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Collision Avoidance for Multi-UAV Based on Geometric Optimization Model in 3D Airspace

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Abstract This paper presents a collision avoidance (CA) algorithm for cooperative unmanned aerial vehicles (UAVs) sharing three-dimensional airspace. The method based on geometric optimization model aims to provide feasible optimal trajectory for the selected UAV, with a local optimization scope at operational level. It generates optimal flight trajectory by the objective function (the integration equation of distance, time and track adjustment costs) in response to a set of restrictions (performance, state and distance constraints) reducing the solution space. The CA maneuver has been validated with various simulations, owning such advantages as optimization spending a minimal cost, robustness considering the global traffic situation, scalability possessing explicit coordinates of waypoints and efficiency in implementing different tests of tuning parameters.

Keywords Multi-UAV · Collision avoidance · Geometric optimization · 3D · Objective function

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الخلاصة

تعرض هذه الورقة العلمية خوارزمية تفادي الاصطدام للمركبات التعاونية دون طيار (الطائرات دون طيار) التي تشترك في المجال الجوي ثلاثي الأبعاد. ويستند الأسلوب إلى نموذج تحسين هندسي يهدف إلى توفير مسار أمثل عملي للمركبات التعاونية دون طيار المختارة ، مع نطاق تحسين محلي عند المستوى التشغيلي. وهو يولد مسار الرحلة الأمثل من خلال دالة الهدف (معادلة تكامل المسافة والوقت وتكاليف تعديل المسار) استجابة لمجموعة من القيود (معوقات الأداء ، والحالة ، والمسافة) التي تقلل من مساحة الحل. وقد تم التحقق من صحة مناورة الخوارزمية الجينية مع محاكات مختلفة، وامتلاك مثل هذه المزايا بوصفه تحسيناً على نحو إنفاق أقل تكلفة ممكنة، ويتميز بالمتانة مع الأخذ في الاعتبار حالة المرور العالمي ، والتدرجية التي تمتلك الإحداثيات الواضحة لنقاط الطريق ، والكفاءة في تنفيذ الاختبارات المختلفة لضبط المعلمات.

1 Introduction

The importance of unmanned aerial vehicles (UAVs) in civil and military applications is growing worldwide [1]. UAVs are superior to manned aircraft evidently due to owning a series of features, and the most prominent is unobstructed implementation of high-risk and long-endurance missions. At operational level, the integration of UAVs into safety flight system is one of the most challenging topics. Consequently, an operational collision avoidance technique is a prerequisite.

There are some existing operational collision avoidance (CA) systems which are currently in use for piloted aircraft, and they can conduct to be deployed on UAVs, such as traffic alert and collision avoidance system [2] [TCAS, developed by the Federal Aviation Administration (FAA)], airborne separation assurance system [3] [ASAS, developed at the National Aerospace Laboratory (NLR) in Netherland], autonomous operations planner and future ATM concepts evaluation tool [4] (AOP and FACET, both belong to NASA). A comprehensive and meticulous survey of various techniques ranging from abstract concepts to prototype system has being used and evaluated in laboratories [5,6]. Numerous typical methodologies applied to avoid collision mainly include genetic negotiation algorithm [7], Monte Carlo approach [8], geometric algorithm [9], optimization theory [10], probabilistic approach [11], game theory [12], evolutionary algorithm [13] and semi-definite programming [14]. A comprehensive and meticulous survey of these multifarious algorithms can be found in the literature [15]. No matter which approach is employed, it guarantees minimum separation between UAVs. For free flight, it is a very essential task to understand geometric relations between the pair UAVs in a conflict [16].

With the development of communication technology, nowadays, the CA tends to be accomplished by using real-time data link between UAVs. Thus, for 3D modeling in real condition, it is possible that this paper considers position and velocity as the input data to resolve encounters. Under the constraints of performance, state and distance, the geometric optimization model (GOM) which is presented in this paper grounds the generation of optimal flight trajectories on the objective function which integrates the distance, time and track adjustment costs. The CA strategy is confirmed to be available and efficient to minimize the global costs of the UAV adjustments. In particular, the proposed GOM provides explicit coordinates of amended waypoints, so the detailed information could be used for monitoring the trajectory of each UAV and other expanding analyses.

This paper is organized as follows: Sect. 2 states the problem background; Sect. 3 describes the details of the proposed approach; Sect. 4 presents a scenario simulation and its results; and finally, conclusions and future work are summarized in Sect. 5.

