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Multi-objective Optimization of Process Parameters in Wire Electric Discharge Machining of Ti-6-2-4-2 Alloy

M. P. Garg · Ajai Jain · Gian Bhushan

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Abstract The present paper identifies the different process parameters that effect the cutting speed and surface roughness in wire electrical discharge machining of Titanium-6-2-4-2 (HSTR aerospace alloy), which is so far not reported in the literature. It also identifies optimal process parameter combination for simultaneous optimization of cutting speed and surface roughness to be presented as a guideline for machining of Titanium-6-2-4-2. Box–Behnken design and response surface methodology are used to plan and analyse the experiments. Six control factors viz. Pulse on time, pulse off time, peak current, spark gap set voltage, wire feed, and wire tension are chosen as process parameters to study the performance of the process in terms of cutting speed and surface roughness. The recommended optimal parameter combinations have been verified by conducting confirmation experiments.

Keywords WEDM · Ti 6-2-4-2 alloy · Box–Behnken designs · Response surface methodology · Multi objective optimization

M. P. Garg (✉)
Department of Mechanical Engineering, M.M.E.C. Mullana, Ambala, Haryana, India
e-mail: mpgargacad@gmail.com

A. Jain · G. Bhushan
Department of Mechanical Engineering, N.I.T., Kurukshetra, Haryana, India
e-mail: ajaijain12@gmail.com

G. Bhushan
e-mail: aroragian@gmail.com

الخلاصة

تحدد هذه الورقة العلمية معاملات العملية المختلفة التي تؤثر في سرعة القطع وخشونة السطح في قطع أسلاك التفريغ الكهربائية من التيتانيوم-6-2-4-2 (سبيكة الفضاء HSTR) التي لم تسجل - حتى الآن - في الأدب. وهي تحدد أيضا مزيج معاملات العملية المثلى لتحسين سرعة القطع وخشونة السطح في وقت واحد لتقديمها كموجه في قطع التيتانيوم-6-2-4-2. وقد استُخدم تصميم صندوق بينكين ومنهجية استجابة السطح لتخطيط التجارب وتحليلها ، واختيرت ستة عوامل سيطرة بمعنى: النبض في الوقت المحدد، والنبض قبالة الوقت، والذروة الحالية، وجهد مجموعة فجوة الشرارة وتغذية وتوتر السلك كمعاملات عملية لدراسة أداء العملية من حيث سرعة القطع وخشونة السطح. وتم التحقق من تركيبات المعامل الأمثل الموصى بها عن طريق إجراء تجارب التثبت.

1 Introduction

Titanium and its alloys possess many superior properties such as chemical inertness, high strength and stiffness at elevated temperatures, high specific strength, oxidation resistance, and corrosion resistance. These properties have led to their wide and diversified range of applications in aerospace, automobile, chemical plants, refineries, surgical implants and marine applications. However, titanium alloys are susceptible to work hardening during machining, which impairs their machinability [1]. Hence these are referred to as “difficult to machine” materials. Although conventional machining of Ti and its alloys requires forces slightly greater than those needed to machine steel, it is more difficult due to the following reasons:

1. Titanium is poor conductor of heat (thirteen times less conductive than Aluminum). Therefore, the heat generated by cutting action does not dissipate rapidly and most of the heat is concentrated on tool cutting edge and face. Moreover, these alloys have a strong alloying tendency

with the tool materials at tool operating temperatures. This leads to processes like galling, welding, and smearing along with rapid destruction of cutting tool [2].
2. In machining, due to large shearing angle, thin chips come into contact with a relatively small area on tool face and exert large forces on a small area. These larger forces, coupled with friction developed by the chips as these pass over the cutting area, result in larger increase in heat over a small localized portion of cutting tool. This leads to shorter tool life. Thus, with increase in cutting speed, tool life drastically decreases [3].
3. Titanium is more "springy" than steel, i.e. work piece tends to move away from tool unless heavy cuts are maintained. Slender parts tend to deflect under tool pressure and it can cause chatter, tool rubbing and hence lead to tolerance problems [3].
4. High-speed machining of titanium alloys is only possible while performing light finishing cuts. Titanium has an upper speed limit, i.e. 300–350 surface feet per minute, beyond which the tool turns red [4].
5. During machining Ti 6-2-4-2 with Tungsten Carbide inserts with a rake angle of 10°, deformation of metal increases with increase in speed. The depth of the deformed layer and extent of deformation depends upon cutting parameters such as cutting speed, tool geometry, etc. [5].

Therefore, there is a crucial need for reliable and cost-effective machining processes for titanium alloys. Many researchers have been searching for effective methods to machine titanium alloys by conventional and non-conventional machining processes [6].

Wire Electric Discharge Machining (WEDM) is found to be an extremely potential electro thermal process in the field of machining of conductive materials and is widely used in manufacturing of cam wheels, stators for stepper motors, press tools, dies, etc. It is a thermoelectric process which erodes materials from work piece by a series of discrete sparks between the work and the tool electrode immersed in a dielectric medium. These electrical discharges melt and vaporize small amounts of work material, which are then ejected and flushed away by the dielectric. The success of a manufacturing process depends upon the selection of appropriate process parameters which in turn play a significant role in ensuring quality of the product to reduce the manufacturing cost and to increase productivity [7]. In general, the data of machining parameters provided by the machine tool manufacturers do not meet the operator requirements and sometimes even do not provide efficient guidelines to manufacturing engineers [8]. The selection of optimum machining parameter combinations for obtaining higher cutting efficiency and other dimensional accuracy characteristics is a challenging task in WEDM due to the presence of large number of process variables and complicated stochastic process mechanism. Performance of WEDM is dependent upon a number of control factors such as pulse on time, pulse off time, servo voltage, and peak current, etc. A slight variation in these process parameters may influence the process in a complex way [7]. Hence, there is a need to find out optimum parameter settings to achieve maximum process yield criteria such as maximization of cutting speed for different class of materials.

