hat{W}(\Theta \to v_i^1) + \sum_{\forall rp \in rePATH} \hat{W}(rp)$;
   /* Compute the global trust value of the recommendation solution *appIRS* */
7. *appIRS*:- $\langle parG_1, rePATH, appIRS(parG_1, G_2)\rangle$;
8. Return *appIRS*;

End.

In the following part, we theoretically prove that the lower bound ε of *appIRS* equals 0.63. That is the global trust value of *appIRS* is at least 63 % of that obtained for the exact optimal solution *optIRS*, which is shown in Theorem 2.

Theorem 2 *Assume that there exists two groups* $G_1 = \{v_1^1, \ldots, v_{n_1}^1\}$ *and* $G_2 = \{v_1^2, \ldots, v_{n_2}^2\}$ *in the social network and the exact optimal solution of information recommendation from* G_1 *to* G_2 *is optIRS. optIRS covers* $\lambda_1 (1 \le \lambda_1 \le n_1)$ *members in* G_1 *which are denoted as* $optG_1 = \{v_1^1, \ldots, v_{\lambda_1}^1\}$, *and its global trust value is* $optIRS(optG_1, G_2)$. *While the approximate optimal solution appIRS produced by the AAORS method covers* $\lambda_2 (1 \le \lambda_2 \le n_1)$ *members in* G_1 *that are denoted as* $appG_1 = \{u_1^1, \ldots, u_{\lambda_2}^1\}$, *and its global trust value is* $appIRS(appG_1, G_2)$. *Then we can have:*

$$\frac{appIRS(appG_1, G_2)}{optIRS(optG_1, G_2)} \ge 0.63.$$

Proof Assume the order that *appIRS* chooses λ_2 members in G_1 is $v_1^1, v_2^1, \ldots, v_{\lambda_2}^1$, and for each member $v_i^1 \in appG_1$, selecting it can gain the trust value a_i. Meanwhile, we assume that the order of *optIRS* chooses λ_1 members in G_1 is $u_1^1, u_2^1, \ldots, u_{\lambda_1}^1$ and for each member $u_i^1 \in optG_1$, selecting it can gain the trust value b_i. Then we can have:

$$appIRS(appG_1, G_2) = \sum_{i=1}^{\lambda_2} a_i \quad \text{and} \quad optIRS(optG_1, G_2) = \sum_{i=1}^{\lambda_1} b_i.$$

For each pair of members v_i^1 and u_r^1 in G_1, we let x_{ri} be the common part of v_i^1's trust value and u_r^1's trust value contributing to the information recommendation. Then the following four inequalities can be satisfied: (1) $\sum_{r=1}^{\lambda_1} x_{ri} \le a_i$; (2) $\forall r \in [1, \lambda_1], b_r \le a_1$; (3) $\forall r \in [1, \lambda_1], b_r - x_{r1} \le a_2$; (4) $\forall r \in [1, \lambda_1], b_r - x_{r1} - x_{r2} - \cdots - x_{r(i-1)} \le a_i$. Then according to the above four inequalities, we can get the following λ_2 inequalities.

[1] $optIRS(optG_1, G_2) \le \lambda_1 \cdot a_1$;
[2] $optIRS(optG_1, G_2) \le \lambda_1 \cdot a_2 + a_1$;
...
[λ_2] $optIRS(optG_1, G_2) \le \lambda_1 \cdot a_{\lambda_2} + a_{\lambda_2 - 1} + \cdots + a_2 + a_1$.

It is easy to see that $optIRS(optG_1, G_2)$ is not greater than any one of $\lambda_1 \cdot a_1, \lambda_1 \cdot a_2 + a_1, \ldots, \lambda_1 \cdot a_{\lambda 2} + a_{\lambda 2-1} + \cdots + a_2 + a_1$, and when these λ_2 values are equal, $optIRS(optG_1, G_2)$ will be the greatest. Hence, we can have

$$optIRS(optG_1, G_2) \le \lambda_1 \cdot \left(\frac{\lambda_1}{\lambda_1 - 1}\right)^{\lambda_2 - 1} \cdot a_{\lambda_2}.$$