2 Representation of Cooperative UAV Flight

The problem considered in this paper assumes a group of cooperative UAVs flying to their own destinations, following the planned trajectories and having to avoid collision with other UAVs in an efficient real-time communication. For simplicity purposes, trajectories are discussed in an ideal outdoor scenario without weather perturbations.

A conceptual show of the problem is illustrated in Fig. 1. The UAVs share a common airspace in various flight levels, and the separation between them should always be greater than a given safety distance. Missions operated by a number of UAVs will have such significant advantages in different applications as increasing re-configurability and productiveness [15]. However, collision risk coefficient rises at the same time because of the increasing number of UAVs. A UAV can be modeled as a unique point in the space surrounded by a safety volume shape. When taking into account the vertical,

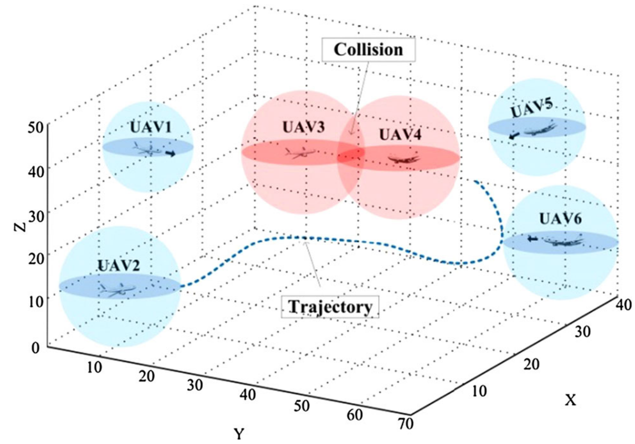

Fig. 1 The conceptual show of multi-UAVs in 3D airspace

lateral and longitudinal minimum distances to turn, the shape to model the UAV should be a 3D sphere. Note that the safety volume space is mainly relevant to the size of the aircraft. The sphere radius (minimum separation) is in direct proportion of it. If the UAVs have the same size, their safety separations would be equal; if not, the value of the distance constraint would be differently set before computation. The planned trajectory can be regarded as a sequence of waypoints that each UAV will follow, containing the initial and goal points.

The objective of CA is to guarantee the safety separation between UAVs and find collision-free trajectories while minimizing the changes of waypoints for each UAV when a collision would occur. Whatever the solution trajectory looks like, the initial and goal points of each UAV should be the same. It is assumed that all UAVs execute at their own constant speed and have same safety separation (minimum radius of turn).

3 Geometric Optimization Model

According to the implementation of airborne collision avoidance systems II (ACAS II) [17] for current manned aircraft, it is economical and safe to provide resolutions in the vertical plane (climb or descent). Therefore, in order to simplify the problem, the approach in this research only considers the change of the vertical direction. The time of executing state changes will not be taken into account, because it is very small in comparison with the duration of CA.

For resolving an encounter, the first problem is that one of the two involved UAVs should be chosen to amend its initial trajectory. To the multiple UAVs in a bounded airspace, they can be prioritized according to the parameters (e.g., tasks importance, speed, performance) while they perform missions. Obviously, the UAV owning the lower priority should respond to the encounter. The priority of *UAVi* ($i = 1, 2, \ldots n$):

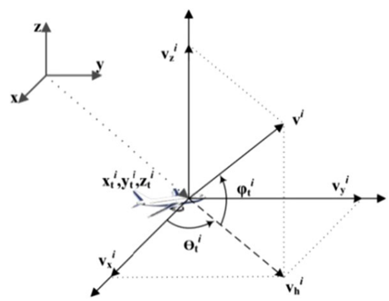

Fig. 2 The UAV dynamic characteristics

$$P_i = a_1 f_t^i + a_2 f_v^i + a_3 f_p^i \tag{1}$$

In this formula, f_t^i is the importance of the trajectory that *UAVi* executes, because different UAVs in charge of their own areas have different importance for the overall missions. The importance could be defined as the levels of $1, 2, \ldots, m$ (m is the most important). f_v^i is relevant to the velocity value of *UAVi*, and it can be defined as the rate $1{:}v_1{:}v_2{:}\ldots{:}v_n$ (1 is the minimum while v_n is the maximum). f_p^i is the difficulty index of amendment depending on the *UAVi* performance which could be set as the levels of $1, 2, \ldots, d$ (d is the most difficult). w_1, w_2, w_3 are weight coefficients, and $a_1 + a_2 + a_3 = 1$. Note that the calculation of priorities can be completed based on the known data before simulation.

