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CORRECTION

Correction to: Multi-twisted additive codes over finite fields

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The purpose of this note is to rectify an error in equation (3.10), which provides the value of the Witt index m_v of the non-degenerate quadratic space $(\mathcal{G}_v, \mathcal{Q}_v)$ over $\mathcal{F}_v$ when q is odd, $v \in \mathcal{J}_1$, $\epsilon_v t$ is even and $\delta \in \{0, *\}$, (see the proof of Lemma 3.5(a) on page 311). In this case, the correct value of the Witt index m_v is given by

$$m_v = \begin{cases} \epsilon_v t/2 & \text{if } \epsilon_v t \equiv 2 \pmod 4 \text{ and } q \equiv 3 \pmod 4; \\ (\epsilon_v t - 2)/2 & \text{if either } \epsilon_v t \text{ is even and } q \equiv 1 \pmod 4 \\ & \text{or } \epsilon_v t \equiv 0 \pmod 4 \text{ and } q \equiv 3 \pmod 4), \end{cases} \tag{3.10}$$

(see page 279 of Huffman (2010) for more details). That is, in this case, the value of the Witt index m_v specified in equation (3.10) of the published version should be flipped, as Theorem 1 of Pless (1968) is not applicable in this case. This error in the value of m_v gives rise to flipping errors in the values of the numbers $\mathfrak{M}_v$ and $\mathfrak{N}_v$ obtained respectively in Lemma 3.5(a) (and hence in the statement of Theorem 3.2) and Lemma 3.8 (and hence in the statement of Theorem 3.3) when q is odd, $v \in \mathcal{J}_1$, $\epsilon_v t$ is even and $\delta \in \{0, *\}$. The values of the numbers $\mathfrak{M}_v$ and $\mathfrak{N}_v$ are correct in the remaining cases. Besides this, this error in m_v gives rise to minor changes in the statement and the proof of Lemma 3.7. The rest of the numbers and results obtained in this paper are correct. All proof techniques are also correct.

Below we list all the changes that will rectify these errors:

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1. Page 309: In the statement of Theorem 3.2, when $v \in \mathcal{J}_1$, q is odd, $\delta \in \{0, *\}$ and $\epsilon_v t$ is even, the value of $\mathfrak{M}_v$ should be replaced by

$$\mathfrak{M}_v = \begin{cases} \sum_{k=0}^{\epsilon_v t/2} \begin{bmatrix} \epsilon_v t/2 \\ k \end{bmatrix}_q \prod_{d=0}^{k-1} \left(q^{\frac{\epsilon_v t - 2d - 2}{2}} + 1 \right) & \text{if } \epsilon_v t \equiv 2 \pmod 4 \text{ and } q \equiv 3 \pmod 4; \\ \sum_{k=0}^{(\epsilon_v t-2)/2} \begin{bmatrix} (\epsilon_v t-2)/2 \\ k \end{bmatrix}_q \prod_{d=0}^{k-1} \left(q^{\frac{\epsilon_v t - 2d}{2}} + 1 \right) & \text{if either } \epsilon_v t \text{ is even and } q \equiv 1 \pmod 4 \\ & \text{or } \epsilon_v t \equiv 0 \pmod 4 \text{ and } q \equiv 3 \pmod 4. \end{cases}$$

In the remaining cases, the values of $\mathfrak{M}_v$ will remain the same as in the original published version.
2. Page 310: In the statement of Lemma 3.5(a), when $v \in \mathcal{J}_1$, q is odd, $\delta \in \{0, *\}$ and $\epsilon_v t$ is even, the value of $\mathfrak{M}_v$ should be replaced by

$$\mathfrak{M}_v = \begin{cases} \sum_{k=0}^{\epsilon_v t/2} \begin{bmatrix} \epsilon_v t/2 \\ k \end{bmatrix}_q \prod_{d=0}^{k-1} \left(q^{\frac{\epsilon_v t - 2d - 2}{2}} + 1 \right) & \text{if } \epsilon_v t \equiv 2 \pmod 4 \text{ and } q \equiv 3 \pmod 4; \\ \sum_{k=0}^{(\epsilon_v t-2)/2} \begin{bmatrix} (\epsilon_v t-2)/2 \\ k \end{bmatrix}_q \prod_{d=0}^{k-1} \left(q^{\frac{\epsilon_v t - 2d}{2}} + 1 \right) & \text{if either } \epsilon_v t \text{ is even and } q \equiv 1 \pmod 4 \\ & \text{or } \epsilon_v t \equiv 0 \pmod 4 \text{ and } q \equiv 3 \pmod 4; \end{cases}$$

In the remaining cases, the values of $\mathfrak{M}_v$ will remain the same as in the original published version.
3. Page 310, the last line: In the proof of case I (i.e., when q is odd) of Lemma 3.5(a), "Further, by Theorem 1 of Pless (1968), we note that …" should be replaced by "Further, by Huffman (2010, p. 279), we note that …".
4. Page 311: Equation (3.10) should be replaced by the following:

$$m_v = \begin{cases} (\epsilon_v t - 1)/2 & \text{if } \epsilon_v t \text{ is odd;} \\ \epsilon_v t/2 & \text{if } \epsilon_v t \equiv 2 \pmod 4 \text{ and } q \equiv 3 \pmod 4; \\ (\epsilon_v t - 2)/2 & \text{if either } \epsilon_v t \text{ is even and } q \equiv 1 \pmod 4 \\ & \text{or } \epsilon_v t \equiv 0 \pmod 4 \text{ and } q \equiv 3 \pmod 4. \end{cases} \tag{3.10}$$

