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ORIGINAL PAPER

A generalized flexibility matrix-based model updating method for damage detection of plane truss and frame structures

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Abstract

Nowadays, many non-destructive damage detection methods for determining the location and severity of damage in the field of health monitoring are considered in order to reduce the cost of maintenance and improve safety and reliability of structure. In this paper, damage specification is obtained by sensitivity-based updating approach. By applying changes on sensitivity matrix and using measured flexibility data, it is concluded that the results of proposed method are more accurate and efficient than the old modal flexibility methods. The mass modeling error and measurement error of flexibility and natural frequency are calculated in order to ensure the accuracy and robustness of proposed method for 2-D finite element truss and frame model. Close index, measuring the performance of the method, and the coefficient of variation, which represents the distribution of response, are used. Compared with Wang method, the proposed method is capable of accurately localizing and quantifying damage in all scenarios.

Keywords Modal flexibility sensitivity · Damage detection · Vibration · Finite element model

1 Introduction

Various structures, such as bridges, dams, and high-rise buildings, have a long-term period of operation. During this period, damage caused by environmental effects impairs the vibration characteristics of the structures, which threatens the useful life of structures. Detecting these damages on the threshold of their formation is one of the most important requirements for optimal maintenance of engineering structures. For this reason, an accurate detection method to determine the damage becomes essential.

These vibration characteristics which are generally measured modal parameters, such as frequencies and mode shapes, are dependent on the physical properties of the structure. Hence, changes in the physical properties, such as stiffness reduction, will cause considerable changes in these modal properties.

Over the past decades, the interest for the issue of health monitoring has increased greatly. Numerous researchers [1–3] have exhibited the precise information of damage detection methodologies in the literature. Law et al. proposed a parallel decentralized damage detection method in which the structure is divided into small zones. Dynamic tests are performed for each zone and the derived responses from the local sensors regarding each zone are used for damage detection. The Newton successive-over relaxation (SOR) is used for updating structural parameters. Using this method, the computational time showed a considerable decline [4].

According to their assertions, damage detection methods typically consist of two major components: static data and vibration-based methods.

The methods based on static data can be used to measure the strain and displacement, while the dynamic methods which one would choose in accordance with the particular dynamic characteristics one was dealing with are the following: natural frequency, mode shape, mode shape curvature, modal strain energy, and flexibility matrix methods.

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It can be stated that the static tests are more accurate than dynamic ones; however, the sensitivity of static methods to the change in structural parameters is low. The static methods only require the stiffness properties, whereas the dynamic methods require the use of mass, stiffness and damping properties.

The measured response is more exact in static methods than the dynamics; moreover, by considering the measurement error, the result of static methods is more dependable than the structural response measured in modal testing.

Static methods have been widely used by many researchers in the field of structural health monitoring. Excellent primary studies of the issue have been done by Gudmundson [5], Sanaeyi et al. [6] and Wang et al. [7]. Bakhtiari-nejad et al. [8] represented a damage detection method based on static test data. By solving non-linear simultaneous equations, the difference between the load vector of the damaged and the intact structure was minimized. Esfandiari et al. [9] proposed a sensitivity-based finite element model updating algorithm to detect changes in stiffness and mass parameters of a structure using strain data applied to a plane truss and a plane frame structure. Abdo [10] presented an analytical study of damage detection, which was carried out based on using changes in displacement curvatures, derived from only a static response, for an overhanging beam and a two-span continuous beam. Ni and Law introduced an approach in which the collected responses from different measurement setups are used and analyzed together. Local damages are directly analyzed by the Pattern Search method and parallel computing strategy. Furthermore, the effects of using a large Generating Matrix on the results and accuracy are evaluated. It is shown that using this method, damage detection of a large-scale structure with short-term tests is viable with using a few sensors [11]. Seyedpoor [12] propounded a crack localization method via an efficient static data-based indicator. According to this study, the Static responses of an Euler–Bernoulli beam were obtained by the finite element modeling. Sanayei et al. [13] described a method for the finite element model updating, based on the load cases and measuring locations of the nondestructive tests. Using Monte Carlo simulation, simultaneous estimation of the stiffness and mass parameters became possible using experimental data.

Another strategy for damage detection is known as dynamic methods. Lin [14] observed that higher modes of the structure have a great proportion in the stiffness matrix rather than lower modes. For this reason, a proper estimation of the stiffness matrix and its variations for detecting the damage is required to measure all, especially higher modes of the structure. Furthermore, in the structural modal test, measurement of higher modes is far more difficult than lower frequency modes. To deal with this practical problem, a series of new methods for detecting damage were introduced, based on flexibility matrix used to estimate the change in the stiffness matrix.

The flexibility matrix is the inverse of the global stiffness matrix of a structure and it can be classified into two categories: the static flexibility and dynamic flexibility. The static flexibility is obtained from a unit force applied to the structure and the other one is derived from the measured modal data. The modal approximation of the flexibility matrix is called the modal flexibility matrix, which can be accurately estimated by the lower modes of a structure [15]. It should be noted that the innovative approach implemented in this study is the parallel use of such two categories.

Many researchers have found modal flexibility parameter alone more sensitive to detecting damages than natural frequencies and mode shapes. Zhao et al. [16] performed a theoretical study, comparing the use of natural frequencies, mode shapes and modal flexibility in structural health monitoring. The result showed that the modal flexibility has been more successful in locating damages. Pandey et al. [17] presented a numerically and experimentally damage-detection method based on changes in the measured flexibility of the structure. The method was not precise in recognizing the case which multiple locations of the structure were damaged. Hence, Yan et al. [18] proposed a damage indicator based on Axial Strain flexibility for a Truss and a five-story steel frame. This method localized multiple damages to the exact members and it was suitable for the cases which baseline data of the intact structure was not available.

Kim [19] proposed a new nondestructive damage evaluation method for a slender beam under an axial force by utilizing dynamically measured modal flexibility.

Among most investigative theories in terms of modal flexibility-based damage detection problems, the vital matter is how to calculate the changes in modal flexibility due to the structural parameters namely sensitivity of modal flexibility. Over the past years, many methods have been suggested to evaluate the sensitivity of flexibility. Li et al. [20] propounded a new approach based on changes in the generalized flexibility matrix used to detect structural damage location and the intensity of damage for a numerical example of a simply supported beam. In such method, the effect of truncating higher-order modes can be substantially reduced. Zhao et al. [21] proposed a method by adding known masses to the structure and then using these new data as well as the original test information, in order that structural damages can be detected using the generalized flexibility perturbation technique. The closed form of modal flexibility sensitivity based on the algebraic Eigen sensitivity method in order to detect the location and extent of structural damages has also been presented by Yan and Ren [22]. The advantage of the method is to eliminate the negative effect of the truncating higher-order modes. Moreover, it could recognize results with sufficient accuracy by employing one or multiple modes and also it could overcome the operational mode shape normalization problems by employing the scaling factors. The sensitive equation of this method is more accurate in terms of mathematical formulation than Yan's method.

Considering the pros and cons of using flexibility methods, damage detection based on this method is applied. In this research, damage identification technique is handled based on a combination of two aforementioned categories. By way of clarification, such technique is based on the concurrent implementation of both static and dynamic flexibilities. In other words, the suggested method named modified modal flexibility is presented, utilizing measured static flexibility data and changing in the sensitivity of modal flexibility matrix.