3.1 Mathematics Description

For the *UAVi* ($i = 1, 2, \ldots, n$), its dynamic characteristics could be described in a Cartesian coordinate system [18] (Fig. 2). The region formed by x and y axes indicates the horizontal plane, and z stands for the altitude.

$$p_t^i = \begin{bmatrix} x_t^i \\ y_t^i \\ z_t^i \end{bmatrix}, \quad v_t^i = \frac{dp_t^i}{dt} = \begin{bmatrix} v_t^i \cos\varphi_t^i \cos\theta_t^i \\ v_t^i \cos\varphi_t^i \sin\theta_t^i \\ v_t^i \sin\varphi_t^i \end{bmatrix}$$

$$\frac{d\varphi_t^i}{dt} = w_t^i, \quad 0 < \theta_t^i < 2\pi, \quad -\frac{\pi}{2} \le \varphi_t^i \le \frac{\pi}{2} \tag{2}$$

The formula defines p_t^i and v_t^i, respectively, as the position and velocity of *UAVi* in 3D; θ_t^i is the course angle, direction of the velocity vector in the $x - y$ plane; φ_t^i is the pitch angle (measured counter-clockwise); and w_t^i is the angular velocity.

Considering simply, each UAV is supposed to keep its own velocity $v_i, i \in \{1, 2, \ldots, n\}$ during cruise flight in the non-segregated airspace, and climb or descend rapidly within a minimum time that could be ignored when an intruder UAV breaks into their safety area.

The *UAVi* trajectory is discretized, and the sampling interval is Δt. Therefore, in the $m{-}th$ moment, related discrete values are $x_i(m), y_i(m), z_i(m), \varphi_i(m)$, m is a natural number. The discrete kinematic model of *UAVi* is:

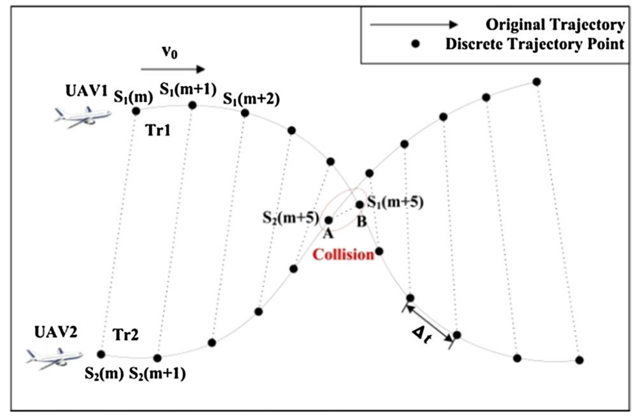

Fig. 3 Representation of discrete trajectory

$$\begin{bmatrix} x_i(m+1) \\ y_i(m+1) \\ z_i(m+1) \\ \varphi_i(m+1) \end{bmatrix} = \begin{bmatrix} x_i(m) + v_i \cos(\varphi_i(m)) \cos\theta_i \cdot \Delta t \\ y_i(m) + v_i \cos(\varphi_i(m)) \sin\theta_i \cdot \Delta t \\ z_i(m) + v_i (\sin\varphi_i(m)) \cdot \Delta t \\ \varphi_i(m) + w_i(m) \cdot \Delta t \end{bmatrix} \tag{3}$$

For describing the problem more conveniently, a nonlinear mapping is defined as $f_i : R^3 \times R \to R^4$, and the two formulas (1), (2) could be written in the form of discrete-time state equation as follows:

$$s_i(m+1) = f(p_i(m), \varphi_i(m)), \quad i \in \{1, 2, \ldots, n\} \tag{4}$$

In the formula, $s_i(m)$ is the state vector of *UAVi* at time m, including the location $(x_i(m), y_i(m), z_i(m))$ and pitch angle $\varphi_i(m)$.

Figure 3 shows an example of two pairwise-processed trajectories composed of several discrete points, respectively. Dotted lines with corresponding time indicate the 4D coordinates. The two UAVs are in the same constant velocity v_0, while the distance between two adjacent points is equal to $v_0 \cdot \Delta t$. As the vortex is dynamically generated with the non-stop movement of each UAV, the envelope shape could be described as a cylinder if we consider the real and continuous trajectory instead of the discrete one. Hence, there would be a collision between the two original trajectories Tr1 and Tr2, because point B is in the scope of vortex influence made by *UAV1* at point A in the $m + 5$ moment.

