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A Frequency Domain PID Controller Design Method Using Direct Synthesis Approach

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Abstract In this study, the PID controller design method based on direct synthesis approach for achieving the desired set-point or load-disturbance response is proposed. The PID controller is derived using an approximate frequency-response-matching criteria. A simple criterion has been also provided to choose the frequency points for matching of the proposed PID controller with the desired direct synthesis controller. It is a unified approach which deals with broad class of processes including integrating and inverse response, and it is directly applicable to any order of process with time delay. The ideal controller based on the direct synthesis approach has been directly approximated to the PID controller in desired frequency range. Therefore, the proposed method is free from model reduction in high-order process to low-order process and also rational approximation of the time-delay term e^{-sL}. The advantage of method is illustrated through examples taken from the literature and compared with some of the well-known methods.

Keywords Direct synthesis design · PID controller · Process control · Inverse response · Integrating process · Higher-order process

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1 Introduction

The proportional–integral–derivative (PID) controller is the most widely accepted controller in the industrial applications, especially in process industries due to advantageous cost-to-benefit ratio. It is well proven in terms of simplicity in control structure, easy to understand, low cost and easy to maintain as well as satisfactory performance. Numerous tuning methods have been proposed in the last few decades that cover various aspects of the control performance requirements such as set-point response, controller output, robust operation and load-disturbance rejection. These methods differ in complexity, flexibility and amount of process knowledge requirement against the level of performance obtained. They are well documented in books [1,2] and in review paper [3].

There are variety of controller tuning approaches reported in the literature; of them, two are widely used for the controller tuning; and one may use open-loop or closed-loop plant tests. Most tuning approaches are based on open-loop plant information—typically, the plant's gain (k), time constant (τ) and time delay (L).

The internal model control (IMC) [4] is a popular technique with improved robustness where the user can specify the performance in terms of a single parameter, i.e., the desired closed-loop time constant. If required, then one can compute a PID controller in the classical feedback configuration from the IMC configuration. Although the IMC design method is mainly used for the low-order processes, it could be applicable for the high-order process where one has to reduce high-order process into low-order process using model reduction technique. To obtain the PID controllers, the Maclaurin series expansion of the equivalent classical feedback controller derived from the IMC configuration has been used [5,6], whereas the Laurent series expansion has been used by Panda [7,8]. Based on the IMC principle, Shamsuzzoha and Lee have used 1/2 Pade approximation of the dead-time term to obtain the PID controller in cascade with a lag/lead compensator [9,10]. Vijayan and Panda [11] have proposed a PID controller based on the IMC scheme using double feedback loops with the inner loop for the purpose of stabilizing the unstable process. Another IMC-based approach is proposed by Wang et al. [12] which is based on minimization of frequency response error in the region of the bandwidth of the desired closed-loop system.

In frequency domain, model-matching technique can be applied for obtaining the PID controller without model reduction in the high-order process. Wang et al. [13] used one or two (for PI/PID, respectively) specific frequency points related to the settling time of the desired control system, whereas Wang et al. [14] used multiple frequency points for matching purpose and an optimization technique for the solution. In the direct synthesis (DS) approach [15], the design is based on a desired closed-loop transfer function. Several researchers [16–18] have utilized the DS approach for the PID controller design for stable and unstable processes. Similar to IMC approach, model reduction is required in DS approach as well because it is based on low-order model.

The main alternative of the above-mentioned open-loop approach is to use closed-loop experiments. It requires very little information about the process to obtain controller setting. Recently, several authors [19–22] have proposed modified tuning methods based on closed-loop experiments, and resulting controller gives consistently better performance and robustness for a broad class of processes.

Recently, Alcantara et al. [23] has addressed the model-based tuning of PI/PID controller based on the robustness/performance and servo/regulator trade-offs. The study suggests how to shift each compromise based upon constraint for several types of processes. They have extended the preliminary design concept of balanced autotuning, which was published earlier [24–26]. K-SIMC method, a modification of SIMC rule has been proposed recently by Lee et al. [27]. Torrico et al. [28] proposed a new and simple design for the filtered Smith predictor (FSP), which belongs to a class of dead-time compensators (DTCs) and allows the handling of stable, unstable and integrating processes. Recently Ravi and Thyagarajan [29] have suggested a simple and straightforward procedure for designing a non-adaptive decoupling-based decentralized PI control scheme and adaptive decoupling-based decentralized PI control scheme using regime-based multi-model adaptive control strategy for the TCTILS.

It should be emphasized that the most of the aforementioned tuning methods is based on the low-order model and approximation of the time-delay term. As a result, model reduction is required to deal with such kind of processes that may eventually have performance and robustness limitation.

Therefore, in this study, a DS-based frequency domain approximate model-matching method has been proposed for the PID controller design. The desired reference model for both the set-point and load-disturbance response is selected. The obtained DS controller is approximated to the PID controller by matching the frequency response of the two controllers. For the purpose of matching, two low-frequency points are chosen which shows an overall matching over the effective range of frequency response. The method is free from any restriction on the structure and complexity of the process, and hence, process reduction for high-order process and the approximation of the time-delay term is not required. It is applicable for different types of stable and integrating processes with and without non-minimum phase zero and time delay. The simulation results of the proposed method are comparable with other well-known methods.

The rest of the paper is organized as follows: The mathematical preliminaries used in the design method have been discussed in Sect. 2; in Sect. 3, the proposed design method; Sect. 4 is devoted for discussion; and in Sect. 5, simulation study followed by conclusion in Sect. 6.