A sensitivity-based finite element model updating technique using flexibility data is presented. By this approach, unknown structural parameters should be updated to minimize the difference between the measured data and the analytical model in order to detect the damage location and severity in the structural elements afterward. The damage is considered based on linear stiffness reduction. The equation of motion is linear with no damping consideration. Therefore, the stiffness is reduced by the decrease in Young's modulus in percent. The main idea of the proposed method is the direct use of measured frequency of damaged structure which changes in modal flexibility sensitivity matrix. In other words, the derivative of frequency which applies in one part of modal flexibility sensitivity equation and in the residual static mode of modal derivatives is replaced with the measured frequency of damaged structure. Flexibility method owing to the quiddity or content of the formula is more sensitive to low-frequency modes. Hence, due to practical drawbacks of measuring all modes, only first eight modes of the structure are calculated. By applying changes and comparing to the previous methods, the proposed method shows an appropriate performance in detecting the location and severity of damage. It is demonstrated that the updated results derived from the proposed method are robust against incompleteness, noise polluted data, and measurement and mass modeling error. Besides, by increasing the noise of proposed method, the results still converge and show the best estimation of damage in structure. In addition to considering noise in the proposed method, the sensitivity of results to the intensity of noise is detected in a way that by adjusting noise rate, the damage detection via the aforementioned technique and the results' convergence are assessed.

2 Theory

2.1 Modal flexibility sensitivity

For a linear, undamped structural system, the equation of motion is given by

$$[M]\{\ddot{u}(t)\} + [K]\{u(t)\} = \{P(t)\}, \tag{1}$$

where $[K]$ and $[M]$ are the global stiffness and mass matrices of the intact structure, respectively. $\{P(t)\}$ and $\{u(t)\}$ are the vectors of applied forces and responses. The steady-state harmonic response of the Eq. (1) to the harmonic excitation according to dynamic of structures sources is

$$u = (-\omega^2[M] + [K])^{-1}P = G(\omega)P, \tag{2}$$

where the $G(\omega)$ is called dynamic flexibility matrix. It is a dynamic generalization of the static flexibility matrix, $G(0) = K^{-1}$ (ω, excitation frequency, is equal to zero) [23]. By multiplying both sides by eigenvector and its transpose, the Eq. (2) yields:

$$\varphi^{\mathrm{T}} G^{-1} \varphi = \varphi^{\mathrm{T}} [K] \varphi. \tag{3}$$

For a multi-degree of freedom structure, if the mode shape vector $\{\varphi_r\}$ is normalized, $\{\varphi_r\}[M]\{\varphi_r\} = 1$ the flexibility matrix $[F]$ can be expressed for (the) first few low-frequency modes (N) as:

$$[K]^{-1} = [F] = \sum_{r=1}^{N} \frac{1}{\lambda_r} \{\varphi_r\}\{\varphi_r\}^{\mathrm{T}} = \sum_{r=1}^{N} [F]_r. \tag{4}$$

Equation (4) shows that the modal contribution to the flexibility matrix decreases as frequency increases. On the other hand, flexibility rapidly converges to a good approximation with a few low-frequency modes [15].

Flexibility matrix can be determined by two ways: using either dynamic or static data. Measuring the static tests is more accurate than dynamic ones; however, the sensitivity of static methods to the change in the structural parameters is low. The static methods only require the stiffness properties, whereas the dynamic methods require the use of mass, stiffness and damping properties.

The response derived from static methods is more exact than dynamic ones; moreover, considering the measurement error, static results are more reliable than modal testing results. In this paper, flexibility matrix is determined based on static data.

The derivative of modal flexibility with respect to the structural parameter "p" is called modal flexibility sensitivity described by

$$S_F = \frac{\partial [F]}{\partial p} = \sum_{r=1}^{N} \frac{\partial [F]_r}{\partial p} = \sum_{r=1}^{N} \frac{\partial}{\partial p}\left(\frac{1}{\lambda_r}\{\varphi_r\}\{\varphi_r\}^{\mathrm{T}}\right). \tag{5}$$

According to Eq. (5), for gaining the modal flexibility sensitivity of the structure (S_F), the rth modal flexibility has to be calculated. Expanding Eq. (5) yields:

$$\frac{\partial [F_{ij}]_r}{\partial p} = \frac{\partial}{\partial p}\left(\frac{1}{\lambda_r}\{\varphi_{ri}\}\{\varphi_{rj}\}^{\mathrm{T}}\right) = \left[\frac{1}{\lambda_r^2}\frac{\partial \lambda_r}{\partial p}\{\varphi_{ri}\}\{\varphi_{rj}\}^{\mathrm{T}} + \frac{1}{\lambda_r}\frac{\partial\{\varphi_{ri}\}}{\partial p}\{\varphi_{rj}\}^{\mathrm{T}} + \frac{1}{\lambda_r}\{\varphi_{ri}\}\frac{\partial\{\varphi_{rj}\}^{\mathrm{T}}}{\partial p}\right]. \tag{6}$$

The numerator of this derivative is the product of modal coefficients at points i, $\{\varphi_{ri}\}$, and j, $\{\varphi_{rj}\}$, of one mode, respectively. The denominator is the corresponding frequency, λ_r. It is known from Eq. (6) that the rth modal flexibility $[F_{ij}]_r$, at the ith point under the unit load at the point j [i.e., the left side of Eq. (6)] is equal to the sum of three terms related to the each rth mode identified [i.e., the right side of Eq. (6)]. If the derivatives of the rth eigenvalue and eigenvector are known, the rth modal flexibility sensitivity can be evaluated.

According to the right side of Eq. (6), the eigenvalue and eigenvector derivatives, which have been presented by Fox [24], are:

$$\frac{\partial \lambda_r}{\partial p} = \{\varphi_r\}^{\mathrm{T}}\frac{\partial [K]}{\partial p}\{\varphi_r\} - \lambda_r\{\varphi_r\}^{\mathrm{T}}\frac{\partial [M]}{\partial p}\{\varphi_r\}, \tag{7}$$

$$\frac{\partial\{\varphi_r\}}{\partial p} = \sum_{k=1}^{n} c_{rk}\{\varphi_{rk}\}, \tag{8}$$

$$c_{rk} = \begin{cases} \dfrac{-\{\varphi_{rk}\}^{\mathrm{T}}\left[\dfrac{\partial [K]}{\partial p} - \lambda_r\dfrac{\partial [M]}{\partial p}\right]\{\varphi_{rk}\}}{\lambda_k - \lambda_r} & k \neq r \\ -\dfrac{1}{2}\{\varphi_{rk}\}^{\mathrm{T}}\dfrac{\partial [M]}{\partial p}\{\varphi_{rk}\} & k = r \end{cases}. \tag{9}$$

Wang [25] presented a method for eigenvector sensitivity which is used in this paper.

$$\frac{\partial\{\varphi_r\}}{\partial p} = \frac{\partial\{\varphi_{r0}\}}{\partial p} + \sum_{k=1}^{n} a_{rk}\{\varphi_{rk}\}, \tag{10}$$

$$\frac{\partial\{\varphi_{r0}\}}{\partial p} = -[K]^{-1}\left[\frac{\partial [K]}{\partial p} - \frac{\partial \lambda_r}{\partial p}[M] - \lambda_r\frac{\partial [M]}{\partial p}\right]\{\varphi_r\}, \tag{11}$$

$$a_{rk} = \begin{cases} \dfrac{\dfrac{-\lambda_r}{\lambda_k}\{\varphi_{rk}\}^{\mathrm{T}}\left[\dfrac{\partial [K]}{\partial p} - \lambda_r\dfrac{\partial [M]}{\partial p}\right]\{\varphi_{rk}\}}{\lambda_k - \lambda_r} & k \neq r \\ -\dfrac{1}{2}\{\varphi_{rk}\}^{\mathrm{T}}\dfrac{\partial [M]}{\partial p}\{\varphi_{rk}\} & k = r \end{cases}. \tag{12}$$

The derivatives of eigenvector for both Fox and Wang methods for complete mode shapes have the same results; however, when the mode shapes are incomplete, Wang's results localize the damage more accurately than Wang's method.