3.2 Objective Function with Constraints

In order to generate a new trajectory for CA, we always hope that each UAV could spend the expense as little as possible, reach its target as soon as possible, and adjust its track as few times as possible. The objective function should include three aspects, as follows:

(1) Distance cost

At time m, the distance from current location to destination $D_i(m)$ of *UAVi* could be approximately calculated by Euclidean distance formula.

$$D_i(m) = \left\| p_i(m) - p_i^d \right\|_2 = \Big[(x_i(m) - x_i^d)^2 + (y_i(m) - y_i^d)^2 + (z_i(m) - z_i^d)^2 \Big]^{1/2} \tag{5}$$

In this formula, $p_i^d = (x_i^d, y_i^d, z_i^d)^T$ is the coordinate of *UAVi* destination.

(2) Time cost

At time m, the flight time of *UAVi* from the initial point to the current location is $m \cdot \Delta t$. The total flight time $T_i(m)$of *UAVi* could be estimated using the following formula:

$$T_i(m) = m \cdot \Delta t + D_i(m)/v_i \tag{6}$$

(3) Track adjustment cost

Normally, if *UAVi* keeps straight flight without direction amendment, it is more economical. At time m, the track adjustment cost of *UAVi* $C_i(m)$ could be defined as the change value of pitch angle.

$$C_i(m) = \begin{cases} |\varphi_i(m) - \varphi_i(m-1)| & if\ m \geq 1 \\ 0 & if\ m = 0 \end{cases} \tag{7}$$

In summary, the objective function is the unity of the three indicators at time m:

$$O_i(m) = w_1 D_i(m) + w_2 T_i(m) + w_3 C_i(m) \tag{8}$$

In this formula, w_1, w_2, w_3 are, respectively, the corresponding weight coefficients of the three indicators. Different weights reflect different requirements in the CA. For instance, if $w_2 > w_3$, it means that the shortest time is more important than the least fuel consumption. Note that a normalization process is needed in the computation.

In the period $[m, m+N-1]$, the *UAVi* trajectory is amended to avoid the collision, while its track points sequence and corresponding pitch angles sequence, respectively, are

$$\begin{aligned} P_i &\triangleq [p_i(m), p_i(m+1), \ldots, p_i(m+N-1)]^T \\ \Psi_i &\triangleq [\varphi_i(m), \varphi_i(m+1), \ldots, \varphi_i(m+N-1)]^T \end{aligned} \tag{9}$$

Thus, the objective function of progressive optimization for *UAVi* in this period is:

$$\begin{aligned} O_i &= \min \sum_{r=0}^{N-1} O_i(m+r) \\ &= \min \left(\sum_{r=0}^{N-1} (w_1 D_i(m+r) + w_2 T_i(m+r) \right. \\ &\quad \left. + w_3 C_i(m+r)) \right), \quad r \in [0, N-1] \end{aligned} \tag{10}$$

The function O_i aims to obtain the track points sequence P_i and corresponding pitch angles sequence Ψ_i in the period $[m, m+N-1]$. In the process of the optimization solution, it needs to meet the following constraints:

(1) Performance constraint

Because of the performance constraints of *UAVi*, its change in the value of pitch angle should meet a specific value.

$$\begin{bmatrix} 1 \\ -1 \end{bmatrix} C_i(m) \leq \begin{bmatrix} B \\ -A \end{bmatrix}, \quad m \geq 1 \tag{11}$$

In this equation, the known numerical values A and Brepresent the variation range of the pitch angle, while A indicates the minimum and B indicates the maximum.

