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Parameter Estimation of Planar Robot Manipulator Using Interval Arithmetic Approach

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Abstract Parameter and state estimation problems are encountered when modeling processes that involve uncertain quantities to be estimated from measurements. The aim of this paper was to show the interval arithmetic approach as the suitable tool to solve problems of estimating the parameters of nonlinear systems in a bounded-error context. Perturbations are assumed bounded but otherwise unknown. This approach computes outer (or inner) approximations of the set of all parameters. An example of planar robot manipulator is presented to illustrate the effectiveness and potential of an interval approach in parameter's estimation. A simulation is conducted to compare these estimates in terms of mean squared.

Keywords Parameter estimation · Interval analysis · Least squares method · Gaussian noise

الخلاصة

تتم مواجهة مشاكل تخمين المعلّمة والحالة عند نمذجة عمليات تتضمن قيماً غير دقيقة يجب تقديرها من القياسات. وتهدف هذه الورقة العلمية إلى إظهار نهج حساب الفترة بوصفها أداة مناسبة لحل مشاكل تخمين معلمات أنظمة غير خطية في سياق له حدود من الخطأ. ويتم الافتراض بأن الاضطرابات لهـا حدود ولكن دون ذلك تكون مجهولة. ويحسب هذا النهج التقديرات الخارجية (أو الداخلية) لمجموعة كل المعلمات. وقد تم عرض مثال هو عبارة عن مناول روبوت مستوى لتوضيح فعالية وإمكانية نهج الفترة فـي تخمين المـعلمات. وقد تم تنفيذ محـاكاة لمقارنة هذه التخمينات بدلالة متوسط التربيع.

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1 Introduction

When modeling a physical process, there are always discrepancies between the model and real behavior due to simplifications and neglected effects. Moreover, the parameters of the real process are never rigorously constant and may vary all along the time around a mean value. Thus, the process state estimation will strongly depend on these variations which are unknown and thus may be considered as uncertainties [1]. Introduction of uncertain aspects in the analysis, generally, is obtained from a probabilistic description of the uncertain model parameters. However, the classical probabilistic methods are subject to important theoretical and practical [2]. An alternative method to describe uncertainties is provided by interval analysis, which assumed that all uncertain quantities (perturbation, measurement noise, and parameter) are bounded and belong to known sets. Bounded errors may encompass significant structural errors that cannot be accounted for by random variables or noise errors without specifying any statistical properties. Parameter estimation is a common problem in many areas of process modeling, both in online applications such as real-time optimization and in offline applications such as the modeling of reaction kinetics and phase equilibrium. The goal is to determine values of model parameters that provide the best fit to measured data. The reliability of numerical results depends on the level of uncertainties associated with the system's parameters and errors due to numerical schemes used to obtain approximate solutions.

Applications of interval methods have been explored in finite element analysis to model systems with parametric uncertainties and to account for the impact of truncation error on the solutions.

2 Estimation Theory

Parameters and state estimated from experimental measures are usually obtained within a stochastic framework in which known distribution laws are associated with interferences and noisy measurements. Oppositely, in the bounded error, context measures and modeling errors are supposed to be unknown but to stay within known and acceptable bounds.

Errors between measured and predicted outputs may rely on many factors, among them: limited sensors accuracy, interferences, noise, structured uncertainties, some are quantifiable, and some are not. When we model an industrial product, the model brings in certain number of parameters, representing generally measurable physical quantities (lengths, coefficients of proportionality). Their values are either identified from experiments or specified by the designer.

The models are then used to realize simulations allowing forecasting the answer of the product. During this process, the sources of uncertainties are numerous. First of all, the models are only representations simplified by products or phenomena which they represent. So, when we identify the value of a parameter from measures, this one can be rarely determined in an unambiguous way. However, all these uncertain quantities have an influence on the results of the simulations and contribute a discrepancy between the real and simulated answers. To size their products exactly, most of the industrialists try to quantify and to master this contribution.