2.2 Damage detection based on proposed method

The modal flexibility parameters, from the aspect of measured degree of freedom (DOF), consist of two sets of data: one from the intact structure denoted by I, and another from the damaged structure denoted by D. The change in flexibility is defined as:

$$\Delta[F] = [F_{\mathrm{D}}] - [F_{\mathrm{I}}]. \tag{13}$$

Any changes in design parameters of the structure will influence on modal parameters, thereby on modal flexibility. These changes can be shown by first-order Taylor's series as:

$$\Delta[F] = \frac{\partial [F]}{\partial p}\Delta p. \tag{14}$$

Substituting Eq. (14) into Eq. (13), one can formulate the inverse problem of model error identification into a linear set of equation as:

$$\frac{\partial [F]}{\partial p}\Delta p = [F_{\mathrm{D}}] - [F_{\mathrm{I}}]. \tag{15}$$

The right side of the equation can be obtained from modal measurement and analysis. The Eq. (14) can be completed according to the Eq. (5) as:

$$\Delta[F] = \frac{\partial [F]}{\partial p}\Delta p = \sum_{r=1}^{N}\frac{\partial [F]_r}{\partial p}\Delta p = \sum_{r=1}^{N}\frac{\partial}{\partial p}\left(\frac{1}{\lambda_r}\{\varphi_r\}\{\varphi_r\}^{\mathrm{T}}\right)\Delta p. \tag{16}$$

By extracting the Eq. (16) and differentiating λ_r and $\{\varphi_r\}$ in the rth mode shape as well as considering Eq. (14), Eq. (16) would be as follows:

$$\Delta[F_{ij}]_r = \frac{1}{\lambda_r^2}\Delta\lambda_r\{\varphi_{ri}\}\{\varphi_{rj}\}^{\mathrm{T}} + \frac{1}{\lambda_r}\Delta\{\varphi_{ri}\}\{\varphi_{rj}\}^{\mathrm{T}} + \frac{1}{\lambda_r}\{\varphi_{ri}\}\Delta\{\varphi_{rj}\}^{\mathrm{T}}. \tag{17}$$

To calculate the changes in modal flexibility based on Eq. (17), the changes in mode shapes and frequencies are required to be known. In this paper, the frequencies associated with the damaged structure are used for considering the changes in frequencies. To consider the changes in mode shapes, as it was previously mentioned, Wang's method is used. Wang's equation comprises two parts: initial mode shape and the residual part. In such equation, the derivative of the frequency is observed. In this paper, the frequencies of the damaged structure are used in lieu of the implicit equation (i.e., Wang's). As a result, the Eqs. (6) and (11) can be changed to:

$$\Delta[F_{ij}]_r = \left[\frac{1}{\lambda_r^2}(\lambda_{rd}-\lambda_r)\{\varphi_{ri}\}\{\varphi_{rj}\}^{\mathrm{T}} + \frac{1}{\lambda_r}\Delta\{\varphi_{ri}\}\{\varphi_{rj}\}^{\mathrm{T}} + \frac{1}{\lambda_r}\{\varphi_{ri}\}\Delta\{\varphi_{rj}\}^{\mathrm{T}}\right], \tag{18}$$

$$\begin{aligned}\Delta\{\varphi_{r0}\} &= -[K]^{-1}\left[\frac{\partial[K]}{\partial p}\Delta p - (\lambda_{rd}-\lambda_r)[M]\Delta p - \lambda_r\frac{\partial[M]}{\partial p}\Delta p\right]\{\varphi_r\}\\ \Delta\{\varphi_{r0}\} &= -[K]^{-1}\frac{\partial[K]}{\partial p}\Delta p\{\varphi_r\} + [K]^{-1}(\lambda_{rd}-\lambda_r)[M]\Delta p\{\varphi_r\}\\ &\quad + [K]^{-1}\lambda_r\frac{\partial[M]}{\partial p}\Delta p\{\varphi_r\}.\end{aligned} \tag{19}$$

The stiffness matrix of the structure is defined as [26]:

$$[K] = [A][P][A]^{\mathrm{T}}, \tag{20}$$

where the $[A]$ and $[p]$ are defined as stiffness connectivity matrix and diagonal matrix of assembled stiffness eigenvalues for the entire structure. In this paper, it is assumed that structural damage does not cause mass variation; however, it brings about only a reduction in the structural stiffness. In other words, mass distribution of the intact and damaged structures remains unchanged. As a result, by substituting Eqs. (17–19) into Eq. (15), the improved modal flexibility sensitivity of the structure is expressed by

$$\tilde{S} = \sum_{r=1}^{N}\left[\frac{1}{\lambda_r}\left(-[K]^{-1}\frac{\partial[K]}{\partial p}\{\varphi_{ri}\} + \sum_{k=1}^{n}a_{rk}\{\varphi_k\}\right)\{\varphi_{rj}\}^{\mathrm{T}} + \frac{1}{\lambda_r}\{\varphi_{ri}\}\left(-[K]^{-1}\frac{\partial[K]}{\partial p}\{\varphi_{rj}\} + \sum_{k=1}^{n}a_{rk}\{\varphi_k\}\right)^{\mathrm{T}}\right]. \tag{21}$$

By solving the Eq. (21), the damage location and its severity can be obtained. The procedure of the proposed method can be summarized as follows:

1. Creating the finite element model of intact structure
2. Obtaining the measured flexibility data and Eigen properties of intact and damaged structure
3. Updating parameters of intact structure and solving the equation considering mass modeling and measurement errors in order to estimate the unmeasured modes of damaged structure and to detect and locate damage

2.3 Mass modeling error

In many real cases, the damage cannot have a significant effect on the mass matrices of the structure. In other words, the mass matrix of a structure is less likely to be changed by damage phenomenon. However, the inaccuracies may not be unexpected for the assumed mass parameters of the intact and damaged structures which may lead the stiffness parameter estimation results to be less accurate. As a result of such inaccuracies, some deviations in the stiffness parameter identification are likely to appear. Sensitivity equations may be negatively affected because of the potential errors in the eigenvectors of the undamaged structure which contribute to the construction of sensitivity equations and are caused by the inaccurate assumption of mass properties.

In this study, with the aid of numerical simulations, it is indicated that despite considering mass modeling errors, robustness is still observable in the parameter estimation process. The stability and robustness of the proposed method against mass modeling errors can be proved by the accurate estimation of variations in the stiffness parameter and low values for the results' COV.

2.4 Measurement error

The model updating methods by which the variations of the flexibilities are considered are limited to the cases that there is an access to the exact flexibly' data of the damaged structures, regardless of the accuracy and computational issues [27]. Therefore, a low level of random errors (up to 2% of random errors) is considered by researchers in order to evaluate robustness of these kinds of model updating methods against measurement errors. Environmental noises caused by ambient loads, or unstable positions of sensors can have an impact on the extracted flexibilities; therefore, experiencing a higher level of measurement error cannot be unexpected.

Measurement of natural frequencies is accurate using sophisticated accelerometers. This fact has led some researchers to assume noise-free natural frequencies for model updating. Sensitivity matrix for evaluation of the stiffness parameters is established by implementation of the measured natural frequencies. Nonetheless, sensitivity equations are highly sensitive to and affected by any unpredicted errors in natural frequencies

3 Numerical results

A simulated two-dimensional truss and frame structures are used as case studies in order to evaluate the ability of this improved method. It should be mentioned that the imposed load is a kind of impulse; as a result, in the simulation study, the load is considered as a concentrated load imposed to some specific degrees of freedom which are wisely selected by engineering judgment.