Ti 6-2-4-2 is used in the manufacturing of engine ducts, exhausts nozzles, fans, automotive valves, gas turbine blades as well as engine discs [9]. The catalogue supplied by the manufacturer with WEDM machine does not recommend any parameter setting for machining of Ti 6-2-4-2 material. In the present paper, an attempt is made for multi objective optimization of process parameters of WEDM to optimize the cutting speed and surface roughness for rough cut using Box–Behnken designs and response surface methodology. Table 1 shows the chemical composition of Ti 6-2-4-2 material used for experiment work and Table 2 illustrates its mechanical properties.

2 Literature Review

This section presents the review of literature on machining of Ti alloys as well as other materials using WEDM. Kuri-

Table 1 Chemical composition of Ti 6-2-4-2

Element	Al	Sn	Zr	Mo	Si	Fe	O_2	C	N_2	H_2	Ti
Max. weight (%)	6	2	4 6	2	0.13	0.25	0.15	0.08	0.05	0.0125	85.3

Table 2 Mechanical properties at room temperature [9]

Ultimate tensile strength (MPa)	Yield strength (MPa)	Elongation (%) at break	Hardness (Rockwell)	Hardness (Vickers)	Modulus of elasticity (GPa)	Shear strength (MPa)	Shear modulus (GPa)
1,010	1,080	3	34	318	120	45.5	300

akose and Shunmugam [10] optimized process parameters of WEDM for machining of Ti-6Al-4V by non-dominated sorting genetic algorithm. The sorting procedure employed a fitness assignment scheme which prefers non-dominated solutions and used a sharing strategy which preserves diversity among the solutions. None of the solution in the pareto-optimal set was better than any other solution in the set. Sarkar et al. [6] developed a second-order mathematical model for surface roughness, dimensional shift, and cutting speed in WEDM of γ-TiAl in trim cutting operation using response surface methodology. The residual analysis and experimental results indicated that the proposed models could adequately describe the performance indicators within the limits of the factors that are being investigated. Liao and Yu [11] studied the effect of specific discharge energy on WEDM characteristics of Ti-6Al-4V and Inconel-718. A quantitative relation between machining characteristics and machining parameters is derived. It was observed that two most significant factors affecting the discharge energy (η) are discharge-on time (pulse on time) and servo voltage. Moreover, discharge-on time and work piece height have a significant effect on machined groove width. Porous and Zoboruski [12] investigated volumetric efficiency in WEDM of Ti 6Al4V taking into consideration both process parameters and material properties. A semi-empirical model is developed to examine variation of WEDM efficiency by using different wire materials, different process parameters by application of dimensional analysis. Saha et al. [13] analysed WEDM of tungsten carbide cobalt composites. Second-order multi-variable regression model and a feed-forward back-propagation neural network model are developed to correlate input process parameters with the process response parameters, namely cutting speed and surface roughness. It was observed that neural network architecture provided the best results prediction although the proposed regression model was adequate and acceptable. Liu et al. [14] studied behaviour of wire electric discharge machining of Al_2O_3 particle reinforced aluminium alloy 6061. The effect of machining voltage, current, pulse duration, and electrolyte concentration on material removal rate was investigated. The results suggested that for achieving the highest MRR, applied current is the most influential among current, pulse duration, and electrolyte concentration. Khanna and Singh [15] investigated the effect of process parameters on WEDM of cryogenic treated D3 tool steel. The authors established mathematical models to highlight parametric influence on cutting rate. Taguchi's L27 array was used for plan of experimentation and analysis of variance was utilized to explore the effect of process parameters. Shah et al. [16] investigated seven different machining parameters in addition to varying the material thickness on the machining responses such as material removal rate, kerf, and surface roughness of tungsten carbide samples machined by WEDM. The design of experiments was based on Taguchi orthogonal designs with eight control factors at three levels. Results showed that material thickness has little effect on material removal rate and kerf. Jangra et al. [17] presented the optimization of surface roughness (Ra) and dimensional lag (μm) of WEDM using Taguchi and Grey Relational Analysis on **D3** tool steel. Taguchi's L_{18} Orthogonal Array was used to conduct experiments. Sadeghl et al. [18] discussed effects of process parameters on surface roughness (Ra) and metal removal rate in WEDM of AISI D5 steel alloy. Regression is used to model the process and Tabu search algorithm is opted for optimization. It was found that discharge current and pulse interval are more influential on MRR and surface roughness than open circuit voltage. Rao and Pawar discussed development of mathematical models using response surface modelling for correlating the inter relationship of various WEDM parameters and responses, namely machining speed (mm/min) and surface roughness (Ra). Artificial bee colony (ABC) technique is then applied to find optimal combination of process parameters to achieve a maximum cutting speed for a desired value of surface finish. It was observed that convergence rate of ABC algorithm is very high and it can be used effectively in the optimization of multi-variable problems [7,19].

Literature review clearly indicates that a few works are reported on WEDM of Titanium and its alloys and there is no study on the WEDM of Ti 6-2-4-2 alloy. As Ti 6-2-4-2 alloy is used in engineering applications and WEDM is used for their manufacturing, there is need to optimize process parameters of WEDM for machining of Ti 6-2-4-2. Thus, in the present study, an attempt is made to optimize process parameters of WEDM for machining of same alloy using Response Surface Methodology, in particular Box–Behnken designs to develop an empirical relationship between different process parameters namely pulse on time (TON), pulse off time (TOFF), peak current (IP), spark gap set voltage (SV), wire feed (WF), wire tension (WT), and output responses, namely cutting speed (CS) and surface roughness (SR). Here CS denotes average speed (mm/min) with which wire cut the work piece material and SR carries its usual meaning and is measured in Ra.

