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Numerical Analysis of Buckling Behavior of Concrete Piles Under Axial Load Embedded in Sand

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Abstract Piles that have been placed in loose and week soils and at the same time are under heavy loads can be in danger of buckling. These piles can be particularly found under bridges and pier where scouring makes the buckling an inevitable issue. Being aware of this fact, lots of experimental investigations have been done to study the complex buckling behavior of piles. But the results suffered from the experiments' limitations. On the other hand, very few numerical investigations have been done to study buckling behavior of piles, particularly under axial loads. In this research three-dimensional finite element analyses have been performed to study the buckling behavior of fully and partially embedded concrete piles. Investigations have been done only for sandy soils and only the axial load was applied to piles. Results of the numerical model have been verified by previous experimental results. Sensitivity analysis has been done for different pile parameters such as total and unsupported length of pile, and different soil parameters such as internal friction angle, unit weight and module of elasticity. Eventually, effect of each parameter on the buckling behavior of piles has been presented and discussed.

Keywords Buckling capacity · Concrete piles · Axial load · Numerical modeling

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الخلاصة

يمكن للمواسير الموضوعة في تربة فضفاضة وضعيفة التي هي في الوقت نفسه تحت أحمال ثقيلة أن تكون في خطر من الالتواء. وهذه المواسير يمكن العثور عليها بخاصة تحت الجسور والرصيف حيث تجعل النفايات مسألة الالتواء أمرا لا مفر منه. وإدراكا لهذه الحقيقة، فقد تم القيام بكثير من التحقيقات التجريبية لدراسة سلوك الالتواء المعقد للمواسير. ولكن النتائج عانت من قيود التجارب. من ناحية أخرى هناك عدد قليل جدا من التحقيقات العددية التي تم القيام بها لدراسة سلوك الالتواء المعقد للمواسير، لا سيما تحت الأحمال المحورية. وقد تم في هذا البحث إجراء تحليلات العنصر المحدود ثلاثية الأبعاد لدراسة سلوك التواء مواسير خرسانة مرسخة جزئيا وبالكامل. وأجريت التحقيقات فقط للتربة الرملية وفقط عند تطبيق حمولة محورية للمواسير. وتم التحقق من نتائج النموذج العددي من قبل النتائج التجريبية السابقة، وحُللت الحساسية لمعاملات الماسورة المختلفة مثل إجمالي الطول غير المدعم للماسورة، ومعاملات التربة المختلفة مثل زاوية الاحتكاك الداخلي، ووحدة الوزن ومعامل المرونة . وفي نهاية المطاف تم عرض تأثير كل معامل في سلوك التواء المواسير ومناقشته.

1 Introduction

Piles are slender structural elements that transfer the structure loads from loose and weak upper layers of soil to dense and strong deeper layers. Because of heavy loads, piles are in danger of buckling. Buckling can be defined as sudden deformation of structure due to reaching the critical load. The buckling behavior of piles is so complicated and at the same time has a significant effect on the instability of a structure. This makes it essential to conduct a comprehensive investigation on the buckling behavior of piles. Vertical piles embedded in loose soils are exposed to buckling risk [1]. On the other hand, long piles are unavoidable in off-shore structures and bridges. A significant length of these piles is not surrounded by soil and this makes them vulnerable for buckling. As was mentioned before, several studies have been performed to investigate the buckling behavior of piles and several efforts have also been done to apply these results to designing problems. An approximate procedure for treating the problem of bending and buckling of partially embedded piles was developed [2]. The procedure was developed with the use of theoretically correct solutions applicable to a partially embedded pile subjected to moment, shear, and axial loads, when acting separately. It was shown that a partially embedded pile may be represented as free-standing with a fixed base at a depth below the ground surface equal to a relationship which involves a subgrade modulus constant with depth and one an involving subgrade modulus increasing linearly with depth. Other researchers proposed finite difference computer software that was able to calculate the buckling load of fully and partially embedded piles [3]. A model for evaluating the critical buckling capacity of long slender friction piles with lateral soil support included was developed based on the concept of subgrade reaction [4]. However, the lateral force-deflection ($P-y$) behavior is assumed to be linear. A parametric study was conducted to demonstrate the effect of ω value, defining the distribution of the horizontal subgrade reaction, on the evaluated buckling capacity. In the case of the free top and fully embedded condition with embedment length (h) greater than 10 m, a 59 % increase in the buckling capacity (P_{cr}) was predicted as ω was increased from 0 (constant horizontal subgrade reaction distribution) to 1 (linearly increasing horizontal subgrade reaction). Results also indicated that the boundary conditions at the pile tip have a minimal effect on $P\backslash\mathrm{dc}\backslash\mathrm{dr}$ when the nondimensional embedded length (h') exceeded 3.3 for the free top, 5.6 for the fixed-sway top, and 7.6 for the pinned-top condition. The stability of piles that are supported laterally along part of their entire length by a layered elastic medium was also investigated [5]. The investigation was conducted for the case in which each layer had the general power distribution of the coefficient of horizontal subgrade reaction. The potential energy method was used to develop the model. Results showed that unsupported length of the pile has much more effect on buckling load than other parameters such as stiffness of soil, depth of soil layer, and pattern of load distribution.

Building codes and common design practices generally assume that the lateral support provided by the soil is sufficient to prevent buckling of fully embedded piles. As small diameter grouted piles (micropiles) have evolved from relatively low capacity friction piles to current applications that include high capacity elements. The issue of potential buckling of these very slender piles was revisited for the case of embedment in soft soils [6]. Pile buckling loads obtained from a semi-empirical relationship were compared to the allowable loads permitted by proposed codes and design guidelines. It was concluded that buckling is generally not a concern for the most common types of micropile design, but there are some designs permitted under the codes and design guidelines for which buckling may be a controlling design factor. A simple approach to predict the buckling capacity of axially loaded partially embedded slender reinforced concrete piles using the Davisson and Robinson method was presented [7]. The flexural stiffness (EI) equation of the slender concrete column permitted by ACI building code was adopted as such for concrete piles. Results of experimental investigations carried out on axially loaded slender concrete piles in a sand medium for the various combinations of unsupported length and coefficient of subgrade modulus were compared with the proposed approach. The analyses indicated the nearness between the theoretical predictions and the experimental results. The importance of bending–buckling interaction in seismic design of piles in liquefiable soils using numerical techniques was investigated [8]. A pseudo-static analysis was performed using a well-documented case history, where the pile–soil interaction was modeled as a beam on nonlinear Winkler foundation (BNWF). The buckling analysis showed that the pile was safe against pure buckling during full liquefaction. However final results showed that if a pile is designed for bending and buckling criteria separately and safe for these individual design criteria, it may fail due to their combined effect. A fuzzy system to estimate the overall and local buckling behavior of cylindrical tubular members under monotonic axial compression was developed [9]. To train and test the fuzzy system, numerical data obtained from the finite element analyses was utilized. For this aim, a degenerate-continuum shell element which accounts for material and geometric nonlinearity was employed. The proposed fuzzy system is capable of tracing the complete load-shortening relation and provides a tool for faster analysis.