To do it, a usual initiative consists on the one hand in multiplying the essays on the product, and on the other hand in oversizing this one by taking into account dispersal observed experimentally, to assure at best the conformity of the answer with the requirements. However, the oversizing and the essays are expensive practices, and the industrialists try to minimize their employment by resorting massively to numeric simulations.

Usually, most of the solvers used in the industry are deterministic that is they appeal to models the parameters of which have unique values and supply a unique result on. They do not thus allow directly taking into account the uncertainties. When the value of uncertainties is considered, we have to add additional constraints to find the exact solution [3]. For that reason, interval analysis provides efficient tools at modeling the uncertainties, and simulating their influence on the answer, [4,5]. In this paper, the problem of parameter set estimation from bounded-error data is expressed using interval analysis, and the results of simulation on a robot planar are represented.

2.1 Parameter Bounding

In the bounded-error context, the sensors used for data measurements are frequently characterized with a prior maximum measurement error. Under the hypothesis of additive noise, and relating to the definition given by [6–8], parameter set estimation consists with all observed data and a priori error bounds. The response of an unknown system can be expressed as follows:

$$f : R^m \to R, \text{excited by } t_k \in R^m \quad y_k = f(t_k), \quad k = 1, \ldots, N \tag{2.1}$$

The system can be static and depend only on the current value of the input, or the system can be dynamic and depend on previous values of the input and output. Actual system response can be given by the following:

$$y_{mk} = y_k + e_{mk}, \quad k = 1, \ldots\ldots, N \tag{2.2}$$

where e_{mk} denotes the additive error and N is the number of observations. The unknown system can be modeled using a parametric function $f_p : R^{m\times n} \to R$, parameterized on $p \in R^n$ so that an estimate of the response equals

$$\hat{y}_k = \hat{f}(t_k, p) \tag{2.3}$$

and the error between the unknown system and the model is called modeling error and is defined as

$$e_{ok} = y_k - \hat{y}_k \tag{2.4}$$

and the error between the measured observation and model is equal to

$$e_k = y_{mk} - \hat{y}_k \tag{2.5}$$

from (2.4) and (2.5) the estimation error due to the measurement noise and modeling error is expressed by the following:

$$e_k = e_{mk} + e_{ok} \tag{2.6}$$

2.1.1 Bounded-Error Parameter Set Estimation

Bounded error is a data structure where an array is used; the array dimension's is bounded. There is a lower and an upper value. In bounded-error parameter set estimation, the measurement error and the modeling error are assumed bounded with $|e_{mk}| \le E_{mk}$ and $|e_{ok}| \le E_{ok}$, respectively. Therefore, the error between the model and the observed response is also bounded. The bound is given by

$$\left| y_{mk} - \hat{y}_k \right| \le E_k \quad \text{where } E_k = E_{mk} + E_{ok} \tag{2.7}$$

From these, parameter set estimation from bounded-error data is a set of feasible parameters consistent with data and known bounds on the error, which can be expressed by the following:

$$p_k = \{p : \left| y_{mk} - \hat{f}(t_k, p) \right| \le E_k\} \tag{2.8}$$

As previously mentioned, algorithms that compute parameter set bounds from bounded-error data have many advantages, a minimal number of assumptions are made regarding the error, and parameters with lower uncertainty have bounding sets with smaller volumes than parameters with greater uncertainty and global optimal parameters are contained in the feasible set [4].

3 Interval Analysis

In this paper, uncertainties and numerical errors are treated as interval quantities. The main principle of interval analysis is to replace every real number by an interval enclosing it and whose bounds are representable by the computer [9–11]. For instance, π can be represented by the interval [3.14159, 3.14160] if 6 significant radix-10 digits are used. Data known with some degree of uncertainty can also be represented, for instance, data measured with bounded measurement errors. Interval vectors and interval matrices are vectors and matrices with interval components u. The major advantage of this arithmetic is the fact that every result is guaranteed.