3 Experiment Set Up and Design

The experiments are performed on a four-axis Electronica Sprintcut 734 CNC Wire cut machine manufactured by Electronica India Limited, Pune (India). A diffused brass wire of 0.25 mm diameter (Nikunj HH) is used as tool material and deionized water as dielectric. A rectangular plate of Ti 6-2-4-2 measuring 200 mm × 200 mm × 20.4 mm is taken as work piece. A 10 mm × 10 mm rectangular cut is taken on the work piece. Figure 1 shows the path followed by wire. The wire enters the work piece at point O (5, 0). It moves along OABCDO and exits the work piece from O (5, 0). CNC code

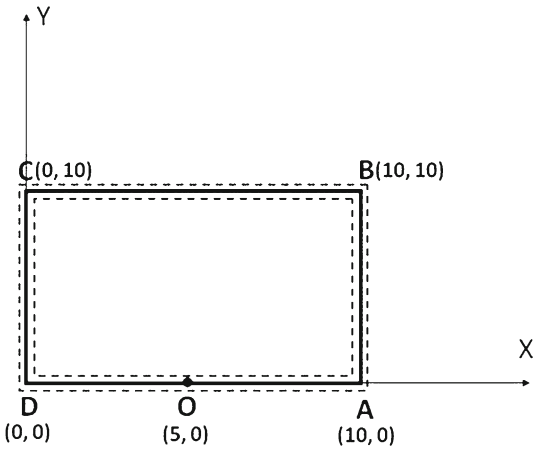

Fig. 1 Work piece profile

for cutting is generated using ELAPT software supplied by the manufacturer. It is important to mention that wire offset is set at zero during machining.

In order to compute average cutting speed, instantaneous cutting speed is noted at a distance of 2.5, 5 and 7.5 mm from the initiation of cut along a particular axis. This ensures that data of cutting speed are noted only when the cutting is properly stabilized. Instantaneous cutting speed data are directly displayed on the monitor of machine tool and mean of the readings provides average cutting speed. Surface roughness of the work piece that is cut from the plate is measured using portable Surface Roughness tester Mitutoya SJ-301. It is measured perpendicular to direction of cut at eight places taken along OABCDO and average of these readings is chosen as SR.

This study utilizes six process parameters viz. TON, TOFF, IP, SV, WF, and WT which are chosen on the basis of literature review as well as preliminary study [20]. The present study considers these parameters at three levels. Table 3 provides details of these process parameters as well as other fixed parameters that are used in the present work. It is important to mention that TON, TOFF as well as WT values mentioned in the brackets in Table 3 show the actual settings in μs and g, respectively, whereas the values outside the brackets indicate the machine control panel settings.

The present work utilizes Box–Behnken experimental design approach as it plans experiments within identified search space (assuming α = 1). In WEDM, loss of productivity occurs due to wire breakage. Thus, if the pilot study is carried out before actual experiment work, then range of parameter combinations, where wire breakage takes place, can be identified and isolated. Box–Behnken design approach is in contrast to central composite design which tends to explore the space beyond the identified search space (Taking α > 1, depending upon number of independent parameters), where chances of wire breakage are quite high. Moreover, Box–Behnken designs are rotatable or nearly rotatable second-order designs based upon three-level incomplete factorial designs. Output response (y) in Box–Behnken design can be modelled as follows [21]:

$$Y = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \beta_{ii} x_i^2 + \sum_{i<j} \beta_{ij} x_i x_j, \quad (1)$$

where x_i, x_j, x_k is input or independent process parameters, $\beta_0, \beta_{ii}, \beta_{ij}$ is regression coefficients, and ε is random error.

Construction of Box–Behnken design (BBD) for six control factors is based upon partially balanced incomplete block designs. The factorial structure for BBD is 2^3. Each row of the design matrix (DX) given below involves eight design points in addition to a row of all factors at central levels. So, in total, 54 design points are generated [22]. In $D(X)$, A, B, C, D, E and F refer to six control factors and +1, −1 and 0 depict high, low and zero level of control factors, respectively.

$$D(X) = \begin{vmatrix} A & B & C & D & E & F \\ \pm1 & \pm1 & 0 & \pm1 & 0 & 0 \\ 0 & \pm1 & \pm1 & 0 & \pm1 & 0 \\ 0 & 0 & \pm1 & \pm1 & 0 & \pm1 \\ \pm1 & 0 & 0 & \pm1 & \pm1 & 0 \\ 0 & \pm1 & 0 & 0 & \pm1 & \pm1 \\ \pm1 & 0 & \pm1 & 0 & 0 & \pm1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{vmatrix}$$

Table 3 Fixed and control factors with their settings/levels

Control factors	Coded factors	Levels machine units (actual)			Units	Fixed factors	Description
		I	II	III			
TON	A	112 (0.7)	115 (0.85)	118 (1.0)	μs	Wire	HH Brass Ø 0.25 mm
TOFF	B	48 (22)	52 (30)	56 (38)	μs	Size of work piece	Square, 10 × 10 × 20.4 mm
IP	C	140	170	200	amp	Dielectric conductivity	15–20 mho
SV	D	35	45	55	volts	Servo feed	2,050
WF	E	6	8	10	m/min	Dielectric pressure	7 kg/cm^2
WT	F	4 (500)	6 (700)	8 (1,000)	g	Dielectric temperature	24 °C