2 Mathematical Preliminaries

Consider a real function $\boldsymbol{f(x)}$ with derivatives $\boldsymbol{f^{(i)}(x), i} \in [\mathbf{1}, \boldsymbol{n}]$ and in some region around the point $\boldsymbol{x} = \boldsymbol{x_0}$. Let the value of $\boldsymbol{f(x)}$ be given for the distinct real numbers $\boldsymbol{x_i}$, where $\boldsymbol{x_i} = \boldsymbol{x_0} + \boldsymbol{hi}$, and $\boldsymbol{h} > 0$. Using the notation of calculus of divided differences, we may define $\boldsymbol{f[x_0]} = \boldsymbol{f(x_0)}$ and the following divided differences of arguments 2 to $\boldsymbol{n} + \mathbf{1}$

$$\begin{aligned} f[x_0x_1] &= (f[x_0] - f[x_1])/(x_0 - x_1) \\ f[x_0x_1x_2] &= (f[x_0x_1] - f[x_1x_2])/(x_0 - x_2) \\ &\vdots \\ &\vdots \\ f[x_0x_1 \ldots .x_k] &= (f[x_0x_1 \ldots x_{k-1}] \\ &\quad - f[x_1x_2 \ldots .x_k])/(x_0 - x_k) k \in [1, n] \end{aligned} \tag{1}$$

Suppose that the interval (a, b) bounded by the greatest and least of $x_0, x_1, \ldots, x_n$, the function $f(x)$ of the real variable x and its first $(n-1)$ derivatives are finite and continuous and that $f^n(x)$ exist. It may then be shown [30] that:

$$f\,[x_0x_1x_2 \ldots x_n] = h^{-n} \sum_{i=0}^{n} \frac{(-1)^{n-i}}{i!(k-i)!} f(x_i) = \frac{1}{n!} f^{(n)}(\eta) \tag{2}$$

where η lies in the interval $x_0 \leq \eta \leq x_0 + nh$.

Now let $\psi(x)$ be a second real function with finite and continuous derivatives $\psi^{(i)}(x)$ around the point $x = x_0$ such that

$$\psi(x_i) = f(x_i), \quad i \in [0, n] \tag{3}$$

Then, from Eq. (2), $\psi^{(n)}(\xi) = f^{(n)}(\eta)$ where ξ lies in the interval $x_0 \le \xi \le x_0 + nh$. If the parameter h takes a very small nonnegative value, we have [31,32]

$$f^{(i)}(x) = \psi^{(i)}(x), \quad i \in [0, n] \tag{4}$$

Thus, for a suitable small value of the parameter h, for a given $f(x)$ another real-valued function $\psi(x)$ may always be constructed using Eq. (3) so that the approximate relations in (4) are satisfied. The above concept is used in the controller design section for approximation.

3 Controller Design Method

Consider a general process transfer function,

$$G(s) = \frac{N(s)}{D(s)} e^{-sL} \tag{5}$$

where $N(s)/D(s)$ is the rational part of the transfer function and L is the time delay of the process. The unity negative-output feedback control system configuration is considered with a controller $C(s)$ as shown in Fig. 1. $F(s)$ is a set-point filter, which is usually required to improve set-point performance when the controller is designed based on the disturbance rejection. r is the input, e is the error, u is the controller output, d is the disturbance, x is the process input, and y is the output of the process. The main design concept is described below without considering the set-point filter.

The closed-loop transfer function for both the set-point and disturbance rejection are given as

$$\frac{y(s)}{r(s)} = \frac{C(s)G(s)}{1 + C(s)G(s)} \tag{6}$$

$$\frac{y(s)}{d(s)} = \frac{G(s)}{1 + C(s)G(s)} \tag{7}$$

In the direct synthesis method, the controller design is based on the process model and a desired closed-loop transfer function. The controller may be designed either for the desired set-point response or for the load-disturbance response. First, the closed-loop transfer function for the desired set-point response is selected as $G_{\mathrm{r,y}}(s)$. The controller, $C(s)$, which yield the desired set-point response may be obtained from

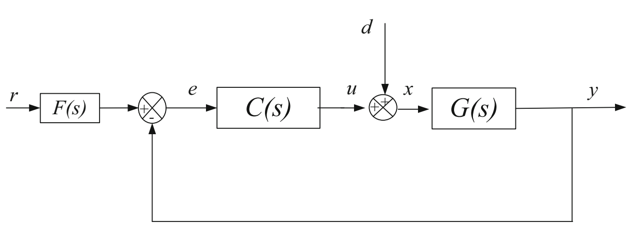

Fig. 1 Block diagram of the classical feedback control systems

Eq. (6) as

$$C(s) = \frac{G_{\mathrm{r,y}}(s)}{G(s)[1 - G_{\mathrm{r,y}}(s)]} \tag{8}$$

Similarly, to achieve the desired load-disturbance response, the closed-loop transfer function $G_{\mathrm{d,y}}(s)$ is selected and the controller $C(s)$ can be obtain from Eq. (7) as

$$C(s) = \frac{1}{G_{\mathrm{d,y}}(s)} - \frac{1}{G(s)} \tag{9}$$

The structure and the order of the controller $C(s)$ depend on the desired closed-loop transfer function and the process model as can be seen from Eqs. (8) and (9). To get a PID controller $C^{\mathrm{PID}}(s)$ from $C(s)$, the researchers usually use a low-order process model such as the first order plus dead time (FOPDT) and second order plus dead time (SOPDT). Then, they used a rational approximation of the time-delay term using the Pade approximation or the power series expansion. In common practice, for the higher-order process, it is first reduced to a suitable low order to apply any design method. The proposed method is free from model reduction in high-order process to low order and rational approximation of the delay term e^{-sL}.