3.1 Truss

The finite element model of the 2-D truss element consists of 25 elements and 21 Kinematics degrees of freedom as shown in Figs. 1 and 2. The basic parameters of the truss element are as follows:

Mass density of 7800 kg/m^3, Young's modulus E = 20 MPa, the cross-sectional areas are given in Table 1.

The excitation is applied at the DOFs 12, 9, 19, 15 and 17 and the measuring points are assumed to be in DOFs 3, 4, 13, 16, 14, 15, 21 and 20. Four damage scenarios that are different in the number, location, and severity of damaged elements in order to investigate the precision of the proposed method are summarized in Table 2. Also, in order to consider the measurement errors, 30 sets of 2 and 0.5% random noises have been applied to the flexibility and natural frequency of the damaged structure, respectively. Since this method is model based and uses the analytical model of intact structure, an error associated with modeling the mass of numerical model compared to the real mass of intact structure should be considered. For considering mass modeling error, 5% random noise has been applied to the mass of intact structure. As it was previously mentioned, difficulties in calculation of all modes, caused by the sensitivity of the equation to low-frequency modes, lead to considering only first eight modes

To determine accuracy of the proposed method and effectively compare the validity of responses, the damage indicator coefficient of variation is used. The COV of the parameters closer to zero indicates that the responses are impervious to noises. As it can be seen in the figures, COV is less than 33%. The results of damage detection using the proposed formulation are depicted in Figs. 3, 4, 5, 6, 7, 8, 9 and 10.

The aim of identifying the location and severity of damage in all scenarios is to understand the sensitivity trend of the noise associated with flexibilities and frequencies, when the proposed method is applied. According to the results, in the presence of modeling errors, damage is predicted to occur in some intact elements. Hence, these errors reduce the accuracy of the method; nevertheless, this method is still acceptable in terms of accuracy.

Table 1 Cross-sectional area and density of truss elements

Element's number	Cross-sectional area (m^2)
01-6	18E−4
07-12	15E−4
13–17	10E−4
18–25	12E−4

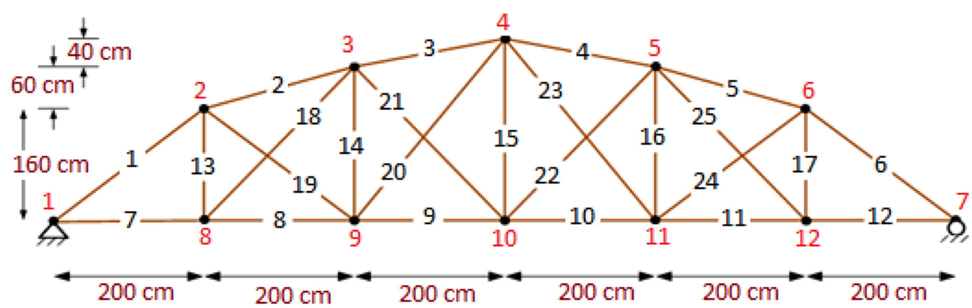

Fig. 1 Geometry of truss model

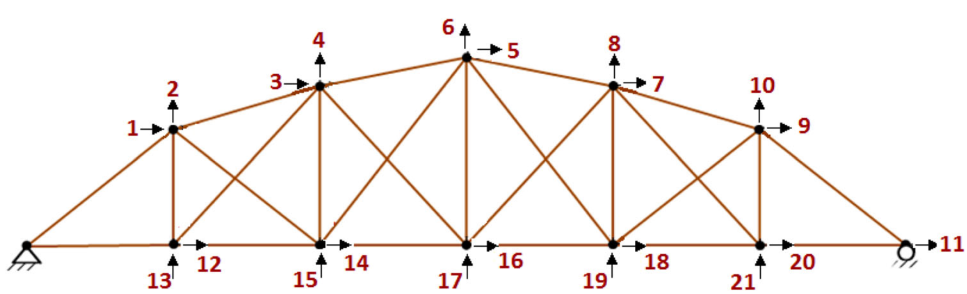

Fig. 2 Degrees of freedom of truss model

Table 2 Damage cases for truss

Damage scenarios					Flexibility noise	Frequency noise	Mass noise
1	Elements	3	17	19	0.02	0.005	0.05
	Damage (%)	40	50	50			
2	Elements	13	15	17	0.02	0.005	0.05
	Damage (%)	50	40	30			
3	Elements	17			0.02	0.005	0.05
	Damage (%)	70					
4	Elements	13	25		0.02	0.005	0.05
	Damage (%)	50	40				

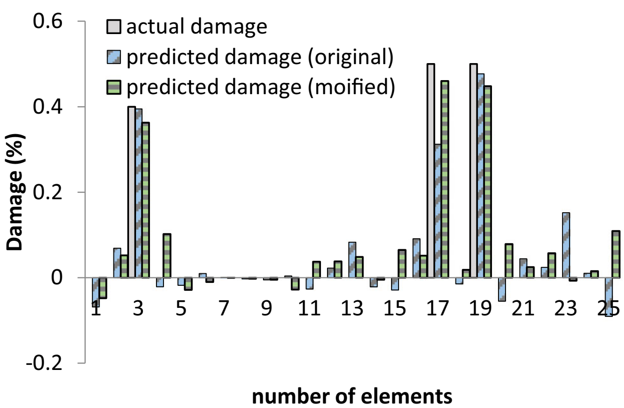

Fig. 3 Actual and predicted damage of case 1 for original modal flexibility and modified modal flexibility

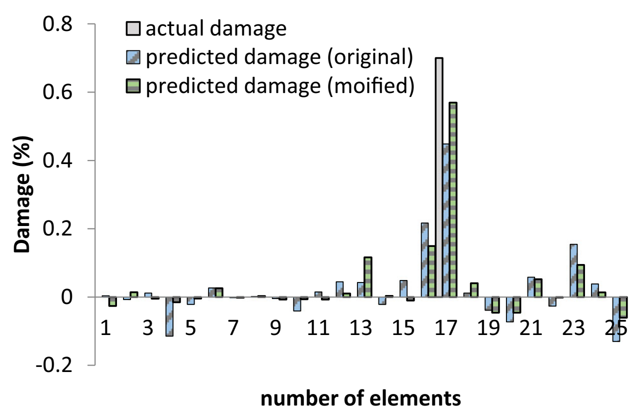

Fig. 5 Actual and predicted damage of case 2 for original modal flexibility and modified modal flexibility

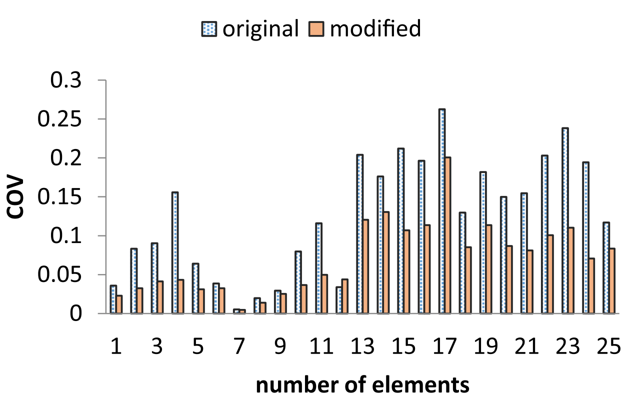

Fig. 4 Coefficient of variation of case 1 for original modal flexibility and modified modal flexibility