On the other hand, when these λ_2 values are equal, $\forall i \in [1, \lambda_2 - 1]$, $\lambda_1 \cdot a_{i+1} - (\lambda_1 - 1) \cdot a_i = 0$. That is $a_i = \lambda_1 / (\lambda_1 - 1) \cdot a_{i+1}$. Then we can have

$$appIRS(appG_1, G_2) = \sum_{i=0}^{\lambda_2 - 1} \left(\frac{\lambda_1}{\lambda_1 - 1}\right)^i \cdot a_{\lambda_2}.$$

Thereby we can have the following inequality.

$$\frac{appIRS(appG_1, G_2)}{optIRS(optG_1, G_2)}$$
$$\geq 1/k \cdot \sum_{i=0}^{\lambda_2-1} \left(\frac{\lambda_1}{\lambda_1 - 1}\right)^{i-\lambda_2+1}$$
$$\geq 1/k \cdot \left(1 + \frac{\lambda_1 - 1}{\lambda_1} + \cdots + \left(\frac{\lambda_1 - 1}{\lambda_1}\right)^{\lambda_2-1}\right)$$
$$\geq 1 - \left(\frac{\lambda_1 - 1}{\lambda_1}\right)^{\lambda_2}.$$

Then when λ_1 and λ_2 approach $+\infty$, $1 - \left(\frac{\lambda_1-1}{\lambda_1}\right)^{\lambda_2}$ will approach $1 - 1/e$ and we can get the following inequality.

$$\frac{appIRS(appG_1, G_2)}{optIRS(optG_1, G_2)} \geq 1 - 1/e.$$

That is,

$$\frac{appIRS(appG_1, G_2)}{optIRS(optG_1, G_2)} \geq 0.63.$$

Therefore, Theorem 2 is correct.

The AAORS method has polynomial time complexity, which is shown in Theorem 3.

Theorem 3 *Given two groups $G_1 = \{v_1^1, \ldots, v_{n_1}^1\}$ and $G_2 = \{v_1^2, \ldots, v_{n_2}^2\}$ in the social network, the approximate optimal solution appIRS from G_1 to G_2 produced by our AAORS method has the polynomial time complexity:*

$$O\left(n_1 \times n_2 + (n_1 + n_2)^{e/(e-1)} + (n_1 + n_2)\log(n_1 + n_2)\right)$$
$$\approx O\left(n_1 \times n_2 + (n_1 + n_2)^{1.58} + (n_1 + n_2)\log(n_1 + n_2)\right).$$

Proof In AAORS, the time cost of constructing the weighted directed acyclic graph $G = (N, E, W)$ is $O(n_1 + n_2)$ and requires a time cost of $O(n_1 + n_2)$ to construct the new weighted directed acyclic graph $G' = (N', E', W')$ based on G. Meanwhile according to [16], a time cost of $O((n_1 + n_2)^{e/(e-1)} + (n_1 + n_2)log(n_1 + n_2))$ is required for building the extend Steiner tree $\overline{ESTree}(\hat{N}, \hat{E}, \hat{W})$ on G' with the EST-A algorithm. Furthermore, it takes $O(n_1 + n_2 + n_1 \times n_2)$ for $\overline{ESTree}(\hat{N}, \hat{E}, \hat{W})$ to produce the approximate optimal recommendation solution *appIRS*. Therefore, the total time cost for AAORS is equal to $O(n_1 \times n_2 + n_1 + n_2 + (n_1+n_2)^{e/(e-1)} + (n_1+n_2)\,log(n_1+n_2) + n_1 \times n_2 + n_1 + n_2) = O(n_1 \times n_2 + (n_1 + n_2)^{e/(e-1)} + (n_1 + n_2)log(n_1 + n_2))$ while we can get the total time cost $O(n_1 \times n_2 + (n_1 + n_2)^{1.58} + (n_1 + n_2)log(n_1 + n_2))$ with e=2.72 substituted. So Theorem 3 is correct.

