

# Performance of Cooperative Spatial Multiplexing SISO/MIMO Communication Systems with Constellation Rearrangement technique

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**Abstract** Cooperative spatial multiplexing technique proposed to simplify the transmitting and receiving process on the relay nodes while providing significant energy savings. This paper investigates the performance of constellation rearrangement scheme when used in conjunction with cooperative spatial multiplexing MIMO/SISO relaying networks. During the MIMO relaying process, instead of having a single antenna for relay we have multiple antennas for relay by using additional information from MAC layer. The idea is to use the optimized constellation so that minimum squared Euclidean distance between different branches is maximized. Through extensive numerical search, we obtain the best constellation rearrangement scheme. Maximum likelihood detector is required in the receiver but the computational complexity of the receiver does not change because the proposed constellation will be saved in a table at the transmitter this search done by using arrival information from MAC layer.

**Keywords** Cooperative spatial multiplexing (C-SM) · Constellation rearrangement (CoRe) · Multiple input multiple output (MIMO) · Single input single output (SISO) · Maximum likelihood (ML) · Squared euclidean distance (SED) · Symbol error rate (SER) · Orthogonal

transmit diversity (OTD) · Singular value decomposition (SVD) · Medium access control (MAC)

## الخلاصة

تم اقتراح تقنية تعاونية مكانية متعددة لتبسيط عملية الإرسال والاستقبال على العقد المتناوبة مع توفير وفورات كبيرة للطاقة. تبحث هذه الورقة العلمية في أداء مخطط إعادة ترتيب برج عندما يستخدم بالاقتران بشبكات مرحل MIMO/SISO تعاونية مكانية متعددة. وفي أثناء عملية ترحيل MIMO، بدلاً من وجود هوائي واحد للمرحل، فلدينا هوائيات متعددة للمرحل. والفكرة هي استخدام البرج الأمثل بحيث يتم تكبير الحد الأدنى لمربع المسافة الإقليدية بين مختلف الفروع. ومن خلال البحث العددي واسع النطاق استطعنا الحصول على أفضل مخطط إعادة ترتيب برج. ومكتشاف احتمال الحد الأقصى في المستقبل هو مطلوب، ولكن التعقيد الحسابي في المتلقي لا يتغير لأنه سيتم حفظ البرج المقترح في جدول في جهاز الإرسال.

## 1 Introduction

Ad-hoc wireless networks are based on multi-hop communications, where the information from the source to the destination is relayed via other mobiles. The ad-hoc networks do not have fixed infrastructure, so this relaying operation is essential to overcome the path loss incurred over large distances. Multi-hop ideas are also utilized in cellular and wireless local area networks (LAN) to provide higher quality of service, power savings and extended coverage. Information theory of multi-hop communication dates back to the relay channel model, which contains a source, a destination and a relay whose goal is to facilitate information transfer from the source to the destination. The relay channel was first introduced by Van der Meulen [1] and investigated extensively by Cover and El Gamal [2]. Cover and El Gamal provided a number of relaying strategies, found achievable regions and provided upper bounds to the capacity of a general relay channel. They also provided an expression for the capacity of the degraded relay channel, in which the communication channel between the source and the relay is physically

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better than the source-destination link. The capacity of the general relay channel is still unknown. Motivated by the recent interest in multi-hop, a number of recent papers investigate the use of multiple relays. Some relevant references include [3–8]. The impact of relays in conventional communication systems has a phenomenal affect. Relays not only provide improved average SER by providing added diversity branches but also enhance system capacity. Wireless communication links suffer from severe degradation due to varying channel conditions. In long haul communication system, it becomes costly to transmit signal directly from transmitter to receiver because of power and infrastructure constraints. Relays were proposed as a solution to subdue this obscurity [9]. In relaying systems, relay acts as an intermediate node between transmitter and receiver. The signal that arrives at relay may contain interference and noise. Relays increase signal strength and re-transmit it to the destination. At the destination, signals received from relays and source are combined using conventional combining techniques for example maximum ratio combining (MRC). In this way, relays improve the average SER of the system. More sophisticated schemes have been proposed at relays namely as amplify and forward (AF), decode and forward (DF) and coded and forward (CF) scheme [10].

Constellation rearrangement is also an interesting technique which has been employed recently in [8, 11–14] to achieve significant gain in term of BER. In constellation rearrangement (CoRe), different bits to symbol mapping are done to maximize minimum squared Euclidean distance (SED) between neighboring symbols. Wengerter et al. [11] have studied the constellation rearrangement with respect to hybrid automatic repeat-request (ARQ) in Code division multiple access (CDMA) systems.

In this paper, we illustrate the performance of the modified cooperative spatial multiplexing (C-SM) SISO Relaying networks for a fixed and mobile relay with different constellation rearrangement using 16- and 64-QAM modulation schemes. This paper investigates the performance of constellation rearrangement scheme when used in conjunction with cooperative spatial multiplexing MIMO/SISO Relaying networks. In MIMO relaying, instead of having a single antenna at relay we have multiple antennas at relay. The idea is to use optimized constellation so that minimum squared Euclidean distance between different branches is maximized. Through extensive numerical search, we propose the best constellation rearrangement scheme. The wireless multipath channel is assumed to be slowly Rayleigh flat fading and uncorrelated between different branches. Maximum likelihood detector is required in the receiver but the computational complexity of the receiver does not change because we save this constellation in a table in transmitter.

Relays can be further classified in to fixed and mobile relays. As the name implies, fixed relays are deployed by the

service provider in strategic locations while mobile relays can be provided by the service provider or can be idle user terminals (UTs) that help other UTs. In this paper, we address both fixed and mobile relays. Since fixed relays are installed at strategic locations, Line-of-sight transmission between the base station (BS) and the relay station (RS) can be achieved in most cases. Consequently, from the physical layer prospective, the difference between fixed and mobile RSs is that, in the former, the link from BS to RS is reliable and can be assumed to be error free, for all practical purposes. However, the error free assumption does not hold in the case of mobile RSs. In mobile relays, the errors made at the RS propagate to the destination. This undesirable phenomenon is called error propagation and it is a limiting factor for the SER performance. According to what was said, in context of mobile relays, we consider average signal to noise ratio (SNR) between BS–UT and RS–UT to be equal and average SNR between BS–RS more than other links. Also, in context of fixed relays, we consider average SNR between BS–RS, BS–UT and RS–UT to be equal.

In cooperative SISO relaying systems, we obtain our results in two modes as follows:

1. When average SNR of the link between base station to user terminal (BS–UT) is equal to average SNR of the link between relay station to user terminal (RS–UT) for different Constellation Rearrangement schemes using 16- and 64-QAM modulation.
2. When average SNR of the link between base station to user terminal (BS–UT) is not equal to average SNR of the link between relay station to user terminal (RS–UT) for different Constellation Rearrangement schemes using 16- and 64-QAM modulation.

To have a closer look at the effect of error propagation on the average SER for the different CoRe schemes, the average SER is plotted as a function of the average SNR of the link between base station to relay station (BS–RS) while fixing average SNR of the link between base station to user terminal (BS–UT) and average SNR of the link between relay station to user terminal (RS–UT) equal to 15 and 25 dB, respectively.

Finally, we compare cooperative spatial multiplexing MIMO relaying networks with Constellation Rearrangement and Cooperative spatial multiplexing SISO Relaying networks with constellation rearrangement.