3.1 Basics Tools

An interval $[X] = [\underline{x}, \overline{x}]$ is a closed and connected subset of R; it may be characterized by its lower and upper bounds $\underline{x}$ and $\overline{x}$ or equivalently by its center $c([x]) = \frac{(\underline{x}+\overline{x})}{2} = m(x)$ and width $w([x]) = \overline{x} - \underline{x}$.

A point interval or degenerate interval is an interval contains a single real number x with $\overline{x} = \underline{x}$.

Arithmetical operations on intervals can be defined by $\forall \circ \in \{+, -, *, /\}, [x] \circ [y] = \{x \circ y \,|x \in [x], y \in [y]\}$. Obtaining an interval corresponding to $[x] \circ [y]$ is easy for the first three operators as follows:

$$[x] + [y] = \left[\underline{x} + \underline{y}, \overline{x} + \overline{y}\right], [x] - [y] = \left[\underline{x} - \overline{y}, \overline{x} - \underline{y}\right],$$

$$[x] * [y] = \left[\min(\overline{x}\overline{y}, \overline{x}\underline{y}, \underline{x}\overline{y}, \underline{x}\underline{y}), \max(\overline{x}\overline{y}, \overline{x}\underline{y}, \underline{x}\overline{y}, \underline{x}\underline{y})\right]$$

For division, when $0 \notin [y]$,

$[x] / [y] = \left[\min(\overline{x}/\overline{y}, \overline{x}/\underline{y}, \underline{x}/\overline{y}, \underline{x}/\underline{y}), \max(\overline{x}/\overline{y}, \overline{x}/\underline{y}, \underline{x}/\overline{y}, \underline{x}/\underline{y})\right]$ and *extended intervals* have to be introduced when $0 \in [y]$, see [11].

More generally, the *interval counterpart* of a real-valued function is an interval-valued function defined as

$$f([x]) = [\{f(x) \,|x \in [x]\}], \text{ or as } f(S) \text{ for } S = [x].$$

where $[S]$ denotes the *interval hull* of a set S, i.e., the smallest interval that contains it. Interval enclosures for continuous elementary functions are easily obtained. For monotonic functions, only computations on bounds are required.

$$\exp([x]) = \left[\exp(\underline{x}),\ \exp(\underline{x})\right],$$
$$\log([x]) = \left[\log(\underline{x}), \log(\underline{x})\right] \text{ if } \underline{x} > 0$$

For non-monotonic elementary functions, such as the trigonometric functions, algorithmic definitions are still easily obtained. For instance, the interval square function can be defined by

$$[x]^2 = \left\{ \begin{matrix} \left[0, \max(\underline{x}^2, \overline{x}^2)\right], if\ 0 \in [x] \\ \left[\min(\underline{x}^2, \overline{x}^2), \max(\underline{x}^2, \overline{x}^2)\right] else \end{matrix} \right\}$$

For more complicated functions, it is usually no longer possible to evaluate their interval counterpart, hence the importance of the concept of *inclusion function*. An inclusion function $[f](.)$ for a function $f(.)$ defined over a domain $D \subset R$ is such that the image of an interval by this function is an interval, guaranteed to contain the image of the same interval by the original function:

$$\forall [x] \subset D,\ f([x]) \subset [f]([x]) \tag{3.1}$$

This inclusion function is *convergent* if $\lim_{w([x])\to 0} w([f]([x])) = 0$ and *inclusion*

Monotonic $if\ [x] \subset [y] \Rightarrow [f]([x]) \subset [f]([y])$.

Various techniques are available for building convergent and inclusion-monotonic inclusion functions. Among them, the simplest is to replace all occurrences of the real variable by its interval counterpart which results in what is called a *natural inclusion function*.

Example 3.1 Consider the function

$f(x) = x^2 - 3(x - \exp(x))$.