In this approach, instead of the process model reduction, the controller $C(s)$ is directly approximated to a PID controller $C^{\mathrm{PID}}(s)$ as

$$C^{\mathrm{PID}}(s) = K_{\mathrm{P}} + \frac{K_{\mathrm{I}}}{s} + K_{\mathrm{D}} s \tag{10}$$

where K_{P}, K_{I} and K_{D} are proportional, integral and derivative gains, respectively. To approximate the $C(s)$ by the PID controller $C^{\mathrm{PID}}(s)$, the frequency response matching of the two controllers is considered as given below

$$C^{\mathrm{PID}}(s)\Big|_{s=j\omega} \cong C(s)|_{s=j\omega} \tag{11}$$

or

$$C_{\mathrm{R}}^{\mathrm{PID}}(\omega) + jC_{\mathrm{I}}^{\mathrm{PID}}(\omega) \cong C_{\mathrm{R}}(\omega) + jC_{\mathrm{I}}(\omega) \tag{12}$$

where

$$C^{\mathrm{PID}}(s)\Big|_{s=jw} = C_{\mathrm{R}}^{\mathrm{PID}}(\omega) + jC_{\mathrm{I}}^{\mathrm{PID}}(\omega)$$

and $C(s)|_{s=jw} = C_{\mathrm{R}}(\omega) + jC_{\mathrm{I}}(\omega)$

Separating the real and the imaginary parts in Eq. (12), one may write

$$C_{\mathrm{R}}^{\mathrm{PID}}(\omega) \cong C_{\mathrm{R}}(\omega) \quad \text{and} \quad C_{\mathrm{I}}^{\mathrm{PID}}(\omega) \cong C_{\mathrm{I}}(\omega) \tag{13}$$

In order to force the equivalence of two real functions, $C_R(\omega)$ and $C_{\mathrm{I}}(\omega)$ with their approximants $C_{\mathrm{R}}^{\mathrm{PID}}(\omega)$ and $C_{\mathrm{I}}^{\mathrm{PID}}(\omega)$, respectively, one may equate appropriate number of initial few terms of the corresponding Taylor series expansions about $\omega = 0$. Thus, to accomplish approximate matching of the left-hand side (LHS) functions in Eq. (13) with the corresponding functions on the right-hand side (RHS), initial

N derivatives of the corresponding functions are equated at $\omega = 0$ to give

$$\left.\frac{d^k}{d\omega^k}\left[C_{\mathrm{R}}^{\mathrm{PID}}(\omega)\right]\right|_{\omega=0} = \left.\frac{d^k}{d\omega^k}\left[C_{\mathrm{R}}(\omega)\right]\right|_{\omega=0} \tag{14}$$

$$\left.\frac{d^k}{d\omega^k}\left[C_{\mathrm{I}}^{\mathrm{PID}}(\omega)\right]\right|_{\omega=0} = \left.\frac{d^k}{d\omega^k}\left[C_{\mathrm{I}}(\omega)\right]\right|_{\omega=0} \tag{15}$$

where, $k \in [0, N-1]$

Now, using the mathematical preliminaries given in earlier section, $C_{\mathrm{R}}^{\mathrm{PID}}(\omega)$ approximately matches $C_{\mathrm{R}}(\omega)$ if

$$\left.C_{\mathrm{R}}^{\mathrm{PID}}(\omega)\right|_{\omega=\omega_k} = \left.C_{\mathrm{R}}(\omega)\right|_{\omega=\omega_k}; \quad k \in [0, N-1] \tag{16}$$

and

$$\left.C_{\mathrm{I}}^{\mathrm{PID}}(\omega)\right|_{\omega=\omega_k} = \left.C_{\mathrm{I}}(\omega)\right|_{\omega=\omega_k}; \quad k \in [0, N-1] \tag{17}$$

where ω_k are sufficiently small positive values around $\omega = 0$. It is clear from Eqs. (16) and (17) that N values of ω give $2N$ linear algebraic equations with the unknown parameters. For 3 numbers of unknowns of the PID controller N is at least equal to 2, and for two low-frequency points ω_0 and ω_1, the following expression is obtained.

$$\boldsymbol{A}\bar{x} = \bar{b} \tag{18}$$

where

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -\frac{1}{\omega_0} & \omega_0 \\ 1 & 0 & 0 \\ 0 & -\frac{1}{\omega_1} & \omega_1 \end{bmatrix}; \quad \bar{x} = \begin{bmatrix} K_{\mathrm{P}} \\ K_{\mathrm{I}} \\ K_{\mathrm{D}} \end{bmatrix}; \text{ and } \bar{b} = \begin{bmatrix} C_{\mathrm{R}}(\omega_0) \\ C_{\mathrm{I}}(\omega_0) \\ C_{\mathrm{R}}(\omega_1) \\ C_{\mathrm{I}}(\omega_1) \end{bmatrix}$$

From Eq. (18), two values of K_{P} are obtained as:

$K_{\mathrm{P1}} = C_R(\omega_0)$; $K_{\mathrm{P2}} = C_{\mathrm{R}}(\omega_1)$

It is observed from various examples that $K_{\mathrm{P1}} \approx K_{\mathrm{P2}}$, and we may take an average of these values.