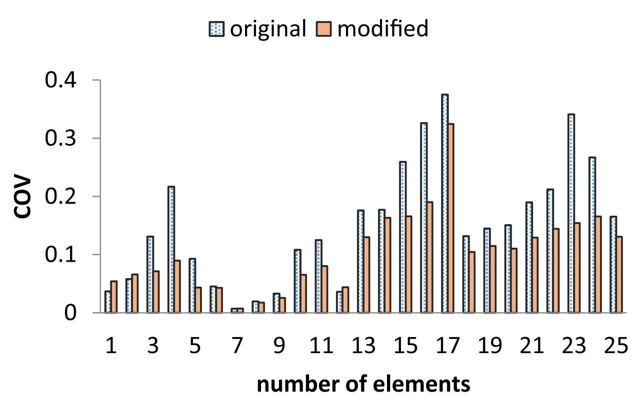

Fig. 6 Coefficient of variation of case 2 for original modal flexibility and modified modal flexibility

To compare the severity and location of the predicted damage derived from the proposed method with the actual damage, a parameter called close index (CI) is defined.

$$\mathrm{CI} = 1 - \frac{\|\delta P_p - \delta P_{\mathrm{actual}}\|}{\|\delta P_{\mathrm{actual}}\|} \quad (22)$$

where δP_p and $\delta P_{\mathrm{actual}}$. are predicted damage and actual damage, respectively. If the damage severity and location are predicted correctly, the CI closes to 1.0. Results of the damage detection process which uses modified modal flexibility method present the CI which is closer to 1.0

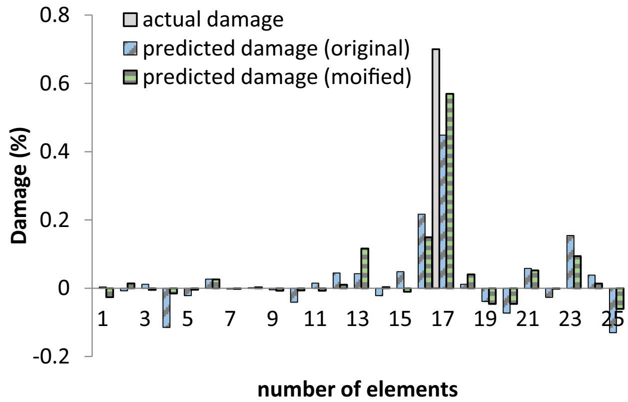

Fig. 7 Actual and predicted damage of case 3 for original modal flexibility and modified modal flexibility

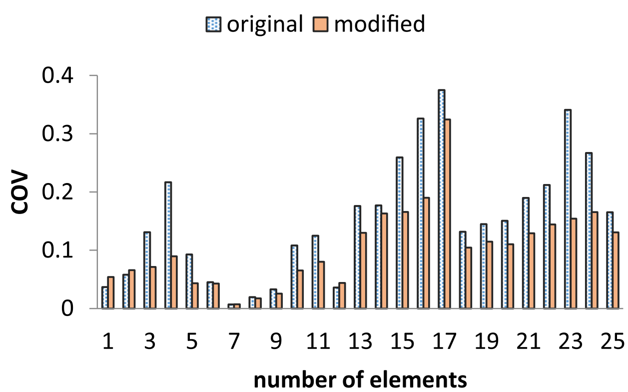

Fig. 8 Coefficient of variation of case 3 for original modal flexibility and modified modal flexibility

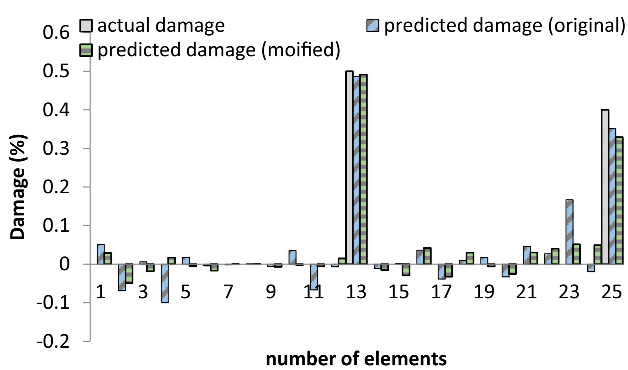

Fig. 9 Actual and predicted damage of case 4 for original modal flexibility and modified modal flexibility

compared to original modal flexibility method. The CI index for truss structure is shown in Table 3.

To evaluate the accuracy of the proposed method, two damage scenarios are randomly elected and assessed, for instance, scenarios number 2 and 3. In such scenarios, the measurement error, or the noise of flexibility, is changed from 0.02 to 0.04; and 0.5% noise is applied to the natural

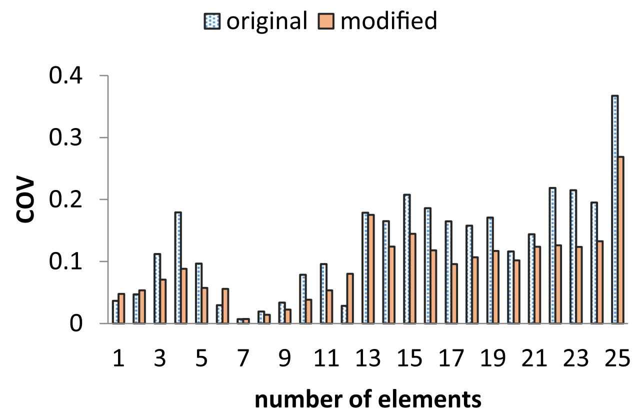

Fig. 10 Coefficient of variation of case 4 for original modal flexibility and modified modal flexibility

frequency of the damaged structure. 5% random noise is applied to the mass of intact structure. It is notable that with the increase in the noise, the accuracy of results is different depending on the various health monitoring techniques; the approach which can provide reliable results in terms of accuracy can be considered as an efficacious technique in the detection of the damage location and severity.

The results derived from modal flexibility and modified modal flexibility are compared. The close index (CI) for original modal flexibility of case 3 is about − 0.0265; whereas, for modified modal flexibility, it is equal to 0.452; moreover, for case 2, this quantity is about 0.424, while, for modified modal flexibility, it is 0.635.

Compared to the proposed method, it is observable that the modal flexibility method is unable to provide converged results and it cannot detect the amount of damage in structure (Figs. 11, 12).

According to Figs. 13 and 14, the COV for the damage cases 1 and 3 with applying measurement errors 4% to the flexibility and 0.5% to the natural frequency of damaged structure and 5% random noise to the mass of intact structure for proposed method is less than 20 and 43%, respectively, whereas, for original modal flexibility, it is less than 60 and 43%. These results indicate that the accuracy of the response of the model with modified modal flexibility method is not affected by the noise.

3.2 Frame

The finite element model of a one-story one-bay aluminum frame consists of 21 two-node beam and column elements and 22 nodes as shown in Fig. 15. A joint of the plane frame can have up to three degrees of freedom (two translational degrees of freedom perpendicular to the beam's axis and a rotational one). The translational degrees of freedom have been measured. The length of each

Table 3 Close index for truss structure

Damage case	Original modal flexibility method	Modified modal flexibility method
1	0.608	0.704
2	0.488	0.643
3	0.383	0.605
4	0.613	0.764

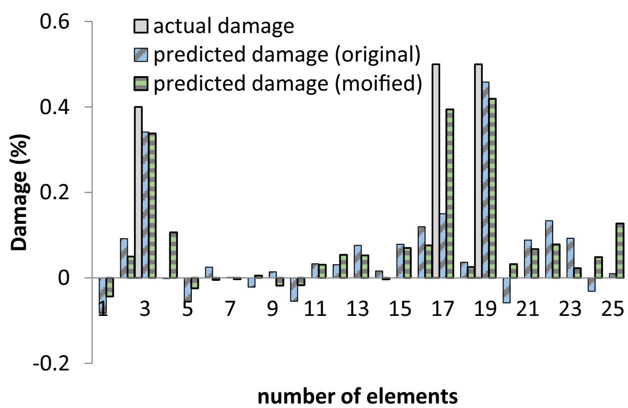

Fig. 11 Actual and predicted damage of case 1 for original modal flexibility and modified modal flexibility