4 Obtain the Recommendation Solution Under the Global Objective Reputation Constraint

In real applications, the global objective reputation should not be lower than a certain threshold ∂. This can ensure the reliability of information sources. For convenience, we refer to the constraint as '∂ constraint'. Thus, the problem definition presented in Sect. 2 can be modified as follows:

Assume there are two groups G_1 and G_2 in social networks whose member sets are, respectively, $G_1 = \{v_1^1, \ldots, v_{n_1}^1\}$ and $G_2 = \{v_1^2, \ldots, v_{n_2}^2\}$. Each member $v_i^1 (1 \leq i \leq n_1)$ in G_1 is associated with a value of objective reputation, denoted as $obj(v_i^1)$, while each member $v_j^2 (1 \leq j \leq n_2)$ in G_2 has the subjective trust towards c_j members $v_1^1, \ldots, v_{c_j}^1$ in G_1 where $c_j \leq n_1$, denoted as $sbj(v_j^2, v_1^1), \ldots, sbj(v_j^2, v_{c_j}^1)$, respectively. Then the optimal recommendation solution *conIRS* satisfying ∂ constraint from G_1 to G_2 can be defined as follows: obtaining $subG_1 \subseteq G_1$ such that for $\forall v_j^2 (1 \leq j \leq n_2) \in G_2$, v_j^2 will receive information from some members in G_1, and ascertaining the recommendation path from members in $subG_1$ to members in G_2 such that $optIRS(subG_1, G_2) = \sum_{v_i^1 \in subG_1} obj(v_i^1) + \sum_{\forall v_j^2 \in G_2} sbj(v_j^2, v_i^1)$ is greatest under the constraint condition of $conIRS(subG_1) = \sum_{v_i^1 \in subG_1} obj(v_i^1) \geq \partial$.

To solve the above problem, we extend the AAORS method and introduce the concept of the index of reputation gain. Assume that *XS* is a subset of G_1 and for each member v_j^2 in G_2, there always exists a member in *XS* who has the recommendation path to v_j^2. That is, *XS* and G_2 form an instance *xsIRS* of recommendation solution space. Thus for a member $\xi \notin XS$ in G_1, its index of reputation gain for *xsIRS* can be expressed as

$$RBinx(\xi) = \frac{xsIRS(XS \cup \{\zeta\}, G_2) - xsIRS(XS, G_2)}{xsIRS(XS \cup \{\zeta\}) - xsIRS(XS)}.$$

Based on the index of reputation gain, we extend AAORS and denote this extended version as X-AAORS. Its basic idea can be described as follows. We first produce two recommendation solutions. One is the approximate optimal recommendation solution *appIRS* produced by AAORS which covers the member set $appG_1 \subseteq G_1$. The other one is *mxIRS* which covers the member set $mxG_1 \subseteq G_1$ satisfying the condition that $\sum_{v \in mxG_1} obj(v)$ is the greatest. Note that mxG_1 can be built as follows: for each $v \in G_2$, we first find the member u from G_1 such that u has the recommendation path to v and has the greatest object reputation value and then add u into mxG_1. Then for each member v of $\subseteq G_1$, X-AAORS creates a list $List(v)$ to store all members receiving information from v. And based on the index of reputation gain, we repeatedly execute the following steps until $conIRS(subG_1)$ is lower than the threshold ∂ given by users: (i) visit $appG_1$ and insert the member *mv* with the greatest index of reputation gain into mxG_1; (ii) create a list $List(mv)$ to store all members from G_2 which has a recommendation path from *mv*; (iii) for each existing list l, delete all members in $List(mv)$ from l; (iv) remove each member whose list is null from mxG_1. Finally, X-AAORS returns the information recommendation solution formed by mxG_1 and G_2 to users. The complete X-AAORS method can be shown below.

Algorithm 2: X-AAORS
Input: $G_1 = \{v_1^1, \ldots, v_{n_1}^1\}$ and $G_2 = \{v_1^2, \ldots, v_{n_2}^2\}$, each member $v_i^1 (1 \le i \le n_1)$ in G_1 is associated with a value of objective reputation, denoted as $obj(v_i^1)$, while each member $v_j^2 (1 \le j \le n_2)$ in G_2 has the subjective trust towards c_j members $v_1^1, \ldots, v_{c_j}^1$ in G_1 where $c_j \le n_1$, denoted as $sbj(v_j^2, v_1^1), \ldots, sbj(v_j^2, v_{c_j}^1)$, respectively. The global objective reputation threshold ∂.
Output: The optimal recommendation solution *appIRS* satisfying the constraint of ∂.