An inclusion function for f, is

$[f]([x]) = [x]^2 - 3([x] - \exp([x]))$

Evaluate $[f]$ *over* $[0, 1]$,

$[f]([0, 1]) = [0, 1]^2 - 3([0, 1] - \exp([0, 1]))$

When the inclusion in (3.1) *becomes*

$= [0, 1] - 3([0, 1] - [1, e]) = [0, 1] - 3([-e, 0])$

$= [0, 1] + [0, 3e] = [0, 3e + 1] \subset [0, 9.16]$

compare with

$f([0, 1]) = [3, -2 + 3e] \subset [3, 6.16]$

of course, $f([0, 1]) \subset [f]([0, 1])$

Equality, the inclusion function is *minimal*. Usually, some pessimism is introduced by the inclusion function, as in Example 3.1.

This pessimism is due to the fact that each occurrence of the interval variable is considered as independent from the others. Various approaches may be considered to reduce pessimism. A first one is to reduce the number of occurrences of the variable by symbolic manipulations.

4 Planar Robot Manipulator Model

Robot manipulators are commonly employed in the wide range of the tasks such as transportation, material handling, loading, welding, miling, and drilling and material assembling. Industrial manipulators are essentially open kinematic chain arm like devices and are generally composed of ternary links interconnected to each other by revolute and prismatic joints. Generally, open kinematic chain robot arms have insufficient mechanical stiffness and exhibit undesired elastic behavior [1]. On the other hand, closed kinematic chain robot arms are preferable due to their high structural stiffness. Slider-crank mechanism-based robot arm has many excellent and superior features of the open and closed kinematic chain robot arm. A *slider-crank* motion generator was designed for the planar four-bar mechanisms(R-RRT). The Slider-Crank is a linkage that transforms linear motion (piston) to circular motion (crank) or vice versa [12]. We recognize it, since it is used in the internal combustion engine, wherein the input force is the gas pressure on the piston. This multibody mechanical system consists of four rigid bodies, which represent the ground, the crank, the connecting rod, and the slider as shown in Fig. 1. The ground, the crank, the connecting rod, and the slider are constrained via ideal revolute joints. The center of mass of each body is considered to be located at the mid-distance of the bodies' total length.

In this paper, we apply the least squares method to estimate the interval lengths of two connecting links of a simulated slider-crank model, for various changes in geometric dimensions. To elaborate a model of a slider-crank system in Cartesian coordinate, the proposition cited below is considered.

In each body i, we associated a base direct orthonormal (xi, yi, zi). The connections and the parameter setting of the various bodies of slider-crank system are the following ones:

A: Connection pivot of axis $(A, \vec{Z}_0)$, we put $(\vec{x}_0, \vec{x}_1) = (\vec{y}_0, \vec{y}_1) = \theta$.

B: Connection pivot of axis $(B, \vec{Z}_0)$, we put $(\vec{x}_2, \vec{x}_1) = (\vec{y}_2, \vec{y}_1) = \theta 1$.

