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Development of an Automatic Calibration Tool Using Genetic Algorithm for the ARNO Conceptual Rainfall-Runoff Model

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Abstract Rainfall-runoff simulation is one of the key steps in hydrology. Conceptual models are frequently used in rainfall-runoff simulation. However, a major difficulty in practice remains on how to optimize the parameters of the model. This is often a time-consuming and labor-intensive task for the modeler when manual calibration is adopted together with employing the knowledge of the model structure and parameters. In this study, an automatic calibration tool was developed to calibrate the ARNO conceptual rainfallrunoff model using the simple genetic algorithm (SGA). SGA is a simple, powerful, and popular optimization method, which explores the search space for the global optimum and has been successfully employed in many optimizations problems. The ARNO model was calibrated automatically for rainfall-runoff simulation of the Pataveh basin, which is a sub-basin of Karun River basin in Iran. The simulation performance of the model was evaluated on the basis of various performance criteria. Efficiency coefficient and coefficient of determination reached values higher than 0.80 during calibration and validation. The values of the remaining performance statistics were acceptable. The results show that this model

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with employed automatic calibration tool can successfully be used for continuous rainfall-runoff simulation.

Keywords Automatic calibration · Conceptual rainfallrunoff models · Fitness function · Genetic algorithm · Karun river

الخلاصية

تُعدّ محاكاة سقوط الأمطار - جريان المياه واحدة من الخطوات الرئيسية في مجال الهيدر ولوجيا حيث كثيرا ما تستخدم النماذج المفاهيمية في محاكاة هطُّول الأمطار - جريان المياه. ومع ذلك، تبقى هَّناك صعوبة كَبيرة في الممارسة من حيث كيفية تحسين معاملات النموذج، وهذه غالبا ما تكون مهمة تستغرق وقتا طويلا ، وتتطلب عمالة مكثفة لصَّانع النموذج عندما يتم معا اعتماد المعايرة اليدوية مع توظيف المعرفة لهيكلية ومعاملات النموذج. وفي هذه الدراسة، تم تطوير أداة المعايرة التلقائية لمعايرة نموذج ARNO لسقوط الأمطار ـ جريان المياه المفاهيمي باستخدام الخوارز ميات الجينية البسيطة (SGA). وتبيّن أن الخوارز ميات الجينية البسيطة هي بسيطة، وقوية، وطريقة تحسين معروفة ، وتستكشف فضاء البحث عن طرق التحسين الأمثل العالمية ، ووقد استخدمت بنجاح في العديد من مشاكل التحسين ونَّمت معايرة نموذج ARNO تلقائيا لمحاكاة سقوط الأمطار -جريان المياه في حوض باتافيه، وهو حوض فرعي من حوض نهر الكارون في ايران وتم تُقييم أداء المحاكاة للنموذج على أساس معايير الأداء المختلفة. وبَّلغ معامل الكفاءة ومعامل التحديد قيَّما أعلى من 0.80 خلال المعايرة والتَّحقق من الصحة. وكانت قيم إحصـاءات الأدَّاء المتبقية مقبولة، وأظهرت النتائج أن هذا النموذج مع أداة المعايرة التلقائية المطبقة يمكن أن يستخدم بنجاح لمحاكاة سقوط الأمطار المستمر حريان المياه

Abbreviations

- *b* A parameter representing spatial distribution of the soil moisture capacity
- *B* Base flow
- B–C Blaney and Criddle method
- *c* Exponent used to represent drainage when saturation is not reached
- *C* A small integer

1 Introduction

1.1 Calibration of Rainfall-Runoff Models

Rainfall-runoff modeling is one of the key steps in scientific hydrology and environmental management [\[1\]](#page-13-0). Many rainfall-runoff models have been developed by hydrologists to model the rainfall-runoff process. These models, among others, are classified as physically based and conceptual [\[2](#page-13-1)]. The physically based models are based on scientifically accepted principles for describing hydrological processes which control the basin responses. These types of models are appealing to some extent as they provide a mathematically idealized representation of the real phenomenon. However, they require huge basin data, which are usually difficult to obtain, and high computational needs [\[3](#page-13-2)].

The other type of models mentioned above is the conceptual ones which consider hydrological processes, perceived to be of importance, as simplified conceptualizations [\[3](#page-13-2)].

They can provide the reality with reasonable accuracy, requiring input data which are readily available for most applications. The parameters of these models are conceptual representations of abstract basin characteristics, and, in general, should be obtained through a calibration process. Compared to physically based models, conceptual models are more frequently used in hydrological applications. However with regard to the calibration process, a major difficulty in practice is how to optimize the parameters of the model. That is because most of these models have a large number of parameters and appropriate parameter set should be found within a large multidimensional parameter space [\[4](#page-13-3)]. Furthermore, the objective function surface is often non-convex, nonlinear, and may have numerous local optima [\[5](#page-13-4)[–7\]](#page-13-5).

The calibration process is performed either manually or automatically, and uses a computer-based technique. Manual calibration employs a try-and-error process of parameter adjustments. This is a time-consuming and labour-intensive task for the modeler unless he has an extensive knowledge regarding the model structure and parameters. Therefore, many modelers resort to automatic calibration.