In the period $[m, m+N-1]$, the performance constraints above could be integrated into an inequality equation:

$$\begin{aligned} q(\varphi_i(m+r)) &= \begin{bmatrix} 1 \\ -1 \end{bmatrix} C_i(m+r) - \begin{bmatrix} B \\ -A \end{bmatrix} \\ &= \begin{bmatrix} 1 \\ -1 \end{bmatrix} (\varphi_i(m+r) - \varphi_i(m+r-1)) \\ &\quad - \begin{bmatrix} B \\ -A \end{bmatrix} \leq 0, \forall r \in [0, N-1] \end{aligned} \tag{12}$$

(2) State constraint

The state and control of each UAV should meet its state equation $s_i(m+1) = f(p_i(m), \varphi_i(m)), i \in \{1, 2, \ldots, n\}$. In the period $[m, m+N-1]$, the state constraint could be integrated into an equality equation:

$$\begin{aligned} & e_i(p_i(m+r)) \\ &= \begin{bmatrix} x_i(m+r) - [x_i(m+r-1) + v_i \cos(\varphi_i(m+r-1)) \cos\theta_i \cdot \Delta t] \\ y_i(m+r) - [y_i(m+r-1) + v_i \cos(\varphi_i(m+r-1)) \sin\theta_i \cdot \Delta t] \\ z_i(m+r) - [z_i(m+r-1) + v_i (\sin\varphi_i(m+r-1)) \cdot \Delta t] \\ \varphi_i(m+r) - [\varphi_i(m+r-1) + \omega_i(m+r-1) \cdot \Delta t] \end{bmatrix} \\ &= 0 \end{aligned} \tag{13}$$

(3) Distance constraint

Distance constraint between the two nearest UAVs is the key for track adjustment and optimization. In the period $[m, m+N-1]$, the distance between the two UAVs in the trajectory plans must not be shorter than d_0, which is the minimum safe distance:

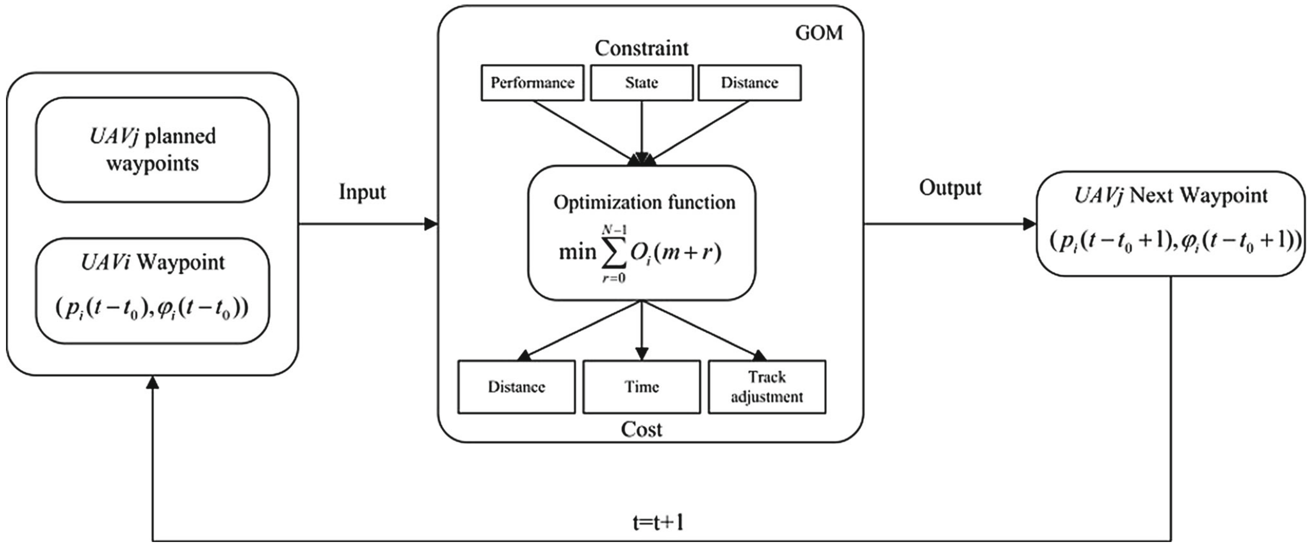

Fig. 4 The framework of the proposed algorithm

$$\begin{aligned} & d_i(p_i(m+r), p_j(m+r)) \\ & \quad = d_0 - \left\| p_i(m+r) - p_j(m+r) \right\|_2 \\ & \quad = d_0 - [(x_i(m+r) - x_j(m+r))^2 + (y_i(m+r) \\ & \qquad - y_j(m+r))^2 + (z_i(m+r) - z_j(m+r))^2]^{1/2} \\ & \quad \le 0,\ j = 1, \ldots, n \text{ and } j \ne i, \forall r \in [0, N-1] \end{aligned} \tag{14}$$