Begin
1. Use AAORS to produce the approximate optimal recommendation solution *appIRS* which covers the member set $mxG_1 \subseteq G_1$;
2. If $appIRS(appG_1) \ge \partial$ Then Return *appIRS*;
3. Else create the recommendation solution *mxIRS* that covers the member set $mxG_1 \subseteq G_1$ satisfying the condition that $\sum_{v \in mxG_1} obj(v)$ is the greatest;
4. If $appIRS(mxG_1) < \partial$ Then
5. Return 'There is no information recommendation solution satisfying the constraintof ∂.';
6. Else
7. $\Upsilon \leftarrow \sum_{v \in mxG_1} obj(v) - \partial$;
8. For $\forall v \in appG_1$ Do
9. Create a list for v;
10. $List(v)$:$-\{v' | v' \in G_2 \wedge$ there exists a recommendation path between v' and $v\}$;
11. While $\Upsilon < 0$ Do
12. $RBinx \leftarrow 0$; $vRB \leftarrow$ NULL;
13. For $\forall \xi \in mxG_1 - appG_1$ Do
14. $RBinx(\xi) \leftarrow \dfrac{appIRS(appG_1 \cup \{\zeta\}, G_2) - appIRS(appG_1, G_2)}{appIRS(appG_1 \cup \{\zeta\}) - appIRS(appG_1)}$;
15. If $RBinx(\xi) > RBinx$ Then $RBinx \leftarrow RBinx(\xi)$; $vRB \leftarrow \xi$;
16. If $\Upsilon + RBinx(\xi) > 0$ Then
17. Create a list $List(vRB)$ for vRB;
18. $List(vRB) \leftarrow \{v | v \in G_2 \wedge$ there exists a recommendation path between v and $vRB\}$;
19. $mxG_1 \leftarrow mxG_1 - \{vRB\}$; $appG_1 \leftarrow appG_1 \cup \{vRB\}$;
20. For each list $List(u)$ except $List(vRB)$ Do
21. $List(u) \leftarrow List(u) - (List(v) \cap List(vRB))$;
22. If $List(u) - List(vRB) = \varnothing$ Then
23. Delete $List(u)$;
24. If $appIRS(appG_1 - \{u\}) - appIRS(appG_1) < 0$ Then
25. $mxG_1 \leftarrow mxG_1 \cup \{u\}$; $appG_1 \leftarrow appG_1 - \{u\}$;
26. $\Upsilon \leftarrow \Upsilon - appIRS(appG_1 - \{u\})$;
27. $\Upsilon \leftarrow \Upsilon + obj(vRB)$;
28. Return *appIRS*;

End.

X- AAORS has the polynomial time complexity, which can be shown in Theorem 4.

Theorem 4 *The X-AAORS method can adjust the recommendation solution produced by the AAORS method in the polynomial time complexity of* $O(n_1^3 + n_2 \times n_1^2)$ *and obtain the optimal recommendation solution satisfying the* ∂ *constraint, where* n_1 *and* n_2 *are the numbers of members in* G_1 *and* G_2*, respectively.*

Proof X-AAORS first takes $O(n_1 \times n_2)$ to create the recommendation solution *mxIRS*. And in order to compute the index of reputation gain, for any two members $u \in G_2$ and $v \in appG_1$, X-AAORS needs $O(n_1 \times (n_1^2 + n_1 \times n_2)) = O(n_1^3 + n_1^2 \times n_2)$ to get the shortest path from v to u. Moreover, the while loop in X-AAORS takes $O(|mxG_1| \times |appG_1| \times |G_2|) = O(n_1^2 \times n_2)$. Therefore, X-AAORS needs $O(n_1 \times n_2 + n_1^3 + n_1^2 \times n_2 + n_1^2 \times n_2) = O(n_1^3 + n_2 \times n_1^2)$

in total to adjust the recommendation solution *appIRS* produced by AAORS. Accordingly, Theorem 4 is correct.

5 Experimental Evaluation

In this section, we evaluate the effectiveness of our AAORS method and its extended version X-AAORS via detailed experiments.