## 2 Singular Value Decomposition

Any MIMO channel is converted into parallel channel using singular value decomposition (SVD) technique. If  $H \in C^{n_r \times n_t}$  is the channel matrix, then by employing SVD it can be written as:

$$H = U \Lambda V^H, \quad (1)$$

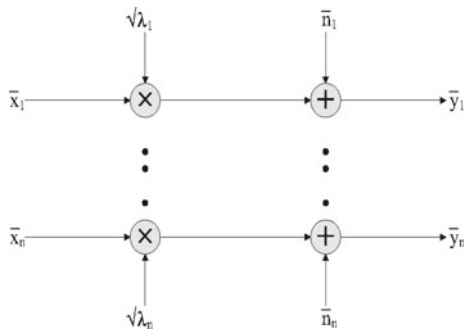


Fig. 1 Parallel SISO Gaussian channel

where  $U \in C^{n_r \times n_r}$  and  $V \in C^{n_t \times n_t}$  are unitary matrices and  $\Lambda \in R^{n_r \times n_t}$  represents the diagonal matrix whose elements are non-negative real numbers. If  $\Lambda$  is not full rank, the diagonal elements are given by  $N = \min(n_r, n_t)$  with off-diagonal elements equal to zero. The elements of  $\Lambda$  are obtained as:

$$H^H H = V \Lambda^H U^H U \Lambda V^H = V \Lambda^2 V^H \tag{2}$$

$$\Lambda = \begin{bmatrix} \sqrt{\lambda_1} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \sqrt{\lambda_N} \end{bmatrix} \tag{3}$$

For detailed mathematical explanations of SVD refer to [15]. Let

$$\bar{Y} = U^H Y \tag{4}$$

$$\bar{X} = V^H X \tag{5}$$

$$\bar{n} = U^H n \tag{6}$$

Then, MIMO antenna system using SVD is converted in to parallel Gaussian fading channel as (Fig. 1):

$$\bar{Y} = \Lambda \bar{X} + \bar{n} \tag{7}$$

### 3 Constellation Rearrangement

CoRe for OTD was introduced in [8]. Such a scheme is also referred to as permutation coding [7]. The basic concept

behind CoRe is explained by following example. Consider a 4-pulse amplitude modulation (PAM) scheme for OTD case as shown in Fig. 2.

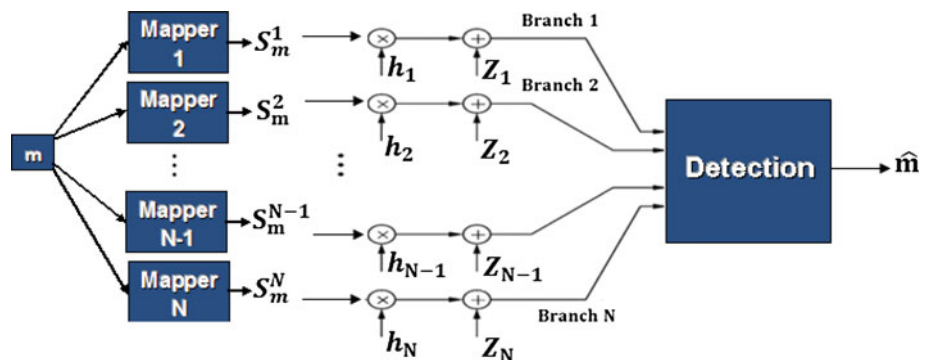
In the case of conventional OTD scheme, an identical symbol is transmitted to all branches. Figure 3 shows conventional OTD branches with gray coding. Figure 4 gives the signal constellation set for conventional transmit diversity. Since same bits to symbol mapping are done on all branches, hence we get one dimensional constellation set at receiver. The constellation points remain one-dimensional while we are using two-dimensional signal space. This scheme can give us diversity gain when each channel undergoes a different fading path. If we increase the number of branches, the signal space will become a hyper-cube but dimension of constellation points will remain the same. This scheme can be improved by employing constellation rearrangement. By rearranging the constellation point as illustrated in Fig. 5, we can increase the minimum SED between the neighbors and utilize both signal space dimensions. In case of 4-PAM with two branches, a relative gain of 4 dB is achieved. To obtain full diversity, we have to transmit unique constellation on each branch. For high level linear modulation schemes, CoRe is done through computer search where we iterate over all possible symbols and over all branches to choose the constellation set which maximizes the minimum squared Euclidean distance between constellation points.

$$d_{\min} = \max \left\{ \min \left\{ |s_m - s_{\hat{m}}|^2 \right\} \right\}, \tag{8}$$

where  $s_m$  is the transmitted symbol and  $s_{\hat{m}}$  denotes the detected symbol.

It is interesting to note that after using constellation rearrangement, the signal spread is increased as shown in Fig. 5. In improved OTD system, same symbols say  $3a, a$  are transmitted to both branches, but the energy corresponding to each symbols is different, with one branch having more energy than the other. Hence CoRe has an effect of equalization of transmitted energy per symbol. This effect is more pronounced in higher order linearly modulated signals like M-QAM. It has been shown in [5,8] that an appropriate choice of bit to symbol mapping on each diversity branch

Fig. 2 Applying CoRe to OTD Systems



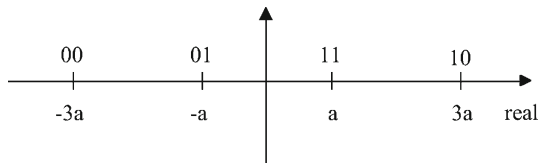


Fig. 3 Signal constellation for 4PAM

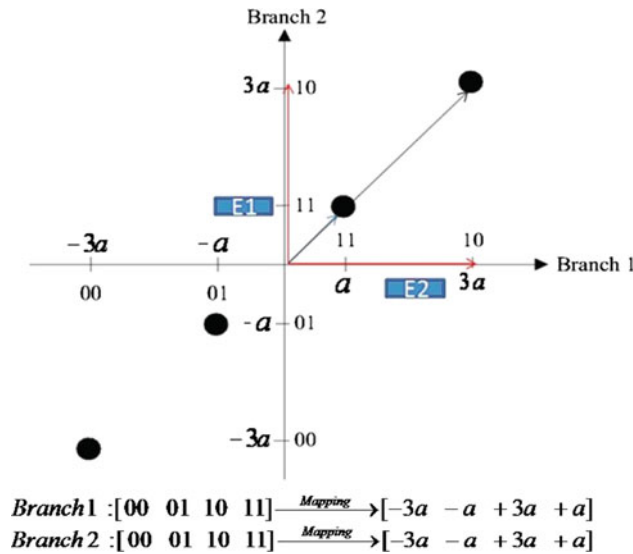


Fig. 4 Signal constellation for conventional OTD with 4-PAM

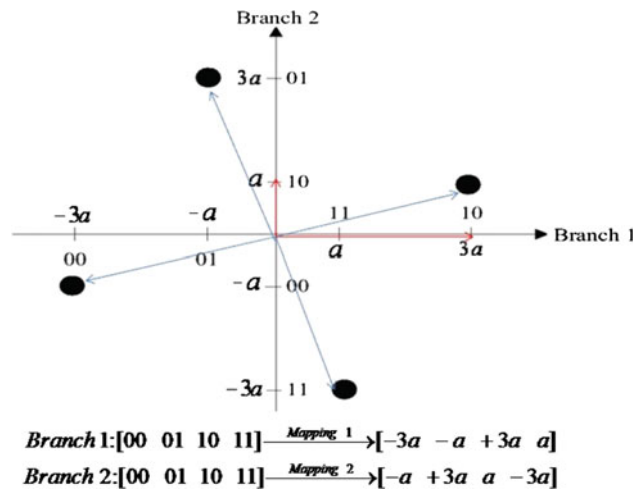


Fig. 5 Signal constellation for OTD with CoRe for 4-PAM

results in an increased Euclidean distance and thus provides better performance (as is evident from Figs. 4, 5).