In automatic calibration technique, a computer-based search algorithm explores the search space. This is fast and more efficient for detailed investigation of the search space for finding the global optimum. Various optimization algorithms have been employed as the basis of automated calibration method.

In general, optimization algorithms can be classified into local and global optimization methods [\[8](#page-13-6)]. The local optimization methods, such as gradient-based optimization algorithms, are efficient for locating the optimum of a uni-modal function. But they are inappropriate for multi-modal functions, because they may be trapped in a local optimum since that does not ensure that the global optimum is found [\[9](#page-13-7)]. In this respect, the possibility of reaching a global optimum largely depends on the location of the starting point of the search [\[10\]](#page-13-8). On the other hand, global optimization methods are especially designed for locating the global optimum.

Research into optimization methods has led to the use of global search methods such as genetic algorithms (GAs) [\[11](#page-13-9)] and shuffled complex evolution (SCE) algorithm [\[5](#page-13-4)]. These methods have been successfully employed in the calibration of conceptual rainfall-runoff models, since the early 1990s [\[2,](#page-13-1)[5](#page-13-4)[,11](#page-13-9)[–15](#page-13-10)]. Performances are improved with these methods because of their superior ability to navigate numerous local optima present in the objective function surface of the conceptual rainfall-runoff model calibration problem [\[4](#page-13-3)]. Although several global optimization methods have been proposed in the literature, there is no general agreement as to which method is the most appropriate [\[4\]](#page-13-3). Genetic algorithm is one of the global optimization methods that has been successfully employed in rainfall-runoff model calibration [\[11](#page-13-9),[13,](#page-13-11)[15](#page-13-10)[–18\]](#page-13-12).

In this study automatic calibration of ARNO conceptual rainfall-runoff model is developed using simple genetic algorithm (SGA), which is a kind of GAs. GAs are simple to operate yet powerful, and are not fundamentally limited by restrictive assumptions about the search space such as continuity and existence of derivatives [\[19\]](#page-13-13). The case study basin is the Pataveh basin, which is a sub-basin of Karun River basin in Iran.

1.2 Arno Rainfall-Runoff Model

The ARNO model, which derives its name from its first application to the Arno River, is a semi-distributed conceptual continuous rainfall-runoff model. The soil moisture balance module of this model is taken originally from the Xinanjiang model $[20]$, in which the spatial distribution of the soil moisture capacity is expressed in the form of a probability distribution function.

Later, the original Xinanjiang model scheme was modified by Todin [\[21](#page-13-15)], by allowing the soil moisture to be depleted not only by evapotranspiration, as in the original Xinanjiang model, but also by drainage into the river network and percolation into the water table.

This modified ARNO model has been extensively used with general circulation models [\[22\]](#page-13-16) and as an operational flood forecasting tool on several basins in different parts of the world [\[23\]](#page-13-17).

To apply the ARNO model to a basin, as any other conceptual model, its parameters must be estimated. Some of the parameters can be estimated as a function of physiographic characteristics of the basin, while others must be estimated via calibration. In summary the soil moisture balance module of the ARNO model, as the most important component of the model, together with its parameters that are subject to calibration are described below [\[23](#page-13-17)]. Also, model assumptions are described in Table [1.](#page-3-0)

The basin surface area (S_T) (excluding the surface extent of water bodies such as reservoirs or lakes) is divided into the impervious area (S_I) and the pervious area (S_P) :

$$
S_{\rm T} = S_{\rm I} + S_{\rm P} \tag{1}
$$

Expressions used for continuous updating of the soil moisture balance are as follows:

$$
S_P = S_T - S_I \tag{2}
$$

By defining S_G as the generic pervious surface area at saturation, and the pervious area that receives precipitation $(S_P =$ $S_T - S_I$, the proportion of pervious area at saturation (x) is:

$$
x = \frac{S_{\rm G}}{S_{\rm T} - S_{\rm I}}\tag{3}
$$

Zhao (1977) showed the following relation holds reasonably well between *x* and the local proportion of maximum soil

Table 1 Assumptions expressed in the soil moisture balance module of the ARNO model [\[23\]](#page-13-17)

moisture content w/w_m , where w is the elementary area soil moisture at saturation and w_m is the maximum possible soil moisture in any elementary area of the basin [\[23](#page-13-17)]:

$$
x = 1 - \left(1 - \frac{w}{w_m}\right)^b \tag{4}
$$

where *b* is a parameter representing spatial distribution of the soil moisture capacity. This is similar to defining the cumulative distribution moisture at saturation, shown the curve in Fig. [1](#page-3-1) [\[23](#page-13-17)], which is:

$$
w = w_{\rm m} [1 - (1 - x)^{1/b}] \tag{5}
$$

In the ARNO model, if the precipitation (P) is larger than the potential evapotranspiration (ET_p) , the actual evapotran-

Fig. 1 Cumulative distribution for the elementary area soil moisture at saturation. Runoff *R* (*shaded area*) generated by an effective meteorological input $M_e > 0$