Generally it can be said that few studies have been performed to investigate the buckling behavior of fully and partially embedded concrete piles. Therefore, in this research, the buckling behavior of concrete piles under axial load embedded in sandy soils has been investigated. Pile and soil have been considered as a system in a way that let the geotechnical properties of the soil interact with the structural behavior of the pile.

2 Problem Definition

The main purpose of this research was to investigate the buckling behavior of concrete piles under axial loads embedded in sandy soils. Two cases of fully and partially embedded singular concrete piles were modeled numerically. Nonlinear soil parameters were considered consistent with common natural values of sandy soil. A small static vertical load was applied to the top of the pile and slowly increased until buckling occurred in the pile. Several analyses were performed to find the effects of different parameters such as total length, unsupported length of partially embedded pile, internal friction angle, modulus of elasticity, etc. on buckling behavior of the pile. To verify the numerical model a comparison was done between its results and the results of the previous experimental studies. Numerical results were also compared to the theoretical prediction of the Euler equation. A good consistency was observed in both cases.

3 Numerical Modeling

The three-dimensional finite element method was used to model the pile-soil system. The soil was assumed isotropic and homogenous with Drucker–Prager yield criterion, for plastic deformations and yielding which is applicable to granular (frictional) material using the outer cone approximation to the Mohr–Coulomb law, as defined by the cohesion value (c), the angle of internal friction (φ), and the dilatancy angle (ψ). The analyses were performed in two steps. First, a static analysis was performed to find and apply the static soil stresses. Then vertical axial load was applied uniformly to the top of the pile and a "nonlinear buckling analysis" was performed to obtain the buckling capacity of piles. Large deformation was assumed in analyses.

3.1 The Geometry of Model

The effects of pile geometry properties on buckling capacity were meant to be investigated in this research. Therefore, various dimensions were considered for pile diameter and length in both fully and partially embedded conditions. The initial results of analyses showed that the embedded length of pile has a significant effect on buckling capacity of fully embedded piles comparing to partially embedded piles. Therefore, piles up to 50 m long were also modeled and studied for fully embedded condition. Table 1 shows a summary of pile lengths used in this study. Three analyses have been performed for each pile length, considering different diameters equal to 0.20, 0.35, and 0.50 m.

3.2 Soil and Pile Properties

In soil-related problems, the first step in simulating a real problem by a numerical method is to simulate realistic soil parameters that correspond to the field condition. However, one of the other primary goals of this research was to study the effect of variation of different soil parameters on the buckling bearing capacity of the pile. Thus three sandy soils with common values for their shear parameters were selected and are shown in Table 2. Table 3 shows the properties that have been used in modeling the concrete pile.

Table 1 Geometric parameters of the pile

Total length of pile, L (m)	Embedded pile length, L_f (m)	Unsupported length of pile, L_u (m)
5	5	0
5	3	2
5	2	3
10	10	0
10	7	3
10	3	7
15	15	0
15	10	5
15	5	10
20	20	0
20	13	7
20	7	13
25	25	0
30	30	0
35	35	0
40	40	0
45	45	0
50	50	0

Table 2 Soil properties

Soil type	E_s (kN/m^2)	γ_s (kN/m^3)	μ_s	C (kN/m^2)	φ^o
Loose sand	10,000	16	0.35	–	30
Medium sand	18,000	18	0.35	–	35
Dense sand	30,000	20	0.35	–	40

Table 3 Pile properties

Material	E_c (kN/m^2)	γ_c (kN/m^3)	ν	f'_c (kN/m^2)
Concrete	21e6	24	0.20	21,000

3.3 Element Type

"SOLID95" element was used for the 3D modeling of both pile and soil medium, defined by twenty nodes and three degrees of freedom at each node: translations in the nodal x, y, and z directions. The element has plasticity, creep, swelling, stress stiffening, large deflection, and large strain capabilities (Fig. 1). Both nodal load and element load can be defined for this element.

Figure 2 shows the "Conta174" element that was used as contact element for modeling the contact between soil and pile. This element is suitable for modeling the surface to surface contacts. The contact behavior was considered as