Hence, to evaluate K_{I} and K_{D}, Eq. (18) may be simplified as

$$\boldsymbol{A}_1\bar{x}_1 = \bar{b}_1 \tag{19}$$

where

$$\boldsymbol{A}_1 = \begin{bmatrix} -\frac{1}{\omega_0} & \omega_0 \\ -\frac{1}{\omega_1} & \omega_1 \end{bmatrix}; \quad \bar{x}_1 = \begin{bmatrix} K_{\mathrm{I}} \\ K_{\mathrm{D}} \end{bmatrix}; \text{ and } \bar{b}_1 = \begin{bmatrix} C_{\mathrm{I}}(\omega_0) \\ C_{\mathrm{I}}(\omega_1) \end{bmatrix}$$

Then, solution of Eq. (19) determines K_{I} and K_{D}. Thus, the parameters of the PID controller are evaluated.

4 Discussion

4.1 Selection of Low-Frequency Points

Good approximation of $C(s)$ by the $C^{\mathrm{PID}}(s)$ results in good matching of the overall designed system with the desired closed-loop system. Thus, frequency points' selection is considered with respect to the desired closed-loop system. However, theoretically, the range of ω is from 0 to ∞ and for such an infinite range, it is meaningless to find ω_k values which are sufficiently small. The small values of frequency points are chosen at around 0.1 % of the bandwidth frequency, where the bandwidth may be treated as an indication to the effective range of frequency response of the desired reference model. Normally, industrial processes show low-pass dynamics in terms of the frequency response, so closer matching in low-frequency zone is more important. Such frequency points for matching have been observed through simulation to give good result for the most of the processes.

4.2 Selection of the Desired Closed-Loop System

In the DS/IMC design methods, the desired closed-loop transfer function is selected based on the closed-loop response speed or effective time constant of closed-loop system. The desired closed-loop transfer function either $G_{\mathrm{r,y}}(s)$ or $G_{\mathrm{d,y}}(s)$ as the case may required to have a time-delay value at least equal to that of the process. From the stability point of view, it is also required to have the zeros of the right half of the s-plane of the process (i.e., non-minimum phase zeros that show the inverse response) in the desired transfer function. Further, for a choice of the desired closed-loop transfer function $G_{\mathrm{d,y}}(s)$ for the load-disturbance rejection response, it is required to have one zero at origin. In the proposed design method, the effective closed-loop time constant is being selected with the consideration of the open-loop system dynamics.

- In addition to FOPDT and SOPDT, the proposed design method is directly applicable to high-order processes that may have over-damped, oscillatory or inverse response dynamics along with dead time. Both the design approaches considering either the desired set-point response or the desired load-disturbance response are applicable for all the cases.
- In case of processes with integrating dynamics, the proposed method, when applied for achieving the desired set-point response, gives the integral constant $K_{\mathrm{I}} = 0$, leading to a PD controller. This may also be shown as discussed below.

A desired reference model of the second order for set-point tracking is considered as given by

$$G_{\mathrm{r,y}}(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} e^{-Ls} \tag{20}$$

Equation (8) may be written as

$$\left(K_{\mathrm{P}} + \frac{K_{\mathrm{I}}}{s} + K_{\mathrm{D}} s\right)\Bigg|_{s=j\omega} \cong \frac{\left(\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}\right)}{\left(\frac{N_p(s)}{sD_p(s)}\right)\left(1 - \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2} e^{-Ls}\right)}\Bigg|_{s=j\omega} \tag{21}$$

Using the first-order Pade approximation of e^{-Ls}, Eq. (21) can be simplified as

$$\frac{\left(K_{\mathrm{D}} s^2 + K_{\mathrm{P}} s + K_{\mathrm{I}}\right)}{s}\Bigg|_{s=j\omega} \cong \frac{D_p(s)[(\omega_n^2 L/2)s + \omega_n^2]}{N_p(s)\left[\left(\frac{L}{2}\right)s^2 + \left(\frac{2\xi\omega_n L}{2} + 1\right)s + \left(L\omega_n^2 + 2\xi\omega_n\right)\right]}\Bigg|_{s=j\omega} \tag{22}$$

In Eq. (22), LHS has one pole at origin (type-1 transfer function), while the RHS has not any pole at origin (type-0 transfer function). Then, for feasible matching, types of both sides of Eq. (22) are to be same. This may be obtained by choosing $K_{\mathrm{I}} = 0$, which makes LHS as type-0. In a similar way, it can be shown for any order of the reference model, K_{I} is to be 0. It is to note that the proposed design method generates the value of K_{I} as 0 by its own procedure.

Thus, the proposed PID controller design for the desired set-point response yields the PD controller for integrating processes. Based on such a PD controller, a good set-point response may be obtained, but it fails to reject the load disturbance. Thus, for the integrating processes, the proposed design method is applied to achieve the desired load-disturbance response.