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spiration (ET_a) is assumed to coincide with the ET_p (i.e. $ET_a = ET_p$, and therefor, an 'effective' meteorological input, *M*e, defined as the difference between precipitation and potential evapotranspiration, equals:

$$
M_e = P - ET_p = P - ET_a > 0 \tag{6}
$$

With reference to Fig. [1,](#page-3-1) the surface runoff R generated by the entire basin is obtained as the sum of two terms; the first is the product of the "effective" input and the percentage of impervious area, and the second one is the average runoff produced by the pervious area, which is obtained by integrating the soil moisture capacity curve, which gives [\[23\]](#page-13-17):

$$
R = \frac{S_{\rm I}}{S_{\rm T}} M_{\rm e} + \frac{S_{\rm T} - S_{\rm I}}{S_{\rm T}} \int_{w}^{M_{\rm e} + w} x(\xi) d\xi \quad \text{if } M_{\rm e} + w < w_{\rm m} \tag{7-1}
$$

$$
R = \frac{S_{\rm I}}{S_{\rm T}} M_{\rm e} + \frac{S_{\rm T} - S_{\rm I}}{S_{\rm T}} \left[M_{\rm e} - \int_{w}^{w_{\rm m}} [1 - x(\xi)] d\xi \right]
$$

if $M_{\rm e} + w \ge w_{\rm m}$ (7-2)

Eqs. [7-1](#page-3-2) and [7-2](#page-3-3) can be expressed in terms of the basin average soil moisture content (*W*) and that at saturation (Wm), which after integration becomes [\[23\]](#page-13-17):

$$
R = M_{e} + \frac{S_{T} - S_{I}}{S_{T}} \left\{ (W_{m} - W) - W_{m} \left[\left(1 - \frac{W}{W_{m}} \right)^{\frac{1}{b+1}} - \frac{M_{e}}{(b+1)W_{m}} \right]^{b+1} \right\}
$$

for $0 < M_{e} < (b+1)W_{m} \left(1 - \frac{W}{W_{m}} \right)^{b+1}$ (8-1)

$$
R = M_{e} + \frac{S_{T} - S_{I}}{S_{T}} (W_{m} - W) \text{ for } M_{e} \ge (b + 1)
$$

$$
\times W_{m} \left(1 - \frac{W}{W_{m}} \right)^{b+1}
$$
 (8-2)

If the precipitation P is smaller than the potential evapotranspiration ET_p , the actual evapotranspiration ET_a is computed as the precipitation *P* plus a quantity which depends upon M_e reduced by the average degree of saturation of the soil [\[23](#page-13-17)]:

$$
ET_a = P + (ET_p - P) \frac{(S_T - S_I)}{S_T} \frac{\left(1 + \frac{W}{W_m} \frac{1}{b}\right) - \left(1 - \frac{W}{W_m}\right)^{\frac{1}{b+1}}}{\left(1 + \frac{1}{b}\right) - \left(1 - \frac{W}{W_m}\right)^{\frac{1}{b+1}}}
$$
\n(9)

These equations, which represent the average surface runoff produced in the sub-basin, must be associated with an equation of state to update the mean water content in the soil. This equation takes the following form [\[23](#page-13-17)]:

$$
W(t + \Delta t) = W(t) + P(t, t + \Delta t) - R(t, t + \Delta t)
$$

$$
-ET_a(t, t + \Delta t) - D(t, t + \Delta t) - I(t, t + \Delta t)
$$
(10)

where, during time step Δt , ET_a is the loss through evapotranspiration; *D* is the loss through drainage; *I* is the percolation loss to groundwater; *P* is the area precipitation; *R* is the surface runoff; $W(t, + \Delta t)$ is the soil moisture content at the end of the time step; and $W(t)$ is the soil moisture content at the beginning of the time step. All the above quantities representing averages over the sub-basin are expressed in millimetres.

The non-linear response of the unsaturated soil to precipitation, represented by the shape of the distribution curve given by Eq. [5,](#page-3-4) is strongly affected by the horizontal drainage and vertical percolation losses. The drainage loss (*D*) is an important quantity to be reproduced in a hydrological model, because on one hand it affects the soil moisture storage and, on the other, it controls the hydrograph recession. Experiences derived from applications suggested the use of the fol-

$$
D = D_{\min} \frac{W}{W_{\text{m}}} \quad \text{for} \quad W < W_{\text{d}} \tag{11-1}
$$
\n
$$
D = D_{\min} \frac{W}{W_{\text{m}}} + (D_{\max} - D_{\min}) \left(\frac{W - W_{\text{d}}}{W_{\text{m}} - W_{\text{d}}} \right)^{c} \quad \text{for} \quad W \geq W_{\text{d}} \tag{11-2}
$$

where c is an exponent of soil moisture variation, D_{max} is the maximum drainage that should be expected when the soil is completely saturated, D_{min} is a drainage parameter, and W_d is the moisture content threshold value [\[23\]](#page-13-17).