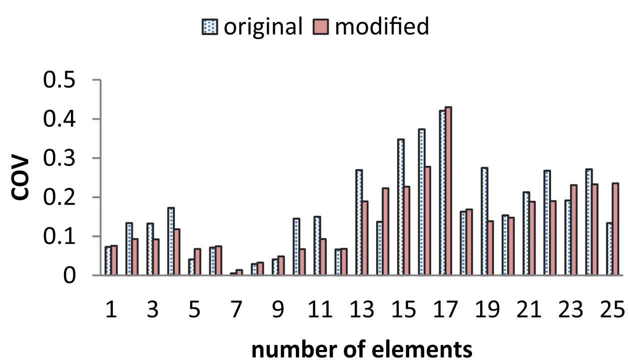

Fig. 12 Coefficient of variation of case 1 for original modal flexibility and modified modal flexibility

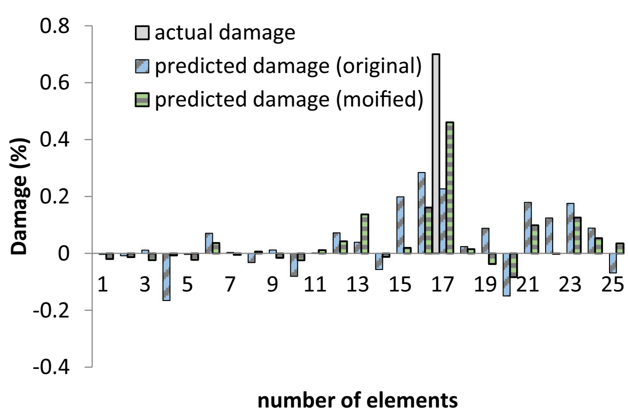

Fig. 13 Actual and predicted damage of case 3 for original modal flexibility and modified modal flexibility

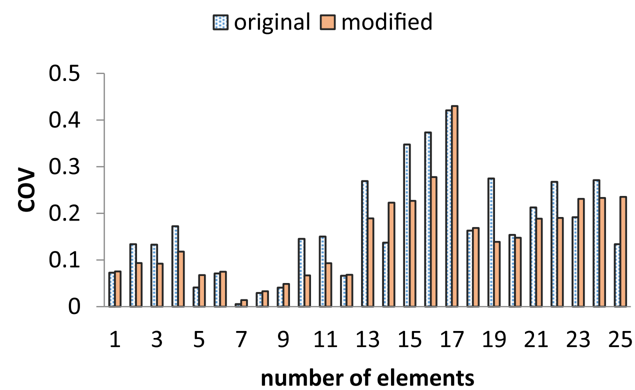

Fig. 14 Coefficient of variation of case 3 for original modal flexibility and modified modal flexibility

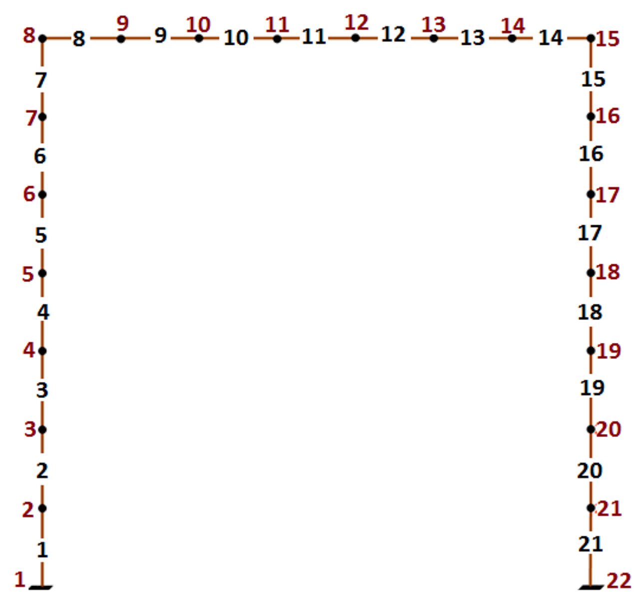

Fig. 15 Geometry of frame model

element is $l = 0.1$ m with cross-section areas equal to $A = 3.6066\mathrm{E}{-4}$ m^2 and moment of inertia of $I = 0.26444\mathrm{E}{-8}$ m^4 and Young's modulus of $E = 0.67$ MPa, and the density equal to $\rho = 2693.9$ kg/m^3.

The excitation is applied at the DOFs 4, 10, 16, 19, 32, 38, 49, 55 and 58 and the measuring points are assumed to be the DOFs 4, 13, 23, 35, 38, 43, 46, 49, 52 and 58. Damage cases with different assumed damage locations and severities in addition to different noises applied to the flexibility and natural frequency of damaged structure and

Table 4 Damage cases for frame

Damage case						Flexibility noise	Frequency noise	Mass noise
1	Elements	2	20			0.02	0.005	0.05
	Damage (%)	40	30					
2	Elements	3	17	19		0.02	0.005	0.05
	Damage (%)	40	50	50				
3	Elements	4	11	19	21	0.02	0.005	0.05
	Damage (%)	40	50	50	40			
4	Elements	11				0.02	0.005	0.05
	Damage (%)	30						

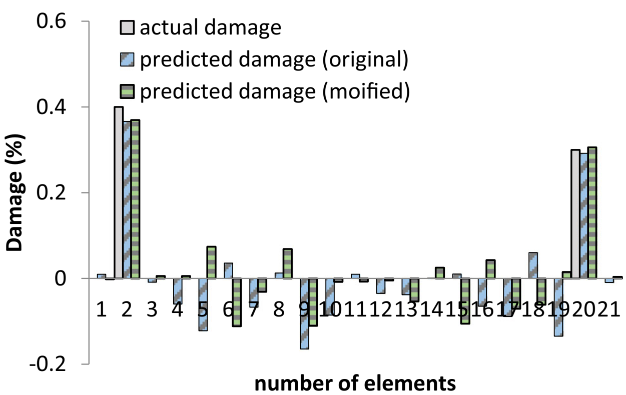

Fig. 16 Actual and predicted damage of case 1 for original modal flexibility and modified modal flexibility

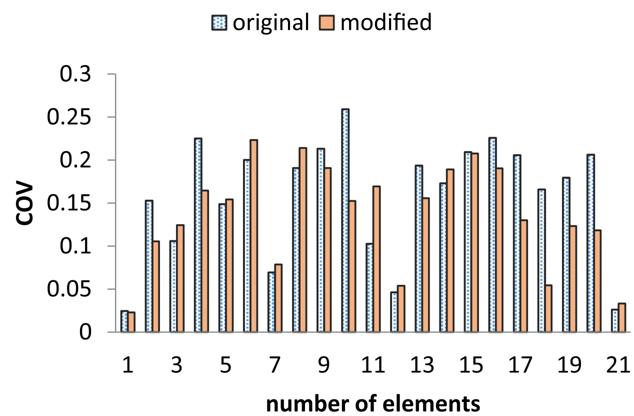

Fig. 17 Coefficient of variation of case 1 for original modal flexibility and modified modal flexibility

the mass of intact structure are summarized in Table 4. Based on the incomplete measurements, only the first eight modes are considered. The COV of frame structure is less than 26%.