### 4 Optimized Constellations

The idea is to use optimized constellation so that minimum squared Euclidean distance between different branches is maximized. Through extensive numerical search, we propose a good constellation rearrangement scheme. Maximum likelihood detector is required in the receiver but the

computational complexity of the receiver does not change because through extensive numerical search, we propose a good constellation rearrangement scheme and save this constellation in a table at the transmitter (Figs. 6, 7, 8).

The extension to higher order modulation for 64-QAM is straight forward. Other constellation rearrangements that we could use from before are as follows:

- (a) Usual CoRe (Fig. 9)
- (b) CoRe 1 (Fig. 10)
- (c) CoRe 2 (Fig. 11)

## 5 Cooperative Spatial Multiplexing SISO Relaying Systems

### 5.1 System Model

The cooperative relaying has recently received significant attention due to its application in wireless networks. It is closely related with multiple input multiple-output (MIMO) system, which has been widely employed to achieve a diversity gain, provide higher quality of service, power savings, extended coverage, and improve reliability in symbol-error rate (average SER). The main challenge of the cooperative relaying is the design of appropriate forwarding strategy at the relay.

The complete system in this paper includes a source, a relay and a destination. The channel between source and destination, source and relay, and relay and destination is modeled as independent Rayleigh fading channels (Fig. 12).

To provide deeper insight into the cooperative relay system, a half-duplex system model is considered here in which the signal is transmitted from the source to both relay and destination in the first time slot. Relay does some processing to the received signal and then transmits the processed data to the destination in the second time slot. During this time, the source keeps silent. Finally, the destination combines the signals from source and relay and then makes detection. This procedure is shown in Fig. 13.

In the first time slot, we can write

$$\begin{aligned} y_{sr}[n] &= a_{sr}x[n] + n_{sr}[n] \\ y_{sd}[n] &= a_{sd}x[n] + n_{sd}[n] \end{aligned} \tag{9}$$

And in the second time slot, we can write

$$y_{rd}[n + 1] = a_{rd}x_r[n] + n_{rd}[n + 1] \tag{10}$$

### 5.2 Cooperative Spatial Multiplexing SISO Relaying Systems Without CoRe

The signal received in the first and second time slot at relay and destination in Cooperative spatial multiplexing SISO Relaying Systems without CoRe is written as:

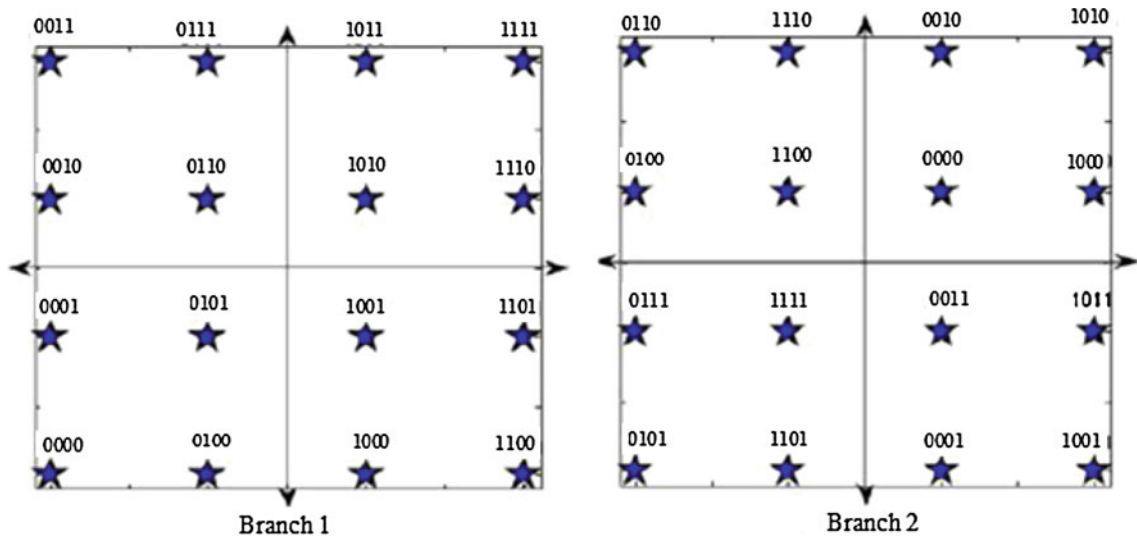


Fig. 6 Optimized constellation set for two 16-QAM branches

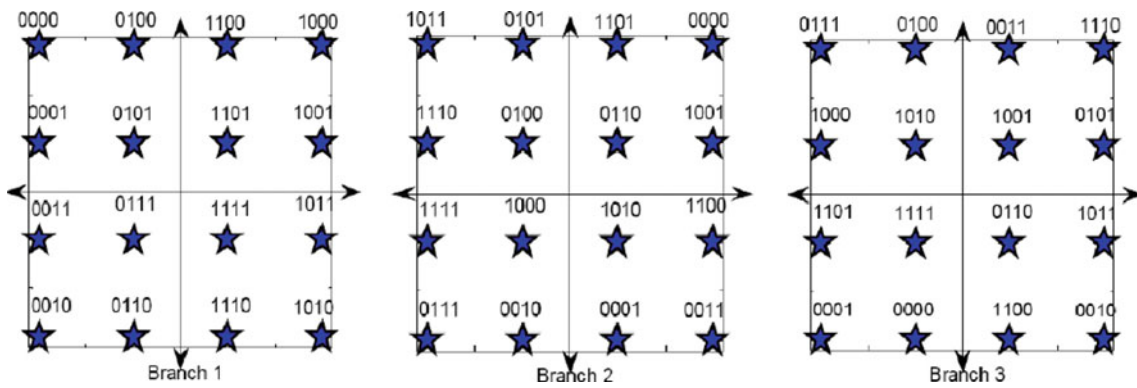


Fig. 7 Optimized constellation set for three 16-QAM branches

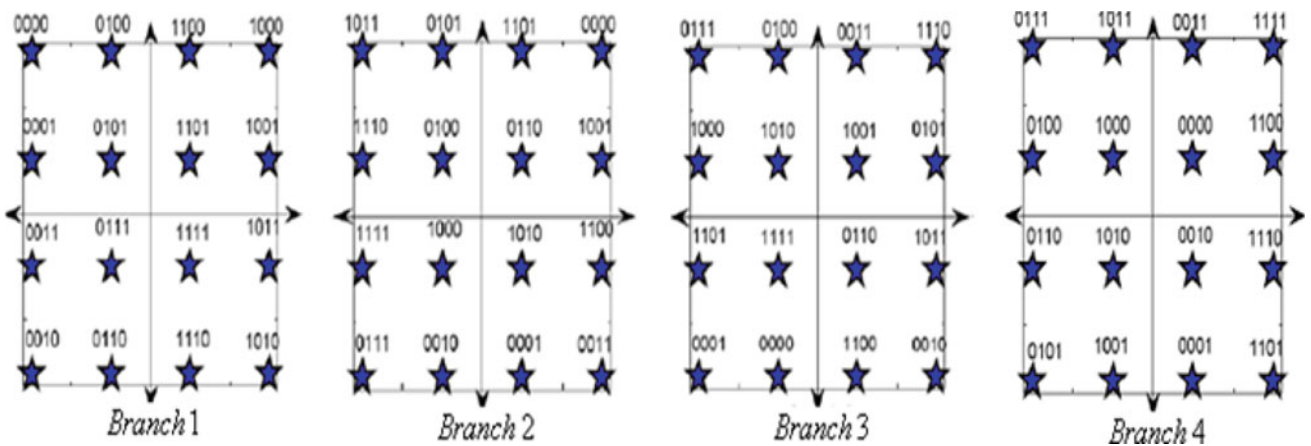


Fig. 8 Optimized constellation set for four 16-QAM branches

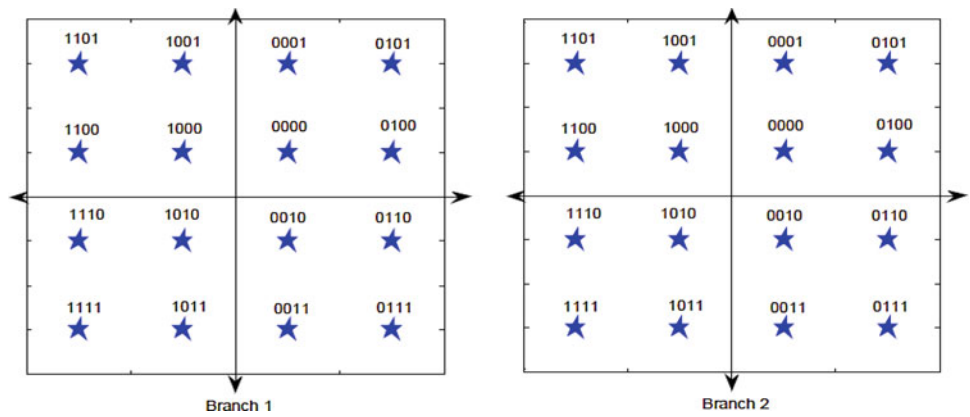
$$\text{time slot 1 : } \begin{cases} y_r = h_{sr} s_m + z_r \\ y_d^1 = h_{sd} s_m + z_d^1 \end{cases} \quad (11)$$

$$\text{time slot 2 : } y_d^2 = h_{rd} s_m + z_d^2 \quad (12)$$

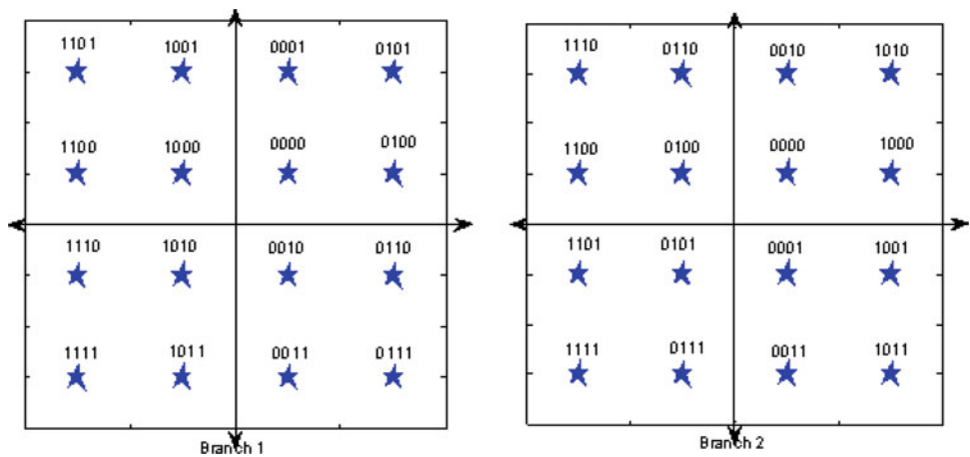
In general, the equation matrix is written as:

$$\begin{bmatrix} y_d^1 \\ y_d^2 \end{bmatrix} = \begin{bmatrix} h_{sd} & 0 \\ 0 & h_{rd} \end{bmatrix} \begin{bmatrix} s_m \\ s_m \end{bmatrix} + \begin{bmatrix} z_d^1 \\ z_d^2 \end{bmatrix} \quad (13)$$

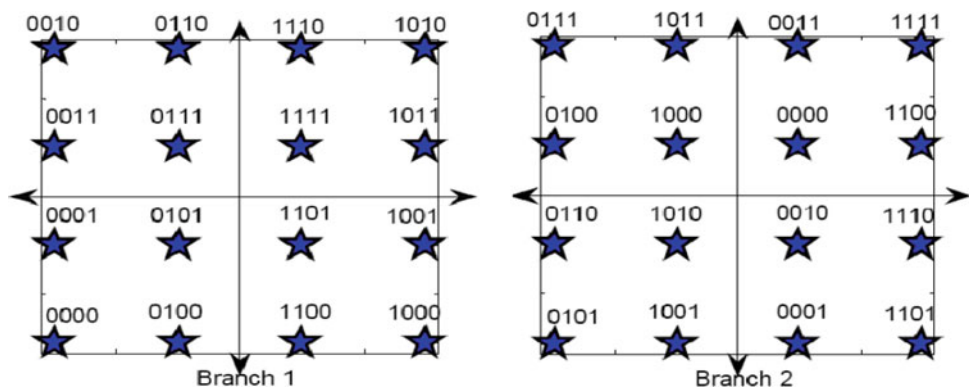
**Fig. 9** Usual constellation set for two 16-QAM branches



**Fig. 10** Constellation Rearrangement 1 set for two 16-QAM branches



**Fig. 11** Constellation Rearrangement 2 set for two 16-QAM branches



$$Y_d = H S_m + Z \tag{14}$$

Relaying Systems with CoRe is written as:

The receiver will use the received vector to detect symbols using maximum likelihood (ML) detection.

$$\text{time slot 1 : } \begin{cases} y_r = h_{sr} s_m^1 + z_r \\ y_d^1 = h_{sd} s_m^1 + z_d^1 \end{cases} \tag{16}$$

$$C = \min_{\hat{m}} (\|Y_d - H S_{\hat{m}}\|^2) \tag{15}$$

$$\text{time slot 2 : } y_d^2 = h_{rd} s_m^2 + z_d^2 \tag{17}$$

### 5.3 Cooperative Spatial Multiplexing SISO Relaying Systems with CoRe

In general, the equation matrix is written as:

The signal received in the first and second time slot at relay and destination in Cooperative spatial multiplexing SISO

$$\begin{bmatrix} y_d^1 \\ y_d^2 \end{bmatrix} = \begin{bmatrix} h_{sd} & 0 \\ 0 & h_{rd} \end{bmatrix} \begin{bmatrix} s_m^1 \\ s_m^2 \end{bmatrix} + \begin{bmatrix} z_d^1 \\ z_d^2 \end{bmatrix} \tag{18}$$

$$Y_d = H S_m + Z \tag{19}$$

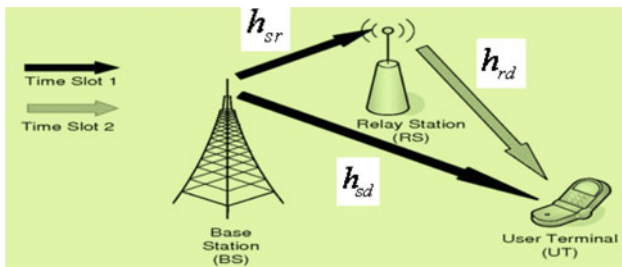


Fig. 12 Cooperative SISO Relaying System

The receiver will use the received vector to detect symbols using maximum likelihood detection.