The percolation loss (*I*), which feeds the groundwater and controls the base flow in the model, varies less significantly over time compared with other terms. Nevertheless, a nonlinear behaviour is also assumed as:

$$
I = 0 \quad \text{for} \quad W < W_i \tag{12-1}
$$

$$
I = I_{\rm s} \frac{W - W_i}{W_{\rm m} - W_i} \quad \text{for} \quad W \ge W_i \tag{12-2}
$$

where W_i represents the moisture content threshold value below which the percolation is negligible, and I_s is a percolation parameter which represents the maximum percolation that should be expected when the soil is completely saturated.

The total runoff per unit area produced by the precipitation *P* is finally expressed by:

$$
R_{\text{tot}} = R + D + B \tag{13}
$$

where *B* is the base flow generated by the presence of a groundwater table fed by the percolation, and can *b* computed by a groundwater module [\[23\]](#page-13-17).

A brief description of ARNO parameters for automatic calibration is given in Table [2.](#page-4-0)

2 Calibration Methodology

The calibration of a conceptual rainfall-runoff model is to find a set of model parameters that provides the best fit between

Table 2 Parameters of ARN

the observed and the simulated runoff. Performance of the conceptual rainfall-runoff models highly depends on proper parameters [\[15\]](#page-13-10). The process of parameter estimation will be essentially an optimization process, in which an objective function will be optimized and the corresponding parameter set will be obtained [\[6](#page-13-18)]. Since conceptual models may have numerous local optima on their objective function surface, it is appropriate to use a global optimization method for their automatic calibration. In this study, SGA is adopted for developing the automatic calibration of the ARNO conceptual rainfall-runoff model.

Genetic algorithms are search procedures based on the mechanics of natural selection and genetics. They combine the concept of survival of the fittest with genetic operators abstracted from nature [\[19\]](#page-13-13). GAs work with populations of individual chromosomes. A chromosome is composed of a set of coded parameters as a feasible solution to the problem [\[24\]](#page-13-19). An initial population is generated randomly in the beginning. Then an iterative process is performed until the termination criteria have been satisfied [\[25\]](#page-13-20). After all the individuals in the population have been evaluated, the genetic operators are applied to produce a new generation. SGA is composed of three operators of reproduction, crossover, and mutation [\[19](#page-13-13)]. A flowchart for the SGA procedure is shown in Fig. [2.](#page-5-0)

Reproduction is a process in which chromosomes are copied according to their fitness function values into a matting pool, a tentative new population, for further genetic operator action. Copying chromosomes according to their fitness values means that chromosomes with a higher fitness have a higher chance of contributing in the next generation. In SGA,

Fig. 2 Flowchart for SGA

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reproduction is implemented in the function selected as a linear search through a roulette wheel with slots weighted in proportion to chromosome fitness values [\[19](#page-13-13)]. In this study, the roulette wheel selection was applied for the reproduction operator.

Crossover is exchanging substrings of two parent chromosomes to generate two offspring chromosomes. The idea behind the crossover is that by exchanging important bits between two chromosomes, new chromosomes that preserve the best material from the parent chromosomes are created [\[26](#page-13-21)]. Simple crossover proceeds with the random selection of two chromosomes from the mating pool, the random selection of a crossover site, and the exchange of substrings from the beginning of the chromosome to the crossover site inclusively with the corresponding subset of the chosen mate [\[19](#page-13-13)].

The mutation is the occasional random alteration of the value of a chromosome position. Mutation is needed because, even though reproduction and crossover effectively search and recombine extant notions, occasionally they may become overzealous and lose some potentially useful genetic material. The mutation operator protects against such an irrecoverable loss [\[19](#page-13-13)].

2.1 Calibration of Arno Model Using SGA

To implement the SGA, the parameters set of the optimization problem should be coded as a specified length string, which is called a chromosome. Also the SGA's parameters of population size, crossover probability, and mutation probability needed to be selected. Considerations have been given to these parameter values as follows.

To construct a chromosome, each parameter should be coded as a finite length substring, which is called a gene. One popular coding method is the binary coding, because it can preserve more information for the GAs [\[15](#page-13-10)]. In binary coding, the relationship between precision of a parameter and length of its gene is defined as:

$$
\pi = \frac{U_{\text{max}} - U_{\text{min}}}{2^l - 1} \tag{14}
$$

where π is the precision of the parameter, U_{max} is the upper limit of the parameter, *U*min is the lower limit of the parameter, and *l* is the length of the gene that represents the parameter. Gene length can be modified to satisfy various level precisions, but the calibration process will become inefficient when more bits comprise each gene [\[26\]](#page-13-21). The suitable precision of each parameter of ARNO model for runoff simulation in the study basin is estimated using a sensitivity analysis in various points in the solution space. Genes are linked together to form a chromosome, which represents a feasible solution. The search space and construction of the chromosome are shown in Table [3.](#page-6-0)