Integrating the objective function and constraints, the GOM is obtained of generating amended trajectories for CA.

$$\begin{aligned} & \min \sum_{r=0}^{N-1} O_i(m+r) \\ & s.t. \\ & \begin{cases} q(\varphi_i(m+r)) \le 0 \\ e_i(p_i(m+r)) = 0 \\ d_i(p_i(m+r), p_j(m+r)) \le 0 \\ \forall r \in [0, N-1],\ j = 1, \ldots, n \text{ and } j \ne i \end{cases} \end{aligned} \tag{15}$$

The model is a rolling optimization strategy which is used to generate the series of amended trajectory points one after another. For instance, at time t, $UAVi$ and *UAVj* would have a collision, and their state information in the period $[t-t_0, t+t_0]$ can be perceived based on the planned trajectory. If $(p_i(t-t_0), \varphi_i(t-t_0))$ and $(p_j(t-t_0+1), \varphi_j(t-t_0+1))$ are known, the next waypoint $(p_i(t-t_0+1), \varphi_i(t-t_0+1))$ of *UAVi* can be calculated by using the GOM. Repeating the process again and again, all the amended discrete waypoints of *UAVi* in the period $[t-t_0, t+t_0]$ will be calculated. Note that t_0 is a predetermined constant derived from the spectrum of UAV type before simulation. Figure 4 illustrates the framework of the proposed algorithm.

In a realistic environment, it will be necessary that the GOM is able to resolve encounters involving more than two UAVs. Considering the situation illustrated in Fig. 5, *UAV1* on the left detects a conflict with *UAV3* and attempts to climb or descend. Neither solution Tr11 or Tr12 is acceptable since it results in a new collision with *UAV2* or *UAV4*.

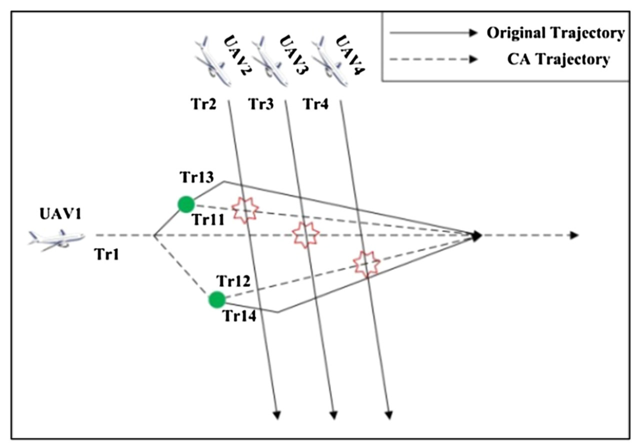

Fig. 5 Geometric optimization for multiple collisions

The GOM is robust enough to consider the global traffic situation simultaneously in a specified time period. For example, if *UAV1* intends to climb, just a distance constraint between *UAV1* and *UAV2* is added to the optimization calculation and then the appropriate trajectory Tr13 is generated. It is similar to the trajectory Tr14 generation when *UAV1* refers to a downward trend. Therefore, the GOM maintains its robustness to this type of problem in multi-UAVs situations.

4 Simulation and Results

Numerous scripts are written in Matlab to put forward the simulation scenario that takes seven UAVs ($UAVi$, $i =$

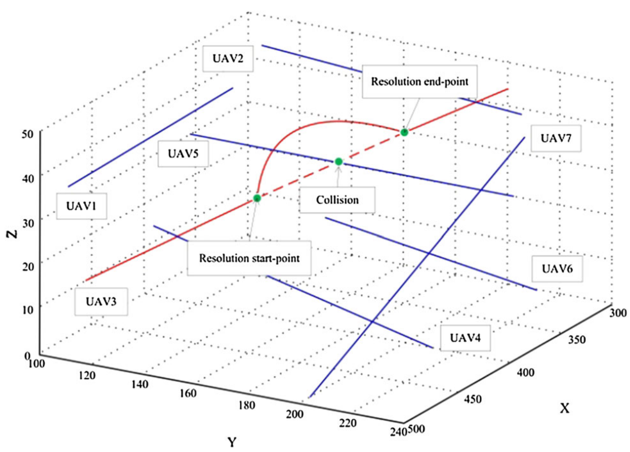

Fig. 6 A snapshot during simulation with a CA between *UAV3* and *UAV5* (the amendment trajectory is labeled *red*)