4.3 Set-Point Filter for Enhanced Servo Response

Achieving a good load-disturbance rejection is usually associated with tight set-point response with overshoot. In such case, a possible solution is to use a set-point filter [1], leading to a two-degree-of-freedom control scheme that does not affect the load-disturbance rejection performance. A simple structure as $F(s) = 1/(\lambda s + 1)$ is considered, where the time constant λ is to be chosen carefully. A small λ results in faster response of the system with improvement in the peak overshoot and the oscillation. Too small λ becomes ineffective ($F(s) \approx 1$). It is observed through simulation that acceptable improvement occurs for $0 < \lambda < t_s/4$, where t_s is the settling time of the set-point response of the control system before employing the filter.

5 Simulation Results

To show the performance of the proposed method, simulation has been conducted on different types of processes, e.g., over-damped, oscillatory, inverse response, integrating dynamics, low order as well as high order with and without time delay. The set-point filter is used in case of aggressive set-point response observed. It is also compared with some of the well-known methods available in the literature. The following robustness and performance parameters are used to evaluate the proposed PID controllers.

- Maximum sensitivity (M_{s}): To evaluate the robustness of a control system, the maximum sensitivity, M_{s}, is considered which is defined as $M_s = \max_{0\le\omega\le\infty} |1/1 + C(j\omega) G(j\omega)|$. Smaller value of M_s is preferred with recommended range of 1.2 – 2.0 [1].
- Integral absolute error (IAE): This indicates a performance measure of the overall step response as given by $\mathrm{IAE} = \int_0^\infty |e(t)|\mathrm{d}t$, where $e(t)$ is the error signal.
- Total variation (TV) of controller output: It is an important parameter for evaluation of manipulated variable $u(t)$, by considering all its up and down moves. It is defined as $\mathrm{TV} = \sum_{i=1}^{\infty} |u_{i+1} - u_i|$, where u_i is the discretized manipulated variable. A lower TV value indicates better smoothness of the controller output [33].

For process $G_1(s)$, various pairs of frequency points are chosen for the design purpose and tabulated in Table 1. It is observed from this table that the values of the frequency points are within a small percentage range (around 0.1 %) of the bandwidth frequency. The variation in obtained controller parameters is insignificant, and resulting difference in performance is almost negligible. In general, the choice of frequency points around 0.1 % of the bandwidth frequency has been observed to give good results for the most of the

Table 1 PID controller parameters considering different frequency point matching for example 1

Cases	ω_0	ω_1	K_{P1}	K_{P2}	K_I	K_D
1	0.01	0.02	1.125	1.125	1.0	0.14
2	0.02	0.04	1.125	1.125	1.0	0.12
3	0.04	0.08	1.125	1.125	1.0	0.12
4	0.1	0.2	1.125	1.125	1.0	0.12
5	0.2	0.4	1.125	1.126	1.0	0.12
6	0.4	0.8	1.126	1.129	1.0	0.12
7	1.0	1.2	1.131	1.137	1.0	0.12
8	1.4	1.8	1.137	1.145	1.0	0.121
9	1.8	2.0	1.145	1.150	1.0	0.124

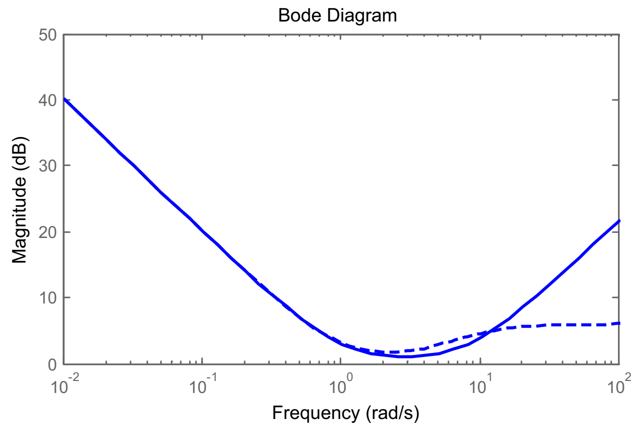

Fig. 2 Bode magnitude plot of controllers for example 1; *solid line*, PID controller $C^{\mathrm{PID}}(s)$; *dotted line*, Ideal controller $C(s)$

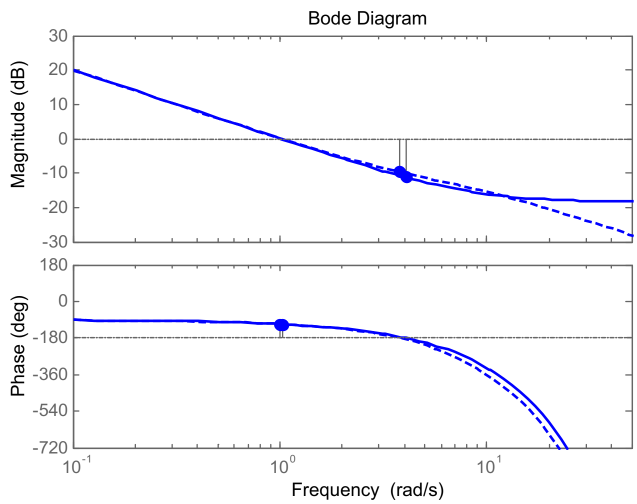

Fig. 3 Bode plot for example 1; *solid line*, open-loop designed system; *dotted line*, open-loop reference model

processes. To show the effectiveness of the approximation, the performance comparison of $G_1(s)$ has been considered in frequency domain. The frequency response of the controller $C(s)$ and the approximated PID controller $C^{\mathrm{PID}}(s)$ is shown in Figure 2, where it has observed of close matching in the low-frequency region. In Fig. 3, the frequency response of the open-loop system and the reference open-loop system has been shown. The result in this case also shows close matching in the low-frequency region. Similar observation has been found for the rest of examples given in Table 2.