$$C = \min_{\hat{m}} (\|Y_d - HS_{\hat{m}}\|^2) \tag{20}$$

### 6 Cooperative Spatial Multiplexing MIMO Relaying Systems

#### 6.1 Systems Model

The cooperative MIMO relaying system is shown in Fig. 14. Variable  $M, N, N_{re}$  denote the number of transmission antennas, number of receiver antennas and number of antennas in the relay station, respectively. The channel between source-relay  $H_{SR} \in C^{M \times N_{re}}$ , relay-destination  $H_{RD} \in C^{N_{re} \times N}$  and

source-destination  $H_{SD} \in C^{M \times N}$  is modeled by uncorrelated flat Rayleigh fading environment. Relays do not operate in full duplex mode because of hardware constraints. Instead, relays operate in half duplex mode, i.e. relay can either listen to its channel or can transmit data but cannot perform both functions at the same time.

#### 6.2 Cooperative Spatial Multiplexing MIMO Relaying Systems Without CoRe

Figure 15 illustrates the cooperative spatial multiplexing MIMO Relaying Systems with two transmission antennas ( $M=2$ ), two receiver antennas ( $N=2$ ) and two relay antennas ( $N_{re} = 2$ ). Performance of this system without Constellation Rearrangement is as follows:

The signal received in the first time slot at the relay and destination is written as:

$$\text{time slot 1 : } \begin{cases} Y_R = H_{SR} S_m + n_R \\ Y_D^{(1)} = H_{SD} S_m + n_D^{(1)} \end{cases}, \tag{21}$$

where  $H_{SR} \in C^{M \times N_{re}}$  and  $H_{SD} \in C^{M \times N}$  are the channel matrix. The noise vectors  $n_R$  and  $n_D^1$  represent noise vectors at the relay and destination and  $S_m$  is transmitted symbol vector.

Fig. 13 Relay system time slots



Fig. 14 Cooperative MIMO relaying system

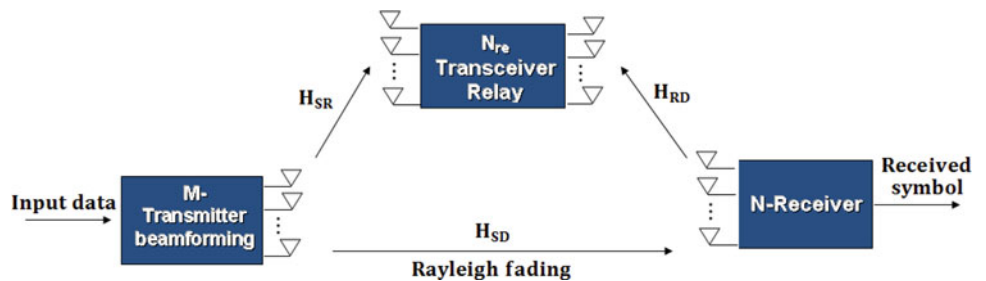
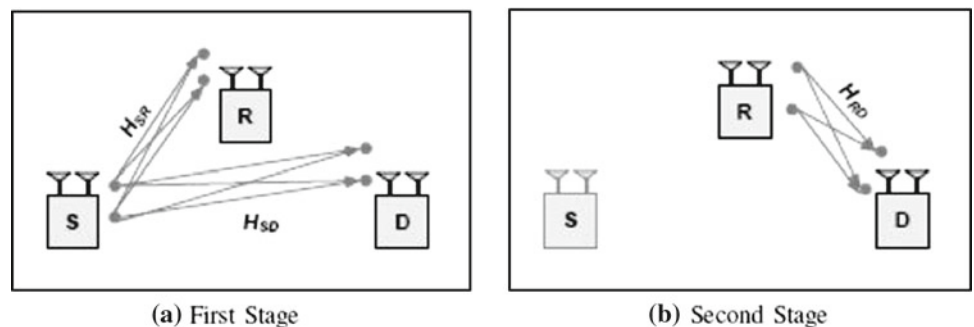


Fig. 15 MIMO Relay system time slots



The signal received in the second time slot at the destination is written as:

$$\text{time slot 2 : } Y_D^{(2)} = H_{RD} S_m + n_D^{(2)}, \tag{22}$$

where  $H_{RD} \in C^{N_{re} \times N}$  is the channel matrix between relay and destination.  $n_D^{(2)}$  is the corresponding noise vector at the destination.  $Y_D^{(2)}$  is received signal vector at the destination. At the receiver, the signals received from the relay and source, are combined together using MRC detection technique. We can write the above two equations in matrix form as below:

$$\begin{bmatrix} Y_D^{(1)} \\ Y_D^{(2)} \end{bmatrix} = \begin{bmatrix} H_{SD} & 0 \\ 0 & H_{RD} \end{bmatrix} \begin{bmatrix} S_m \\ S_m \end{bmatrix} + \begin{bmatrix} n_D^{(1)} \\ n_D^{(2)} \end{bmatrix} \tag{23}$$

where  $Y_D^{(1)} = \begin{bmatrix} y_d^{1(1)} \\ y_d^{2(1)} \end{bmatrix}$ ,  $Y_D^{(2)} = \begin{bmatrix} y_d^{1(2)} \\ y_d^{2(2)} \end{bmatrix}$ ,

$$n_D^{(1)} = \begin{bmatrix} n_d^{1(1)} \\ n_d^{2(1)} \end{bmatrix}, n_D^{(2)} = \begin{bmatrix} n_d^{1(2)} \\ n_d^{2(2)} \end{bmatrix}, S_m = \begin{bmatrix} s_m \\ s_m \end{bmatrix}$$

where  $Y_D^{(1)}$  and  $Y_D^{(2)}$  denote the received signal vectors in the first and second time slot at the destination, respectively.  $y_d^{1(1)}$ ,  $y_d^{2(1)}$  denote the signal received in the first time slot at the destination, in the first and second antenna, respectively.  $y_d^{1(2)}$ ,  $y_d^{2(2)}$  denote the signal received in the second time slot at the destination, in the first and second antenna, respectively.  $n_D^{(1)}$ ,  $n_D^{(2)}$ , denote the noise vectors in the first and second time slot at the destination, respectively.  $n_d^{1(1)}$ ,  $n_d^{2(1)}$  denote the noise received in the first time slot at the destination, in the first and second antenna, respectively.  $n_d^{1(2)}$ ,  $n_d^{2(2)}$ , denotes the noise received in the second time slot at the destination, in the first and second antenna, respectively.

The receiver will use the received vector to detect the symbols using maximum likelihood detection. As shown below:

$$C = \min_{\hat{m}} (\|Y_D - H S_{\hat{m}}\|^2) \tag{24}$$

where  $Y_D = \begin{bmatrix} Y_D^{(1)} \\ Y_D^{(2)} \end{bmatrix}$  and  $H = \begin{bmatrix} H_{SD} & 0 \\ 0 & H_{RD} \end{bmatrix}$

### 6.3 Cooperative Spatial Multiplexing MIMO Relaying Systems with CoRe

The performance of this system as shown in Fig. 15 with Constellation Rearrangement is as follows:

In the first time slot, the source performs constellation rearrangement and then transmits data towards the destination and relays. In the second time slot, the relay detects the received signal and applies the optimized constellation arrangement scheme.