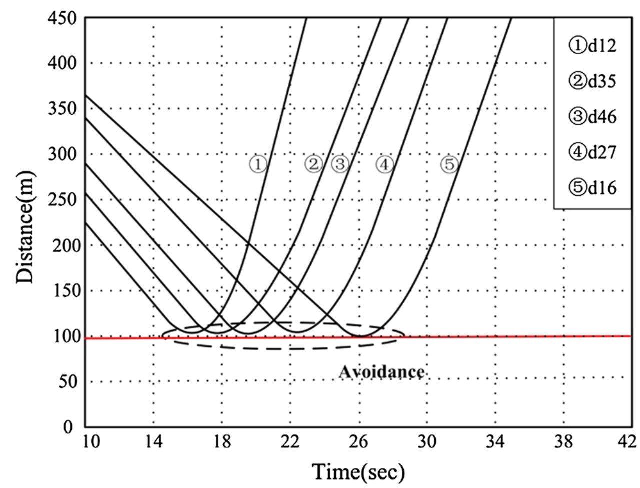

Fig. 7 Relative distance

Table 1 Summary of CA-modified data of *UAV3* (start-point and end-point keep the same)

Position	Original	Modified
Start-point	(372, 125, 27)	(372, 125, 27)
Collision point	(360, 131, 30)	(360, 131, 40)
End-point	(348, 137, 33)	(348, 137, 33)

$1, 2, \ldots, 7)$ from stochastic starting points to randomly selected finishing points. All the UAVs are assumed to head straight toward their goal positions at the speed of 30 m/s. Minimum separation distance d_0 is 100m as defined. The interval time Δt for discrete points is set up as 0.1 s. The corresponding weight coefficients w_1, w_2, w_3 are assumed to be equal, and it means that the costs of distance, time and track adjustment are treated to have the same importance in the optimization process. The minimum variation range of the pitch angle A is -1.047 while the maximum value B is 1.047. In initial simulation, it is assumed that the time of executing state changes will not be taken into account, and each UAV can sense the global information. The computer used for this simulation is an EliteBook laptop with a processor Intel i5 of 2.6 GHz, and 4 GB of RAM which is enough for the memory requirements of algorithm operation and simulation.

Figure 6 illustrates a snapshot during simulation in 3D airspace, and several labels are marked. When *UAV5* flies into the range of *UAV3*, the proposed CA algorithm runs efficiently to ask *UAV3* climb to result the threat (the *UAV3* original trajectory in CA is represented by dotted line while the amended is represented by full line). Table 1 summarizes the main waypoints of *UAV3* (resolution start-point, end-point, and modified collision point) during the CA process.

From this simulation, the results are expected pattern to avoid collisions effectively. Figure 7 shows the relative distances between involved UAVs ($UAVi, i = 1, 2, \ldots, 7$) of collisions in a scenario, which has seven planned discrete trajectories and five potential collisions among them. Note that dij indicates the distance between *UAVi* and *UAVj*. Evidently, it could be concluded that the CA algorithm takes effect because the minimum separation region of each UAV is never intruded. For instance, *UAV1* encounters *UAV2* and *UAV6* separately at different times; however, the safety flight is guaranteed because the distances between them (d12 and d16) are always above the minimum separation distance even in the closest points.

To achieve a perfect performance, different appropriate values should be selected for tuning parameters of the GOM. Thus, several extensional simulations with varying parameters values are constructed in the research. Efficiency is a frequently used performance metric used to measure the different performance of CA algorithm [14]. The total time of all UAVs reaching their own goals is in minimum possible time. The efficiency can be calculated as follows:

$$Efficiency = \frac{1}{n}\sum_{j=1}^{n}\frac{t_j^i}{t_j^a} \tag{16}$$

In this formula, n is the number of UAVs. $t_j^a \geq t_j^i$, t_j^i is the planned time of UAV_j while t_j^a is the actual time. Figure 8 illustrates the efficiency with the tuning parameters of safety separation and sense range. Sense range indicates the sensing distance capability of UAV.

It shows that efficiency is rising with the increasing sensor radius. More information brings into correspondence with better efficiency, due to two adjacent collisions would be avoid in one trajectory amendment. Restricting the sensor range makes a UAV ignoring neighbors out of the range and takes extra cost of CA between the neighbor UAVs. It has the similar intuition to the safety separation changing, because more neighboring collisions would be considered as the safety separation increasing in the same sensor range.