In the simulation study, wide range of processes has been considered to show the effectiveness of the proposed method. To design PID controller for stable process, a desired set-point response model is considered, while for an integrating process, a desired load-disturbance response model is considered. The desired reference model for both the set-point and the load-disturbance rejection with selected frequency points for all the cases is listed in Table 2. Set-point filter of the first order is considered in case of aggressive set-point response.

The closed-loop performance is evaluated by introducing a unit step change in both the set point and load disturbance, i.e., ($y_s = 1$ and $d = 1$). The proposed PID controller is compared with other well-known methods, and the controller parameters, including the performance and robustness matrix (IAE, M_s and TV) are listed in Table 3. The simulation results of examples 1, 3, 6, 7 and 9 are shown in Figs. 4, 5, 6, 7 and 8. It clearly shows that the proposed method gives both smaller overshoot and faster disturbance rejection while maintaining the set-point performance in most of the cases. From above analysis, it seems that the proposed method constantly gives either better closed-loop response or comparable with other well-known methods.

Figure 9 shows the manipulated variable (MV) response for $G_3(s)$ as the representative case. The response of the MV of the proposed method is smooth and better in comparison with other methods. As mentioned earlier, TV is a good measure of the smoothness of a signal, and the TV value of all 9 processes is given in Table 3.

Table 2 Process models and reference models for the simulation study

Process	Reference model	ω_0, ω_1 rad/s
$G_1(s) = \frac{e^{-0.5s}}{s+1}$	$G_{\mathrm{r,y}}(s) = \frac{e^{-0.5s}}{0.5s+1}$	0.02,0.04
$G_2(s) = \frac{2(-3s+1)e^{-0.5s}}{(2s+1)(s+1)}$	$G_{\mathrm{r,y}}(s) = \frac{-3s+1}{4s+1}e^{-0.5s}$	0.2,0.4
$G_3(s) = \frac{e^{-0.4s}}{(s^2+s+1)(s+3)}$	$G_{\mathrm{r,y}}(s) = \frac{e^{-0.4s}}{4.5s+1}$	0.002,0.004
$G_4(s) = \frac{e^{-4s}}{s(s+1)}$	$G_{\mathrm{d,y}}(s) = \frac{120se^{-4s}}{(5s+1)(15s+1)}$	0.001,0.002
$G_5(s) = \frac{0.0506}{s}e^{-6s}$	$G_{\mathrm{d,y}}(s) = \frac{4S}{(5S+1)(6.3S+1)}e^{-6s}$	0.001,0.002
$G_6(s) = \frac{0.547(-0.418s+1)e^{-0.1s}}{s(1.06s+1)}$	$G_{\mathrm{d,y}}(s) = \frac{(-0.418s+1)s}{(s+1)(2s+1)}e^{-0.1s}$	0.001,0.002
$G_7(s) = \frac{e^{-8s}}{(2s+1)^3(s+1)^2}$	$G_{\mathrm{r,y}}(s) = \frac{e^{-8s}}{10s+1}$	0.002,0.004
$G_8(s) = \frac{e^{-2.2s}}{(4s^2+2.8s+1)(s+1)^2}$	$G_{\mathrm{r,y}}(s) = \frac{e^{-2.2s}}{(1.2s+1)^4}$	0.003,0.006
$G_9(s) = \frac{1}{(s+1)^{20}}$	$G_{\mathrm{r,y}}(s) = \frac{1}{(s+1)^{20}}$	0.001,0.002

Table 3 Performance comparison for nominal process model

Process	Method	K_P	K_I	K_D	M_S	Set-point response		Load-disturbance response	
						IAE	TV	IAE	TV
$G_1(s)$	Proposed	1.12	1.0	0.12	1.44	1.05	2.25	1.0	1.0
	Shamsuzzoha and Lee [6]	1.08	1.02	0.11	1.44	1.08	2.18	0.99	1.0
	IMC-PID	1.11	0.88	0.11	1.44	1.12	2.10	1.26	1.0
$G_2(s)$	Proposed	0.212	0.066	0.176	1.32	7.575	0.59	20.52	3.21
	Jeng and Lin [34]	0.191	0.063	0.127	1.26	7.973	0.51	20.98	2.22
	Chen et al. [35]	0.21	0.067	0.134	1.33	7.563	0.50	20.68	2.33
$G_3(s)$	Proposed	0.74	0.612	0.827	1.14	4.903	2.99	1.634	0.99
	Wang et al. [14]	1.96	1.75	3.74	1.69	2.799	4.78	0.821	1.34
	Ho [36]	1.60	0.41	0	1.68	7.314	3.77	2.438	1.24
$G_4(s)$	Proposed[a]	0.20	0.0083	0.357	1.30	14.49	0.17	120.1	1.81
	Ali and Majhi [37]	0.19	0.0084	0.53	1.31	14.75	0.54	119.7	1.81
	SIMC [33]	0.13	0.0039	0.126	1.32	17.28	0.32	256.0	1.55
$G_5(s)$	Proposed[b]	3.50	0.16	6.62	1.34	16.04	3.42	5.99	2.59
	Ali and Majhi [37]	3.39	0.17	9.96	1.35	17.53	15.03	6.029	3.17
	Chidambaram and Sree [38]	4.06	0.15	10.97	1.42	16.61	24.13	6.665	4.73
$G_6(s)$	Proposed[c]	3.518	1.00	1.95	1.28	2.061	4.71	1.044	3.93
	Shamsuzzoha and Lee [39][d]	2.43	0.667	1.786	1.32	2.567	2.53	1.542	2.93
	Gu et al. [40] [e]	2.088	0.541	1.436	1.27	2.706	2.05	1.889	2.64
$G_7(s)$	Proposed	0.543	0.055	2.30	1.24	18.45	1.36	18.19	1.08
	Yang et al. [41]	0.63	0.060	1.745	1.30	18.52	1.66	16.69	1.22
	IMC-PID [42]	0.56	0.063	1.316	1.26	18.92	1.48	17.10	1.15
$G_8(s)$	Proposed	0.558	0.143	0.857	1.23	8.362	1.30	7.151	1.05
	Yang et al. [41]	0.73	0.170	1.467	1.31	8.216	1.84	6.152	1.31
	IMC-PID	0.58	0.124	0.650	1.26	8.462	1.34	8.014	1.11
$G_9(s)$	Proposed	0.525	0.055	1.66	1.27	23.33	1.56	20.02	1.25
	Yang et al. [41]	0.62	0.052	2.21	1.33	23.77	1.68	19.85	1.26
	IMC-PID [42]	0.55	0.05	1.87	1.29	23.72	1.43	22.53	1.11