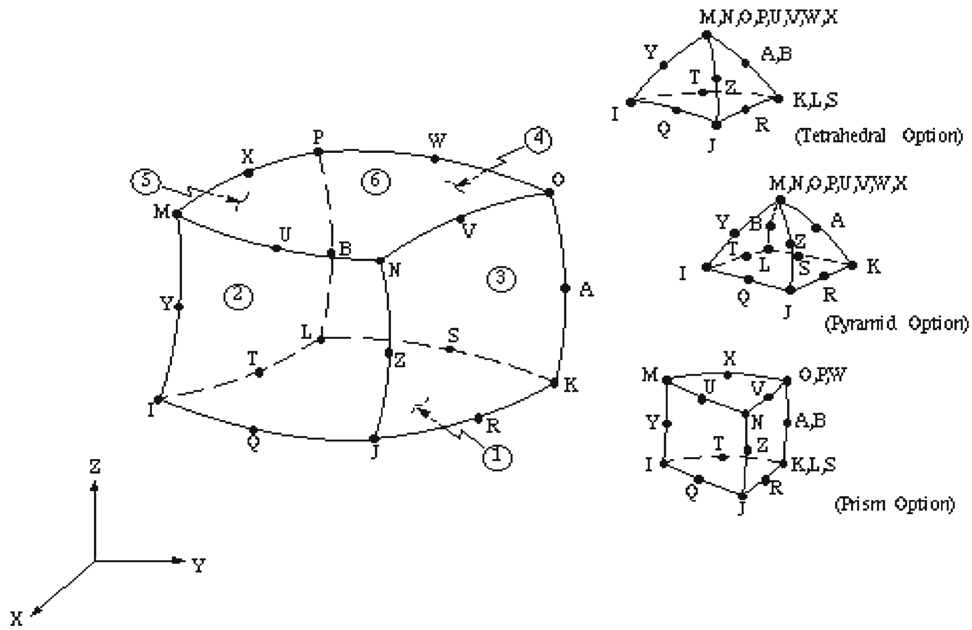

Fig. 1 Three-dimensional twenty-noded element (SOLID95) used in the finite element mesh

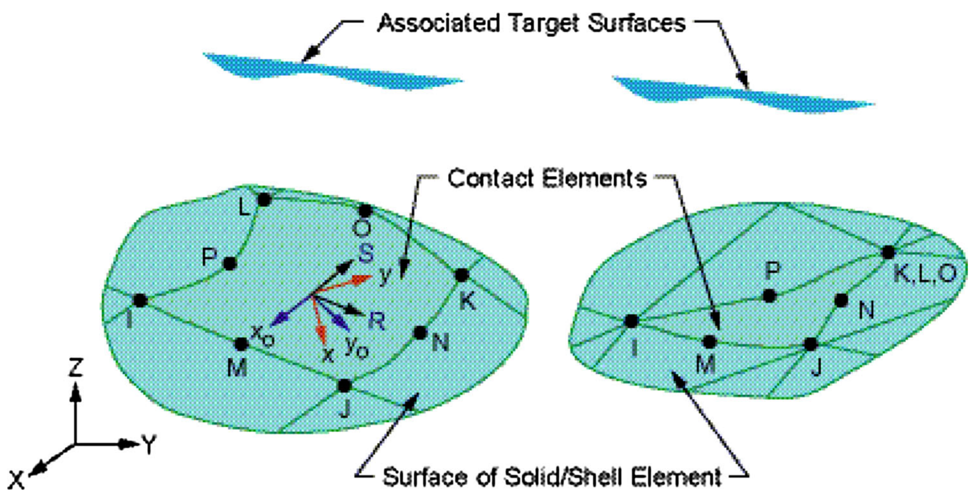

Fig. 2 "Conta174" element used as contact element between soil and pile

flexible to flexible contact which allows both surfaces to slide and have friction. Since piles are much stiffer than soil, they are considered as target surface and soil surface assumed as to be contact surface.

3.4 Boundary Condition

Being symmetric in geometry, boundary, and loading, only half of the actual model is sufficient to study buckling behavior of piles. Although the symmetric plane were restrained in X and Y direction, the inner nodes were observed to undergo significant displacements in different directions. Boundary edges parallel to X axis were restricted along Y direction and boundary edges parallel to Y axis were restricted along X direction. The displacements perpendicular to the plane of symmetry are neglected owing to the symmetric nature of the problem (Fig. 3). The hard stratum underlying the soil layer could be bedrock, hard clay, or sand stone, which are all much stiffer than the overlying soil and would practically restrict the displacement of the soil unless there is an adequate depth for stresses and strains to dissipate. Thus, a range of depth is checked through trial and error to obtain the minimum model depth to prevent restriction of displacement. It is, therefore, reasonable to assume that this stratum represents a rigid boundary and the base of the model is fixed. However, it should be mentioned that although considering a smaller model and extracting stresses and strains from boundary elements would reduce the size of the model and would cause analyses to run faster, the area that deforms and stress propagates in the model was aimed to be observed to obtain some practical idea about the distance of propagation of the stresses and deformations.

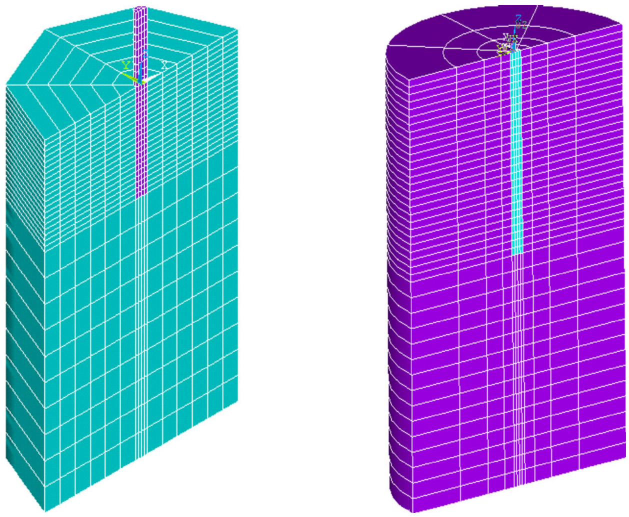

Fig. 3 The geometry of finite element model for both fully and partially embedded cases

3.5 Meshing

To obtain more accurate results, elements are kept very small near the pile, increasing gradually in size while moving away from the pile. In order to compare the results of different analyses, the nodes must be placed at the identical position in all models. Therefore, manual and mapped mesh is employed. The geometry of FEM, meshing method and boundary conditions have been shown in Fig. 3.

4 Verification of Numerical Model

One of the most outstanding experimental studies on buckling behavior of the piles was considered to verify the numerical model [7]. Two different pile-soil systems were modeled numerically in a way that all soil and pile parameters were exactly same as the experimental model [7]. These parameters have been shown in Table 4. A theoretical method (Euler equation, Eq. 1) was also used to calculate and predict the buckling load of modeled piles. Table 5 shows that the results of three approaches were consistent:

$$P_{cr} = \frac{\pi^2 EI}{(KL_e)^2}, \tag{1}$$

where E is the module of elasticity of the concrete pile, I is the inertial moment of the pile's section, L_e is the effective length of pile, and K is the coefficient of effective length, depending on support conditions.