In the same time slot at the relay, if data are detected correctly, then it is transmitted towards the destination otherwise relays remain idle. Slow flat Rayleigh fading environment is

considered, so that the user experiences the same channel conditions in two consecutive time slots. The signal received in the first time slot at the relay and destination is written as:

$$\text{Time Slot 1 : } \begin{cases} Y_R = H_{SR} S_m^{(1)} + n_R \\ Y_D^{(1)} = H_{SD} S_m^{(1)} + n_D^{(1)} \end{cases} \tag{25}$$

The signal received in the second time slot at the destination is written as:

$$\text{Time Slot 2 : } Y_D^{(2)} = H_{RD} S_m^{(2)} + n_D^{(2)} \tag{26}$$

$$\begin{bmatrix} Y_D^{(1)} \\ Y_D^{(2)} \end{bmatrix} = \begin{bmatrix} H_{SD} & 0 \\ 0 & H_{RD} \end{bmatrix} \begin{bmatrix} S_m^{(1)} \\ S_m^{(2)} \end{bmatrix} + \begin{bmatrix} n_D^{(1)} \\ n_D^{(2)} \end{bmatrix} \tag{27}$$

where  $S_m^{(1)} = \begin{bmatrix} s_m^1 \\ s_m^1 \end{bmatrix}$ ,  $S_m^{(2)} = \begin{bmatrix} s_m^2 \\ s_m^2 \end{bmatrix}$ ,

where  $S_m^{(1)}$ ,  $S_m^{(2)}$  denote transmitted symbol vector in transmitter and relay when using constellation rearrangement, respectively.

The receiver will use the received vector to detect the symbols using maximum likelihood detection.

$$C = \min_{\hat{m}} (\|Y_D - H S_{\hat{m}}\|^2) \tag{28}$$

## 7 Numerical Results

To verify the conclusions made by observing the augmented signal constellation, we examine the average SER results for different schemes.

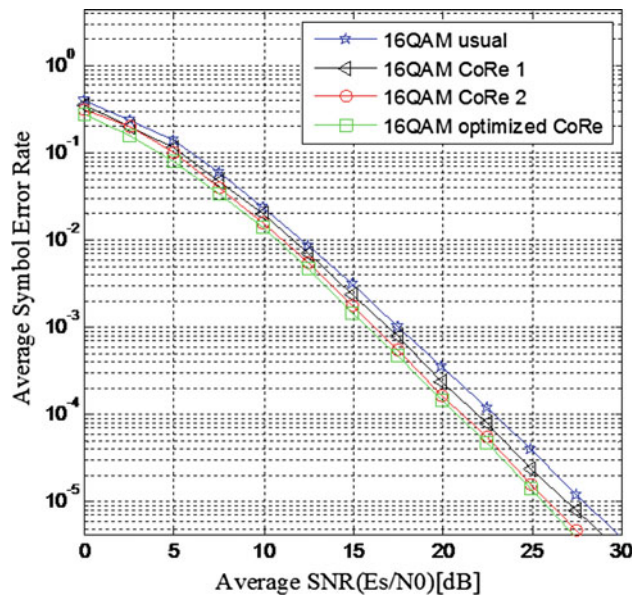
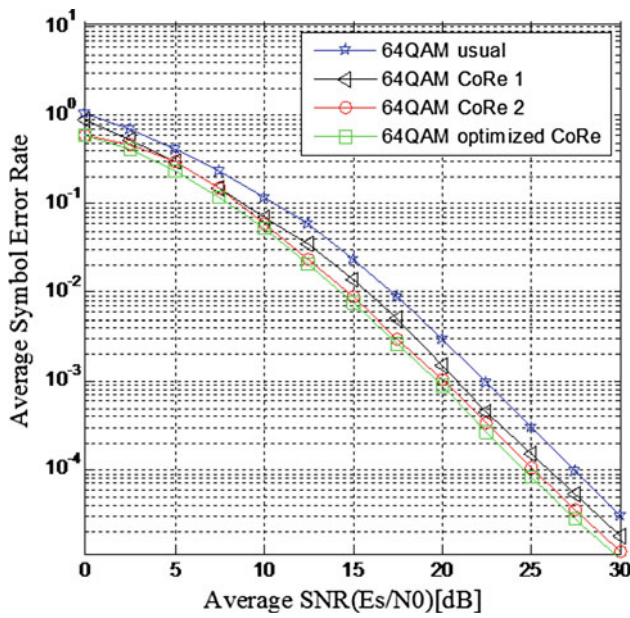


Fig. 16 Average SER performance of different CoRe schemes in fixed relay network using 16-QAM



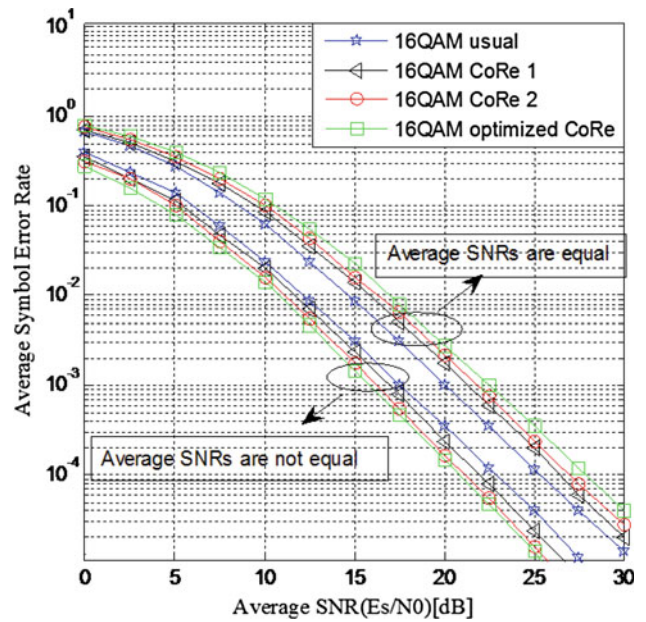


**Fig. 17** Average SER performance of different CoRe schemes in fixed relay network using 64-QAM

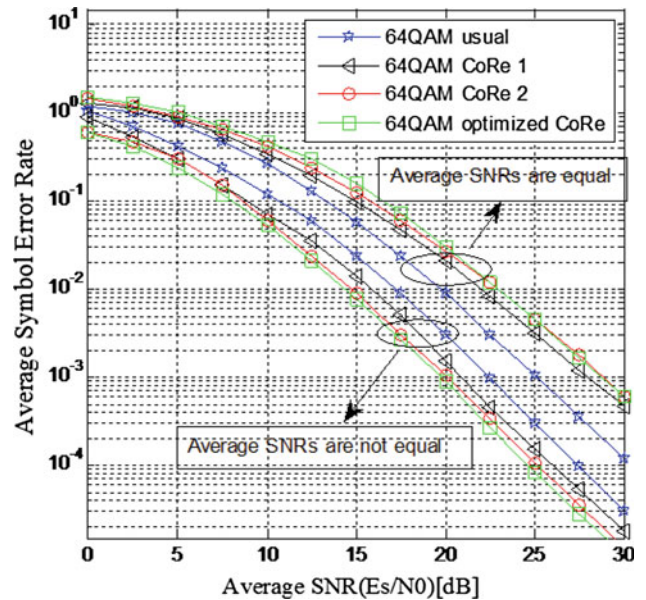
In Figs. 16 and 17, we plot the SER curve for a single fixed relay network with equal average SNR of the link between BS–UT and average SNR of the link RS–UT for different CoRe schemes using 16- and 64-QAM, respectively. In both figures, although all schemes achieve the same diversity order of 2, the optimized CoRe achieves the highest SNR gain. For the 16-QAM case, at an average SER of  $10^{-3}$ , the optimized CoRe achieves gains of 2, 1.25, and 0.5 dB over the usual CoRe, CoRe 1 and CoRe2 schemes, respectively. For the 64-QAM case, at an average SER of  $10^{-3}$ , the optimized CoRe achieves gains of 3, 1.5 and 0.5 over the usual CoRe, CoRe 1 and CoRe2 schemes, respectively. These are significant gains, considering the simple processing involved to achieve them.