The set-point filter is used in some of the cases to improve servo response as given below,
[a] $F(s) = 1/12s + 1$; [b] $F(s) = 1/15s + 1$; [c] $F(s) = 1/15s + 1$;
[d] $F(s) = \frac{1.0963s+1}{2.7528s^2+3.6543s+1}$; [e] $F(s) = \frac{1.1599s+1}{2.6597s^2+3.8664s+1}$

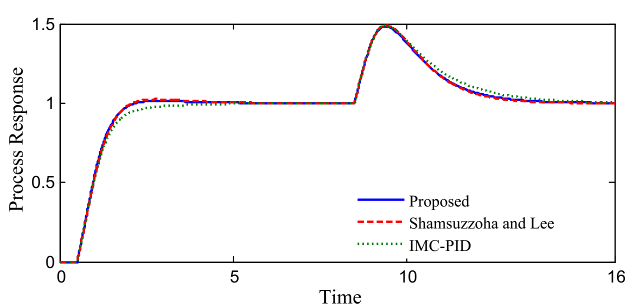

Fig. 4 Process responses of the first order with time-delay process $G_1(s) = \frac{e^{-0.5s}}{s+1}$, set-point change at $t = 0$; load disturbance of magnitude 1 at $t = 8$

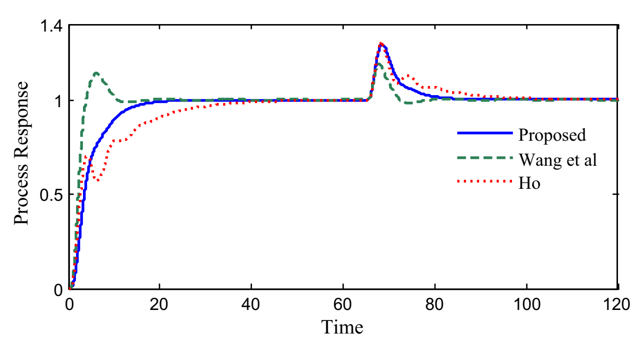

Fig. 5 Process responses of third order with time-delay process $G_3(s) = \frac{1}{(s^2+s+1)(s+3)}e^{-0.4s}$, set-point change at $t = 0$; load disturbance of magnitude 1 at $t = 60$

The performance of the nominal case has been compared and shown both in figures and Table 3; it is also worth to analyze the robustness of the controller evaluated by inserting a perturbation uncertainty in gain and dead time. To show the closed-loop response of the model mismatch, a case has

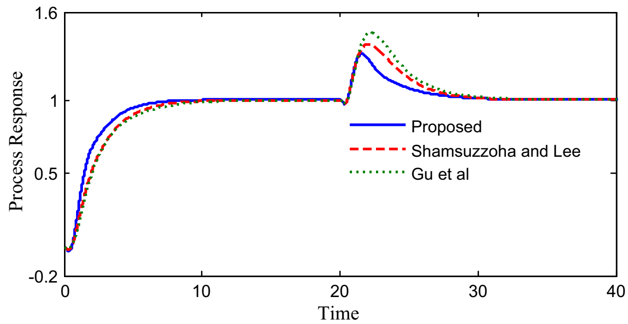

Fig. 6 Process responses of the second order with positive zero and time-delay integrating process $G_6(s) = \frac{0.547(-0.418s+1)e^{-0.1s}}{s(1.06s+1)}$, set-point change at $t = 0$; load disturbance of magnitude 1 at $t = 20$