Table 3 Search space and coding of ARNO model parameters into binary digits

| Parameter | $W_{\rm m}$ | b | D_{max} | D_{\min}^* | W_d^* | \boldsymbol{c} | $I_{\rm s}$ | W_i |
|----------------------------|-------------|--------|------------------|------------------|--------------|------------------|-------------|----------|
| Lower limit | 50 | 0.01 | $\overline{0}$ | $\mathbf{0}$ | $\mathbf{0}$ | 1.5 | θ | θ |
| Upper limit | 600 | 0.5 | 10 | D_{max} | $W_{\rm m}$ | 3 | 3 | W_m |
| Length of gene | 8 | 6 | 7 | 6 | 6 | 5 | 6 | 6 |
| Randomized binary digits | | | | | | | | |
| for each gene (in | | | | | | | | |
| first generation) | 01101011 | 001010 | 1101011 | 010011 | 111101 | 00011 | 010111 | 111001 |
| Total Length of chromosome | 50 | | | | | | | |
| Randomized binary digits | | | | | | | | |
| for a chromosome | | | | | | | | |
| | | | | | | | | |

∗ Search space for this parameter in SGA, is defined as [0,1], and for evaluation the parameter value is calculated by multiplying the decoded value (in domain of [0,1]) by considering the upper limit parameter value. For example, if decoded value of D_{min} (in domain of [0,1]) is 0.5 and D_{max} is 5, then D_{min} is 2.5 (=0.5 \times 5)

Population size (*N*) is defined as the number of chromosomes in a population. If the population size is too small, then the SGA may converge prematurely to a local optimum. On the other hand, if the population size is too large, then the algorithm will be too slow. The typical size of the population can range from 20 to 1,000 [\[27](#page-13-22)[,28](#page-13-23)] and the population size within a range of 200 to 500, related to the chromosome length (*L*), is appropriate for many GA applications [\[29\]](#page-13-24). In this study, a sensitivity analysis was performed to find the appropriate value of *N* for calibration of the ARNO model. In sensitivity analyses, *N* from 300 to 700 are considered.

Crossover probability (P_c) controls the frequency of the crossover operation. P_c is the probability that a chromosome from the mating pool will be chosen for crossing over with another selected chromosome. If P_c is too large, the high quality chromosomes could be prematurely destroyed and the improvement of the population quality could be influenced. If P_c is too small, then the exploration rate and searching efficiency can be very low [\[30](#page-13-25)]. The probability of crossover is usually in the range of 0.5–1.0 [\[26](#page-13-21)]. In this study a sensitivity analysis was performed to find the appropriate value of P_c . In sensitivity analyses, P_c from 0.5 to 0.9 are considered.

Mutation probability (P_m) is the probability of a single bit for mutation in each generation. If P_m is too small, then new gene segment cannot be induced and the algorithm may be trapped in a local optimum; if *P*^m is too big, then a large number of good chromosomes may be lost and the genetic evolution degenerates into a random search [\[30](#page-13-25)]. *P*^m is usually in the range of 0.01–0.1 and guidelines for computing *P*_m are: $1/L \ge P_m \ge 1/n$ [\[26](#page-13-21)]. In this study a sensitivity analysis was performed to find the appropriate value of *P*m. In sensitivity analyses, *P*^m from 0.002 to 0.1 are considered.

When calibration parameters were input and calibration started, the calibration procedure automatically proceeds until the optimized parameter achieved (according Fig. [1\)](#page-3-1).

2.2 Objective Function

Many objective functions have been proposed and used in the literature for parameter estimation in rainfall-runoff models. The choice of an objective function is a subjective decision which influences parameter estimates and the performance of the model. One of the commonly adopted objective functions for calibrating a rainfall-runoff model is coefficient of efficiency (CE) or the Nash–Sutcliffe coefficient [\[31](#page-14-0)]. Servat and Dezetter [\[32\]](#page-14-1) found CE to be the best objective function for reflecting the overall fit of a hydrograph. CE can be defined as normalized measure of the sum of square error:

$$
CE = 1 - \frac{\sum_{i=1}^{n} (Q_{obs}(t) - Q_{sim}(t))^{2}}{\sum_{i=1}^{n} (Q_{obs}(t) - Q_{obs})^{2}}
$$
(15)

where $Q_{obs}(t)$ and $Q_{sim}(t)$ are the average daily observed and simulated flow, respectively, *Q*obs is the average observed flow over the period, and *n* is number of calibration days.

Loukas et al. [\[33](#page-14-2)] employed the following objective functions for calibrating a daily rainfall-runoff model on two basins in British Columbia.

$$
EOPT = CE - \left| 1 - \frac{V_{\text{sim}}}{V_{\text{obs}}} \right| \tag{16}
$$

where *V*sim and *V*obs are the simulated and observed flow volumes, respectively.

In this study, EOPT was first used in model calibration. Results showed that wet years are simulated well, whereas dry years simulated poorly.

In CE, the differences between the observed and simulated values are calculated as squared values, so EOPT tends to simulate wet years better than dry years. But within each year, because frequency of low flows, in the study basin, is higher than peak flows, EOPT performs well in the overall hydrograph. So, the objective function for the calibration

procedure was set as:

$$
OBF = \sum_{j=1}^{K} EOPT_j
$$
 (17)

where EOPT $_i$ is the EOPT in year, j and K is total number of calibration years.