Table 4 Design matrix and results for Wire EDM output response

Standard order	Run order	Control factors						Responses	
		TON	TOFF	IP	SV	WF	WT	CS (mm/min)	SR (μm)
7	1	112	56	170	55	8	6	0.35	1.69
50	2	115	52	170	45	8	6	0.714	1.79
36	3	115	56	170	45	10	4	0.52	1.86
51	4	115	52	170	45	8	6	0.6271	1.77
29	5	112	52	170	35	10	6	0.635	1.79
41	6	112	52	140	45	8	4	0.554	1.65
40	7	115	56	170	45	10	8	0.556	1.885
2	8	118	48	170	35	8	6	1.355	2.3
23	9	115	52	140	55	8	8	0.547	1.87
48	10	118	52	200	45	8	8	0.925	2.125
12	11	115	56	200	45	6	6	0.547	1.81
49	12	115	52	170	45	8	6	0.722	1.91
32	13	118	52	170	55	10	6	0.698	1.92
30	14	118	52	170	35	10	6	1.1	2.132
39	15	115	48	170	45	10	8	0.896	1.952
25	16	112	52	170	35	6	6	0.647	1.777
31	17	112	52	170	55	10	6	0.45	1.615
9	18	115	48	140	45	6	6	0.925	2.04
4	19	118	56	170	35	8	6	0.874	2.09
13	20	115	48	140	45	10	6	0.92	1.868
45	21	112	52	140	45	8	8	0.538	1.695
17	22	115	52	140	35	8	4	0.815	1.79
19	23	115	52	140	55	8	4	0.58	1.815
24	24	115	52	200	55	8	8	0.61	1.97
8	25	118	56	170	55	8	6	0.518	1.96
47	26	112	52	200	45	8	8	0.562	1.72
38	27	115	56	170	45	6	8	0.563	1.967
14	28	115	56	140	45	10	6	0.512	1.8
46	29	118	52	140	45	8	8	0.839	2.15
21	30	115	52	140	35	8	8	0.848	1.946
43	31	112	52	200	45	8	4	0.577	1.855
16	32	115	56	200	45	10	6	0.592	2.01
6	33	118	48	170	55	8	6	0.906	2.01
34	34	115	56	170	45	6	4	0.556	1.74
44	35	118	52	200	45	8	4	0.916	2.075
28	36	118	52	170	55	6	6	0.74	2.105
20	37	115	52	200	55	8	4	0.6275	1.93
3	38	112	56	170	35	8	6	0.458	1.71
27	39	112	52	170	55	6	6	0.485	1.66
37	40	115	48	170	45	6	8	0.935	2.046
42	41	118	52	140	45	8	4	0.827	1.937
15	42	115	48	200	45	10	6	0.961	1.98
5	43	112	48	170	55	8	6	0.554	1.722
53	44	115	52	170	45	8	6	0.688	1.88
22	45	115	52	200	35	8	8	0.886	1.962
52	46	115	52	170	45	8	6	0.725	2.012

Table 4 continued

Standard order	Run order	Control factors						Responses	
		TON	TOFF	IP	SV	WF	WT	CS (mm/min)	SR (μm)
35	47	115	48	170	45	10	4	0.904	1.92
33	48	115	48	170	45	6	4	0.96	1.96
54	49	115	52	170	45	8	6	0.72	1.96
26	50	118	52	170	35	6	6	1.054	2.122
1	51	112	48	170	35	8	6	0.82	1.82
10	52	115	56	140	45	6	6	0.563	1.91
11	53	115	48	200	45	6	6	0.974	1.942
18	54	115	52	200	35	8	4	0.761	1.9

Table 4 summarizes various parameter combinations for 54 experiments as well as run order. It is planned to carry out one replication of each experiment and thus total 108 experiments are to be conducted.

4 Results and Discussion

Experiments are conducted on Electronica 4 axis Sprintcut-734 CNC Wire Cut machine. Each time the experiment is performed, a particular set of parameter combination is chosen and work piece is cut as shown in Fig 1. Table 4 summarizes the results obtained for 54 experiments with one replication. The CS and SR, indicated in Table 4, are average of values obtained in both runs.

Results obtained are analysed using Design Expert 6.0 software and discussed in the following sections and sub sections as per output response.

5 Effect of Control Factors on Cutting Speed

To identify the effect of control factors on CS, it is desirable to find goodness of fit of the given data. Quadratic model for CS is recommended by the Design expert 6.0 software. Table 5 summarizes analysis of variance (ANOVA) for the quadratic model at 95 % confidence level.

The above mentioned data indicate that F value of the model is 242.08 and corresponding P value is less than 0.001. which implies that quadratic model is significant at 95 % confidence level. Moreover, lack of fit of 0.54 implies that it is not significant relative to pure error. The value of R^2 of 0.9825 indicates that 98.25 % of variation of CS is attributed to control factors and only 1.75 % of total variation cannot be explained by the quadratic model. This indicates that the accuracy and general ability of the polynomial model is good. Moreover, predicted value of R^2 of 0.97 is in reasonable agreement with adjusted R^2 of 0.9784, which depicts a high correlation between observed and predicted values. Figure 2 shows the normal probability plot of residuals for CS. It clearly indicates that errors are normally distributed as most of the residuals are clustered around straight line. It is observed that regression model is fairly well fitted with observed values. Adequate precision measures signal-to-noise ratios. A value greater than 4 is desirable. The ratio of 74.793 indicates an adequate signal which suggests that the quadratic model can be used to navigate in the design space. Insignificant terms are eliminated using backward elimination and the following surface equation in actual factors is obtained Table 6:

$$\begin{aligned} \text{CS} = & -24.85563 + (0.29637 \times \text{TON}) + (0.12237 \times \text{TOFF}) \\ & + (6.53472\text{E-}004 \times \text{IP}) + (0.1454 \times \text{SV}) \\ & + (0.060880 \times \text{WT}) + (1.52323\text{E-}003 \times \text{TOFF}^2) \\ & - (3.15625\text{E-}003 \times \text{TON} \times \text{TOFF}) \\ & - (1.66667\text{E-}003 \times \text{TON} \times \text{SV}) \\ & + (7.84375\text{E-}004 \times \text{TOFF} \times \text{SV}) \\ & - (1.30312\text{E-}003 \times \text{SV} \times \text{WT}) \end{aligned} \tag{2}$$

Equation 2 shows that main effects TON, TOFF, IP, SV, Two Factor Interaction between TON and TOFF, TON and SV, TOFF and SV, SV and WT and quadratic function of TOFF have significant effects on CS and can be used to predict CS within limits of control factors.

