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Cognitive Opportunistic DF Relay Networks with Interference and Multiple Primary Receivers Using Orthogonal Spectrums

Anas M. Salhab · Salam A. Zummo

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Abstract Most of existing papers on cognitive relay networks with multiple primary user (PU) receivers consider the scenario where the PU receivers utilize the same spectrum band. In this paper, we consider a new scenario where the PU receivers utilize orthogonal spectrum bands and the spectrum of a PU receiver whose channel enhances the performance of the secondary system is shared with the secondary user (SU) nodes. Using orthogonal spectrum bands in cellular networks aims to reduce the interference between users as in the downlink transmission where orthogonal frequency bands are used by the base station in transmitting the data for the different users. In this paper, we study the outage performance of cognitive opportunistic decode-and-forward relaying operating in the secondary network with multiple PU receivers and in the presence of interference from a PU transmitter. A closed-form expression is derived for the outage probability with all system links following Rayleigh distribution. Furthermore, to get more insights about the system behavior, the performance is studied at the high signal-to-noise ratio (SNR) regime where approximate expressions for the outage probability, diversity order, and coding gain are obtained. Monte Carlo simulations are given to validate the achieved results. Main findings illustrate that with fixed interference power, the diversity order of the secondary system equals the number of relays and it is not affected by the number of PU receivers. Also, results show that the number of PU receivers affects the system performance through affecting only the coding gain. Unlike the existing papers where the same spectrum band is assumed to be shared by the PU receivers, our findings demonstrate that increasing the number of PU receivers in the proposed scenario enhances the system performance. Finally, results illustrate that when the interference at the SU relays or at SU destination or at both scales with SNR, the system achieves a zero diversity order and a noise floor appears in the results due to the effect of interference on the system performance.

Keywords Opportunistic decode-and-forward relay · Cognitive network · Interference · Rayleigh fading

الخلاصة

أغلب الدراسات المتناولة لشبكات الـ Cognitive Radio ذي المرحّلات و متعدّدة المستخدمين الإبتدائيّين تدرس فقط السيناريو الذي يُستخدم فيه حُزمة تردّدية واحدة لجميع المستخدمين الإبتدائيّين. سيناريو آخر مهم وممكن تناوله هو السيناريو الذي يستخدم فيه المستخدمون الإبتدائيّين حُزَم تردّدية متعامدة بهدف تقليل أو منع التداخل بين قنوات المستخدمين. في النظام المقترح بهذه الدراسة ، الحُزمة التردّديه للمُستقبل الإبتدائي الذي قناته تعطي أفضل آداء للنظام الثانوي وتحقّق شرط التداخل بين القنوات يتم مشاركتها مع المستخدمين الثانويّين.

لقد تمّت - في هذه الدراسة - دراسة وتقييم آداء نظام Cognitive Radio من نوع Decode-and-Forward مع وجود تداخل بين القنوات يسبّبه مُرسِل إبتدائي على المستخدمين الثانويّين ، حيث تمّ حساب صيغة جبرية لإحتمال سقوط النظام مع فَرَض كافّة قنوات النظام وفقاً لنموذج التلاشي من نوع Rayleigh. بالإضافة إلى ذلك تمّت دراسة الآداء عند القيم العالية جداً لـ SNR ، حيث تمّ حساب صيغة جبرية بسيطة وتقريبية لإحتمال سقوط النظام. أيضاً ، تمّ التأكّد من صحّة النتائج من خلال مقارنتها بالـ Monte-Carlo simulations.

A. M. Salhab (✉) · S. A. Zummo
Electrical Engineering Department, King Fahd University of Petroleum and Minerals, Dhahran 31261, Saudi Arabia
e-mail: salhab@kfupm.edu.sa

S. A. Zummo
e-mail: zummo@kfupm.edu.sa

وأظهرت النتائج أنّ الـ diversity order للنظام المدروس بوجود تقنية الـ Cognitive Radio هي نفسها بدون وجود هذه التقنية وأنّ عدد المُستقبلين الإبتدائيّين يؤثّر فقط في الـ coding gain للنظام. كما وبيّنت النتائج أنّه خلافاً للأنظمة التي يُستخدم فيها حُزمة تردّدية واحدة للمّستقبلين الإبتدائيّين ، بزيادة عدد المُستخدمين الإبتدائيّين في النظام المقترح يتحسّن آداء النظام. أخيراً ، أظهرت النتائج أنّه في حالة أنّ قدرة إشارة التداخل عند المرحّلات ، أو المستقبل أو كليهما تزداد بزيادة الـ SNR للإشارات المرغوبة ، تَصِل قيمة الـ diversity order للنظام إلى القيمة صفر وذلك بسبب أثر التداخل بين القنوات على آداء النظام.

1 Introduction

Spectrum sharing or cognitive radio has been proposed to improve the spectrum resource utilization efficiency [1]. The underlay scenario is among the various cognitive radio paradigms that have been proposed in [2]. This paradigm allows users in a secondary cell (secondary or cognitive users) to utilize the frequency bands of users in a primary cell (primary or licensed users) only if the interference is below a certain threshold. Along with the cognitive radio networks, the relay networks have been recently presented to deal with the problem of multipath fading in wireless systems by providing diversity [3]. This technique proved itself also as an efficient tool in widening the coverage area and in reducing the need for high-power transmitters in wireless networks. The cognitive relay networks (CRNs) are currently a hot area of research for a large number of researchers.

Closed-form expressions for outage probability of opportunistic decode-and-forward (DF) CRNs were evaluated in [4] assuming Rayleigh fading channels. Recently, the outage performance of DF CRNs with the Nth best-relay selection scheme was presented in [5]. In such scheme, the second or even the Nth best relay is selected to forward the source message to destination which makes this scheme efficient in conditions where the best relay is busy in some scheduling or load balancing duties in other parts of the network. Most recently, the outage performance of opportunistic amplify-and-forward (AF) and DF CRNs with multiple secondary destinations and in the presence of direct link was studied in [6]. The performance of selection DF and AF CRNs with collaborative distributive beamforming was recently studied in [7]. The outage and error rate performances of an underlay fixed-gain AF CRN with reactive relay selection were evaluated in [8]. Among the relays which satisfy the interference constraint, the relay with the best end-to-end (e2e) channel is selected to forward the source message to destination. As an extension to the previous works on AF CRNs, the outage performance of an AF CRN with multiple primary users was recently presented in [9].

The effect of interference from PU transmitter on the performance of secondary systems with single SU relay was studied in [10–12]. Particularly, Xu et al. [12] derived closed-form expression for the outage probability of DF CRNs with interference from primary user. The performance of CRNs with multiple SU relays and interference from primary user was recently studied in [13–15]. Particularly, in [15], a closed-form expression was derived for the outage probability of proactive DF CRNs with interference from primary user and assuming Rayleigh fading channels. In the proactive DF networks, the relay with the e2e signal-to-noise ratio (SNR) is selected to forward the source message to destination. A drawback of this scheme is the huge amount of overhead it requires for obtaining the channel information of all nodes each transmission time [16]. In the area of CRNs with single SU relay and multiple PU receivers, Tran et al. [17] derived in a closed-form expression for the outage probability and an approximate expression for the ergodic capacity assuming Nakagami-m fading channels. Recently, the performance of CRNs with single SU relay and multiple PU transceivers was investigated in [18] assuming interference-limited scenario. Asymptotic expressions were derived for the outage probability and symbol error rate assuming multiple antennas per each node and Nakagami-m channels. The outage probability performance of CRNs with multiple SU relays and multiple primary receivers was studied in [19]. Most recently, the performance of opportunistic DF CRNs with multiple PU receivers and transmitters and distributed beamforming was studied in [20]. The interference caused by primary user on secondary users can be mitigated using various techniques. Wang et al. [21] presented two methods for mitigating interference in multi-user two-way relay networks with distributed beamforming. The first method nulls out every interference at every user, and the second one treats the interferences at each user as a whole and nulls the power of the total interferences. Another important problem that could be studied in future is the outdated channel state information and its impact on the performance of cognitive relay networks [22].

In most of the existing papers on CRNs with multiple PU receivers, the PU receivers were assumed to utilize the same spectrum band, and hence, the spectrum of PU receiver whose channel causes the minimum interference with the SU nodes is assigned to the SU transmitters. Another important scenario which can be widely seen in practice is the one where the PU receivers utilize orthogonal spectrum bands as in frequency division multiple access (FDMA)-based cellular networks. In such networks, the downlink transmission from a primary base station to the different users in its cell is conducted over orthogonal spectrum bands. The main aim of using orthogonal frequency bands was to avoid or reduce the interference between users. In such scenario, the SU transmitters will need to share the spectrum with one of the PU receivers, and at the same time, the SU receivers will be corrupted by interference from the primary base station. An important application of the proposed scenario could be in GSM cellular systems where the primary and secondary users are assigned the system resources using the FDMA/time division multiple access (TDMA) technique. The proposed scenario could be seen also in long-term evolution (LTE) networks where the orthogonal FDMA (OFDMA) technique is used in the downlink transmission in which different subchannels and bands are assigned for different users. Another application is in IEEE 802.22 wireless regional area networks (WRANs) where the OFDMA is a candidate access method for these networks. The importance of addressing this scenario where the PU receivers utilize orthogonal spectrum bands and its applicability motivated us to contribute in this area of research.