5.1 Experimental Setting

In the experiments, the numbers of members of G_1 and G_2 vary from 1×10^4 to 5×10^4. Objective reputation values of members in G_1 are generated to satisfy the Gauss distribution [21], and we normalize these objective reputation values in [0,1] for convenience. The mean number of recommendation paths between G_1 and G_2 varies from 10 to 50. That is for each member u in G_2, there exists 10 to 50 members in G_1 on average which recommend information to u. Subjective trust values of members in G_2 towards the ones in G_1 are also generated to satisfy the Gauss distribution. Also we normalize these subjective trust values in [0, 1].

The computers in experiments are configured with a quad-core i5-3450 CPU, 4G memory and 500G hard disk. The operating system is CentOS Linux 6.4. All methods are compiled using JDK 1.6.

5.2 Experimental Evaluation for the AAORS Method

In this section, we evaluate the effectiveness of the AAORS method. The two compared methods are (i) OPTIMAL: get the exact optimal solution of information recommendation from G_1 to G_2 exhaustively with exponential time complexity. (ii) GREEDY: for each member v in G_2,obtain the most trustworthy member in G_1 and thus generate the greedy recommendation solution.

The experiments divide into three groups: (1) The number of members in G_2 is fixed to 3×10^4, and the mean number of recommendation paths between G_1 and G_2 is fixed to 30, while the number of members in G_1 varies from 1×10^4 to 5×10^4. (2) The number of members in G_1 is fixed to 3×10^4, and the mean number of recommendation paths between G_1 and G_2 is fixed to 30, while the number of members in G_2 varies from 1×10^4 to 5×10^4. (3) The numbers of members in G_1 and members in G_2 are fixed to 3×10^4, while the mean number of recommendation paths between G_1 and G_2 varies from 10 to 50. Figures 3 and 4 shows the experimental results.