Figure 3 shows interaction effect of TON and TOFF on cutting speed. It clearly shows that CS attains a peak value 1.13 mm/min at a higher value of TON (118) and a lower value of TOFF (48). This is attributed to the fact that a high value of TON and corresponding lower value of TOFF result in sparking for longer duration of time and leads to higher release of spark energy causing faster erosion of material. Figure 3 also indicates that CS attains a minimum value of 0.416 mm/min at a low value of TON (112) and at a high value of TOFF (56). This is due to the fact that lower value of TON with a higher value of TOFF results in impinge-

Table 5 ANOVA for response surface of reduced quadratic model of cutting speed

Source	Sum of squares	Degree of freedom	Mean square	*F* value	*P* > *F*	
Model	2.0578	10	0.2057	242.0796	<0.0001	Significant
A (TON)	0.7079	1	0.7079	832.8252	<0.0001	Significant
B (TOFF)	0.8441	1	0.8441	993.0152	<0.0001	Significant
C (IP)	0.0092	1	0.0092	10.8506	0.002	Significant
D (SV)	0.4233	1	0.4233	498.0102	<0.0001	Significant
F (WT)	0.0004	1	0.0004	0.5664	0.4558	Not significant
B^2	0.0079	1	0.0079	9.3166	0.0039	Significant
AB	0.0114	1	0.0114	13.5003	0.0007	Significant
AD	0.04	1	0.04	47.0553	<0.0001	Significant
BD	0.0078	1	0.0078	9.2641	0.004	Significant
DF	0.0054	1	0.0054	6.39250	0.0152	Significant
Residual	0.0365	43	0.0008			
Lack of fit	0.0293	38	0.0007	0.54	0.874	Not significant
Pure error	0.0071	5	0.0014			
Cor total	2.0943	53				

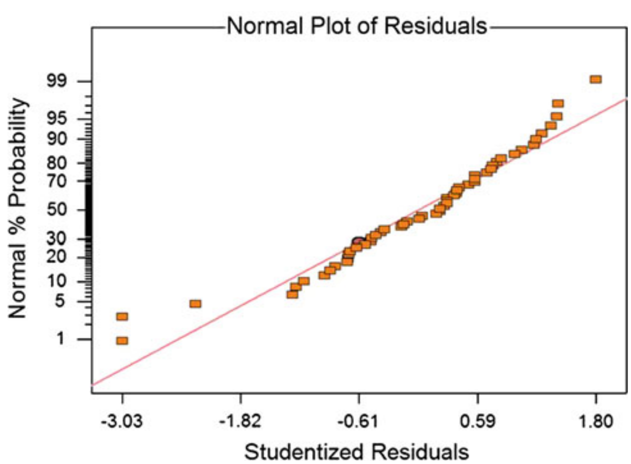

Fig. 2 Normal probability plot of residuals for cutting speed

Table 6 Model summary statistics for cutting speed

Std. dev.	0.029156	R^2	0.982547
Mean	0.724752	Adjusted R^2	0.978488
C.V. (%)	4.022871	Predicted R^2	0.970066
PRESS	0.062694	Adequate precision	74.7932

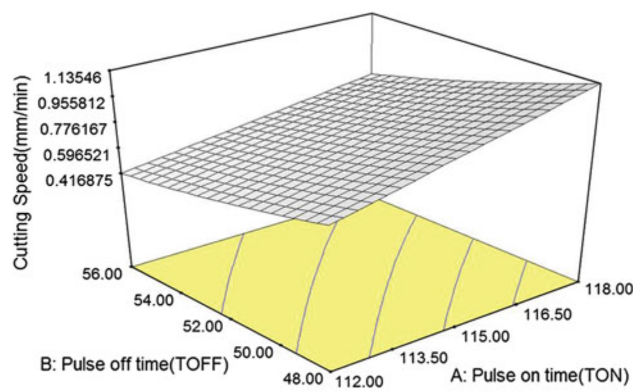

Fig. 3 Interaction effect of pulse on time and pulse off time on cutting speed

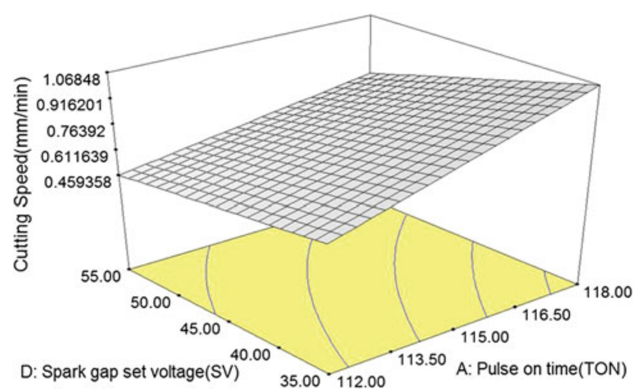

Fig. 4 Interaction effect of pulse on time and spark gap set voltage on cutting speed

ment of spark on the work piece for lesser duration of time that leads to less amount of release of spark energy causing slower erosion of metal.

Interaction effect of TON and SV is shown in Fig. 4. It shows that CS achieves a maximum value of 1.068 mm/min at a higher value of TON (118) and at a lower value of SV (35). This is due to the fact that higher value of TON produces discharge energy for longer duration of time. A lower value of SV reduces the spark gap. Both factors act synergetically to produce large discharge energy on the work piece resulting into faster erosion of material. At lower value of TON

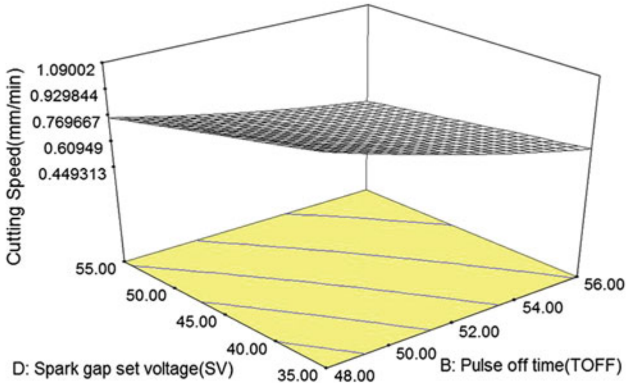

Fig. 5 Interaction effect of pulse off time and spark gap set voltage on cutting speed