To the best of our knowledge, the scenario and performance of opportunistic DF CRNs with multiple PU receivers using orthogonal spectrum bands and interference from PU transmitter have not been presented yet. The contributions of our paper over the existing studies can be summarized in the following points: (1) we propose and study the performance of the new scenario of cognitive opportunistic DF relay network where the multiple PU receivers utilize orthogonal spectrum bands and in the presence of interference from PU transmitter; (2) the proposed scenario is applicable in the FDMA-based cellular systems where the primary base station utilizes orthogonal spectrum bands in the downlink transmission with its different users. Using orthogonal frequency bands aims to avoid or reduce the interference between the various users; (3) in contrast to the opportunistic proactive DF relaying scheme where the two hop channels of relays are required to be estimated each transmission time, the studied opportunistic DF partial relay selection scheme requires only the estimation of relays second hop channels each transmission time. In this scheme, among the relays who succeeded in decoding the source message in the first phase of communication, the relay with the best second hop channel's SNR is selected in the second phase of communication to forward a re-encoded version of the source message to the destination; (4) in order to get more insights about the system behavior, we study the performance at the high SNR regime where approximate expressions for the outage probability, diversity order, and coding gain are derived and analyzed. In the analysis, the cumulative distribution function (CDF) of first hop channels' signal-to-interference plus noise ratio (SINR) of the SU relays conditioned on the statistics of channels from the SU source to the PU receivers is first derived. This CDF is then used along with the CDF of second hop channels' SINR of the SU relays conditioned on the statistics of channels from these relays to the PU receivers to derive an exact closed-form expression for the outage probability of the studied system. A full description of how the proposed scenario works is also provided in this paper.

This paper is organized as follows. Section 2 presents the system and channel models and a description of the proposed scenario. The exact outage performance is evaluated in Sect. 3. Section 4 provides the asymptotic outage performance. Some simulation and numerical results are presented and discussed in Sect. 5. Finally, conclusions are given in Sect. 6.

2 System and Channel Models

Consider a dual-hop spectrum-sharing relay network consisted of one secondary user (SU) source S, K DF SU relays R_k $(k = 1, \ldots, K)$, one SU destination D, M primary user (PU) receivers P_m $(m = 1, \ldots, M)$ using orthogonal frequency bands, and one PU transmitter P_{Tx} as shown in Fig. 1. All nodes are assumed to be equipped with single antenna, and the communication is assumed to operate in a half-duplex mode. Also, a downlink transmission is assumed to be conducted between the PU transmitter or base station and its PU receivers. The SU users need to share the spectrum with the PU receiver whose channel results in a best performance for the secondary system. At the same time, the K SU relays and destination will be corrupted by interference from the PU transmitter or base station P_{Tx}. The direct link is assumed to be in a deep fade, and hence, it is neglected in the analysis. Also, we assume block fading model where the channel coefficient stays constant over an entire block of communication. The communications take place in two phases. In first phase, the SU source sends its message x to K relays under a transmit power constraint which guarantees that the interference with the selected PU receiver $\mathsf{P}_{\mathsf{Sel}}$ does not exceed a threshold $\mathcal{I}_{\mathsf{p}}$. To satisfy the PU interference constraint and result in a best performance for the secondary system, the SU source S must transmit at a power given by $P_{\mathsf{s}} = \mathcal{I}_{\mathsf{p}}/\min_m |g_{\mathsf{s},m}|^2$, where $g_{\mathsf{s},m}$ is the channel coefficient of the $\mathsf{S} \rightarrow \mathsf{P}_m$ link. The received signal at the kth relay can be expressed as

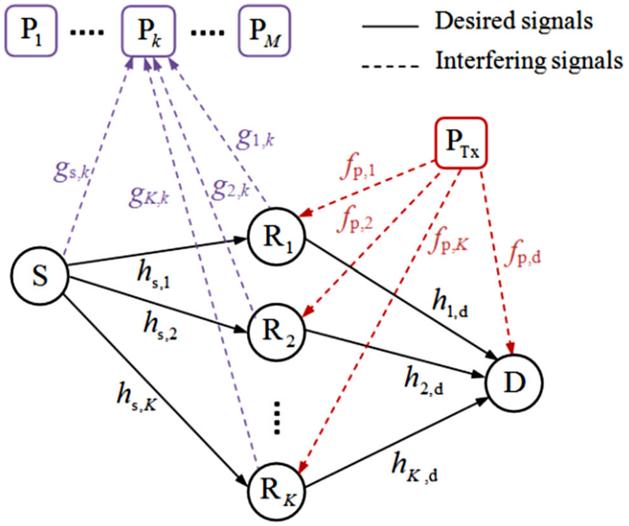

Fig. 1 Cognitive opportunistic DF relay network with interference and multiple PU receivers

$$y_{\mathsf{s},k} = \sqrt{P_\mathsf{s}}\, h_{\mathsf{s},k} x_0 + \sqrt{P_\mathsf{p}^1}\, f_{\mathsf{p},k} x_\mathsf{p} + n_k, \tag{1}$$

where $h_{\mathsf{s},k}$ is the channel coefficient of the $\mathsf{S} \rightarrow \mathsf{R}_k$ link, x_0 is the transmitted symbol from the SU source S with $\mathbb{E}\{|x_0|^2\} = P_0$, $f_{\mathsf{p},k}$ is the channel coefficient of the $\mathsf{P}_{\mathsf{Tx}} \rightarrow \mathsf{R}_k$ link, x_p is the transmitted symbol from the PU transmitter P_{Tx} with $\mathbb{E}\{|x_\mathsf{p}|^2\} = P_\mathsf{p}^1$, where the superscript 1 is used to denote that this is the transmitted power at the first communication phase, $n_k \sim \mathcal{CN}(0, N_0)$ is an additive white Gaussian noise (AWGN), and $\mathbb{E}\{\cdot\}$ denotes the expectation operation. Let us define $h_{k,\mathsf{d}}$, $g_{k,m}$, and $f_{\mathsf{p},\mathsf{d}}$ as the channel coefficients of the $\mathsf{R}_k \rightarrow \mathsf{D}$, $\mathsf{R}_k \rightarrow \mathsf{P}_m$, $\mathsf{P}_{\mathsf{Tx}} \rightarrow \mathsf{D}$ links, respectively. All channel coefficients are assumed to follow the Rayleigh distribution, that is, the channel powers denoted by $|g_{\mathsf{s},m}|^2$, $|h_{\mathsf{s},k}|^2$, $|h_{k,\mathsf{d}}|^2$, $|g_{k,m}|^2$, $|f_{\mathsf{p},k}|^2$, and $|f_{\mathsf{p},\mathsf{d}}|^2$ are exponentially distributed random variables (RVs) with mean powers $\mu_{\mathsf{s},m}$, $\Omega_{\mathsf{s},k}$, $\Omega_{k,\mathsf{d}}$, $\mu_{k,m}$, $\beta_{\mathsf{p},k}$, and $\beta_{\mathsf{p},\mathsf{d}}$, respectively.

Using (1), the SINR at the kth relay can be written as

$$\gamma_{\mathsf{s},k} = \frac{\frac{\mathcal{I}_\mathsf{p}}{N_0}\frac{|h_{\mathsf{s},k}|^2}{W_1}}{\frac{P_\mathsf{p}^1}{N_0}|f_{\mathsf{p},k}|^2 + 1} = \frac{X_{\mathsf{s},k}}{Y_1 + 1} = \frac{X_{\mathsf{s},k}}{Z_1}, \tag{2}$$

where $W_1 = \min_m |g_{\mathsf{s},m}|^2$, $X_{\mathsf{s},k}$, Y_1, and Z_1 are some RVs used for an easy follow of the paper.

Let C_L denote a decoding set defined by the set of active relays that could have correctly decoded the source message in first phase of communication. It is defined as

$$\begin{aligned} C_L &\triangleq \left\{ k \in \mathcal{S}_r : \frac{1}{2}\log_2\left(1 + \gamma_{\mathsf{s},k}\right) \geq R \right\} \\ &= \left\{ k \in \mathcal{S}_r : \gamma_{\mathsf{s},k} \geq 2^{2R} - 1 \right\}, \end{aligned} \tag{3}$$

where $\mathcal{S}_r$ is a set of L relays and R denotes a fixed spectral efficiency threshold.