Since SGA requires non-negative fitness function, the sigma truncation scaling technique was used to transform the OBF values into the scaled fitness values [\[34](#page-14-3)]:

$$
f' = F(\text{OBF} - (\mu_{\text{OBF}} - \text{C}\sigma_{\text{OBF}}))
$$
\n(18)

where f' is the scaled fitness, μ_{OBF} and σ_{OBF} are the average and standard deviation of the OBF of all the chromosomes in the population, respectively,*C* is a small integer, and function *F* is expressed as:

$$
F(x) = \begin{cases} x, & x > 0 \\ 0, & \text{otherwise} \end{cases}
$$
 (19)

This technique Eq. [18](#page-7-0) also improves the convergence speed of the genetic algorithm. Use of *F (*OBF) as the fitness function for SGA has two drawbacks. First, the convergence speed in the early generations is high. This brings about an increasing diversity loss rate in population which may cause premature convergence. Second, in the late stages the convergence speed is slow. This brings about an excessive convergence time. Use of Eq. [18](#page-7-0) resolves these problems.

Since populations in SGA is random based, the worst chromosomes in the population may disturb $\mu_{\rm OBF}$ and $\sigma_{\rm OBF}$. Thus, the fitness function of the calibration procedure is defined as:

$$
f' = F(OBF - (\mu_{OBF(90 \%)} - C\sigma_{OBF(90 \%)}))
$$
 (20)

where $\mu_{\text{OBF}(90 \%)}$ and $\sigma_{\text{OBF}(90 \%)}$ are the average and standard deviation of the OBF values of 90 % of the best chromosomes in the population, respectively.

2.3 Model Performance Criteria

Six performance criteria are adopted to evaluate the simulation performance of the calibrated model. They are: CE, coefficient of determination (R^2) , percentage error of mean discharge ($E\overline{Q}$), percentage error of mean annual peak discharges ($E\overline{Qp}$), 5-percentage error of standard deviation (ESD), and percentage error of skewness (ESkew). They are defined as:

$$
CE = 1 - \frac{\sum_{t=1}^{n} (Q_{obs}(t) - Q_{sim}(t))^{2}}{\sum_{t=1}^{n} (Q_{obs}(t) - Q_{obs})^{2}} \tag{21}
$$
\n
$$
R^{2} = \frac{\left(\sum_{t=1}^{n} (Q_{obs}(t) - Q_{obs})(Q_{sim}(t) - Q_{sim})\right)^{2}}{\sum_{t=1}^{n} (Q_{obs}(t) - Q_{obs})^{2} \sum_{t=1}^{n} (Q_{sim}(t) - Q_{sim})^{2}} \tag{22}
$$

$$
E\overline{Q} = \frac{\overline{Q_{\rm sim}} - \overline{Q_{\rm obs}}}{\overline{Q_{\rm obs}}} \times 100
$$
 (23)

$$
E\overline{Qp} = \frac{\overline{Qp_{\rm sim}} - \overline{Qp_{\rm obs}}}{\overline{Qp_{\rm obs}}} \times 100
$$
 (24)

$$
ESD = \frac{SD(Q_{sim}) - SD(Q_{obs})}{SD(Q_{obs})} \times 100
$$
 (25)

$$
ESkew = \frac{Skew(Q_{sim}) - Skew(Q_{obs})}{Skew(Q_{obs})} \times 100
$$
 (26)

where $Q_{obs}(t)$ and $Q_{sim}(t)$ are, respectively, observed and simulated runoff, $\overline{Q_{\text{obs}}}$ and $\overline{Q_{\text{sim}}}$ are, respectively, average observed and simulated runoff, *n* is the number of data elements, $\overline{Qp_{\text{obs}}}$ and, $\overline{Qp_{\text{sim}}}$ are, respectively, mean annual observed and simulated peak discharges, *m* is the number of years in the considered period, SD(*Q*obs) and SD(*Q*sim) are, respectively, standard deviation of observed and simulated runoff, and Skew(*Q*obs) and Skew(*Q*sim) are, respectively, skewness of observed and simulated runoff.

3 Application

3.1 Study Area

The study area is Pataveh basin, located in the Karun River basin in south-west of Iran between latitudes 30◦ and 31◦ N and longitudes $51°$ to $52°$ E (Fig. [3\)](#page-7-1). Area of the basin is 2,800 $km²$ and elevation ranges from 1,540 to 4,300 m (a.m.s.l.).

There is one meteorological station in the basin that measures daily precipitation, and maximum and minimum temperature. The station is located in Yasuj at an elevation of

Fig. 3 Location of the Pataveh basin and the hydro-meteorological stations

1,821 m (a.m.s.l.). Some 29 years (1973–2001) of data are available. Also, there is a synoptic station in the vicinity of the basin at 1,831 m (a.m.s.l.) with 17 years (1987–2003) of record. Furthermore, there are five daily rainfall stations located in lowlands of the basin. Daily streamflow data are recorded by Pataveh hydrometric station.

3.2 Data Processing

The results of rainfall-runoff modeling can be more dependent on the quality of the input data than on the model [\[35](#page-14-4)]. Nathan and McMahon [\[36](#page-14-5),[37\]](#page-14-6) calibrated the SFB model on 168 basins in southeastern mainland Australia. In 37 % of the situations, calibration results were too poor to be considered acceptable. They found that the model was robust, and the poor calibration results were generally associated with basins in which the quality of input data was poor. Boughton and Chiew [\[38\]](#page-14-7) calibrated the AWBM on 331 basins across Australia. They reported that problems with the data resulted in the discarding of some data sets. As a result of the data problems, calibration on 33 % of the basins was not acceptable. Such studies emphasize that in rainfall-runoff modeling, careful attention to the quality of input data is required.