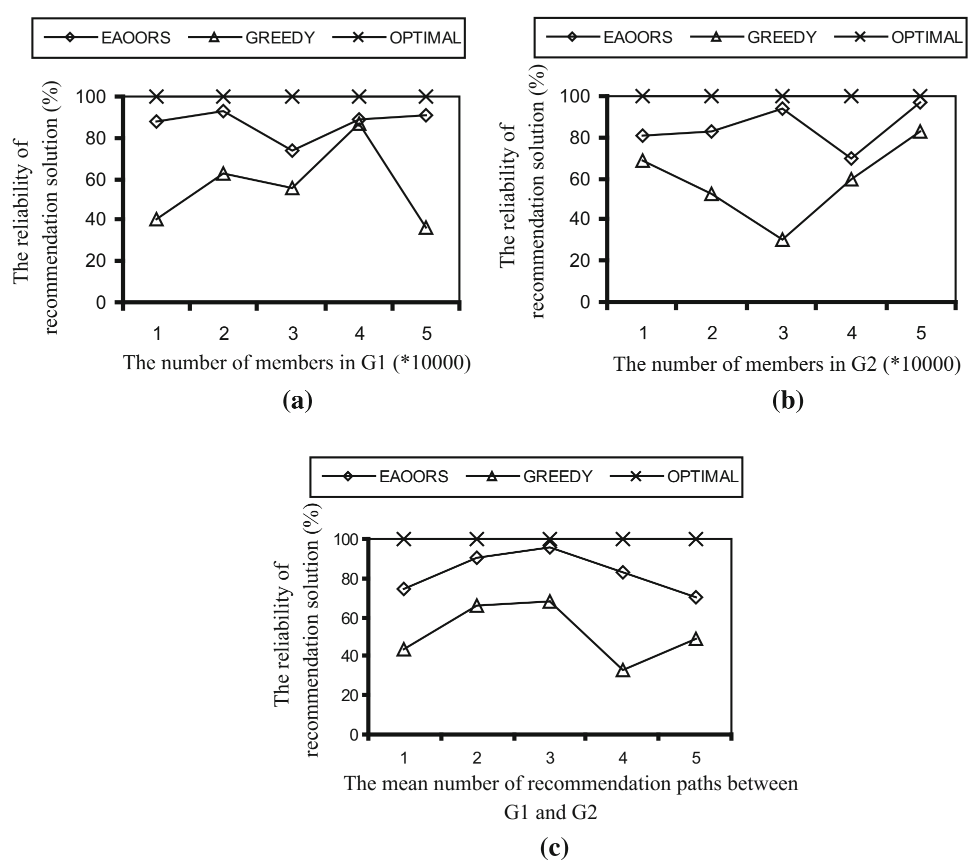
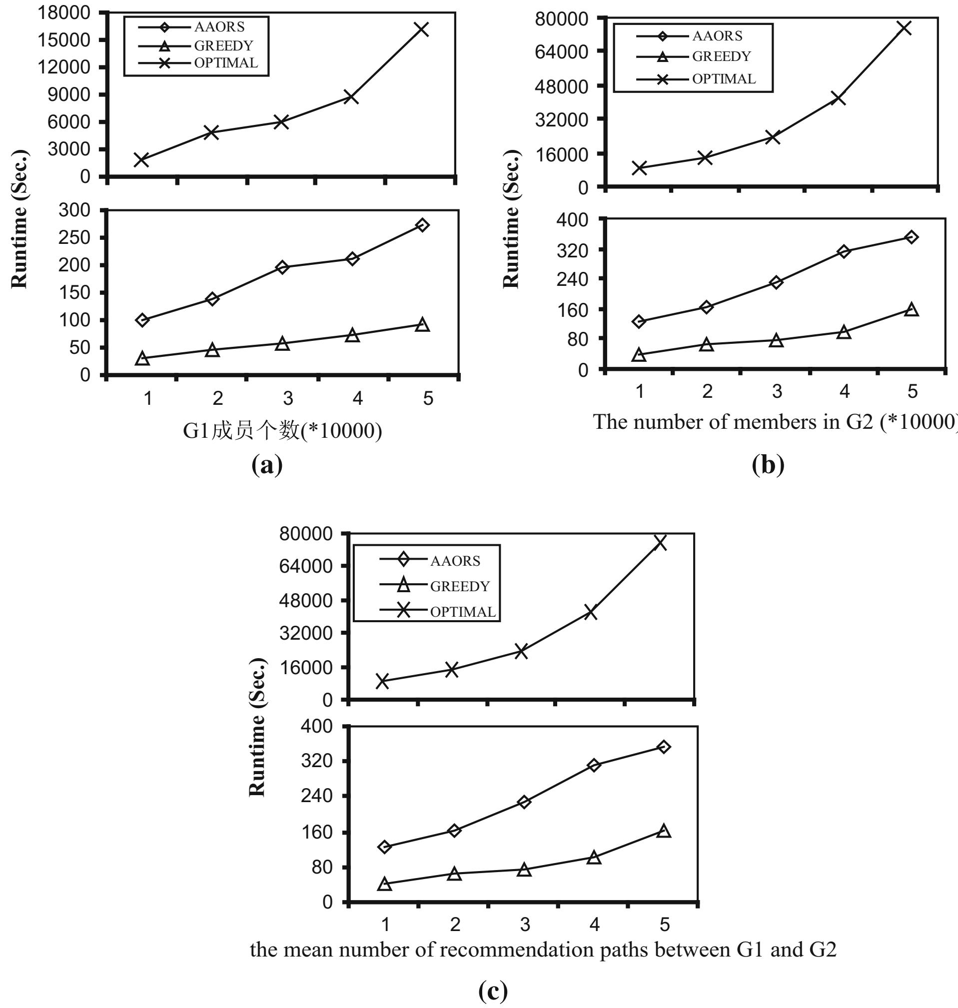


Fig. 3 Experimental evaluation for the reliability of recommendation solutions produced by three methods. **a** The first group of experiments. **b** The second group of experiments. **c** The third group of experiments

Fig. 4 Experimental evaluation for the time cost of three methods. **a** The first group of experiments. **b** The second group of experiments. **c** The third group of experiments