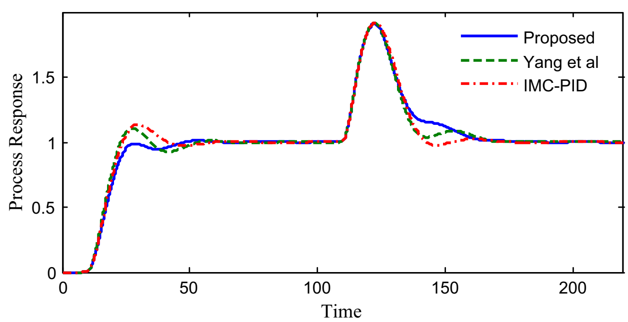

Fig. 7 Process responses of fifth order with time-delay process $G_7(s) = \frac{e^{-8s}}{(2s+1)^3(s+1)^2}$, set-point change at $t = 0$; load disturbance of magnitude 1 at $t = 100$

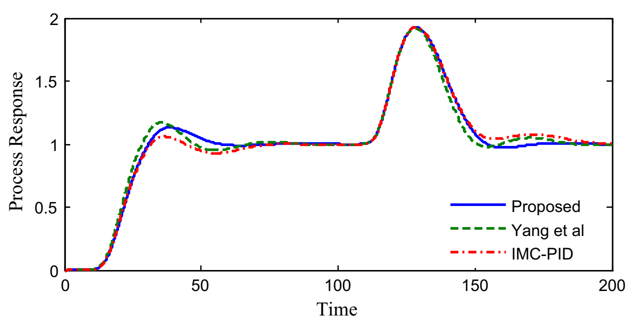

Fig. 8 Process responses of 20th order process $G_9(s) = \frac{1}{(s+1)^{20}}$, set-point change at $t = 0$; load disturbance of magnitude 1 at $t = 100$

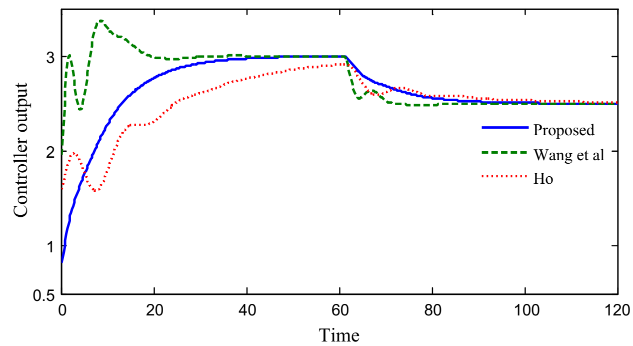

Fig. 9 Controller output of third order with time-delay process $G_3(s) = \frac{1}{(s^2+s+1)(s+3)}e^{-0.4s}$, set-point change at $t = 0$; load disturbance of magnitude 1 at $t = 60$

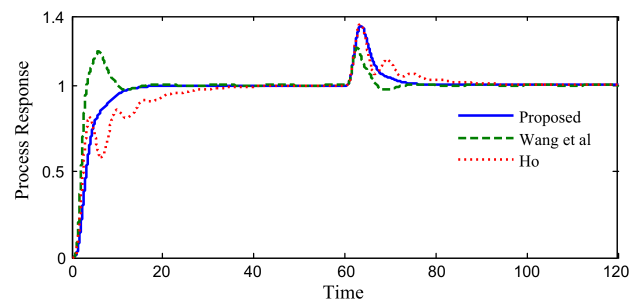

Fig. 10 Process output of the third order with time-delay perturbed process $G_3(s) = \frac{1}{(s^2+s+1)(s+3)}e^{-0.4s}$, set-point change at $t = 0$; load disturbance of magnitude 1 at $t = 60$

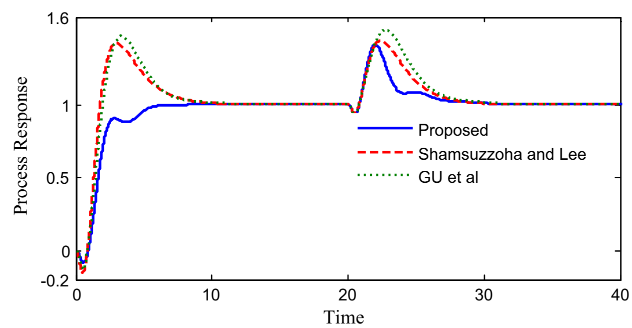

Fig. 11 Process output of the second order with positive zero and time-delay perturbed process $G_6(s) = \frac{0.547(-0.418s+1)e^{-0.1s}}{s(1.06s+1)}$, set-point change at $t = 0$; load disturbance of magnitude 1 at $t = 20$

been selected for 20 % uncertainty in both the process gain and dead time simultaneously. The simulation results for the plant-model mismatch are given in Figs. 10 and 11 for both the servo and regulatory problems. It should be mentioned that the controller settings used in simulation are those calculated for the process with nominal process parameters. The performance and robustness indices clearly demonstrate the comparable robust performance of the proposed controller design.

In the proposed design method, both the selected frequency points is sufficiently low values which emphasis for closer matching in the low-frequency region. It ensures achievement of the desired steady-state specification and consequently good transient response for stable and integrating processes with inverse response.

6 Conclusions

In this study, a simple DS-based PID controller design method for industrial processes have been proposed. Initially DS control scheme is obtained for a given process, and further, it is converted to a PID controller by an approximate frequency-response-matching method. Two frequency points are required for matching the frequency response, and an effective criterion has been provided for choosing such frequency points. The method is free from model reduction in high-order process to low-order process and rational approximation of the delay term e^{-sL}. The design procedure has acceptable computation burden to obtain the PID controller settings. The important feature of the proposed methodology is that it deals with stable and integrating process in a unified way. The proposed method shows comparable results with other well-known methods.

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