In the current study, the reliability of Pataveh stream flow records was controlled by comparing its standardized records with the standardized records of one upstream and one downstream hydrometric station. Only 9 years (1979, 1981, 1983, 1985, 1994, 1995, 1996, 1998, 2001) of reliable records were identified. Figure [4](#page-8-0) shows 1 year of reliable record and 1 year of unreliable record.

Many methods have been developed to estimate evapotranspiration from different climatic variables. The FAO Penman–Monteith method is recommended in FAO Irrigation and Drainage Paper 56 for determining reference evapotranspiration (ET0). This method closely approximates grass ET0 and provides consistent ET0 values in all regions and climates. This method has been recommended for all regions across Iran [\[39](#page-14-8)].

The FAO Penman–Monteith method (P–M) requires radiation, air temperature, air humidity and wind speed data. For the meteorological station, located near the centre of the basin, only air temperature is available. Thus, PET for basin can be estimated using a temperature-based method which requires local calibration to achieve satisfactory results [\[40](#page-14-9)]. The selection and calibration of a temperature-based method was accomplished in comparison to the FAO P–M developed for the synoptic station, located in the neighborhood of the basin. In the synoptic station, all of the required data for FAO P–M method are available. The synoptic station has similar climatic and geographic conditions to the meteorological station (Table [4\)](#page-8-1).

Three frequently used temperature-based methods were compared with the FAO P–M method. They were Blaney and Criddle (B–C) [\[41\]](#page-14-10), Thornthwaite (Th) [\[42\]](#page-14-11), and Hargreaves and Samani (H–S) [\[43](#page-14-12)] methods. As shown in Table [5,](#page-9-0) the Hargreaves and Samani method has the highest cor-

Fig. 4 Comparison of a year with unreliable record (**a**) with a year with reliable records (**b**) in Pataveh station

Table 5 Comparison of temperature-based methods and P–M method in the synoptic station (1987–2003)

| Method: | $P-M$ | $H-S$ | Th | $B-C$ |
|----------------------|-------|-------|------|-------|
| Average ET (mm/year) | 1.460 | 3.550 | 836 | 1.545 |
| R^2 with P–M | | 0.96 | 0.89 | 0.94 |

Table 6 Summary of climatic and hydrologic characteristics of the Pataveh basin over the calibration and validation time periods

relation with the FAO P–M method in the synoptic station. This method is represented by a linear relationship, such as the one suggested by Allen et al. [\[40\]](#page-14-9). The regression relationship between H–S and P–M methods resulted in:

$$
ET0_{P-M} = 0.3827 \times ET0_{H-S} - 0.0535
$$
 (27)

where $ET0_{P-M}$ is the Penman-Monteith ET0 (mm/month) and $ET0_{H-S}$ is the Hargreaves and Samani ET0 (mm/month). Then the ET0 in the meteorological station was calculated using the H–S method and converted to equivalent FAO P– M ET0 using Eq. [27\)](#page-9-1). The potential evapotranspiration from the basin surface is determined by considering the ET0 and surface condition of the basin such as ground cover and plant density [\[40\]](#page-14-9).

(a)

0.82

Fig. 6 sensitivity to **a** crossover probability, **b** mutation probability, and **c** population size

The rainfall recorded by the meteorological station was assumed to be representative of the average of the basin since other rainfall gauges are located in lowlands of the basin.

Fig. 5 Reliably observed streamflow used for the calibration and validation of the ARNO model

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Averages and ranges of climatic and hydrologic data of the Pataveh basin over the calibration and validation time periods are presented in Table [6.](#page-9-2)

3.3 Sensitivity Analyses

A series of sensitivity analyses were performed to find appropriate parameters of SGA for calibration of the ARNO model [\[26\]](#page-13-21). According to the proposed range of the SGA parameters in literature (as mentioned above), Sensitivity to crossover probability is performed using a population size and a mutation probability of 500 and 0.05, respectively, and crossover probability values of 0.5, 0.7, 0.8, and 0.9. Sensitivity to mutation probability is performed using a population size and a crossover probability of 500 and 0.7, respectively, and mutation probability values of 0.002, 0.01, 0.05, and 0.1. Also, sensitivity to population size is performed using a mu**Table 7** Results of the calibrated model parameters

tation probability of 0.01 and a crossover probability of 0.7 and population size values of 300, 500, and 700.

4 Results and Discussion

The ARNO model linked to the SGA was applied to the Pataveh basin. As shown in Fig. [5,](#page-9-3) 9 years of reliable records were used for calibration and validation of the model. The first 5 years were used for calibration and the remaining 4 years were allocated for validation. The whole record included both dry and wet years.