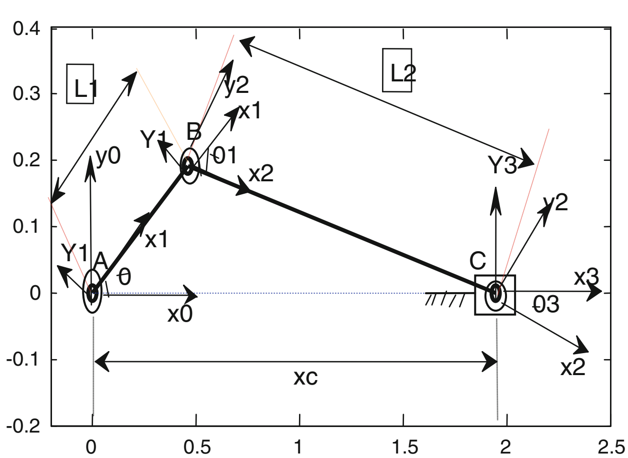

Fig. 1 Planar slider crank

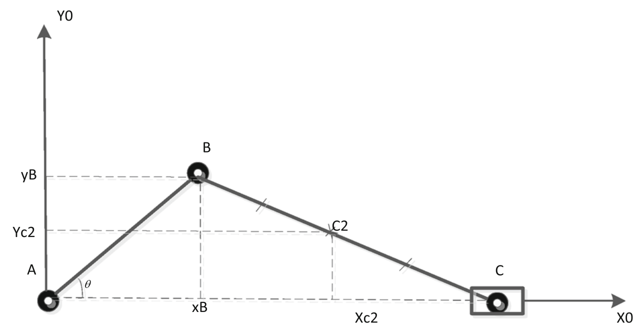

Fig. 2 The midpoint C2 in the Cartesian reference frame

C: Connection pivot of axis $(C, \vec{Z}_0)$, we put $(\vec{x}_3, \vec{x}_2) = (\vec{y}_3, \vec{y}_2) = \theta 3$, and Connection slippery pivot of axis $(C, \vec{x}_3)$, we put $AC = x_C$.

A Cartesian reference frame xOy is selected, as illustrated in the Fig. 2. The joint A is in the origin of the reference frame, that is, $A \equiv 0$. The vectors loop of planar slider-crank scheme yields the following equations:

$$\vec{AB} + \vec{BC} = \vec{AC} \tag{4.1}$$

The coordinates of the joint B are as follows:

$$x_A = AB\cos(\theta),\ y_B = AB\sin(\theta) \tag{4.2}$$

The unknowns are the coordinates of the joint $\boldsymbol{C}$, x_C and y_C. The joint C is located on the horizontal axis $y_c = 0$.

The connecting rod BC has a general plane motion: translation along the x-axis, translation along the y-axis, and rotation about the z-axis. The mass center of link BC is located at $C2$. The mass center of the link BC is the midpoint of the segment BC, $x_{C2} = \frac{x_B + x_C}{2}$, $y_{C2} = \frac{y_B + y_C}{2}$.

The lengths of the segments AB and BC are L1 and L2, respectively. Using Pythagoras' theorem for the Bx_BC right-angled triangle as illustrated in Fig. 2, the following relations can be written:

$$(x_C - x_B)^2 + (y_C - y_B)^2 = BC^2, \tag{4.3}$$

As mentioned above,

$$y_C = 0, \|AB\| = L1 \quad \text{and} \quad \|BC\| = L2,$$

The equation (4.3) can be rewritten as follows:

$$(x_C - x_B)^2 + (y_B)^2 = (L2)^2, \tag{4.4}$$

From (4.2),

$$y_B = AB\sin(\theta) = L1\sin(\theta),\ x_B = L1\cos(\theta),$$

And X_C can be expressed as

$$X_C = L1\cos\theta + \sqrt{((L2)^2 - (L1\sin\theta)^2)}. \tag{4.5}$$

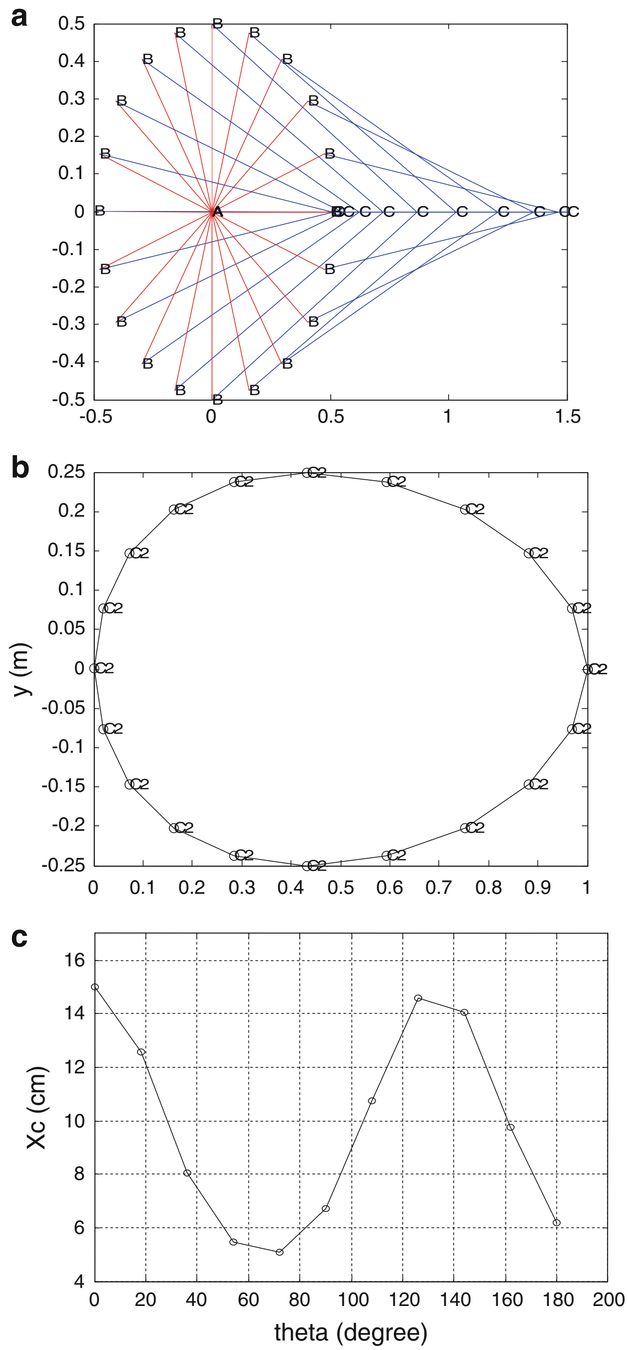

Fig. 3 **a** Graphic of the mechanism. **b** Closed path described by the point C2. **c** Path described by C (theta)

Equation (4.5) is the kinematic equation for the slider-crank mechanism, the values of L1, L2 are unknown parameters, and the input parameter ⊖ is known for kinematic analysis.

The mechanism and the closed path described by the point $C2$ (the center of mass of BC) for a complete rotation of the driver link AB, and for desired values of parameters are shown in Fig. 3a in the plane $(o\vec{x}, o\vec{y})$, and in Fig. 3b in the plane$(o\vec{y}, o\vec{\theta})$. Figure 3c shows the closed path described by the point C in the plane $(o\vec{x}, o\vec{\theta})$.

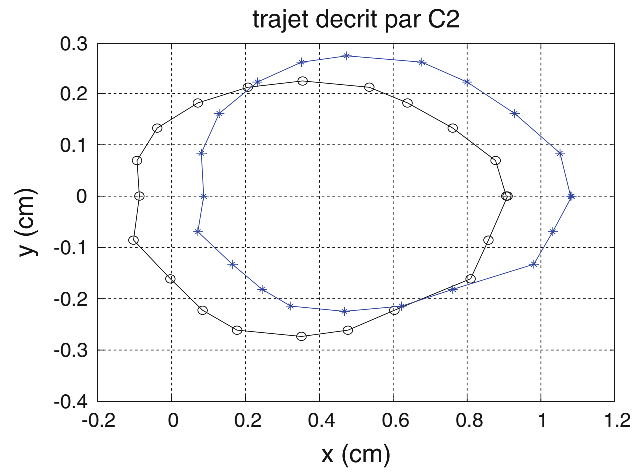

Fig. 4 Closed path described by interval

Interval analysis allows taking into account changes in geometric dimensions, caused by manufacturing variation, revealing the resulting variations in the system's configuration from its nominal configuration. The crank and connecting links L1 and L2 are allowed to be variable in length; this small change is expressed by $\Delta L = \Delta L1 = \Delta L2$. Therefore, L1, L2 can be represented as follows: $L1 = [L1 - \Delta L, L1 + \Delta L]$, $L2 = [L2 - \Delta L, L2 + \Delta L]$.

To illustrate what proceeds. We suppose that the error between the unknown system and the model is equal to zero. Thus, from the equation (2.4), we conclude that $y_k = X_C$.