In the second phase and after decoding the received message, a relay with the best second hop channel's SNR is selected from C_L to forward the re-encoded copy of the SU source message to the SU destination. In order to satisfy the PU interference constraint and result in a best performance for the secondary system, the transmit power at R_l must be $P_{\mathsf{R}_l} = \mathcal{I}_\mathsf{p} / \min_m |g_{l,m}|^2$. The SINR at the destination resulting from the lth relay signal can be written as

$$\gamma_{l,\mathsf{d}} = \frac{\frac{\mathcal{I}_\mathsf{p}}{N_0}\frac{|h_{l,\mathsf{d}}|^2}{W_2}}{\frac{P_\mathsf{p}^2}{N_0}|f_{\mathsf{p},\mathsf{d}}|^2 + 1} = \frac{X_{l,\mathsf{d}}}{Y_2 + 1} = \frac{X_{l,\mathsf{d}}}{Z_2}, \tag{4}$$

where P_p^2 is the transmitted power of the interferer at the second communication phase, $W_2 = \min_m |g_{l,m}|^2$, $X_{l,\mathsf{d}}$, Y_2, and Z_2 are some RVs used for an easy follow of the paper. Equivalently, the relay with the best $X_{l,\mathsf{d}}$ is selected to forward the source signal to destination since the denominator is common to the SINRs from all relays belonging to C_L.

The proposed cognitive opportunistic DF relaying scenario works as follows. At the beginning of first communication phase, the SU source obtains the channel information of the PU receivers ($g_{\mathsf{s},m}, m = 1, \ldots, M$) by either a direct reception of pilot signals from primary users [23], or by exchange of channel information between primary and secondary users through a band manager [24]. Using the estimated channels, the SU source knows which spectrum band to share with the PU and determines its transmit power using $P_\mathsf{s} = \mathcal{I}_\mathsf{p} / \min_m |g_{\mathsf{s},m}|^2$. Then, the SU source sends its message to the K SU relays through which each relay calculates its first hop SINR ($\gamma_{\mathsf{s},k}, k = 1, \ldots, K$) using (2) and then compares it with the outage threshold γ_{out}. The relays whose SINRs are greater than γ_{out} are called active relays. At the beginning of second communication phase and through sensing of pilot signals from PU receivers, the active relays obtain their channel information ($g_{k,m}, k = 1, \ldots, L; m = 1, \ldots, M$). Using the estimated channels, each active relay knows which spectrum band to share with the PU and determines its transmit power using $P_{\mathsf{R}_l} = \mathcal{I}_\mathsf{p} / \min_m |g_{l,m}|^2$. Then, each active relay sends a training sequence to the SU destination through which the SU destination calculates the SNR received from the relays ($X_{l,\mathsf{d}}, l = 1, \ldots, L$).[1] To avoid interference between relays while transmitting their training sequences, they can be coordinated to transmit in a time division duplex manner. This is a feasible assumption in TDMA systems. Using the calculated SNRs and according to the opportunistic relay selection criterion, the destination sends a positive acknowledge or a 1-bit feedback to the relay who has the largest SNR asking him to start forwarding a re-encoded version of the source message to it.[2] It is important to mention here that the interference channels at the relays need to be locally estimated by the relay nodes as a step in decoding the source message before it is being forwarded to the destination. This helps in calculating the SINRs ($\gamma_{\mathsf{s},k}, k = 1, \ldots, K$) of the relays to compare with the outage threshold γ_{out} when finding the set of active relays C_L.

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[1] It is worthwhile to mention here that the relay selection in the second hop is performed using the numerators of SINRs received at the destination from the active relays ($X_{l,\mathsf{d}}, l = 1, \ldots, L$). This is because the interference at the destination is common to the SNRs of all relays belonging to C_L.

[2] We are assuming that the channels of the second hop transmission do not change while a decision on which relay is selected is made [25]. This is a valid assumption in relay networks where the data communication is conducted in slow movements and where the channels can be estimated.

3 Exact Outage Performance

In this section, we evaluate exact closed-form expressions for the outage probability of the studied system for the independent non-identically distributed (i.n.i.d.) generic case of relay second hop channels. The outage probability is defined as the probability that the SINR at D goes below a predetermined outage threshold γ_{out}, i.e., $P_{\mathsf{out}} = P_{\mathrm{r}}[\gamma_{\mathsf{D}} \leq \gamma_{\mathsf{out}}]$, where $P_{\mathrm{r}}[\cdot]$ denotes the probability operation. Let C_L be a decoding subset with a number of L active relays (i.e., cardinality $|C_L| = L$), then

$$Pr\left[C_L\right] = \prod_{l \in C_L} P_{\mathrm{r}}\left[\gamma_{\mathsf{s},l} \geq u\right] \prod_{m \notin C_L} P_{\mathrm{r}}\left[\gamma_{\mathsf{s},m} < u\right], \tag{5}$$

where $u = (2^{2R} - 1)$. The outage probability for the studied system can be written as

$$\begin{aligned} P_{\mathsf{out}} &\triangleq \mathrm{P_r}\left[\frac{1}{2}\log_2\left(1+\gamma_{\mathsf{D}}\right) < R\right] \\ &= \sum_{L=0}^{K}\sum_{C_L} \mathrm{P_r}\left[\gamma_{\mathsf{D}} < u|C_L\right] \mathrm{P_r}\left[C_L\right], \end{aligned} \tag{6}$$

where the internal summation is taken over all of $\binom{K}{L}$ possible subsets of size L from the set with the K relays. In order to evaluate (6), we need first to derive $P_{\mathrm{r}}[\gamma_{\mathsf{D}} < u|C_L]$ and $P_{\mathrm{r}}[C_L]$.

First, we find the term $P_{\mathrm{r}}[C_L$ which can be obtained by first deriving the CDF of $\gamma_{\mathsf{s},k}$. This CDF conditioned on W_1 can be obtained using

$$P_{\mathrm{r}}\left[\gamma_{\mathsf{s},k} < u|W_1\right] = \int_1^{\infty} f_{Z_1}(z) \underbrace{\int_0^{uz} f_{X_{\mathsf{s},k}}(x|W_1)\mathrm{d}x}_{F_{X_{\mathsf{s},k}}(uz|W_1)} \mathrm{d}z. \tag{7}$$

The probability density function (PDF) of Z_1 can be directly obtained from the PDF of Y_1 which is given for Rayleigh fading channels as $f_{Y_1}(y) = \alpha_{\mathsf{p},k}\exp(-\alpha_{\mathsf{p},k}y)$, where $\alpha_{\mathsf{p},k} = (\bar{\gamma}_{\mathrm{r}}^{I}\beta_{\mathsf{p},k})^{-1}$ and $\bar{\gamma}_{\mathrm{r}}^{I} = \frac{P_{\mathrm{p}}^{1}}{N_0}$. Using transformation of RVs, the PDF of Z_1 can be easily obtained as $f_{Z_1}(z) = \alpha_{\mathsf{p},k}\exp(-\alpha_{\mathsf{p},k})\exp(-\alpha_{\mathsf{p},k}z)$. The CDF of $X_{\mathsf{s},k}$ conditioned on W_1 can be obtained as

$$F_{X_{\mathsf{s},k}}\left(x|W_1\right) = 1 - \exp\left(-\lambda_{\mathsf{s},k}W_1 x\right), \tag{8}$$

where $\lambda_{\mathsf{s},k} = 1/\left(\Omega_{\mathsf{s},k}\frac{\mathcal{I}_{\mathrm{p}}}{N_0}\right)$. Upon substituting $f_{Z_1}(z)$ and (8) in (7) and after doing the integration, we get

$$P_{\mathrm{r}}\left[\gamma_{\mathsf{s},k} < u|W_1\right] = 1 - \alpha_{\mathsf{p},k}\frac{\exp\left(-\left(\lambda_{\mathsf{s},k}W_1 u\right)\right)}{\left(\lambda_{\mathsf{s},k}W_1 u + \alpha_{\mathsf{p},k}\right)}. \tag{9}$$