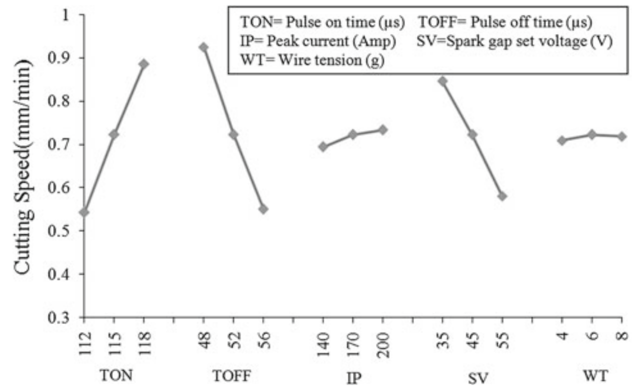

Fig. 6 Main effect of control factors at three levels on cutting speed

(112) and higher value of SV (55), cutting speed is drastically reduced to 0.4593 mm/min. This is due to the fact that with lower value of TON (112), discharge is produced for shorter duration of time and, therefore, small amount of discharge energy is impinged on the work piece. Higher value of SV (55) widens the spark gap, therefore reducing the number of sparks occurring on work piece per unit time. When both the factors act at these levels, it leads to slower erosion of material. Hence less cutting speed is achieved.

Figure 5 shows combined effect of TOFF and SV. It shows that higher cutting speed (0.9298 mm/min) is obtained at lower level of SV (35) and lower level of TOFF (48). Small TOFF means less time duration during which current is off, i.e. more number of discharges per second, which implies that metal erosion will be more and hence faster cutting speed results. The variation of CS with smaller value of SV is attributed to the reasons cited earlier.

Table 5 shows that a weak interaction is also present between SV and WT. This interaction plays a little part in contribution to CS as indicated by a smaller F value (6.3925) and corresponding larger P value (0.01).

Figure 6 shows the effect of main factors on CS. It indicates that main effect of control factors follow the same trend as illustrated in case of interactions. The main effect of WT is included in the analysis to obey the principle of hierarchy although it appears insignificant from the Fig. 6.

6 Effect of Control Factors on Surface Roughness

The fit summary of Design Expert 6.0 recommended that 2FI (two factor interaction) model is statistically significant for surface roughness analysis. Table 7 presents ANOVA for 2FI model of SR. The associated P value for the model is lower than 0.05, which indicate that the model is significant. Moreover, lack of fit term is not significant as desired. Table 8 shows that value of R^2 and Adjusted R^2 are 0.8522 and 0.8260, respectively, which implis that the model provides an excellent relation between control factors and response (SR). Figure 7 shows the normal probability plot of residuals for SR. Most of the residuals are falling on the straight line, implying that the errors are normally distributed. It shows that the regression model is fairly well fitted with the observed values. During preparation of the model non-significant terms are eliminated by backward elimination. The main effects of TON, TOFF, IP, SV, WF and WT; interaction effects between TON and WT as well as IP and WF are found to be statistically significant for the analysis. Following response surface equation in actual factors is obtained:

$$\begin{aligned} SR = {} & 2.28046 + (0.014514 \times \text{TON}) \\ & - (0.011750 \times \text{TOFF}) \\ & - (7.54444\text{E-}003 \times \text{IP} - 4.466\text{E-}003 \times \text{SV}) \\ & - (0.19140 \times \text{WF}) - (0.82790 \times \text{WT}) \\ & + (7.35417\text{E-}003 \times \text{TON} \times \text{WT}) \\ & + (1.08333\text{E-}03 \times \text{IP} \times \text{WF}) \end{aligned} \quad (3)$$

Figure 8 shows the interaction plot of IP with WF. It indicates that with increase in IP, the SR increases. This is because an increase in peak current (200 A) causes an increase in pulse discharge energy at the discharge points in the cutting zone. With increase in discharge energy, larger chunks of material are removed from work piece surface. It leads to the creation of large sized micro cavities on the surface, which is associated with more surface roughness [23]. With an increase in wire feed, there is a considerable decrease in the surface roughness. This is in agreement with the findings of Ramakrishna and Karunamoorthy [24]. Surface roughness reduces with increase in WF as shown in Fig. 8. Each spark will be produced between a comparatively fresh piece of wire and the work piece, thereby producing stable discharge conditions which lead to lower surface roughness. On the contrary, when the wire feed is less (6 m/min), the same portion of wire experiences multiple sparks. This causes deformation of the wire and leads to unstable cutting conditions which increase the surface roughness.

Table 7 ANOVA for response surface of reduced 2FI model

Analysis of variance table (Partial sum of squares)						
Source	Sum of squares	Degree of freedom	Mean square	*F* value	*P* > *F*	
Model	0.9557	8	0.1195	32.443	<0.0001	Significant
A (TON)	0.7427	1	0.7427	201.6944	<0.0001	Significant
B (TOFF)	0.053	1	0.053	14.3971	0.0004	Significant
C (IP)	0.0272	1	0.0272	7.3872	0.0093	Significant
D (SV)	0.0479	1	0.0479	13.0031	0.0008	Significant
E (WF)	0.005	1	0.005	1.3624	0.2493	Not significant
F (WT)	0.0305	1	0.0305	8.291	0.0061	Significant
AF	0.0156	1	0.0156	4.2299	0.0455	Significant
CE	0.0338	1	0.0338	9.1788	0.004	Significant
Residual	0.1657	45	0.0037			
Lack of fit	0.1211	40	0.003	0.3391	0.9767	Not significant
Pure error	0.0446	5	0.0089			
Cor total	1.1215	53				