With L1 and L2 are intervals, X_c can be rewritten as an interval with lower and upper borne, respectively:

$$[X_C]=\left[\underline{x}_c, \overline{x}_c\right]=[L1+\Delta L, L1-\Delta L]\cos\theta + \sqrt{(([L2+\Delta L, L2-\Delta L])^2-([L1+\Delta L, L1-\Delta L]\sin\theta)^2)}. \quad (4.6)$$

Actual system response is defined as follows:

$$X_{C-mk} = \{X : X \in S\},$$
$$S = [X_C] + \text{WGN}.$$

WGN: is white Gaussian noise and generate random number value which is uniformly distributed in the interval (0, 1) [13]. From these, parameter set estimation from bounded-error data is a set of feasible parameters consistent with data and known bounds on the error, which can be expressed by the following: $Lk = \left\{L : \left|X_{C-mk} - (L1\cos(\theta) + \text{sqrt}((L2)^2 - (L1\sin(\theta)^2))\right| \le Ek\right\}$ where $L = [L1; L2]$.

The parameters of the model are intervals of the width of which equals twice the value of tolerance applied to each segments. Figure 4 shows the closed path described by the point C2, the mass center of link BC in the Cartesian reference frame *xOy,* when an addition of noise and the tolerance of segments are considered.

Table 1 Values of the parameters after simulation

Description	Length (cm)	Tolerance ΔL (cm)	$L \pm \Delta L$	Estimate value $\hat{L}$	$W[\hat{L}]$
Crank [$L1$]	5	0.2	[4.8, 5.2]	[4.99, 5]	0.01
Connecting rod [$L2$]	10	0.2	[9.8, 10]	[10, 10]	0
Crank [$L1$]	5	0.5	[4.5, 5.5]	[4.9, 5]	0.1
Connecting rod [$L2$]	10	0.5	[9.5, 10.5]	[10, 10]	0
Crank [$L1$]	25	0.2	[24.8, 25.2]	[24.99, 25]	0.01
Connecting rod [$L2$]	40	0.2	[39.8, 40.2]	[40, 40]	0

Parameter estimation is the problem of finding the values of the unknowns of a mathematical model for simulating a complex system. In our case, unknowns are the bounded parameters, L1 and L2. In the literature [4], the fitting of y to experimental data is often implemented by iterative methods for nonlinear regression analysis, which compute “best-fit” shapes, for instance, the point least squares method or the minmax interval approximation as defined in [14].

The lower bound and upper bound of interval data form two of point data. By applying point least squares approximation to them separately, we obtain two point estimations. These two estimations can form interval estimation. This method has been reported and applied in [14].

Lsqcurvefit (least squares curves fit) is a function in MATLAB, appropriate to resolve parameters estimation problems by point least squares method, and we have the choice, in medium dimension, between techniques of Gauss Newton and of levenberg as well as two choices of technics of linear research. And in large dimension, the algorithm is of type reliable region, subproblems being resolved by an algorithm of gradient.

5 Simulation Results

In order to clearly show the effectiveness and potential of an interval approach in parameter's estimation, a geometric properties of the slider-crank mechanism cited below are used to estimate the parameters. The interval of the estimated parameters is designed by $\hat{L}$.

Therefore, WGN = 0.5rand.

The rand is a function in MATLAB and generates arrays of random numbers whose elements are uniformly distributed in the interval (0, 1).

From this, the bound of WGN is: $\|\text{WGN}\| \leq 0.5$.

By applying the point least squares method to the model presented in Fig. 4, the intervals of the estimated parameters are given in the Table 1 seen below:

From the result of simulation cited above, nominal value of parameters $L1$ and $L2$ belong to the intervals of the estimated parameters, and the widths of these intervals are more less than the width of the uncertainty interval as cited in Sect. 4.

6 Conclusion

We have described here the least squares method for reliably solving interval parameter estimation problems. Approach provides a mathematical and computational guarantee that the global optimum in the parameter estimation problem is found. We applied the technique here to several data sets, in which the model parameters were used as initial guess.

On the other hand, this becomes inaccurate when the initial guess value is far from the desired one.

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