Assuming i.n.i.d. channels between the SU source and the PU receivers, the CDF and PDF of W_1 are, respectively, given by

$$\begin{aligned} F_{W_1}(w) &= 1 - \prod_{m=1}^{M}\left(1 - F_{|g_{\mathsf{s},m}|^2}(w)\right) \\ &= 1 - \exp\left(-\sum_{m=1}^{M}\zeta_{\mathsf{s},m}w\right), \\ f_{W_1}(w) &= \sum_{m=1}^{M}\zeta_{\mathsf{s},m}\exp\left(-\sum_{m=1}^{M}\zeta_{\mathsf{s},m}w\right), \end{aligned} \tag{10}$$

where $\zeta_{\mathsf{s},m} = 1/\mu_{\mathsf{s},m}$. Now, averaging over the PDF of W_1 using $\int_0^{\infty} P_{\mathrm{r}}[\gamma_{\mathsf{s},k} < u|W_1]f_{W_1}(w)\mathrm{d}w$, and with the help of [26, Eq. (3.352.2)], we get

$$\begin{aligned} P_{\mathrm{r}}\left[\gamma_{\mathsf{s},k} < u\right] = 1 &+ \alpha_{\mathsf{p},k}\left(\frac{\sum_{m=1}^{M}\zeta_{\mathsf{s},m}}{\lambda_{\mathsf{s},k}}\right) \\ &\times \exp\left(\alpha_{\mathsf{p},k}\left(1 + \frac{\sum_{m=1}^{M}\zeta_{\mathsf{s},m}}{\lambda_{\mathsf{s},k}u}\right)\right) \\ &\times \mathrm{Ei}\left(-\alpha_{\mathsf{p},k}\left(1 + \frac{\sum_{m=1}^{M}\zeta_{\mathsf{s},m}}{\lambda_{\mathsf{s},k}u}\right)\right)u^{-1}, \end{aligned} \tag{11}$$

where $\mathrm{E_i}(\cdot)$ is the exponential integral defined by [26, Eq. (8.211.1)]. Upon substituting (11) in (5), the term $P_{\mathrm{r}}[C_L]$ can be calculated.

Now, we derive the first term in (6) which is $P_{\mathrm{r}}[\gamma_{\mathsf{D}} < u|C_L]$. With opportunistic or best-relay selection scheme, the CDF of γ_{D} conditioned on C_L, W_2 can be obtained using

$$P_{\mathrm{r}}\left[\gamma_{\mathsf{D}} < u|C_L, W_2\right] = \int_1^{\infty} f_{Z_2}(z) \underbrace{\int_0^{uz} f_{X_{\mathrm{Sel}}}(x|W_2)\mathrm{d}x}_{F_{X_{\mathrm{Sel}}}(uz|W_2)} \mathrm{d}z, \tag{12}$$

where $f_{\mathrm{Sel}}(x|W_2)$ is the PDF of the best relay conditioned on W_2.

The PDF of Z_2 can be directly obtained from the PDF of Y_2 which is given for Rayleigh fading channels as $f_{Y_2}(y) = \alpha_{\mathsf{p,d}}\exp(-\alpha_{\mathsf{p,d}}y)$, where $\alpha_{\mathsf{p,d}} = (\bar{\gamma}_{\mathsf{d}}^{I}\beta_{\mathsf{p,d}})^{-1}$, where $\bar{\gamma}_{\mathsf{d}}^{I} = \frac{P_{\mathrm{p}}^{2}}{N_0}$. Using transformation of RVs, the PDF of Z_2 can be easily obtained as $f_{Z_2}(z) = \alpha_{\mathsf{p,d}}\exp(-\alpha_{\mathsf{p,d}})\exp(-\alpha_{\mathsf{p,d}}z)$. The CDF of X_{Sel} conditioned on W_2 can be written as

$$F_{X_{\mathrm{Sel}}}(x|W_2) = \prod_{l=1}^{L} F_{X_{l,\mathsf{d}}}(x|W_2), \tag{13}$$

where $F_{X_{l,\mathrm{d}}}(x|W_2)$ is given by

$$F_{X_{l,\mathrm{d}}}(x|W_2) = 1-\exp\left(-\lambda_{l,\mathrm{d}}W_2x\right), \tag{14}$$

where $\lambda_{l,\mathrm{d}} = 1/(\Omega_{l,\mathrm{d}}\frac{\mathcal{I}_\mathrm{p}}{N_0})$.

Upon substituting (14) in (13) and applying the identity

$$\prod_{l=1}^{L}(1-q_l) = \sum_{l=0}^{L}\frac{(-1)^l}{l!}\sum_{n_1,\dots,n_l}^{L}\prod_{t=1}^{l}q_{n_t}, \tag{15}$$

with $\sum_{n_1,\dots,n_l}^{L}$ being a short hand-notation for $\sum\cdots\sum_{\substack{n_1=\cdots=n_l=1\\ n_1\neq\cdots\neq n_l}}$, (13) can be rewritten as

$$F_{X_{\mathrm{Sel}}}(x|W_2) = \sum_{l=0}^{L}\frac{(-1)^l}{l!}\sum_{n_1,\dots,n_l}^{L}\prod_{t=1}^{l}\exp\left(-\lambda_{n_t,\mathrm{d}}W_2x\right). \tag{16}$$

Upon substituting (16) in (12), and after simple manipulations, we get

$$P_\mathrm{r}\left[\gamma_\mathrm{D} < u|C_L, W_2\right] = \alpha_{\mathrm{p,d}}\sum_{l=0}^{L}\frac{(-1)^l}{l!}\sum_{n_1,\dots,n_l}^{L} \times\prod_{t=1}^{l}\frac{\exp\left(-\left(\lambda_{n_t,\mathrm{d}}W_2u+\alpha_{\mathrm{p,d}}\right)\right)}{\left(\lambda_{n_t,\mathrm{d}}W_2u+\alpha_{\mathrm{p,d}}\right)}. \tag{17}$$

Assuming i.n.i.d. channels between the SU relays and the PU receivers, the CDF and PDF of W_2 are, respectively, given by

$$\begin{aligned} F_{W_2}(w) &= 1-\prod_{m=1}^{M}\left(1-F_{|g_{n_t,m}|^2}(w)\right)\\ &= 1-\exp\left(-\sum_{m=1}^{M}\zeta_{n_t,m}w\right),\\ f_{W_2}(w) &= \sum_{m=1}^{M}\zeta_{n_t,m}\exp\left(-\sum_{m=1}^{M}\zeta_{n_t,m}w\right), \end{aligned} \tag{18}$$

where $\zeta_{n_t,m} = 1/\mu_{n_t,m}$.
Averaging over the PDF of W_2 using $\int_0^\infty P_\mathrm{r}[\gamma_\mathrm{D} < u|C_L, W_2]\, f_{W_2}(w)\mathrm{d}w$, and with the help of [26, Eq. (3.352.4)], we get

$$\begin{aligned} P_\mathrm{r}\left[\gamma_\mathrm{D} < u|C_L\right] = &-\alpha_{\mathrm{p,d}}\sum_{l=0}^{L}\frac{(-1)^l}{l!}\sum_{n_1,\dots,n_l}^{L}\\ &\times\prod_{t=1}^{l}\left(\frac{\sum_{m=1}^{M}\zeta_{n_t,m}}{\lambda_{n_t,\mathrm{d}}}\right)\\ &\times\exp\left(\frac{\sum_{m=1}^{M}\zeta_{n_t,m}\alpha_{\mathrm{p,d}}}{\lambda_{n_t,\mathrm{d}}u}\right)\\ &\times\mathrm{E_i}\left(-\frac{\sum_{m=1}^{M}\zeta_{n_t,m}\alpha_{\mathrm{p,d}}}{\lambda_{n_t,\mathrm{d}}u}\right)u^{-1}. \end{aligned} \tag{19}$$

Having the terms $P_\mathrm{r}[C_L]$, $P_\mathrm{r}[\gamma_\mathrm{D} < u|C_L]$ being obtained, a closed-form expression for the outage probability in (6) can be achieved.

4 Asymptotic Outage Performance

Due to complexity of the achieved expressions in previous sections, it is hard to get more insights about system performance. Therefore, we see it is important to derive simple expressions where more insights about the system behavior can be achieved.