Then, we consider another case where the network has a single mobile relay. In Figs. 18 and 19, the average SER curve for different CoRe schemes is plotted using 16- and 64-QAM, respectively. In both figures, we consider cases when the average SNRs between these links (BS–RS, BS–UT, RS–UT) are equal to  $(i, i, i)$  and  $(i + 25 \text{ dB}, i, i)$  where  $i$  is a value for SNR.

For both the 16- and 64-QAM, it is seen that the performance of all CoRe schemes is worse than the usual scheme when the average SNRs are  $(i, i, i)$ . Indeed, the optimized CoRe has the worst performance, since it does not assume Gary-coding in the transmission made by the BS, which amplifies the effect of the error propagation. However, the situation is reversed for the case when the average SNRs are  $(i + 25 \text{ dB}, i, i)$ . In this case, the BS–RS link is reliable enough to minimize the effect of error propagation. Never-



**Fig. 18** Average SER performance of different CoRe schemes in mobile relay network, using 16-QAM



**Fig. 19** Average SER performance of different CoRe schemes in mobile relay network, using 64-QAM

theless, the gain achieved by the optimized CoRe for the case of mobile relay is less than the case of fixed relay. For the 16-QAM case, at an average SER of  $10^{-3}$ , the optimized CoRe achieves gains of 1.6, 1 and 0.35 over the usual CoRe, CoRe 1, and CoRe2 schemes, respectively. For the 64-QAM case, at an average SER of  $10^{-3}$ , the optimized CoRe achieves gains of 2.5, 1.1, and 0.35 over the usual CoRe, CoRe 1, and CoRe2 schemes, respectively.

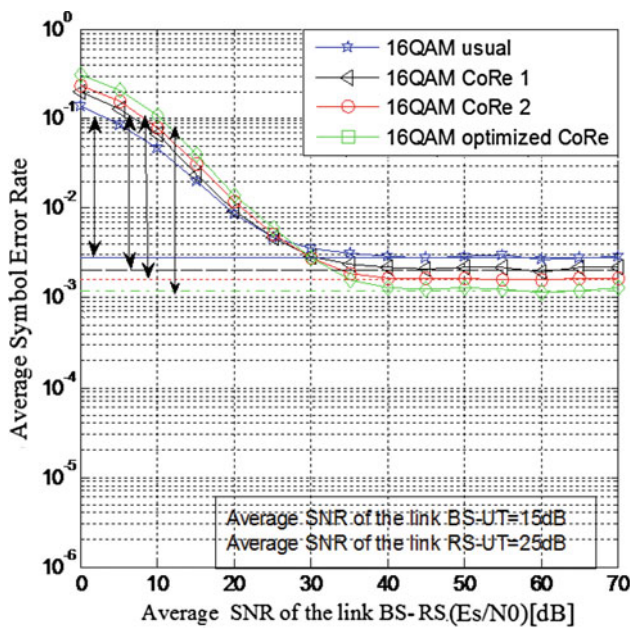


Fig. 20 Average SER performance of different CoRe schemes in mobile relay network as a function of average SNR of the link between BS–RS, using 16-QAM

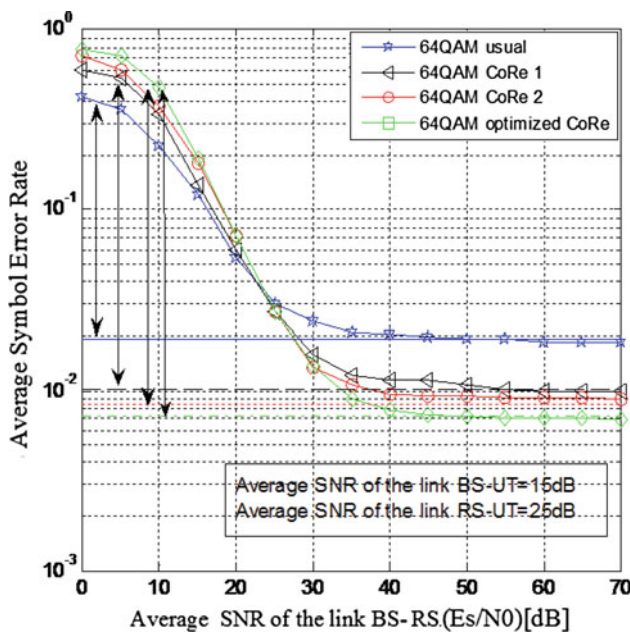


Fig. 21 Average SER performance of different CoRe schemes in mobile relay network as a function of average SNR of the link between BS–RS, using 64-QAM

To have a closer look at the effect of error propagation on the average SER for the different CoRe schemes, the average SER is plotted as a function of the average SNR of the link between BS and RS while fixing average SNR of the link between BS–UT and average SNR of the link between RS–UT equal to 15 and 25 dB, respectively. This is shown

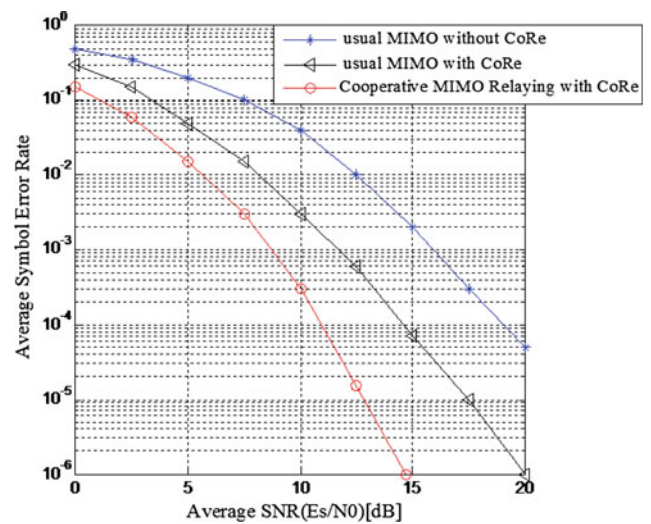


Fig. 22 Average symbol error rate for cooperative  $2 \times 2$  MIMO relay with 16-QAM

in Figs. 20 and 21 for the case of 16- and 64-QAM, respectively. In both figures, it is seen that the optimized CoRe is the most sensitive scheme to error propagation as it suffers from the highest degradation in average SER, compared to the fixed relay case. More importantly, in order for any CoRe scheme to have better performance than the usual scheme, average SNR of the link between BS–RS must be greater than a threshold value. For the 16-QAM case, the threshold values are 25, 26.5, and 27 for CoRe 1, CoRe 2, and the optimized CoRe, respectively. For the 64-QAM case, the threshold values are 21, 22.7, and 24 for CoRe 1, CoRe 2, and the optimized CoRe, respectively. The same observations were made for different values of average SNR of the link between BS–UT and average SNR of the link between RS–UT. However, the threshold values were different which hints that the threshold is a function of both average SNR of the link between BS–UT and average SNR of the link between RS–UT.