In Fig. 3, we take OPTIMAL as the benchmark since its recommendation solution is the exact optimal. And we let the reliability of recommendation solution of OPTIMAL equal 100 %. From Fig. 3, we can see that the reliability of the recommendation solution produced by AAORS approaches OPTIMAL's, while the reliability of recommendation solution produced by GREEDY is relatively poor. It is mainly because (i) AAORS uses the extended Steiner tree to get the approximate optimal recommendation solution so that the reliability of the solution can be well guaranteed. (ii) For each member v in G_2, GREEDY obtains the member in G_1 that is the most trust for v and generates the final greedy recommendation solution. And this recommendation solution is easy to fall into the local optimal problem. For example, in Fig. 3a, when there are 1×10^4 members in G_1 and 3×10^4 members in G_2, the reliability of recommendation solution produced by AAORS is 93.6 % of OPTIMAL's, while the reliability of recommendation solution produced by GREEDY is only 30.3 % of OPTIMAL's. And in Fig. 3c, when there are 40 recommendation paths between G_1 and G_2 on average, the reliability of recommendation solution produced by AAORS is 82.9 % of OPTIMAL's, while the reliability of recommendation solution produced by GREEDY is only 33.2 % of OPTIMAL's. Furthermore, we can see from Fig. 3 that under each experimental setting, the reliability of recommendation solution produced by AAORS is not <70 % of OPTIMAL's, which validates the correctness of Theorem 2 proposed in Sect. 3.

Although the reliability of the exact recommendation solution produced by OPTIMAL is slightly higher than AAORS's, in Fig. 4 we can find that OPTIMAL take enormous time in the experiments. This is mainly because for obtaining the exact optimal solution, OPTIMAL involves visiting all instances in the space of recommendation solutions and thus has an exponential time cost, while AAORS only needs the polynomial time complexity to return the approximate optimal recommendation solution and its runtime is close to the one of GREEDY. For example, in Fig. 4a, when there are 5×10^4 members in G_1, the runtime of AAORS is only 1.7 % of OPTIMAL's, while the runtime of GREEDY

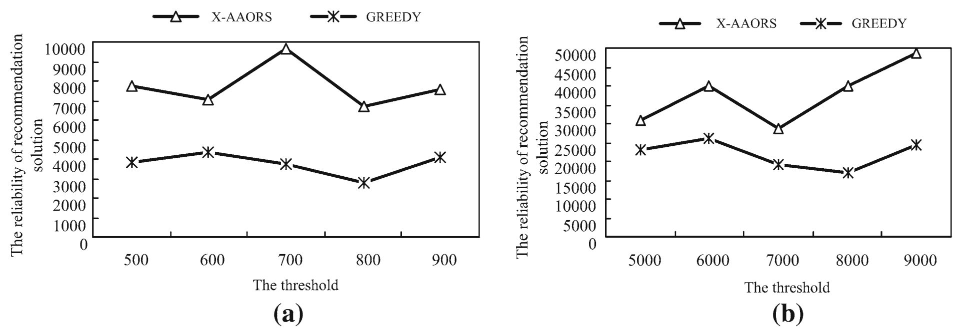


Fig. 5 Experimental evaluation for the reliability of recommendation solutions produced by two methods. **a** The first group of experiments. **b** The second group of experiments

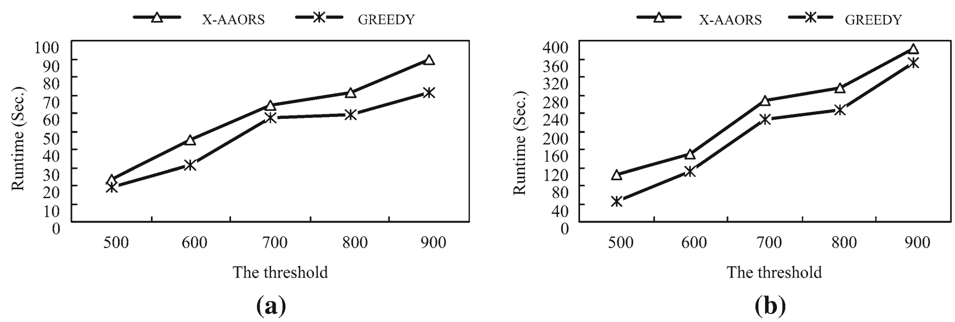


Fig. 6 Experimental evaluation for the time cost of two methods. **a** The first group of experiments. **b** The second group of experiments

is only 0.56 % of OPTIMAL's. And in Fig. 4c, when there are 50 recommendation paths between G_1 and G_2 on average, the runtime of AAORS is only 0.45 % of OPTIMAL's, while the runtime of GREEDY is only 0.34 % of OPTIMAL's.