Table 5 Comparison between the results of different approaches

Pile-soil system	$P_{cr-\text{Numerical}}$ (kN)	$P_{cr-\text{Experimental}}$ (kN)	$P_{cr-\text{Theoretical}}$ (kN)
1	87.00	72.44	87.44
2	89.22	79.43	90.72

Where h and b are the length and width of rectangular section of concrete pile, L_u is the unsupported length of the pile, L is the total length of the pile, f_{cc} is the concrete cube compressive strength, D_r is the relative density of the soil, E is the modulus of elasticity of soil, and φ is the internal friction angle of the soil.

Where $P_{cr-\text{Numerical}}$ is the buckling load obtained from numerical analysis, $P_{cr-\text{Experimental}}$ is the experimental buckling load [7], and $P_{cr-\text{Theoretical}}$ is the theoretical predicted buckling load (Eq. 1). As it can be seen, a clear consistency exists between the results of all three methods. The small difference between the numerical results and the experimental results [7] can be due to sample preparation in the experimental work. To prepare the sample in the experimental model the soil was placed and compacted in a box in different layers [7]. This could cause a layered, non-isotropic soil. The main goal of this research, however, was to model the isotropic sand. The minor non-isotropic condition in the experiment [7] might have caused the small difference.

5 Results and Discussions

The buckling behavior of the concrete piles, embedded in the sandy soil, has been studied in this research. More than 300 numerical and analytical analyses were performed and the effect of different parameters has been investigated.

5.1 Effective Length and Point of Fixity

As can be seen in Fig. 4 the total length of the pile will not buckle before the failure; but some part of its length which is known as effective length (L_{cr}), will deform. Effective length is described as the distance between the top of the pile and the point of fixity. The point of fixity has been observed in both fully and partially embedded piles. In fully embedded piles, the point of fixity would be seen in lower depths while in partially embedded piles the point of fixity is almost always close to the ground. Therefore, it can be assumed that for

Table 4 Soil and pile characteristics

Pile-soil system	Pile characteristics					Soil characteristics		
	h (m)	b (m)	L (m)	L_u (m)	F_{cc} (N/mm^2)	D_r (%)	E (kN/m^2)	φ (°)
1	0.04	0.05	2.20	1.10	29.92	50	15,000	35
2	0.04	0.05	2.20	1.10	28.63	70	35,000	40

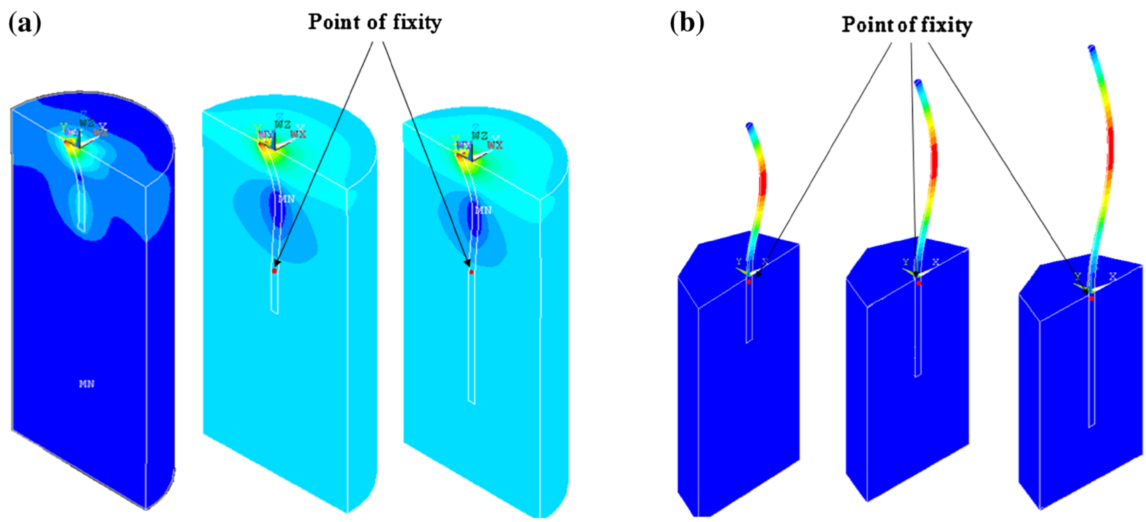

Fig. 4 Buckling deformation and point of fixity, **a** fully embedded pile with different length, **b** partially embedded pile with different length

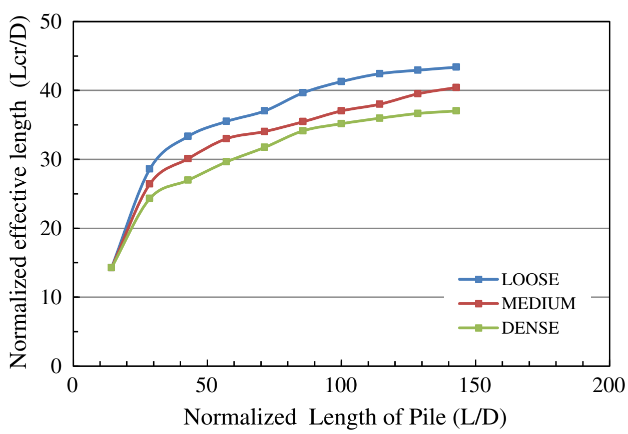

Fig. 5 Variation of normalized effective length versus normalized length of piles, fully embedded in sandy soil

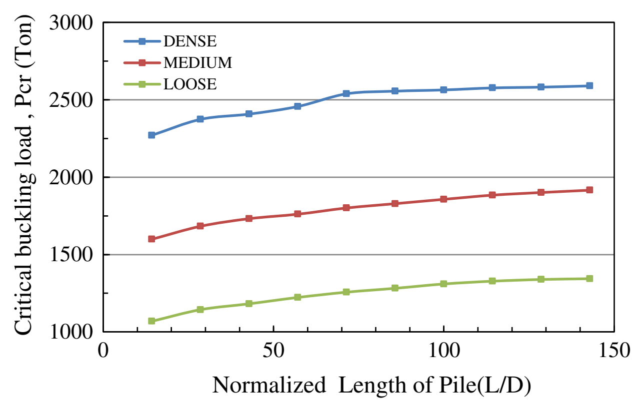

Fig. 6 Variation of critical buckling load versus normalized length of pile, fully embedded in different soils

partially embedded piles, effective length (L_{cr}) is equal to unsupported length (L_u). Thus it can be said that the unsupported length of a pile behaves like a free-fixed column. In fully embedded piles the location of the point of fixity would depend on the variation of geometric parameters of the pile and the resistive parameters of soil. Figure 5 shows the variation of normalized effective length versus normalized length of pile. Normalization has been done to the diameter of the piles (D). It is important to study the effective length of the piles because in some cases the bearing capacity of a pile would decrease significantly due to the buckling in the effective length of the pile. This has been discussed fully in the following sections.