At high SNR values, the outage probability can be expressed as $P_\mathrm{out} \approx (G_\mathrm{c}\mathrm{SNR})^{-G_\mathrm{d}}$, where G_c denotes the coding gain of the system and G_d is the diversity order of the system. In the upcoming analysis, the relays are assumed to have identical second hop channels ($\lambda_{1,\mathrm{d}} = \cdots = \lambda_{K,\mathrm{d}} = \lambda_{\mathrm{r,d}}$) and identical $\mathsf{R} \to \mathsf{P}_m$ links ($\zeta_{1,1} = \cdots = \zeta_{1,M} = \zeta_{1,\mathrm{p}}$), and ($\zeta_{1,\mathrm{p}} = \cdots = \zeta_{K,\mathrm{p}} = \zeta_{\mathrm{r,p}}$). Also, the channels between the SU source and PU receivers $\mathsf{S} \to \mathsf{P}_m$ are assumed to be identical ($\zeta_{\mathrm{s},1} = \cdots = \zeta_{\mathrm{s},M} = \zeta_{\mathrm{s,p}}$). As $\frac{\mathcal{I}_\mathrm{p}}{N_0} \to \infty$, the CDF in (8) simplifies to $F_{X_{\mathrm{s},k}}(x|W_1) \approx \lambda_{\mathrm{s},k}W_1\gamma$. Upon substituting this CDF in (7) and following the same procedure as in Sect. 3, the CDF $P_\mathrm{r}[\gamma_{\mathrm{s},k} < u]$ which is a part of the term $P_\mathrm{r}[C_L$ can be obtained at high SNR values with the help of [26, Eq. (3.381.3)] as

$$P_\mathrm{r}\left[\gamma_{\mathrm{s},k} < u\right] \approx \frac{\exp(\alpha_{\mathrm{p},k})}{\alpha_{\mathrm{p},k}}\left(\frac{\lambda_{\mathrm{s},k}}{M\zeta_{\mathrm{s,p}}}\right)\Gamma(2,\alpha_{\mathrm{p},k})u. \tag{20}$$

In evaluating $P_\mathrm{r}[\gamma_\mathrm{D} < u|C_L]$, as $\frac{\mathcal{I}_\mathrm{p}}{N_0} \to \infty$, the CDF in (14) simplifies for the identical case to $F_{X_{\mathrm{r,d}}}(x|W_2) \approx \lambda_{\mathrm{r,d}}W_2\gamma$. Upon substituting this CDF in (12) and following the same procedure as in Sect. 3, the term $P_\mathrm{r}[\gamma_\mathrm{D} < u|C_L]$ can be obtained at high SNR with the help of [26, Eq. (3.351.2)] and [26, Eq. (3.351.3)] as

$$P_\mathrm{r}[\gamma_\mathrm{D} < u|C_L] \approx L!(\alpha_{\mathrm{p,d}})^{-L}\left(\frac{\lambda_{\mathrm{r,d}}}{M\zeta_{\mathrm{r,p}}}\right)^L u^L. \tag{21}$$

The transmit power of the PU transmitter can be assumed to be fixed or it can be assumed to be dependent of and scaling with the transmit power of the SU transmitters. These two cases were considered in [27]. The case where the transmit power of the PU transmitter is fixed was considered in [10,12]. In practice, the network where the interference power scales with the desired user power is called symmetric network, whereas, the network where the interference power is fixed and not related to the desired user power is called asymmetric network [28]. The first case could reflect the situation where the secondary user is located far away from the primary transmitter or interferer. In this case, the interference caused by the primary transmitter on the secondary user will be very small and can be assumed to be constant. In such condition, the secondary user can increase or decrease its transmit power according to his requirements and in an independent way from the primary transmitter power. The second case could reflect the situation where the secondary user and primary transmitter or interferer are located closely and near the edge of their cells. In this case, the interference from the interferer on the secondary user can be strong and even harmful on the performance of the secondary system. In such case, the interference power could take a wide range of values and will be affecting the operating conditions of the secondary user. In order to capture these effects, the value of the interference power is taken to be dependent on the transmit power of the secondary user. These two cases are considered in the following analysis.

Case 1 $\bar{\gamma}^I_{\text{r}}, \bar{\gamma}^I_{\text{d}}$ are constants (asymmetric network).
With a simple analysis, we noticed that the outage probability for this case is dominated by the first term in (6) $P_{\text{r}}[\gamma_{\text{D}} < u|C_L]$ which was obtained in (21). For this case, the parameter $\alpha_{\text{p,d}}$ can be approximated by $\alpha_{\text{p,d}} \approx (\bar{\gamma}^I_{\text{d}}\beta_{\text{p,d}})^{-1}$. Recalling that for the identical case, we have $\lambda_{\text{r,d}} = 1/(\Omega_{\text{r,d}}\frac{\mathcal{I}_{\text{p}}}{N_0})$. Hence, the outage probability at high SNR values can be obtained in a simple expression as

$$P^{\infty}_{\text{out}} = \left[(L!)^{-1/L}\frac{\bar{\gamma}^I_{\text{d}}\beta_{f_{\text{p,d}}}M\zeta_{\text{s,p}}\Omega_{\text{r,d}}}{\gamma_{\text{out}}}\frac{\mathcal{I}_{\text{p}}}{N_0}\right]^{-L}. \tag{22}$$

As can be seen from (22), the coding gain of the system is affected by the parameters $\bar{\gamma}^I_{\text{d}}$, $\beta_{\text{p,d}}$, M, $\zeta_{\text{s,p}}$, $\Omega_{\text{r,d}}$, and γ_{out}; while the diversity order equals the number of active relays L. It is worthwhile to mention here that the maximum number of active relays in DF relay networks could reach the total number of relays K and this makes the diversity order of the system in (22) equal K. Furthermore, it can be noticed from (22) that the diversity order of cognitive opportunistic DF relay networks with partial relay selection is the same as that of its non-cognitive counterpart and is independent of the primary cell [29]. With fixed interference power, the interference from primary users will be degrading the performance of the secondary users through effecting the coding gain without affecting the diversity order.

Case 2 $\bar{\gamma}^I_{\text{r}}, \bar{\gamma}^I_{\text{d}}$ are scaling with SNR (symmetric network).
In this case, the interference powers can be expressed as $\bar{\gamma}^I_{\text{r}} = a\frac{\mathcal{I}_{\text{p}}}{N_0}$, $\bar{\gamma}^I_{\text{d}} = b\frac{\mathcal{I}_{\text{p}}}{N_0}$, where a, b are some positive numbers. As the interference at the relays differs than that at the destination, we have the following different subcases; $\bar{\gamma}^I_{\text{r}} = a\frac{\mathcal{I}_{\text{p}}}{N_0}$ or $\bar{\gamma}^I_{\text{d}} = b\frac{\mathcal{I}_{\text{p}}}{N_0}$ or $\bar{\gamma}^I_{\text{r}} = a\frac{\mathcal{I}_{\text{p}}}{N_0}$ and $\bar{\gamma}^I_{\text{d}} = b\frac{\mathcal{I}_{\text{p}}}{N_0}$. With $\bar{\gamma}^I_{\text{r}} = a\frac{\mathcal{I}_{\text{p}}}{N_0}$ and fixed $\bar{\gamma}^I_{\text{d}}$, we noticed that the outage probability for this case is dominated by the second term in (6) $P_{\text{r}}[C_L]$ which can be obtained using the CDF derived in (20). Also, the parameter $\alpha_{\text{p,r}}$ can be approximated by $\alpha_{\text{p,r}} \approx (\bar{\gamma}^I_{\text{r}}\beta_{\text{p,r}})^{-1} = (a\frac{\mathcal{I}_{\text{p}}}{N_0}\beta_{\text{p,r}})^{-1}$. Recalling that for the identical case, we have $\lambda_{\text{s,r}} = 1/(\Omega_{\text{s,r}}\frac{\mathcal{I}_{\text{p}}}{N_0})$. Hence, the outage probability at high SNR values can be obtained in a simple expression as

$$\begin{aligned}P^{\infty}_{\text{out}} &= \left(a\frac{\mathcal{I}_{\text{p}}}{N_0}\beta_{\text{p,r}}\right)\exp\left(1\Big/a\frac{\mathcal{I}_{\text{p}}}{N_0}\beta_{\text{p,r}}\right)\left(\frac{\Omega_{\text{s,r}}\frac{\mathcal{I}_{\text{p}}}{N_0}}{M\mu_{\text{s,p}}}\right)^{-1}\\ &\quad\times\Gamma\left(2, 1\Big/a\frac{\mathcal{I}_{\text{p}}}{N_0}\beta_{\text{p,r}}\right)\gamma_{\text{out}}\\ &= (a\beta_{\text{p,r}})\left(\Omega_{\text{s,r}}/M\mu_{\text{s,p}}\right)^{-1}\gamma_{\text{out}},\end{aligned} \tag{23}$$

where at high SNR, the exponential term and the incomplete Gamma function term in the first line of (23) reach the value of 1. As can be seen from (23), when the interference at the relays has a power that scales with SNR, the diversity gain of the system reaches zero and a noise floor is expected to appear in the results. In such case, the system performance is affected by various parameters such as $\beta_{\text{p,r}}$, $\Omega_{\text{s,r}}$, M, $\mu_{\text{s,p}}$, and γ_{out}.