Table 8 Model summary statistics for surface roughness

Std. dev.	0.060683	R^2	0.852238
Mean	1.904167	Adjusted R^2	0.825969
C.V. (%)	3.186844	Predicted R^2	0.796831
PRESS	0.227845	Adequate precision	22.48188

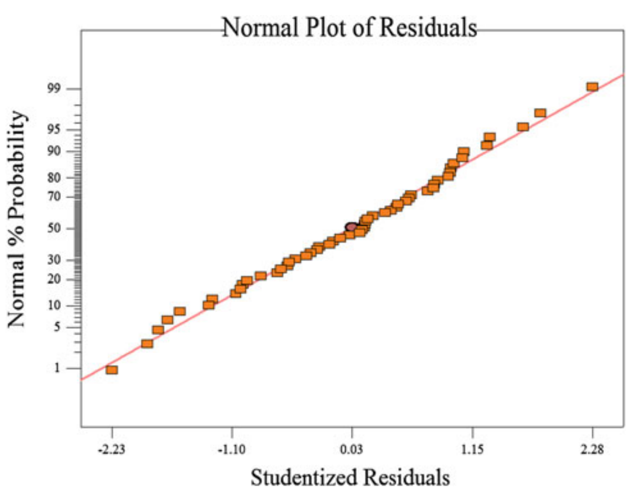

Fig. 7 Normal probability plot of residuals for SR

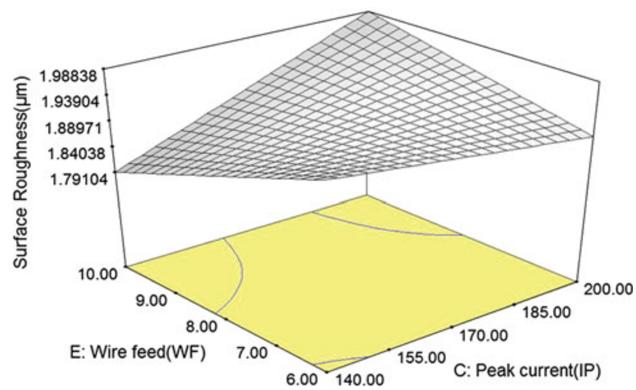

Fig. 8 Interaction effect of wire feed and peak current on surface roughness

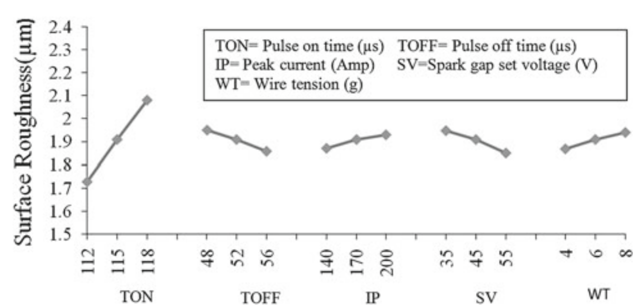

Fig. 9 Main effects of control factors at three levels on surface roughness

Table 7 indicates that an interaction exists between TON and SV. This interaction plays a little part as shown by a large *P* value (0.045) which is closer to 0.05, i.e. *P* value at which a control factor becomes insignificant.

Figure 9 shows the effect of main factors on CS. It indicates that with increase in TON from 112 to 118 and corresponding decrease in TOFF from 56 to 48, discharge will last for a longer duration of time, which leads to high discharge energy that will increase the SR due to increase in depth of craters on work piece surface. The increase in value of SV from 35 to 55 will widen the spark gap. So less discharge energy is impinged on work piece surface per unit time which decreases the SR. The effect of IP on SR follows the same trend as exhibited by the interaction between WF and IP.

7 Multi Objective Optimization of Control Factors

There are two different objectives in the present study, namely cutting speed and surface roughness. Effort is made to find the machine settings to obtain maximum cutting speed and minimum surface roughness simultaneously. This multi objective optimization problem is formulated as multi variable, non-linear optimization problem. The responses CS and SR have been optimized simultaneously using composite desirability optimization method [25]. It makes use of an objective function $D(X)$, called desirability function which transforms an estimated response (y_i) into a an individual desirability function (d_i) that varies over the range $0 \leq d_i \leq 1$. Where $d_i = 1$ represents the ideal case, i.e. response y_i at its goal or target. $d_i = 0$ if one or more responses are outside the acceptable limits. There is also a positive number, weight (w_i) associated with the desirability function. It indicates the relative importance of an objective function in multi objective optimization problems. To reflect the difference in importance of different responses, weights w_i satisfy $0 < w_i < 1$ and $w_1 + w_2 + \ldots w_n = 1$. The optimization is accomplished in following steps:

- Obtaining individual desirability (d) for each response;
- combining individual desirabilities to obtain composite desirability (D) for given weights of CS and SR. Composite desirability is the weighted geometric mean of individual desirability for the responses;
- maximizing the composite desirability and identifying the optimal parameter combinations.

If it is desirable to maximize a response, the individual desirability is calculated as

$$d_i = \left\{ \left(0 \quad y_i < L_i @ \left(y_i - \frac{L_i}{T_i} - @ 1 \quad y_i > T_i \right) L_i \right) w \quad L_i \leq y_i \leq T_i \right.$$

If the target (T_i) is to minimize a response, the individual desirability is calculated as

$$d_i = \left\{ \left(1 \quad y_i < T_i @ \left(U_i - \frac{y_i}{U_i} - @ 0 \quad y_i > U_i \right) T_i \right) w \quad T_i \leq y_i \leq U_i, \right.$$

where L_i = lower limit value of response y_i, U_i = upper limit value of response y_i.