As seen by increasing the pile length, the graphs will first rise with a steep slope that indicates the buckled length of the pile increases rapidly. Then they will become moderate and finally they will reach a constant value. This constant value is actually the length of the point of fixity. Achieving a constant value reveals that even by increasing the pile length, the buckling length of the pile would not increase more than the final constant value. In other words, those parts of pile that are under the point of fixity will not contribute to buckling and increasing pile length; more importantly, it will not increase the buckling bearing capacity of the pile. It can also be seen that the longest and the shortest effective length has happened in loose and dense sand, respectively. This is because piles are less confined in loose sand compared to dense sand and they can displace easier.

5.2 Effect of Increasing Pile Length on the Buckling Behavior of Piles

5.2.1 Fully Embedded Piles

Figure 6 shows the variation of critical buckling load versus normalized length of pile, fully embedded in different soils. As can be seen, by increasing the pile length the critical buck-

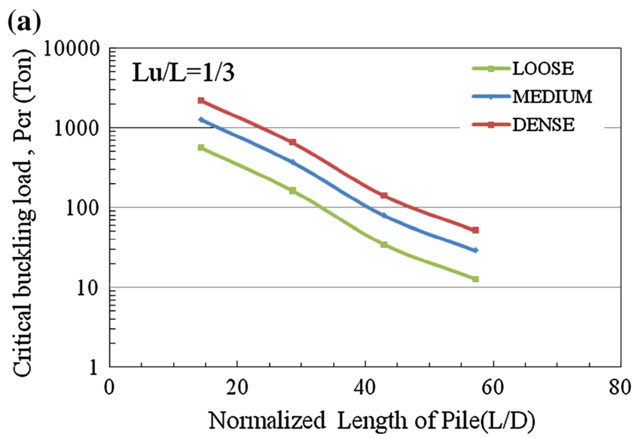

Fig. 7 Variation of critical buckling load versus normalized length of pile partially embedded in different soils. **a** Unsupported length of piles (L_u) is equal to one third of pile length (L), **b** unsupported length of piles is equal to two third of pile length

ling load increases. At first the increment is significant, but by increasing the normalized length, the critical buckling load rises more slowly and eventually it reaches a constant value. Thus it can be concluded that increasing the pile length will increase the critical buckling load. However, after a specific length, increasing the pile length does not have any significant effect on the critical buckling load.

5.2.2 Partially Embedded Piles

Figure 7a, b shows the variation of critical buckling load versus normalized length of pile partially embedded in different soils. The unsupported length of piles is equal to one third and two third of pile length respectively. As can be seen, increasing normalized pile length decreases the critical buckling load. This is because each time increasing the total length of pile also increases the unsupported length of pile. Therefore, the critical buckling load decreases continuously.

5.3 Effect of Soil Density

Effects of soil density on buckling capacity have been shown in Fig. 8a–c. For both fully and partially embedded conditions, increasing soil density increases the critical buckling load of a pile with identical length and diameter. It also decreases the effective length of a pile which means a shorter length of pile deforms. This is because of the existence of more confinement of the pile and consequently more resisting forces against displacement in denser soil.

5.4 Effect of Unsupported Length

Unsupported length is one the most important parameters that has a significant effect on the buckling bearing capacity of partially embedded piles. Increasing the unsupported length of a partially embedded pile significantly decreases the buckling capacity, as seen in Fig. 8. Increasing the unsupported length of a pile from zero (fully embedded pile) to $1/3L$ remarkably decreases the buckling bearing capacity. A greater decrease can also be seen by increasing the unsupported length to $2/3L$. In addition, by increasing L/D and consequently achieving a longer unsupported length, the difference between the critical buckling load (P_{cr}) of three cases of $L_u/L = 0$, 1/3 and 2/3 increases. In fact, the unsupported length of partially embedded pile behaves like a free-fixed column without any lateral confinement. Increasing unsupported length (L_u) means a longer unconfined column which, according to Eq. 1, leads to a considerable decrement of the critical buckling load (P_{cr}).

Figure 9 shows the ratio of the critical buckling load for different unsupported lengths (P_{cr}) to the buckling load of a fully embedded pile (P_{cr-0}) versus the ratio of unsupported length to total length of a pile. The graphs represent the results for loose, medium, and dense soil. By increasing the unsupported length an exponential decrease can be clearly seen for all piles but the 5-meter one. This, of course, is because buckling is not a significant issue in short piles due to their short unsupported length.

As it was expected, less decrease is observed in the buckling capacity of piles embedded in denser soils. This is a result of less confinement in loose sand compared to dense sand.

Table 6 shows the exponential equations that have been extracted from Fig. 9. These equations can predict the buckling capacity of piles with any unsupported length with good precision.

As it was expected, an exponential relationship can be seen between normalized buckling capacity and normalized length in moderate and long piles.