Fig. 7 Distribution of EOPT values in the first and the last generations

Fig. 8 Convergence process of genetic algorithm for **a** best and **b** average EOPT Values

Table 8 Calibration and validation performance of the ARNO model

| | | | | CE R^2 EV (%) EQ _p (%) ESD (%) Eskew (%) |
|--|--|--|------|---|
| Calibration 0.80 0.82 0.34 -8.10 | | | 6.50 | -7.56 |
| Validation 0.82 0.83 -5.38 1.27 | | | 4.27 | 10.70 |

Sensitivity analyses were performed using 2 years of the observed data of the Pataveh basin. Figure [6](#page-9-4) shows the sensitivity to crossover probability, mutation probability, and population size. The results demonstrate that in the proposed range of the SGA parameters, the sensitivity of the maximum fitness values to the P_c , P_m , and N is low; however,

the best set of parameters appear to be $N = 500$, $P_c = 0.7$, and $P_{\rm m} = 0.01$.

The parameters of SGA were set as follows: the chromosome length was 50, population size 500, the probability of crossover 0.7, and the probability of mutation 0.01. The stopping criterion was set to either exceed the number of generations of 50 (a maximum number of model evaluations equal to 25,000), or 10 generations without improvement of EOPT values.

The genetic algorithm achieved an optimal solution, using a population size of 500 over 38 generations. Since no improvement was detected beyond 48 generations, the optimization process was terminated.

Fig. 9 Comparison of the observed and simulated hydrographs during calibration stage

The distribution of EOPT values in the first and last generations are shown in Fig. [7.](#page-10-0) Figure [8](#page-10-1) shows the plots of the best and average EOPT values in each generation.

The results show that the objective function were rapidly impring to converge over the first several generations and were refined over the remaining generations.

The values of calibrated parameters are presented in Table [7.](#page-10-2)

The simulation performance of the ARNO model was evaluated on the basis of six performance criteria as outlined earlier. Calibration and validation results of ARNO model are presented in Table [8.](#page-11-0) CE and R^2 , as the two most appropriate criteria to evaluate the efficiency of modeling, are higher than 0.80 during calibration and validation. The values other performance statistics are also acceptable.

Observed and simulated hydrographs of calibration and validation stages are presented in Figs. [9](#page-11-1) and [10,](#page-12-0) respectively. The model performance can be also evaluated by a visual interpretation of the agreement between the observed and simulated hydrographs. It can be seen in Figs. [9](#page-11-1) and [10](#page-12-0) that a good agreement between observed and simulated hydrographs exists.

The accuracy of the daily rainfall-runoff simulation was also compared with previous works. Zhang and Savenije [\[44](#page-14-13)] applied REWASH model to the Geer River basin in Belgium to simulate the daily rainfall-runoff process. They chose a level of 0.6 for CE as the threshold to discriminate behavioural and non-behavioural models. The model was calibrated and validated using two 2-year data sets. The obtained CE's for calibration and validation phases were 0.68 and 0.65, respectively. Kamali et al. [\[6\]](#page-13-18) considered a CE value greater than or equal to 0.7 as acceptable in daily rainfall-runoff modeling of Smokey River basin in Canada. Evans and Schreider [\[45](#page-14-14)] used CMD-IHACRES model to simulate the daily rainfall-runoff of six basins in Australia. They used 4 years of observed data and achieved CE values between 0.67 and 0.78 during the calibration stage. Loukas et al. [\[46\]](#page-14-15) applied the UBC model to simulate rainfall runoff processes of the Illecillewaet River basin in Canada. The model was calibrated for a period of 20 hydrologic years and the obtained value of CE was 0.93. Nourani and Mano [\[47\]](#page-14-16) applied TOPMODEL to Karun River basin in Iran. They used 2 years of data for model calibration and arrived at a CE value of 0.79 in daily time steps.

Based on the reported literature on daily rainfall-runoff modeling, the performance of the SGA-based automatic calibration of ARNO model in this study (with CE and R^2 higher than 0.80) is found to be acceptable.

Fig. 10 Comparison of the observed and simulated hydrographs during the validation stage

5 Conclusions

In this paper, automatic calibration of ARNO conceptual rainfall-runoff model has been developed. Genetic algorithm, a simple and powerful global search method, was adopted as the basis for automatic calibration. This method is simple without requiring extensive knowledge regarding the model structure and parameters.

A combination of the Nash–Sutcliffe coefficient and percent error of total runoff volume was selected as the objective function. Since SGA required non-negative fitness function and for control of convergence pressure during the optimization process, the sigma truncation scaling technique was used to transform the objective function into the scaled fitness values. The objective function can be arbitrarily exchanged by the user. However, the objective function values should be transformed by a proper fitness function for SGA.

The ARNO rainfall-runoff model was then automatically calibrated in the Pataveh basin, located in Karun River basin in Iran. The genetic algorithm achieved an optimal solution using a population size of 500 over 38 generations. The simulation performance of the ARNO model was evaluated on the basis of six performance criteria. The efficiency coefficient and coefficient of determination, as two most appropriate criteria to evaluate the accuracy of modeling, reached values higher than 0.80 during calibration and validation. The values of the remaining performance statistics, namely error of mean discharge, error of mean annual peak discharges, error of standard deviation, and error of skewness were acceptable. The results showed that SGA-based automatic calibration was successful.

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