Therefore, from the experimental evaluation shown in Figs. 3 and 4, we can conclude that AAORS proposed in this paper can not only balance the reliability of recommendation solution and the running time, but also has a good scalability.

5.3 Experimental Evaluation for the X-AAORS Method

In this section, we evaluate the effectiveness of the X-AAORS method. The compared method is GREEDY. For the approximate recommendation solution $appG_1$ produced by AAORS, GREEDY repeatedly executes the following steps until the global objective reputation value is greater than or equal to the threshold ∂: (i) It obtains the member v in $appG_1$ with the lowest object reputation value; (ii) assume that there exists recommendation paths from v to members in the set $vG_2 \subseteq G_2$, it gets the member v' in G_1–$appG_1$ which has the recommendation paths to vG_2 and has the greatest object reputation value, and then exchanges v with v' in $appG_1$.

The experiments divide into two groups: (1) The numbers of members in G_1 and G_2 are both fixed to 1×10^4, and the mean number of recommendation paths from G_1 to G_2 is set to 10. Under the above experimental setting, we use AAORS to produce the approximate recommendation solution *appIRS* which is the input of X-AAORS and GREEDY. The threshold ∂ varies from 500 to 900. (2) The numbers of members in G_1 and G_2 are both fixed to 5×10^4, and the mean number of recommendation paths from G_1 to G_2 is set to 50. Under the above experimental setting, we use AAORS to produce the approximate recommendation solution *appIRS* which is the input of X-AAORS and GREEDY. The threshold ∂ varies from 5000 to 9000. Figures 5 and 6 shows the experimental results.

From Fig. 5, we can see that after X-AAORS adjusts the recommendation solution, its reputation value is much higher that GREEDY's in each experimental setting. It is mainly because (i) X-AAORS uses the index of reputation gain to slightly adjust the recommendation solution until the global objective reputation value is not less than the threshold ∂. (ii) While GREEDY repeatedly changes the member with the lowest objective reputation value, it is easy to fall into the local optimal problem. For instance, in Fig. 5a, when the threshold ∂ equals 700, the reliability of recommendation solution adjusted by X-AAORS is 9634.2, while the one adjusted by GREEDY is 3746.5, i.e. only 38.9 % of X-AAORS. And in Fig. 5b, when the threshold ∂ equals 800, the reliability of recommendation solution adjusted by X-AAORS is 39,957.4, while the one adjusted by GREEDY is 17,093.8, i.e. only 42.8 % of X-AAORS.

While from Fig. 6, we can see that the time cost of X-AAORS is close to GREEDY's, but slightly higher than GREEDY's. For instance, in Fig. 6a, when the threshold ∂ equals 700, X-AAORS and GREEDY need 64.5 and 57.4 s, respectively. That is the time cost of GREEDY is 89.0 % of the one of X-AAORS. And in Fig. 6b, when the threshold ∂

equals 800, X-AAORS and GREEDY need 297.2 and 248.3 s, respectively. That is, the time cost of GREEDY is 83.5 % of the one of X-AAORS.

Therefore, from the experimental evaluation shown in Figs. 5 and 6, we can conclude that X-AAORS proposed in this paper can not only balance the reliability of adjusted recommendation solution and the running time, but also has a good scalability.

6 Conclusion

Information recommendation between user groups in social networks has been a research hotspot in recent years. However, it is an NP-hard problem to obtain the exact optimal recommendation solution. In this paper, based on the evaluation models of objective reputation and subjective trust, we propose an efficient method AAORS with polynomial time complexity to obtain the approximate optimal recommendation solution. We also theoretically prove that AAORS has the approximate lower bound 0.63. Furthermore, based on the index of reputation gain, we adjust the recommendation solution produced by the AAORS method in polynomial time and thereby rapidly obtain the optimal recommendation solution satisfying the global reputation constraint. The detailed theoretical analyses and extensive experiments demonstrate that our proposed methods are both efficient and effective.

In the future work, it might also be good to compare some real example(s) of social network data sets and show how AAORS can avoid local minima encountered by GREEDY and nearly or always find a ‘robust’ solution which is comparable to the global optimum found by the “brute force” OPTIMAL method.

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