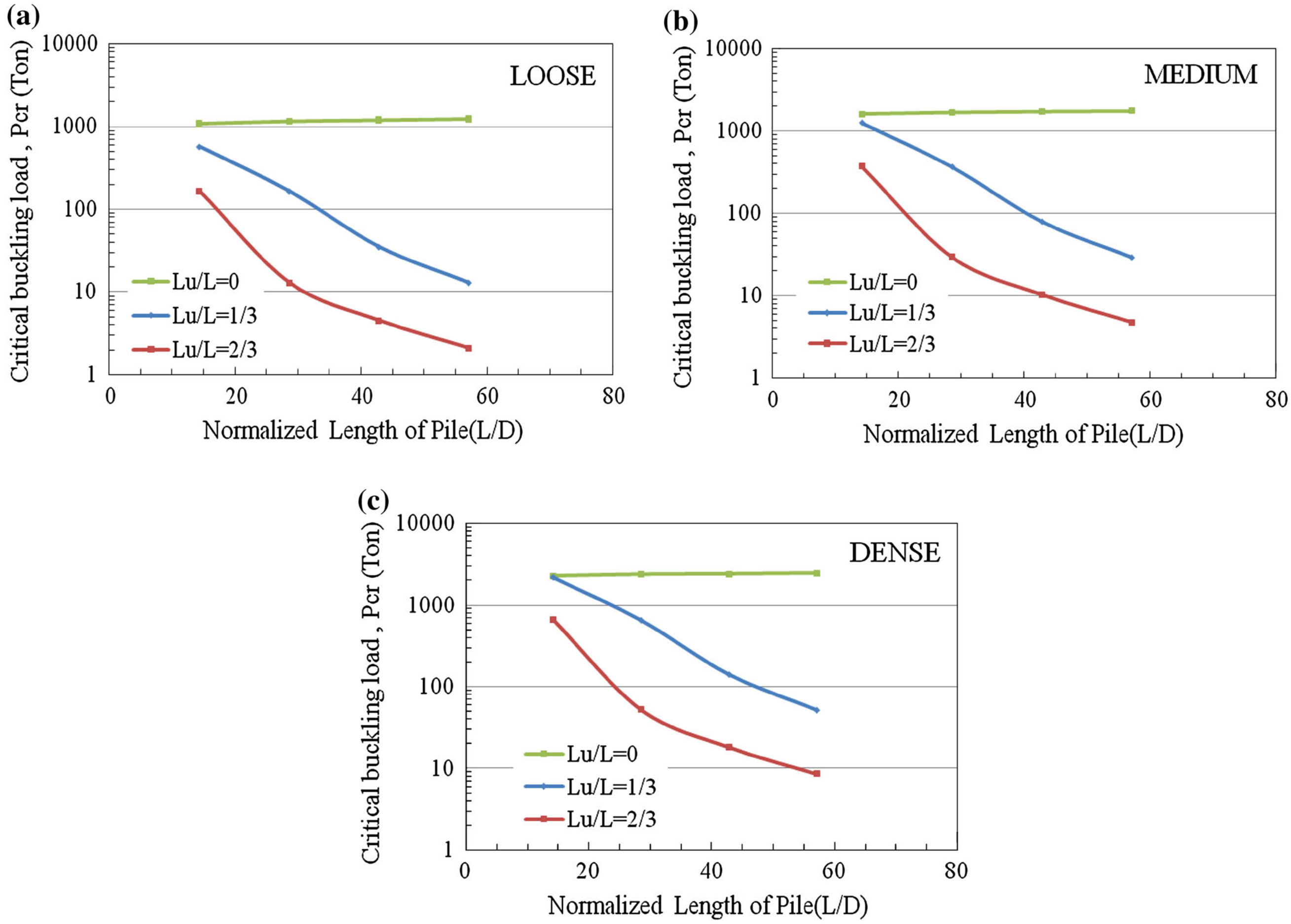

Fig. 8 Variation of critical buckling load versus normalized total length of the pile with three different embedment length, **a** loose soil, **b** medium soil, **c** dense soil

5.5 Comparison Between Buckling Bearing Capacity and Geotechnical Bearing Capacity of a Pile

To investigate the necessity of considering buckling capacity in designing projects, a comparison has been done between the buckling bearing capacity obtained from numerical analyses, and geotechnical bearing capacity calculated by hand. Column charts have been used to present just for piles with a diameter equal to 0.35 m as an example in Fig. 10. The horizontal axis shows pile length and the vertical axis shows the critical load that causes failure in a pile.

It should be noted that geotechnical bearing capacity is defined as the maximum load that a pile-soil system can take before failure happens in the soil surrounding the pile or the soil under the tip of the pile. Depending on the properties of the pile, numerical buckling capacity could be larger or smaller than this.

Equation 2 calculates the geotechnical bearing capacity [10]:

$$Q_U = Q_P + Q_S, \tag{2}$$

where Q_U is the ultimate bearing capacity of the pile, Q_P is the base capacity, and Q_S is the frictional capacity or shaft capacity. Q_P and Q_S can be calculated by Eqs. 3 and 4.

$$Q_P = A_P \cdot q_p = A_P(C \cdot N_C^* + q' \cdot N_q^* - q') \tag{3}$$

$$Q_S = \int_0^{L1} P \cdot f \cdot dz = \pi \cdot D \cdot (1 - \sin\phi) \int_0^{L2} \sigma_v' \cdot \tan\delta \cdot dz, \tag{4}$$

where q' is the effective vertical stress at the tip of the pile, N_q^* is the coefficient of bearing capacity, N_c^* is the coefficient of cohesion, C is the cohesion of the soil, A_P is the cross section of the pile tip, P is the perimeter of the pile section, f is the unit friction resistance at any depth z, L_1 is the pile length over which P and f are taken constant, D is the diameter of the pile, φ is the soil internal friction angle, σ_v' is the effective vertical stress at depth z, and δ is the soil-pile friction angle.

It can be seen that for all fully embedded piles, the buckling bearing capacity is larger than the geotechnical capacity. This means that buckling is not an issue in fully embedded piles because before reaching the buckling capacity, fully embedded piles will fail due to geotechnical failure.