When the interference at the destination scales with SNR $\bar{\gamma}^I_{\text{d}} = b\frac{\mathcal{I}_{\text{p}}}{N_0}$, and the interference at the relays has a fixed power, we noticed that the outage probability for this case is dominated by the first term in (6) $P_{\text{r}}[\gamma_{\text{D}} < u|C_L]$, which was obtained in (21). Also, the parameter $\alpha_{\text{p,d}}$ can be approximated by $\alpha_{\text{p,d}} \approx (\bar{\gamma}^I_{\text{d}}\beta_{\text{p,d}})^{-1} = (b\frac{\mathcal{I}_{\text{p}}}{N_0}\beta_{\text{p,d}})^{-1}$. Recalling that for the identical case, we have $\lambda_{\text{r,d}} = 1/(\Omega_{\text{r,d}}\frac{\mathcal{I}_{\text{p}}}{N_0})$. Hence, the outage probability at high SNR values can be obtained in a simple expression as

$$P^{\infty}_{\text{out}} = \left[(L!)^{-1/L}\frac{\bar{\gamma}^I_{\text{d}}b\beta_{\text{p,d}}M\zeta_{\text{r,p}}\Omega_{\text{r,d}}}{\gamma_{\text{out}}}\right]^{-L}. \tag{24}$$

It is clear from (24) that when the interference at the destination has a power that scales with SNR, the system achieves a diversity gain of zero and the system performance is affected by various parameters including L, $\bar{\gamma}^I_{\text{d}}$, $\beta_{\text{p,d}}$, M, $\zeta_{\text{r,p}}$, $\Omega_{\text{r,d}}$, and γ_{out}.

Finally, when the interference at the relays and the interference at the destination have powers that scale with SNR $\bar{\gamma}^I_{\text{r}} = a\frac{\mathcal{I}_{\text{p}}}{N_0}$, $\bar{\gamma}^I_{\text{d}} = b\frac{\mathcal{I}_{\text{p}}}{N_0}$, the outage probability for this case was shown to be dominated by the second term in (6) $P_{\text{r}}[C_L]$, which can be obtained using the CDF derived in (20). Therefore, the outage probability for this case is similar to that found in (23) where the diversity gain reaches zero and the system behavior is affected by several parameters including $\beta_{\mathrm{p,r}}$, $\Omega_{\mathrm{s,r}}$, M, $\mu_{\mathrm{s,p}}$, and γ_{out}.

5 Simulation and Numerical Results

In this section, we illustrate the validity of the achieved analytical and asymptotic expressions via a comparison with Monte Carlo simulations. We also provide some numerical examples to show the effect of the interference and some system parameters such as number of relays and number of PU receivers on the system performance.

Figure 2 portrays the outage performance versus SNR for different numbers of relays K. We can see from this figure that perfect fitting exists between the analytical and asymptotic results with Monte Carlo simulations. Also, one can see from this figure that with constant interference power, as K increases, the diversity order of the system increases and the system performance enhances. This is clear from the asymptotic results where the diversity order equals K when the interference power is fixed. Also, we can see from this figure that the system can still achieve full diversity gain as SNR increases.

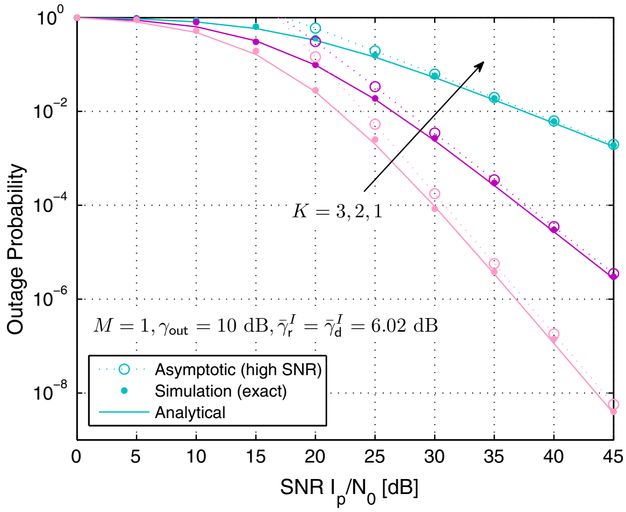

Fig. 2 Outage probability versus SNR of cognitive opportunistic DF relay network with interference and multiple PU receivers for different values of K and $\mu_{\mathrm{s,p}} = 0.9$, $\mu_{\mathrm{r,p}} = 0.1$, $\Omega_{\mathrm{s},1} = 0.6$, $\Omega_{\mathrm{s},2} = 0.7$, $\Omega_{\mathrm{s},3} = 0.8$, $\Omega_{\mathrm{r,d}} = 0.8$, $\beta_{\mathrm{p,r}} = 0.7$, $\beta_{\mathrm{p,d}} = 0.9$

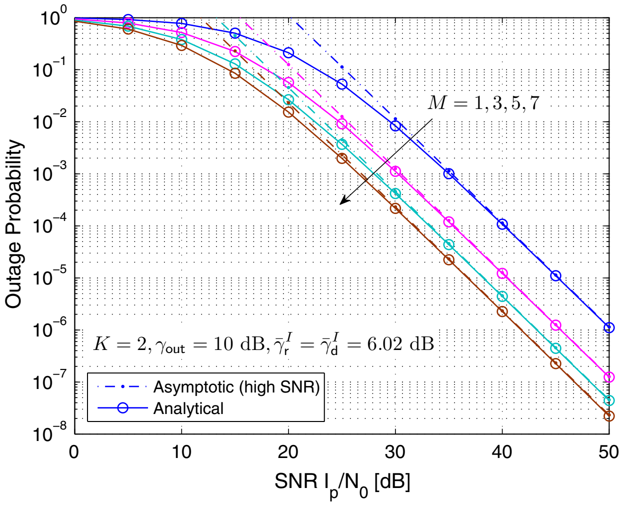

Fig. 3 Outage probability versus SNR of cognitive opportunistic DF relay network with interference and multiple PU receivers for different values of M and $\mu_{\mathrm{s},1} = \cdots = \mu_{\mathrm{s},7} = 1.8$, $\mu_{\mathrm{r},1} = \cdots = \mu_{\mathrm{r},7} = 0.02$, $\Omega_{\mathrm{s},1} = 0.6$, $\Omega_{\mathrm{s},2} = 0.7$, $\Omega_{\mathrm{r,d}} = 0.8$, $\beta_{\mathrm{p,r}} = 0.7$, $\beta_{\mathrm{p,d}} = 0.9$

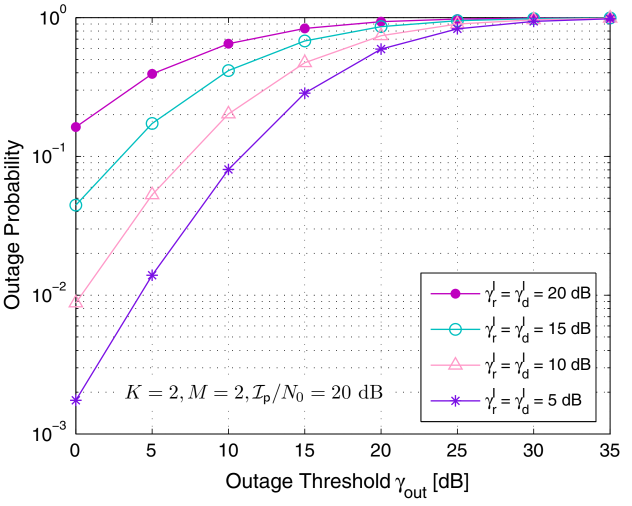

Fig. 4 Outage probability versus γ_{out} of cognitive opportunistic DF relay network with interference and multiple PU receivers for different values of $\bar{\gamma}_{\mathrm{r}}^{I}$, $\bar{\gamma}_{\mathrm{d}}^{I}$ and $\mu_{\mathrm{s},1} = \mu_{\mathrm{s},2} = 1.8$, $\mu_{\mathrm{r},1} = \mu_{\mathrm{r},2} = 0.02$, $\Omega_{\mathrm{s},1} = 0.6$, $\Omega_{\mathrm{s},2} = 0.7$, $\Omega_{\mathrm{r,d}} = 0.8$, $\beta_{\mathrm{p,r}} = 0.7$, $\beta_{\mathrm{p,d}} = 0.9$

The effect of number of PU receivers M on the system performance is studied in Fig. 3. Again, the perfect fitting between the analytical results and asymptotic results is clear in this figure. More importantly, we can see that as M increases, better the achieved performance. This is because having more PU receivers increases the probability to find primary receivers of weak channels and hence, higher the transmit power of the SU transmitters. Furthermore, we can see that the number of PU receivers affects the system performance through affecting its coding gain and not the diversity order. Again, this fact was proved by the asymptotic results.

Figure 4 illustrates the interference effect on the system performance. It portrays the outage behavior versus outage threshold γ_{out} for different values of interference powers $\bar{\gamma}_{\mathrm{r}}^{I}$, $\bar{\gamma}_{\mathrm{d}}^{I}$ when they are equal. As expected, as $\bar{\gamma}_{\mathrm{r}}^{I}$, $\bar{\gamma}_{\mathrm{d}}^{I}$ increase, worse the achieved performance. Also, the continuous increase in γ_{out} results in a unity outage probability.