As it was previously mentioned for the truss elements, the measurement errors are applied to the damaged structure which are equivalent to 4% for the flexibility and 0.5% for the natural frequency; additionally, 5% random noise is applied to the mass of intact structure, so as to estimate the power of the proposed method; then, the results regarding case 1 and 3 of modal flexibility and modified modal flexibility of frame structure are compared. The close index (CI) for original modal flexibility of case 1 is approximately − 0.18, whereas, for modified modal flexibility, this parameter is roughly equal to 0.357. Similarly, the close index (CI) for original modal flexibility of case 3 is about − 0.2, while, for modified modal flexibility, it is 0.696. The COV of the case 1 for original modal flexibility is less than 50% and for modified method it is less than 30%. For the case 2 related to the original modal flexibility, this quantity is less than 110% and less than 38% for the modified method (Figs. 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27).

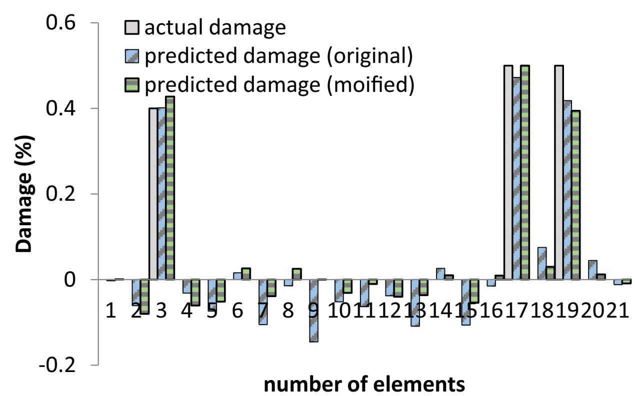

Fig. 18 Actual and predicted damage of case 2 for original modal flexibility and modified modal flexibility

As it is evident from the comparison of the modified modal flexibility (i.e., the proposed method) with the original one in terms of the derived results, the modal flexibility method is incapable of providing converged results in addition to the point that it unable to effectively detect the damage intensity of the structure.

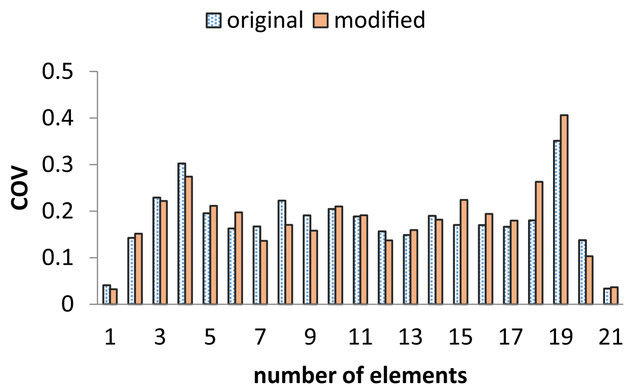

Fig. 19 Coefficient of variation of case 2 for original modal flexibility and modified modal flexibility

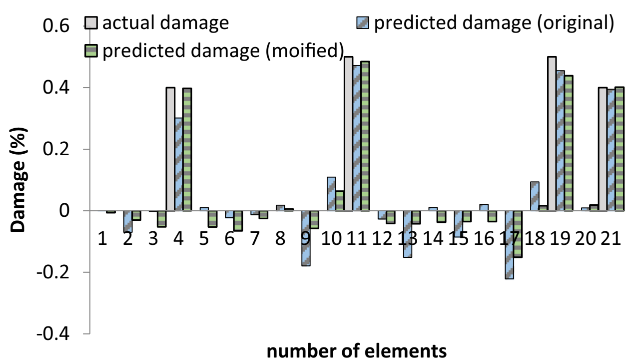

Fig. 20 Actual and predicted damage of case 3 for original modal flexibility and modified modal flexibility

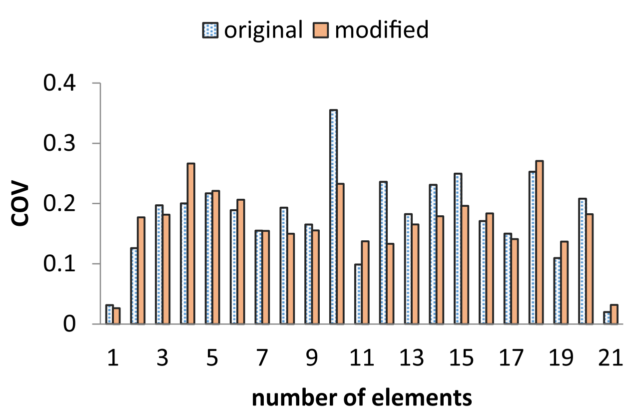

Fig. 21 Coefficient of variation of case 3 for original modal flexibility and modified modal flexibility

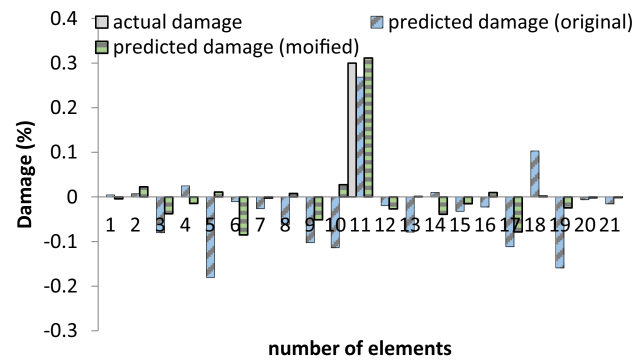

Fig. 22 Actual and predicted damage of case 4 for original modal flexibility and modified modal flexibility

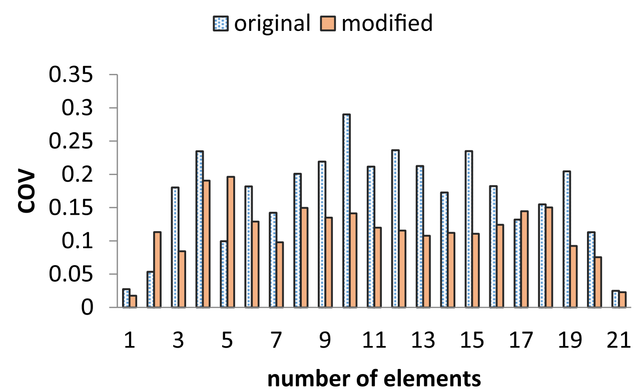

Fig. 23 Coefficient of variation of case 4 for original modal flexibility and modified modal flexibility

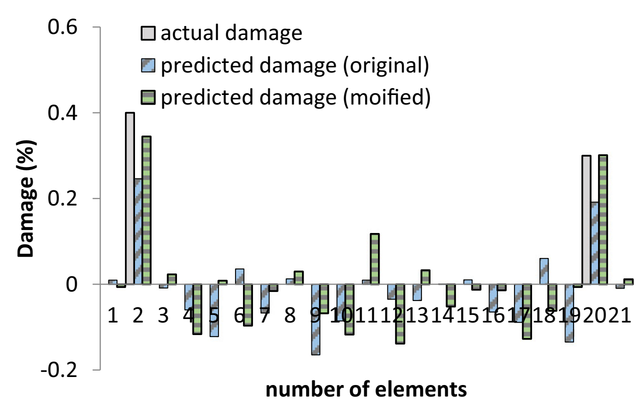

Fig. 24 Actual and predicted damage of case 1 for original modal flexibility and modified modal flexibility

It can be observed from tables that the proposed method predicts damage severity more accurately than the conventional method.

Figure 28 illustrates that by considering eight measured modes, damage charts do not change. In other words, the damaged elements will be unvaried alongside with increasing the measured mode shapes. Thus, requiring only eight mode shapes for damage localization is one of the most important advantages of the proposed method. It depicts the actual and predicted damage detected by the proposed method, considering 5, 6, 7 and 8 measured modes for frame structure. The CI index for frame structure of 4 scenarios is shown in Table 5.