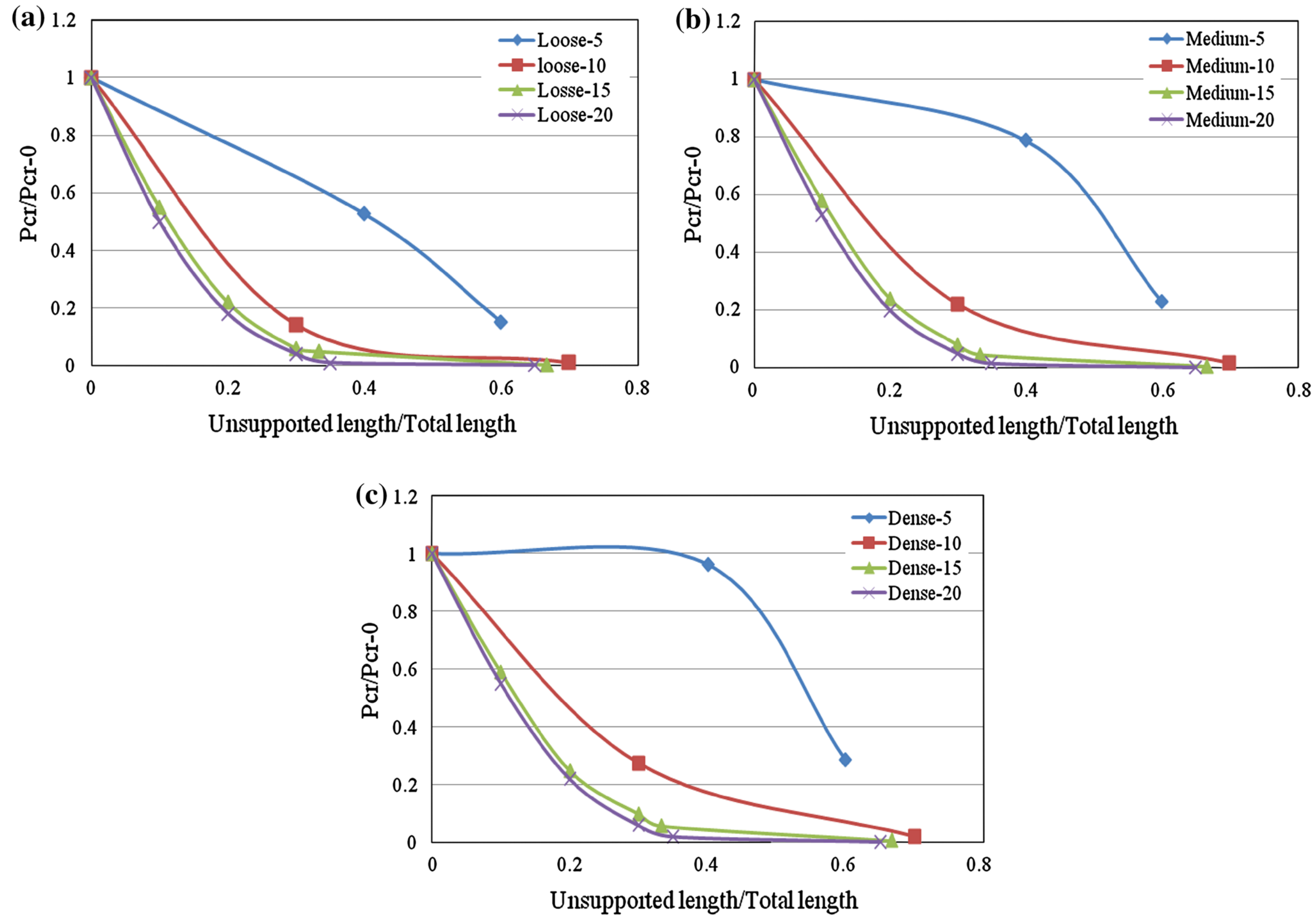

Fig. 9 The ratio of critical buckling load for different unsupported length (P_{cr}) to the buckling load of fully embedded pile ($P_{\mathrm{cr}-0}$) versus the ratio of unsupported length to total length of the pile, **a** loose soil, **b** medium soil, **c** dense soil

Table 6 Relationship between normalized buckling capacity and normalized length

Soil	Total length (m)	Relationship
Loose	10	$\frac{P_{\mathrm{cr}}}{P_{\mathrm{cr}-0}} = \mathrm{e}^{-6.43(L_{\mathrm{un}}/L_{\mathrm{t}})}$
	15	$\frac{P_{\mathrm{cr}}}{P_{\mathrm{cr}-0}} = \mathrm{e}^{-8.512(L_{\mathrm{un}}/L_{\mathrm{t}})}$
	20	$\frac{P_{\mathrm{cr}}}{P_{\mathrm{cr}-0}} = \mathrm{e}^{-10.39(L_{\mathrm{un}}/L_{\mathrm{t}})}$
Medium	10	$\frac{P_{\mathrm{cr}}}{P_{\mathrm{cr}-0}} = \mathrm{e}^{-5.673(L_{\mathrm{un}}/L_{\mathrm{t}})}$
	15	$\frac{P_{\mathrm{cr}}}{P_{\mathrm{cr}-0}} = \mathrm{e}^{-7.972(L_{\mathrm{un}}/L_{\mathrm{t}})}$
	20	$\frac{P_{\mathrm{cr}}}{P_{\mathrm{cr}-0}} = \mathrm{e}^{-9.587(L_{\mathrm{un}}/L_{\mathrm{t}})}$
Dense	10	$\frac{P_{\mathrm{cr}}}{P_{\mathrm{cr}-0}} = \mathrm{e}^{-5.673(L_{\mathrm{un}}/L_{\mathrm{t}})}$
	15	$\frac{P_{\mathrm{cr}}}{P_{\mathrm{cr}-0}} = \mathrm{e}^{-7.972(L_{\mathrm{un}}/L_{\mathrm{t}})}$
	20	$\frac{P_{\mathrm{cr}}}{P_{\mathrm{cr}-0}} = \mathrm{e}^{-9.587(L_{\mathrm{un}}/L_{\mathrm{t}})}$

L_{un} unsupported length, L_{t} total length

The behavior of partially embedded piles is a little more complicated and two general behaviors were observed. Piles with $L_{\mathrm{U}}/L = 1/3$ partially embedded in loose, medium, and dense sand, with a diameter equal to 0.20, 0.35, and 0.50 and with a length longer than 15 m ($L > 15$ m) have a buckling bearing capacity less than the geotechnical bearing capacity. Thus it can be said that when $L_{\mathrm{U}}/L = 1/3$ and the pile is longer than 15 m, attention should be paid to buckling. This is because before reaching the geotechnical bearing load, a pile will buckle and fail. It can also be said that compared to an unsupported length of pile, the diameter of the pile and the relative density of the soil do not have a significant effect on the buckling behavior of a pile. However, increasing the diameter or the density of the soil will increase the buckling capacity, but it would still be less than the geotechnical bearing capacity.