In Figs. 22 and 23, comparison between cooperative  $2 \times 2$  MIMO communication system with constellation rearrangement as shown in Fig. 15 and  $2 \times 2$  MIMO communication system without relay without constellation rearrangement and  $2 \times 2$  MIMO communication system without relay with constellation rearrangement is done for different modulation schemes using 16- and 64-QAM. Simulation results show that for the 16-QAM case, at an average SER of  $10^{-4}$ , the cooperative MIMO communication system with constellation rearrangement achieves gains of 3.8 more than  $2 \times 2$  MIMO communication system without relay with constellation rearrangement. Also for the 64-QAM case, at an average SER of  $10^{-4}$ , the cooperative  $2 \times 2$  MIMO communication system with constellation rearrangement achieves gains of 3.5 dB more than  $2 \times 2$  MIMO communication system without relay with constellation rearrangement.

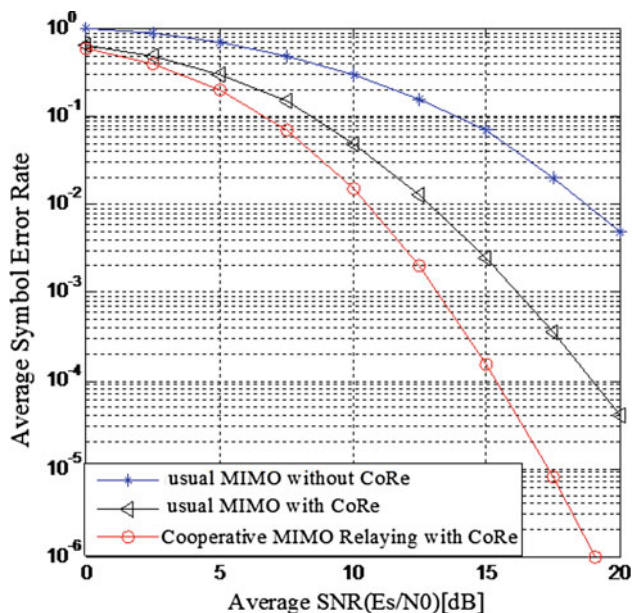


Fig. 23 Average symbol error rate for cooperative  $2 \times 2$  MIMO relay with 64-QAM

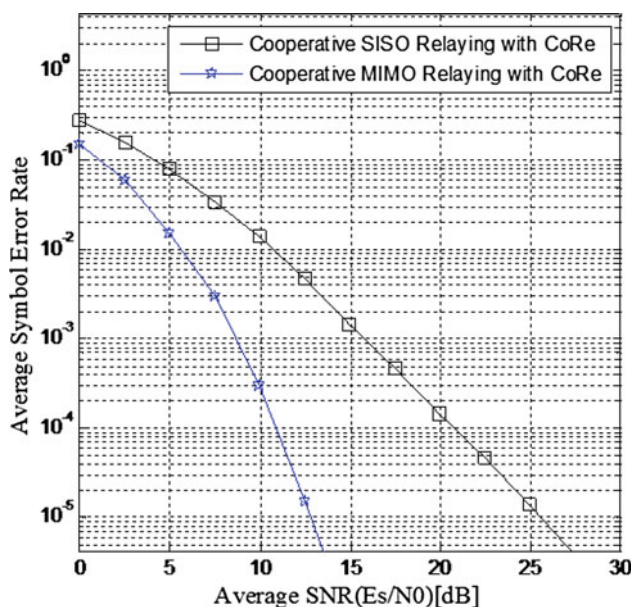


Fig. 24 Average symbol error rate for cooperative relaying system with 16-QAM

In case of 16- and 64-QAM as shown in Figs. 22 and 23 for a cooperative  $2 \times 2$  MIMO communication system with constellation rearrangement, we get a gain of 12.75 dB at average SER of  $10^{-5}$  and 17.25 dB at average BER of  $10^{-5}$ , respectively.

In Figs. 24 and 25, comparison between cooperative  $2 \times 2$  MIMO communication system with constellation rearrangement and cooperative SISO communication sys-

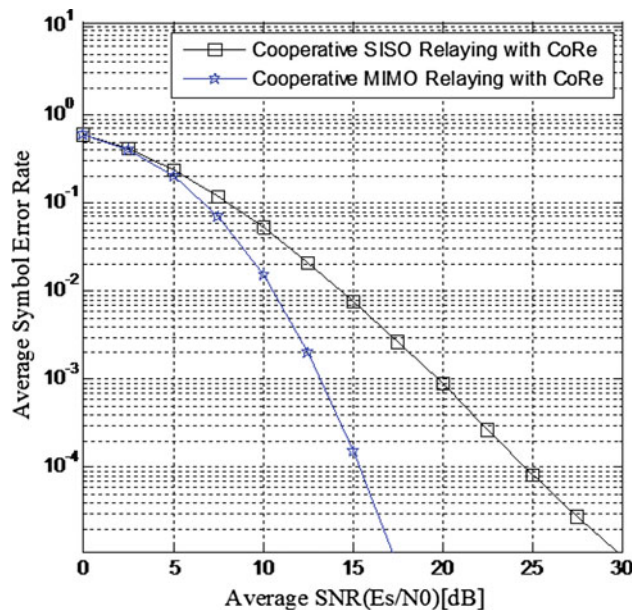


Fig. 25 Average symbol error rate for cooperative relaying system with 64-QAM

tems with constellation rearrangement for different modulation schemes using 16- and 64-QAM is done.

### 8 Conclusions

Constellation rearrangement technique can be used in digital communication systems to decrease SER in spite of slight increase in computational complexity of system. We combined CoRe technique with cooperative spatial multiplexing systems. Simulation results clearly showed improvements of the proposed method in MIMO communication system without relay and MIMO communication system without relay with constellation rearrangement in terms of SER. Finding the best or optimum constellation in the CoRe scheme is now through extensive search method. There is no other search method to find the optimum constellation with low complexity. This issue will be our future research area.

In this paper, we study performance of fixed and mobile relays in Cooperative spatial multiplexing SISO Relaying networks with different constellation rearrangement and different modulation schemes. In the case of fixed relay, we observed that optimize constellation rearrangement has better performance rather than other constellation rearrangements. Also, when the level of modulation increases, the optimized constellation rearrangement would be better. However, in the case of mobile relay, the performance of CoRe could be blamed on the errors made at the RS propagate to the destination.

To have a closer look at the effect of error propagation on the average SER for the different CoRe schemes, the average

SER is plotted as a function of the average SNR of the link BS–RS while fixing average SNR of the link BS–UT and average SNR of the link between RS–UT equal to 15 and 25 dB, respectively.

It is seen that the optimized CoRe is the most sensitive scheme to error propagation as it suffers from the highest degradation in terms of average SER, compared to the fixed relay case. More importantly, for any CoRe scheme to have better performance than the usual scheme, average SNR of the link BS–RS must be greater than the threshold value. Threshold values for the 64-QAM case were lower than 16-QAM case for any CoRe scheme. The same observations were made for different values of the average SNR of the link BS–UT and average SNR of the link RS–UT. However, the threshold values were different which hints that the threshold is a function of both average SNR of the link BS–UT and average SNR of the link RS–UT.

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