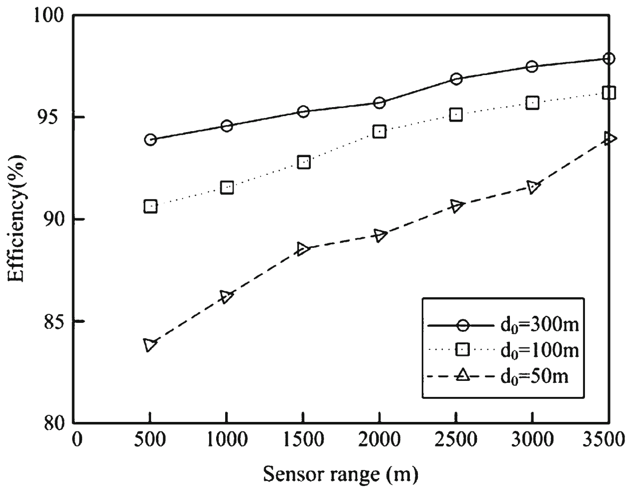

Fig. 8 Efficiencies with different parameters

In the simulation, the discrete waypoints is clearly calculated and recorded, which can be directly used in other data analysis, making for the advantage of expansibility. The explicit coordinates of waypoints forming each original trajectory and its solution trajectory make it easy to examine various performances of the approach and provide complete information.

In telecommunications and software engineering, scalability is the ability of a system, network or process, to handle growing amounts of work in a graceful manner or its ability to be enlarged to accommodate that growth [19]. In realistic scenario, there are several sets of independent hot spots [20] in which there may be a pair of involved UAVs in a threat and other nearby UAVs which could have an effect on the CA process. Since the proposed algorithm is feasible to generate a synthesized CA trajectory in a sub-scenario, the scalability problem is tackled by the independent hot spots based on the interaction between the planned trajectories during a period of time. In particular, these independent sub-scenarios (hot spots) can be processed separately so that it greatly improves the efficiency.

To test the performances of the proposed CA algorithm and compare it with other presented approaches, Satisficing Game Theory-based Algorithm (SGTA) [21] and Reactive Inverse PN Algorithm (RIPNA) [22] which are validated to be practicable for high-density traffic scenarios, are adopted as the contrasts. In this research, the computation time is used to be the comparing performance. Under similar test conditions, along with SGTA and RIPNA, we implement GOM using information available in [20]. The results for low- density (20 aircraft), medium-density (40 aircraft), and high- density (60 aircraft) traffic are recorded in Table 2.

Evidently, GOM has a great computational advantage over the other two algorithms. The average computational times taken in the GOM simulations with various air traffic densities are shorter. Besides, the average computational times taken in the simulations with various air traffic densities are not in the exponential growth. In Table 2, it is clear that scalability is achievable because even high density of multiple UAVs in the scenario airspace could still be treated in a relatively reasonable period of time.

Table 2 Comparison of computation times of SGTA, RIPNA and GOM: test case of radom flights (averaged over 20 runs)

Number of aircraft	Time taken (s)		
	SGTA	RIPNA	GOM
20	673	68	27
40	1,711	200	68
60	3,009	309	189

5 Conclusions

The research and innovation activities on all sorts of UAVs have experienced a significant increase in the last years. To achieve a high level of safety, CA has become an inevitable issue, but also a need to address the problem. For free flight, this paper has presented a CA algorithm for multiple cooperative UAVs, aiming to avoid collision by generating optimal trajectory in 3D airspace. Under some constraint conditions, the GOM based on the essential geometric relations of UAVs generates trajectory waypoints by the objective function. The proposed model has been validated with the results of several simulations.

The main advantages of the proposed algorithm are optimization (generating the optimal flight trajectory with minimal cost), robustness (considering the interaction between the UAVs involved a threat and neighboring UAVs in a specified time period), efficiency (maintaining higher level with varying parameters values) and scalability (handling the growing amounts of UAVs in the airspace). In addition, the presented algorithm provides explicit coordinates of amended waypoints, so it can be applied to different analyses that require detailed information.

Recommendations for future work are as follows: (1) avoiding the collisions by applying different kind of maneuvers like heading change, speed alteration, level modification or a combination of them; (2) considering several disturbances (e.g., wind) in the simulations; and (3) improving the technique to tolerate massive UAVs.

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