The outage performance versus SNR is plotted in Fig. 5 for different values of outage threshold γ_{out}. Two cases are

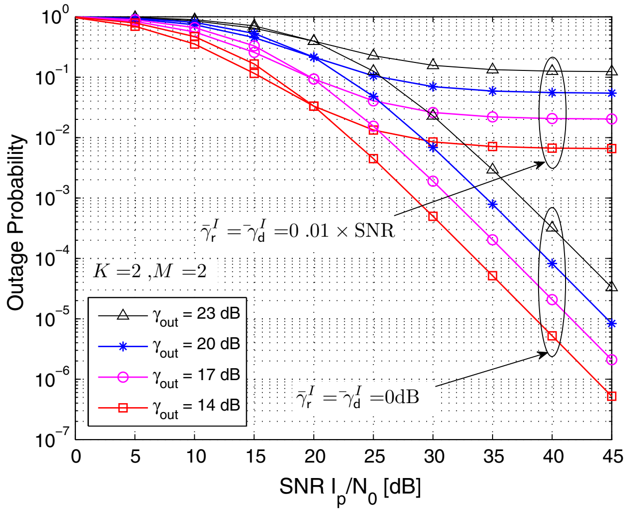

Fig. 5 Outage probability versus SNR of cognitive opportunistic DF relay network with interference and multiple PU receivers for different values of γ_{out} and $\mu_{\mathrm{s},1} = \mu_{\mathrm{s},2} = 0.7$, $\mu_{\mathrm{r},1} = \mu_{\mathrm{r},2} = 0.1$, $\Omega_{\mathrm{s},1} = 0.6$, $\Omega_{\mathrm{s},2} = 0.7$, $\Omega_{\mathrm{r,d}} = 0.8$, $\beta_{\mathrm{p,r}} = 0.7$, $\beta_{\mathrm{p,d}} = 0.9$

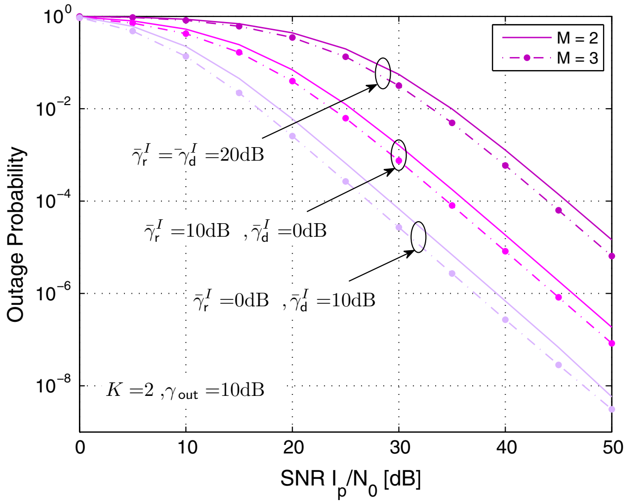

Fig. 6 Outage probability versus SNR of cognitive opportunistic DF relay network with interference and multiple PU receivers for different values of $\bar{\gamma}_{\mathrm{r}}^I, \bar{\gamma}_{\mathrm{d}}^I, M$ and $\mu_{\mathrm{s},1} = \cdots = \mu_{\mathrm{s},3} = 0.7$, $\mu_{\mathrm{r},1} = \cdots = \mu_{\mathrm{r},3} = 0.1$, $\Omega_{\mathrm{s},1} = 0.6$, $\Omega_{\mathrm{s},2} = 0.7$, $\Omega_{\mathrm{r,d}} = 0.8$, $\beta_{\mathrm{p,r}} = 0.7$, $\beta_{\mathrm{p,d}} = 0.9$

shown in this figure: the case where the interference power scales with SNR and the case where the interference power is fixed. We can see that when the interference power scales with SNR, the system diversity gain reaches zero and a noise floor appears in all curves of this figure due to the effect of interference on the system performance. On the other hand, when the interference power is fixed, the system performance keeps enhancing as SNR increases. Also, we see from this figure that the outage threshold is affecting the system behavior through affecting its coding gain and not the diversity order.

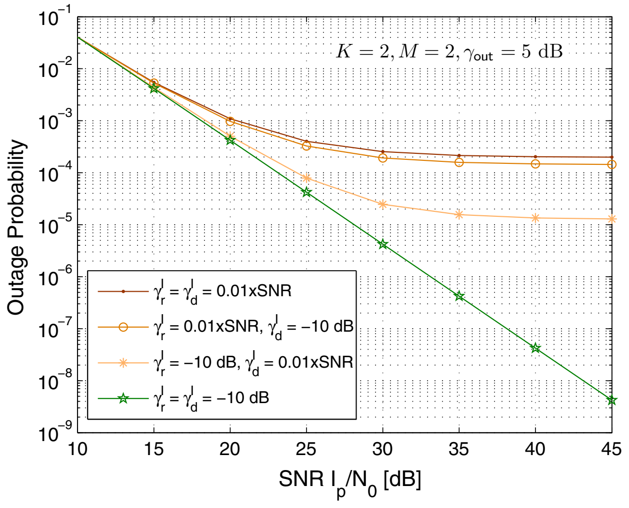

Fig. 7 Outage probability versus SNR of cognitive opportunistic DF relay network with interference and multiple PU receivers for different values of $\bar{\gamma}_{\mathrm{r}}^I, \bar{\gamma}_{\mathrm{d}}^I$ and $\mu_{\mathrm{s},1} = \mu_{\mathrm{s},2} = 0.7$, $\mu_{\mathrm{r},1} = \mu_{\mathrm{r},2} = 0.1$, $\Omega_{\mathrm{s},1} = 0.6$, $\Omega_{\mathrm{s},2} = 0.7$, $\Omega_{\mathrm{r,d}} = 0.8$, $\beta_{\mathrm{p,r}} = 0.7$, $\beta_{\mathrm{p,d}} = 0.9$

Figure 6 shows the outage performance versus SNR for different numbers of PU receivers M and different values of interference powers $\bar{\gamma}_{\mathrm{r}}^I, \bar{\gamma}_{\mathrm{d}}^I$ including the case when they are unequal. The figure aims to compare the interference severity at the relays and destination on the system performance. It is obvious from this figure that the interference at the relay nodes is more severe on the system behavior compared to that at the destination node. This is because the interference at the relays affects the signal on the first hop which is also affecting the re-encoded signal on the second hop. In other words, the signal processing conducted by the relay nodes is negatively affected by the interference and this results in a further degradation in the system performance. Furthermore, the enhancement in system performance due to having more PU receivers is clear in this figure. Clearly, this enhancement in system performance happens in the coding gain and not the diversity order of the system.

The asymptotic behavior of the system is studied in Fig. 7. Two cases are shown in this figure: a system performance with full diversity gain and a system performance with zero diversity gain. The system can achieve full diversity gain only if the interference at the relays and destination is assumed to be fixed and not scaling with SNR. This was summarized in case 1 of the asymptotic analysis section. On the other hand, when the interference at the relays or at the destination or at both scales with SNR, the system reaches zero diversity gain and a noise floor appears in the results due to the effect of interference on the system performance. Furthermore, it can be seen from this figure that the worst performance is achieved when both $\bar{\gamma}_{\mathrm{r}}^I, \bar{\gamma}_{\mathrm{d}}^I$ scale with SNR, as expected. Also, the case where $\bar{\gamma}_{\mathrm{r}}^I$ scales with SNR results in worse performance compared to the case where $\bar{\gamma}_{\mathrm{d}}^I$ scales with SNR as the interference at the relays is more severe on the system

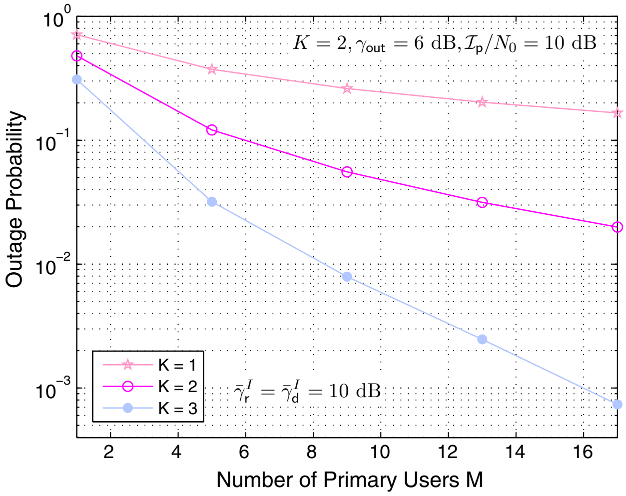

Fig. 8 Outage probability versus M of cognitive opportunistic DF relay network with interference and multiple PU receivers for different values of K and $\mu_{\mathsf{s},1} = \cdots = \mu_{\mathsf{s},17} = 0.7, \mu_{\mathsf{r},17} = \cdots = \mu_{\mathsf{r},3} = 0.1$, $\Omega_{\mathsf{s},1} = 0.6$, $\Omega_{\mathsf{s},2} = 0.7$, $\Omega_{\mathsf{s},3} = 0.8$, $\Omega_{\mathsf{r,d}} = 0.8$, $\beta_{\mathsf{p,r}} = 0.7$, $\beta_{\mathsf{p,d}} = 0.9$

performance compared to that at the destination node. This result was also illustrated in Fig. 6.