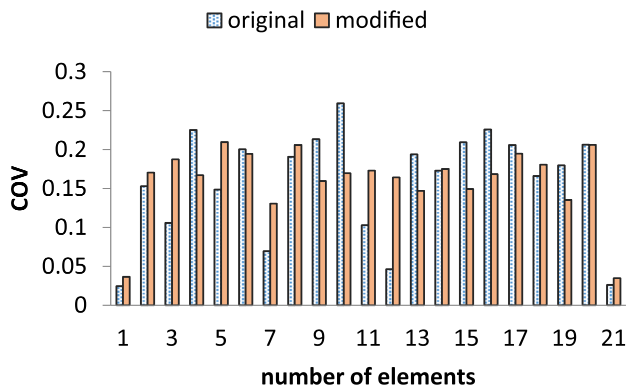

Fig. 25 Coefficient of variation of case 1 for original modal flexibility and modified modal flexibility

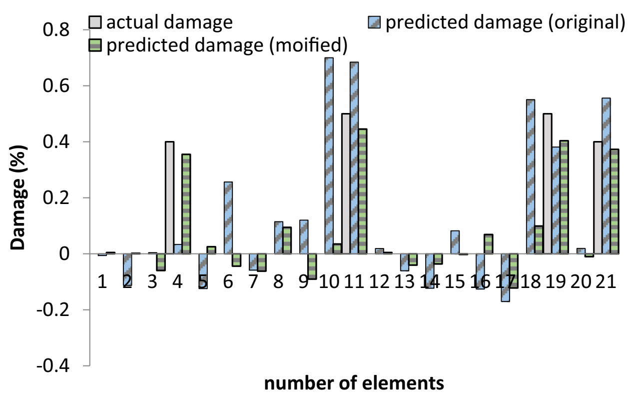

Fig. 26 Actual and predicted damage of case 3 for original modal flexibility and modified modal flexibility

4 Conclusion

In this paper, an efficient damage detection method called modified modal flexibility has been proposed to detect, locate, and determine the intensity of damage in the 2-D truss and frame structures.

This method is based on model-based updating of sensitivity equation, thereby determining the dynamic characteristic and the modal flexibility. The finite element model of intact structure should be developed, then the damage data should be achieved from experiment. Due to the number of sensors essential to be used, difficulty in determining the points of excitation, and the impossibility of measuring all features of the damaged structure, carrying out a real experiment is not a straightforward approach; therefore, in this paper, the location and severity of damage in the damaged structure are detected by model updating of intact structure with the least square solving of the sensitivity equation.

The proposed approach represented in this paper is based on the parallel use of static and dynamic flexibilities.

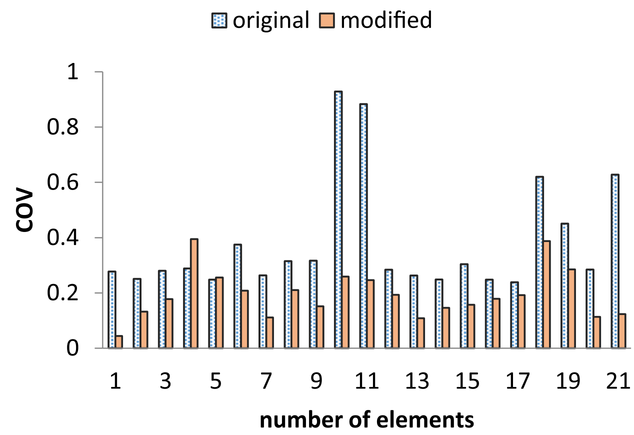

Fig. 27 Coefficient of variation of case 3 for original modal flexibility and modified modal flexibility

Another significant point to consider in this approach is that the frequency data of the damaged structure are implemented in lieu of the implicit derivative of frequency in order to enhance the sensitivity equation. The derivative of the frequency which is observable in both modal flexibility and derivative mode shapes (i.e., Wang's) equations is removed; variation of frequency, including the frequency of the intact and damaged structures, is supplanted. To verify the proposed method, the results of the modified modal flexibility approach are compared to the results derived from the conventional method.

The damage scenarios are considered under the situation that the measured flexibility, frequencies, and the mass of intact structure are contaminated by uncorrelated random noise. Moreover, as it is not plausible to apply excitation to all mode shapes due to the high computational and other operational costs it imposes to the process, measuring all mode shapes is not feasible. Considering such point, in this paper, the effect of number of mode shapes on the results of damage detection is evaluated. According to the derived results, mode shapes number 6, 7, and 8 provide close results; therefore, eight modes are considered in this research.

Two parameters including "close index" and "coefficient of variation" are used in order to evaluate the robustness of the proposed method. The "close index" for the proposed method is closer to 1 than the original method. Moreover, the "coefficient of variation" for the proposed method is less than the conventional approach. Consequently, it is safe to say that the sensitivity equation of the proposed method is more accurate in terms of mathematical formulation than Yan's method. By comparing the numerical results of all damage scenarios with the assumed damages of the truss and frame structures, it can be concluded that the modified modal flexibility method makes damage detection process more convenient

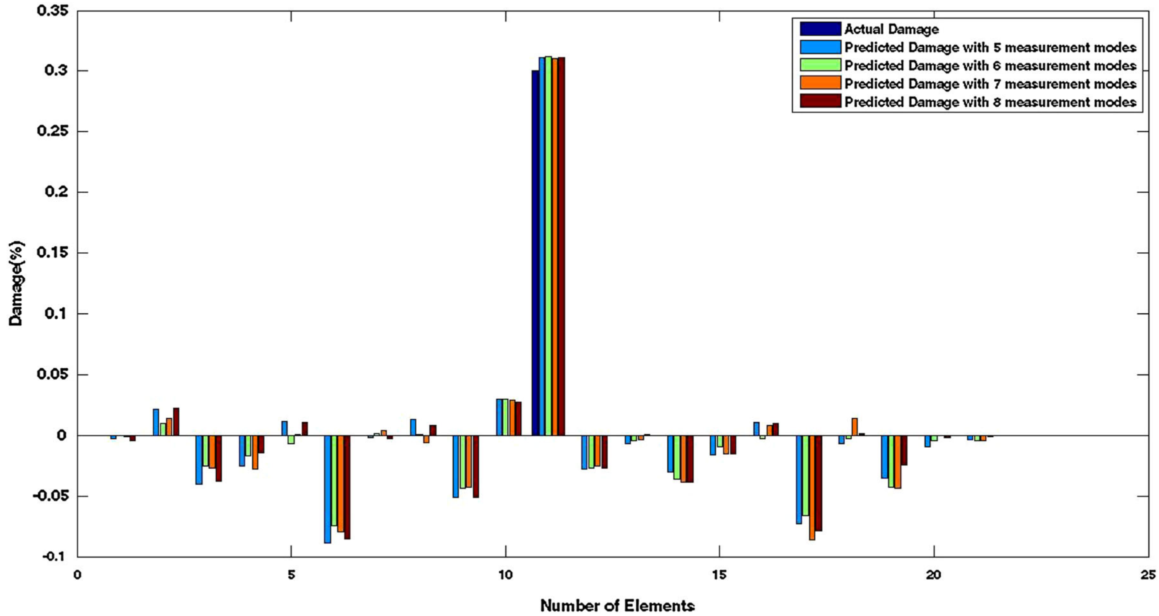

Fig. 28 Actual and predicted damage of frame structure with 5, 6, 7, 8 measured modes

Table 5 Close index for frame structure

Damage case	Original modal flexibility method	Modified modal flexibility method
1	0.378	0.633
2	0.63	0.767
3	0.569	0.744
4	− 0.17	0.501

to evaluate the damage severity and location, with presence of measurements and modeling errors, when the first eight orders of modal data are used. As a suggestion, it would be much more accurate to consider damping in this research and assess its effects on the results separately.

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