When $v \in \mathcal{J}_1$, $\delta \in \{0, *\}$, $\epsilon_v t$ is even, and q is odd, we see that $m_v = \epsilon_v t/2$ if and only if $\epsilon_v t \equiv 2 \pmod 4$ and $q \equiv 3 \pmod 4$, which holds if and only if $(-1)^{\epsilon_v t/2}$ is a non-square in $\mathbb{F}_q$.
5. Page 315, line 3: In the statement of Theorem 3.3, "…when $\delta \in \{0, *\}$, $(-1)^{\epsilon_v t/2}$ is a square in $\mathbb{F}_q$ for each $v \in \mathcal{J}_1$" should be replaced by "…when $\delta \in \{0, *\}$ and q is odd, the element $(-1)^{\epsilon_v t/2}$ is a non-square in $\mathbb{F}_q$ for each $v \in \mathcal{J}_1$".
6. Page 315, lines 8-9: In the statement of Theorem 3.3, when $v \in \mathcal{J}_1$, q is odd and $\delta \in \{0, *\}$, we assumed that $(-1)^{\epsilon_v t/2}$ is a non-square in $\mathbb{F}_q$, which holds if and only if $\epsilon_v t \equiv 2 \pmod 4$ and $q \equiv 3 \pmod 4$. In this case, the value of $\mathfrak{N}_v$ should be replaced by the following:

$$\mathfrak{N}_v = \prod_{a=0}^{(\epsilon_v t/2)-1} \left(q^{\frac{\epsilon_v t - 2a - 2}{2}} + 1 \right) \text{ when } \delta \in \{0, *\}, \epsilon_v t \equiv 2 \pmod 4 \text{ and } q \equiv 3 \pmod 4.$$

In the remaining cases, the values of $\mathfrak{N}_v$ will remain the same as in the original published version.

7. Page 315: In the statement of Lemma 3.7, "(ii) the element $(-1)^{\epsilon_v t/2}$ is a square in $\mathbb{F}_q$ when $\delta \in \{0, *\}$ and $v \in \mathcal{J}_1$" should be replaced by "(ii) the element $(-1)^{\epsilon_v t/2}$ is a non-square in $\mathbb{F}_q$ when $\delta \in \{0, *\}$, q is odd and $v \in \mathcal{J}_1$".
8. Page 315: The third paragraph in the proof of Lemma 3.7 should be replaced by the following:
"When $v \in \mathcal{J}_1$, q is odd and $\delta \in \{0, *\}$, by Lemma 3.4(a), we note that $(\mathcal{G}_v, [\cdot, \cdot]_\delta \restriction_{\mathcal{G}_v \times \mathcal{G}_v})$ is an orthogonal space over $\mathcal{F}_v$. Since $\epsilon_v t$ is even, we see, by (3.10), that the Witt index of $(\mathcal{G}_v, [\cdot, \cdot]_\delta \restriction_{\mathcal{G}_v \times \mathcal{G}_v})$ is $\epsilon_v t/2$ if and only if and $(-1)^{\epsilon_v t/2}$ is a non-square in $\mathbb{F}_q$."
9. Page 315, line 33 (the last paragraph): "…in the case when $\delta \in \{0, *\}$, $(-1)^{\epsilon_v t/2}$ is a square in $\mathbb{F}_q$ for each $v \in \mathcal{J}_1$" should be replaced by "…in the case when $\delta \in \{0, *\}$ and q is odd, $(-1)^{\epsilon_v t/2}$ is a non-square in $\mathbb{F}_q$ for each $v \in \mathcal{J}_1$".
10. Page 316: In the statement of Lemma 3.8, "…that $(-1)^{\epsilon_v t/2}$ is a square in $\mathbb{F}_q$ when $\delta \in \{0, *\}$ and $v \in \mathcal{J}_1$" should be replaced by "…that $(-1)^{\epsilon_v t/2}$ is a non-square in $\mathbb{F}_q$ when $\delta \in \{0, *\}$, q is odd and $v \in \mathcal{J}_1$".
11. Page 316, line 5: In the statement of Lemma 3.8, when $v \in \mathcal{J}_1$, q is odd and $\delta \in \{0, *\}$, we have assumed that $(-1)^{\epsilon_v t/2}$ is a non-square in $\mathbb{F}_q$, which holds if and only if $\epsilon_v t \equiv 2 \pmod 4$ and $q \equiv 3 \pmod 4$. In this case, the value of $\mathfrak{N}_v$ should be replaced by

$$\mathfrak{N}_v = \prod_{a=0}^{(\epsilon_v t/2)-1} \left(q^{\frac{\epsilon_v t - 2a - 2}{2}} + 1 \right) \text{ if } \delta \in \{0, *\}, \epsilon_v t \equiv 2 \pmod 4 \text{ and } q \equiv 3 \pmod 4.$$

In the remaining cases, the values of $\mathfrak{N}_v$ will remain the same as in the original published version.

12. Page 316, line 14: In the proof of Lemma 3.8(a), "From this point on, let $\delta \in \{0, *\}$. Here $(-1)^{\epsilon_v t/2}$ is a square in $\mathbb{F}_q$." should be replaced by "From this point on, let $\delta \in \{0, *\}$. Here when q is odd, we know that $(-1)^{\epsilon_v t/2}$ is a non-square in $\mathbb{F}_q$."
13. Page 316, line 21: In the proof of case I of Lemma 3.8(a), "Further, by Theorem 1 of Pless (1968), we note that …" should be replaced by "Further, by (3.10), we note that …".