Figure 8 studies the impact of number of PU receivers M on the system behavior. It portrays the outage performance versus M for different numbers of relays K. It can be seen from this figure that when M increases, the outage probability decreases, but the slope of the curves depends on the value of K. The highest slope is achieved at the largest value of K, as expected.

6 Conclusion

In this paper, we proposed and evaluated the outage performance of a new scenario in spectrum-sharing opportunistic DF relay networks where the PU receivers are assumed to utilize orthogonal spectrum bands in the presence of interference from PU transmitter. Closed-form expression was derived for the outage probability assuming the i.n.i.d. generic case of relays second hop channels. Furthermore, the system outage performance was evaluated at the high SNR regime where simple expressions for the outage probability, diversity order, and coding gain were derived. Monte Carlo simulations proved the accuracy of the achieved analytical and asymptotic results. Main findings illustrated that with fixed interference power, the diversity order of the secondary system equals the number of relays and it is not affected by the number of PU receivers. Also, results showed that the number of PU receivers affects the system performance through affecting the coding gain. Finally, results illustrated that when the interference at the SU relays or the SU destination or at both scales with SNR, the system reaches a zero diversity gain and a noise floor appears in the results due to the effect of interference on the system performance.

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References

1. Haykin, S.: Cognitive radio: brain-empowered wireless communications. IEEE J. Sel. Areas Commun. **23**, 201–220 (2005)
2. Goldsmith, A.; Jafar, S.; Maric, I.; Srinivasa, S.: Breaking spectrum gridlock with cognitive radios: an information theoretic perspective. Proc. IEEE **97**(5), 894–914 (2009)
3. Laneman, J.N.; Tse, D.N.C.; Wornell, G.W.: Cooperative diversity in wireless networks: efficient protocols and outage behavior. IEEE Trans. Info. Theory **50**(12), 3062–3080 (2004)
4. Hong, J.-p.; Hong, B.; Ban, T.W.; Choi, W.: On the cooperative diversity gain in underlay cognitive radio systems. IEEE Trans. Commun. **60**(1), 209–219 (2012)
5. Zhang, X.; Yan, Z.; Gao, Y.; Wang, W.: On the study of outage performance for cognitive relay networks (CRN) with the Nth best-relay selection in Rayleigh-fading channels. IEEE Wireless Commun. Lett. **2**(1), 110–113 (2013)
6. Guimarães, F.R.V.; da Costa, D.B.; Tsiftsis, T.A.; Cavalcante, C.C.; Karagiannidis, G.K.: Multiuser and multirelay cognitive radio networks under spectrum-sharing constraints. IEEE Trans Veh. Technol. **63**(1), 433–439 (2014)
7. Afana, A.; Asghari, V.; Ghrayeb, A.; Affes, S.: On the performance of cooperative relaying spectrum-sharing systems with collaborative distributed beamforming. IEEE Trans. Commun. **62**(3), 857–871 (2014)
8. Hussain, S.I.; Alouini, M.-S.; Qaraqe, K.; Hasna, M.: Reactive relay selection in underlay cognitive networks with fixed gain relays, In: Proceedings of the IEEE International Conference on Communication (ICC '12), Canada, pp. 1784–1788 (2012)
9. Chen, J.; Li, Z.; Si, J.; Huang, H.; Gao, R.: Outage probability analysis of partial AF relay selection in spectrum-sharing scenario with multiple primary users. Electron. Lett. **48**(19), 1211–1212 (2012)
10. Guan, X.; Yang, W.; Cai, Y.: Outage performance of statistical CSI assisted cognitive relay with interference from primary user. IEEE Commun. Lett. **17**(7), 1416–1419 (2013)
11. Xing, J.; Zhang, X.; Wang, J.; Zhang, Z.; Wang, W.: Performance analysis of cognitive relay networks with imperfect channel knowledge over Nakagami-m fading channels. In: Proceedings of the IEEE Wireless Communication and Network Conference (WCNC '14), Turkey, pp. 851–856 (2014)
12. Xu, W.; Zhang, J.; Zhang, P.; Tellambura, C.: Outage probability of decode-and-forward cognitive relay in presence of primary user's interference. IEEE Commun. Lett. **16**(8), 1252–1255 (2012)
13. Yang, P.; Zhang, Q.; Luo, L.; Qin, J.: Outage performance of underlay cognitive opportunistic multi-relay networks in the presence of interference from primary user. Wirel. Pers. Commun. **74**, 343–358 (2014)
14. Prasad, B.; Roy, S.D.; Kundu, S.: Outage performance of cognitive relay network with imperfect channel estimation under proactive DF relaying. In: Proceedings of the 20th National Conference on Communication, India, February 28–March 2, pp. 1–6 (2014)
15. Wang, X.; Zhang, H.; Gulliver, T.A.; Shi, W.: Outage performance of a proactive DF cognitive relay network with a maximum transmit power limit. J. Inf. Comput. Sci. **10:18**, 5927–5934 (2013)

16. Sagong, S.; Lee, J.; Hong, D.: Capacity of reactive DF scheme in cognitive relay networks. IEEE Trans. Wirel. Commun. **10**(10), 3133–3138 (2011)
17. Tran, H.; Duong, T.Q.; Zepernick, H.-J.: Performance analysis of cognitive relay networks under power constraint of multiple primary users. In: Proceedings of the IEEE Global Communication Conference (GLOBECOM '11), USA, pp. 1–6 (2011)
18. Yeoh, P.L.; Elkashlan, M.; Kim, K.J.; Duong, T.Q.; Karagiannidis, G.K.: Cognitive MIMO relaying with multiple primary transceivers. In: Proceedings of the IEEE Global Communication Conference (GLOBECOM '13), USA, pp. 1978–1983 (2013)
19. Guimarães, F.R.V.; da Costa, D.B.; Benjillali, M.; Tsiftsis, T.A.; Karagiannidis, G.K.: Best relay selection in cooperative spectrum sharing systems with multiple primary users. In: Proceedings of the IEEE International Communication Conference (ICC '13), Hungary, pp. 2661–2665 (2013)
20. Afana, A.; Ghrayeb, A.; Asghari, V.; Affes, S.: Distributed beamforming for spectrum-sharing relay networks under mutual primary-secondary interference. In: Proceedings of the IEEE Wireless Communication and Network Conference (WCNC'14), Turkey, pp.845–850 (2014)
21. Wang, C.; Chen, H.; Yin, Q.; Feng, A.; Molisch, A.F.: Multiuser two-way relay networks with distributed beamforming. IEEE Trans. Wirel. Commun. **10**(10), 3460–3471 (2011)
22. Li, Y.; Yin, Q.; Sun, L.; Chen, H.; Wang, H.-M.: A channel quality metric in opportunistic selection with outdated CSI over Nakagami-*m* fading channels. IEEE Trans. Veh. Technol. **61**(3), 1427–1432 (2012)
23. Ban, T.; Choi, W.; Jung, B.; Sung, D.: Multi-user diversity in a spectrum sharing system. IEEE Trans. Wirel. Commun. **8**(1), 102–106 (2009)
24. Ghasemi, A.; Sousa, E.S.: Fundamental limits of spectrum-sharing in fading environments. IEEE Trans. Wirel. Commun. **6**(2), 649–658 (2007)
25. Bletsas, A.; Khisti, A.; Reed, D.P.; Lippman, A.: A simple cooperative diversity method based on network path selection. IEEE J. Sel. Areas Commun. **24**(3), 659–672 (2006)
26. Gradshteyn, I.S.; Ryzhik, I.M.: Tables of Integrals, Series and Products, 6th edn. Acadamic Press, San Diago (2000)
27. Yang, P.; Luo, L.; Qin, J.: Outage performance of cognitive relay networks with interference from primary user. IEEE Commun. Lett. **16**(10), 1695–1698 (2012)
28. Suraweera, H.A.; Michalopoulos, D.S.; Yuen, C.: Performance analysis of fixed gain relay systems with a single interferer in Nakagami-*m* fading channels. IEEE Trans. Veh. Technol. **61**(3), 1457–1463 (2112)
29. Salhab, A.M.; Al-Qahtani, F.; Zummo, S.A.; Alnuweiri, H.: Outage analysis of *N*th-best DF relay systems in the presence of CCI over Rayleigh fading channels. IEEE Commun. Lett. **17**(4), 697–700 (2013)