If the objective for the response is a target value, then individual desirability is calculated as

$$d_i = [(y_i - L_i)/(T_i - L_i)]^w \quad L_i \leq y_i \leq T_i$$

$$d_i = [(U_i - y_i)/(U_i - T_i)]^w \quad T_i \leq y_i \leq U_i$$

$$d_i = 0 \quad \text{if } y_i < L_i$$

$$d_i = 0 \quad \text{if } y_i > U_i$$

If the importance is same for each response, the composite desirability (D) is the geometric mean of all desirability functions and is given by

$$D = (d_1 \times d_2 \times d_3 \times \cdots \times d_n)^{1/n} = \left(\prod_{i=1}^{n} d_i \right)^{1/n}, \qquad (4)$$

where n = number of responses ($n = 2$, for the present study).

Equation 4 can be extended to consider the possible difference in the importance of different responses by assigning weights to individual response functions.

$$D = ({d_1}^{w_1} \times d_2^{w_2} \times \cdots \times {d_n}^{w_n})^{1/n} \qquad (5)$$

The factor setting with maximum total desirability is considered to be optimal parameter combination.

The aim of present study was to optimize parameters for rough cut planning, so the optimal parameters have been generated by incrementing weights for CS from 0.4 to 0.9 in steps of 0.1. Table 9 shows the impact of weights on predicted CS and SR. In experiment number 1 more priority is given to minimize SR; therefore, weight ($w = 0.6$) was chosen for SR and weight ($w = 0.4$) was selected for CS. In experiment number 6, a weight ($w = 0.9$) is assigned to CS to give it high priority and weight ($w = 0.1$) is assigned to SR. Table 9 shows

Table 9 Optimal control factor levels and corresponding predicted responses

Exp. no.	Weights		Control factors CS (mm/min)						CS (mm/min)	SR (µm)	Desirability
	w_1	w_2	TON	TOFF	IP	SV	WF	WT	Predicted	Predicted	
1.	0.4	0.6	114.75	48	140	37.8	10	4	0.9798	1.823	0.816
2.	0.5	0.5	116.55	48	140.7	38.4	10	4	1.118	1.902	0.816
3.	0.6	0.4	117.87	48	140.0	36.6	9.8	4	1.255	1.971	0.837
4.	0.7	0.3	117.88	48	145.3	35.0	10	4	1.298	2.007	0.862
5.	0.8	0.2	117.94	48	140	35.0	9.9	5.1	1.3098	2.021	0.897
6.	0.9	0.1	118	48	142.5	35.0	10	7.1	1.349	2.113	0.926

Table 10 Confirmation experiments and their results

S. no	TON	TOFF	IP	SV	WF	WT	Cutting speed (mm/min)			Surface roughness (μm)		
							Predicted	Actual	Error (%)	Predicted	Actual	Error (%)
1	114.75	48	140	37.8	10	4	0.9798	0.933	−5.01	1.823	1.77	−2.99
2	116.55	48	140.7	38.4	10	4	1.118	1.062	−5.27	1.902	1.831	−3.87
3	117.87	48	140.0	36.6	9.8	4	1.255	1.195	−5.0	1.971	1.891	−4.23
4	117.88	48	145.3	35.0	10	4	1.298	1.253	−3.59	2.007	1.942	−4.01
5	117.94	48	140	35.0	9.9	5.1	1.3098	1.265	−3.47	2.021	1.958	−3.21
6	118	48	142.5	35.0	10	7.1	1.349	1.28	−5.34	2.113	1.996	−5.86

the considerable difference in CS and SR obtained in the two cases. It is observed that, as the weights given to a particular response increases, there is considerable improvement in that response with a corresponding deterioration in performance of other response functions. Table 9 will provide flexibility and assist the machine operator to select different parameter combinations for achievement of CS and SR, according to the requirements of part drawings. The same methodology can be further extended for generating optimal input control factors settings for different requirements of cutting speed and surface roughness by choosing suitable sets of weights.

Six confirmation experiments with one replication are performed to verify the validity of response surface equations as they are derived from the regression fits. During confirmatory experimentation, values of optimal levels of control factors have been rounded off to nearest machine settings due to the machine limitations. An average of the values obtained in the two runs is considered as actual cutting speed and actual surface roughness. Table 10 shows the results obtained from confirmatory experiments. It reveals that error between the experimental (actual) and predicted values for CS and SR lies within −5.34 to −3.47 % and −2.99 to −5.86 %, respectively, and confirms reproducibility of the experimental conclusions.

8 Conclusions

The present paper investigates the effect of process parameters on cutting speed and surface roughness in WEDM of Ti 6-2-4-2 alloy using Box–Behnken designs. It considers six control factors viz. TON, TOFF, IP, SV, WF and WT at three levels. Empirical relations are obtained for CS and SR using regression analysis. Multi objective optimization of CS and SR is carried out using desirability approach. From the study, the following conclusions are drawn:

(a) The results of ANOVA and comparison of experimental results proved that mathematical models of CS and SR are fairly well fitted with experimental values within 95 % confidence level.
(b) For cutting speed, main effects of TON, TOFF, IP, SV, Two-Factor Interaction between TON and TOFF, TON and SV, TOFF and SV, SV and WT and quadratic function of TOFF play a significant role. Wire tension and Wire feed have negligible effect on cutting speed. In general, cutting speed is found to increase with increase in TON and decrease in TOFF and SV due to higher discharge energy produced in the machining zone.
(c) Surface roughness is effected by the main effects of TON, TOFF, IP, SV, WF and WT as well as interaction effects between TON and WT, IP and WF. Wire feed as a main effect has little impact on surface roughness but its interaction with IP shows a considerable influence on the same.
(d) Weights play a considerable role in the multi- objective optimization. This is evident from the range of predicted values which vary from 0.979 to 1.349 mm/min for cutting speed and 1.823–2.113 μm for surface roughness.
(e) Results of optimization provides a reference to machine tool operator for selection of optimal parameter combinations for simultaneous optimization of CS and SR, depending upon their job requirements.

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