For partially embedded piles with $L_{\mathrm{U}}/L = 2/3$ buckling will become important when the pile is longer than 10 m ($L > 10$ m).

6 Conclusions

A three-dimensional finite element method was employed to study the buckling behavior of fully and partially embedded

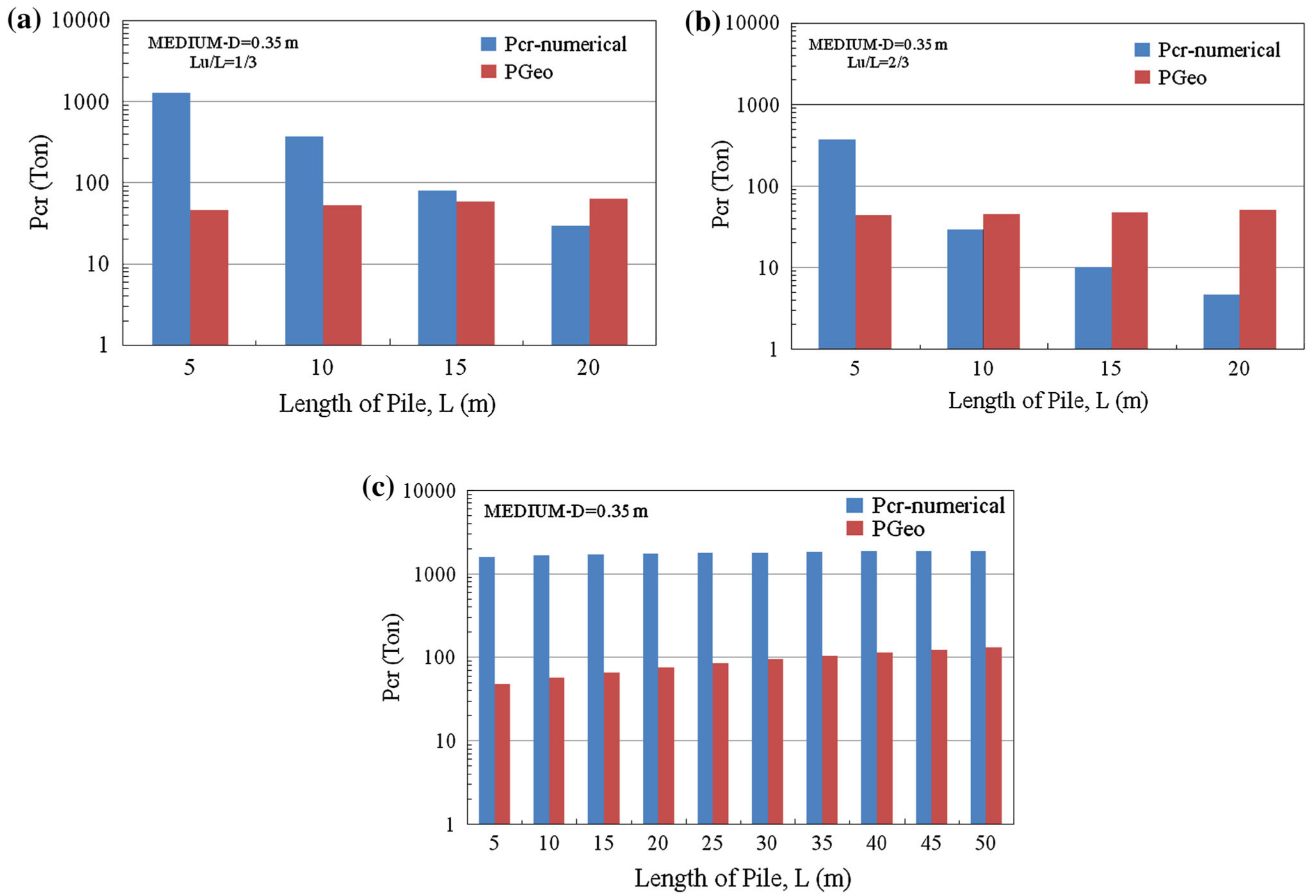

Fig. 10 Comparison between the numerical buckling capacity and geotechnical bearing capacity, **a** partially embedded piles with $L_U/L = 1/3$, **b** partially embedded piles with $L_U/L = 2/3$, **c** fully embedded piles

piles. The effects of unsupported length and point of fixity, total length, and relative density of soil have been studied. A comparison has also been done between all numerical results and geotechnical bearing capacity calculated by hand. The following specific conclusions can be drawn from the study:

1. Total length of the pile does not participate in buckling, but only a specific length which in this research has been called "effective length, L_{cr}" takes part in buckling.
2. The longest and the shortest effective lengths for fully embedded piles happen in loose and dense sand, respectively. This is because of less confinement of the piles in loose sand compared to dense sand.
3. Increasing the length of a fully embedded pile increases its buckling capacity. However, after a specific length, it does not have any significant effect on the buckling capacity. This specific length is in fact the effective length of the pile which is the distance between the top of the pile and its point of fixity.
4. In partially embedded piles, increasing the embedded length does not have a considerable effect on buckling capacity compared to the effect of increasing the unsupported length.
5. For piles with identical lengths, increasing soil density increases the buckling capacity of piles in both fully and partially embedded cases.
6. Unsupported length of a partially embedded pile behaves like a free-fixed column without any lateral confinement. Increasing unsupported length leads to a longer unconfined column and it causes a significant decrease in critical buckling load.
7. Comparing all numerical and analytical results reveals that buckling is not a general issue in fully embedded piles. The dominant failure mechanism is geotechnical failure which is fairly below the buckling bearing capacity.
8. Buckling would become the general failure mechanism in partially embedded piles with $L_U/L = 1/3$ that are longer than 15 m.
9. Partially embedded piles with $L_U/L = 2/3$ that are longer than 10 m generally fail due to buckling. Therefore, considerations must be made for the buckling design of these piles.

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