



Post-earthquake controls and damage detection through structural health monitoring: applications in l'Aquila

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Abstract

The paper presents structural health monitoring (SHM) activities performed on some representative cultural heritage (CH) buildings in the city of l'Aquila after the strong earthquake ($M_w = 6.3$) that struck the Abruzzo region (central Italy) on April 6, 2009. The severity and the extent of damages caused by the earthquake to historical buildings and monument were never reached before in the recent Italian earthquake history. Emergency activities started immediately after the earthquake to protect CH structures, including damage survey and design/implementation of temporary safety measures. Some historic buildings were soon equipped with monitoring systems in order to assess the level of damage and verify the effectiveness of the executed provisional interventions. The paper focuses in particular on two case studies, i.e. the Spanish Fortress and the Civic Tower. The results of preliminary investigations are reported, including damage survey and operational modal analysis for modal parameter identification using ambient vibration tests. 3-year static and dynamic monitoring features, automatically extracted from raw data acquired by continuous monitoring systems, were then processed using a data-driven approach based on regression analysis to filter out the environmental effects. Following this approach data are decomposed into their reversible and irreversible components, the latter being associated with active damaging processes and the residual structural performance of the two buildings assessed.

Keywords Structural health monitoring · Cultural heritage · Modal analysis · System identification · Damage detection

1 Introduction

Structural health monitoring is nowadays increasingly applied in the vulnerability assessment of historic (and cultural heritage) buildings, as a key activity to improve the knowledge on their structural behaviour and identify the presence/activation of possible damage mechanisms. This knowledge allows engineers to carry out with more confidence and only if necessary a strengthening intervention, and it helps to prevent the execution of intrusive repair works, if not justified by an experimentally demonstrated worsening of the structural conditions [1–3].

The application of structural health monitoring (SHM) techniques and methodologies during post-earthquake activities on damaged buildings [4, 5] proved its usefulness in order to: (1) evaluate quantitatively the evolution of identified on-going damage processes; (2) appraise the structural response of the monitored buildings to after-shocks; (3) assess the effectiveness of provisional strengthening interventions and intervene quickly if an

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unsafe displacement pattern is recorded. Monitoring can also be effective when implemented on seriously damaged buildings, if the time schedule for the interventions is difficult to be a priori planned.

Starting from this approach, the authors designed and installed SHM systems on six representative and emblematic CH buildings in L'Aquila (Abruzzo region, central Italy), after the strong earthquake ($M_w = 6.3$) that hit the city on April 6, 2009. Two selected case studies (Spanish Fortress and Civic tower) reported the experience gained on the use of SHM during the emergency activities after a seismic event.

The paper focuses in particular on the application of data-driven approaches to analyse and interpret SHM data and detect the presence of active damage processes. Static and dynamic responses of structures are in fact strictly correlated to the change of environmental factor (temperature, relative humidity) and loading conditions. For these reasons, before applying any damage identification algorithms it is of fundamental importance to filter out the environmental and loading effects from the dynamic response of the structure and from the cyclic behaviour of 'static' features. If this effect is not taken into account in the damage detection process, false-positive or negative damage diagnosis may occur so that vibration-based health monitoring becomes unreliable [6, 7]. One of the methods used to solve this problem is to perform correlation between the extracted features (e.g., modal parameters, strain or displacement data, etc.) and the corresponding environmental conditions (e.g., ambient temperature, relative humidity, etc.) [8–10].

2 Modelling environmental effects: theoretical background

Different methods can be applied to establish relationships between the observed environmental factor and the estimated natural frequencies or other static parameters [11].

A way to do this is to apply regression analysis, able to establish relationships between the observed environmental factors and SHM data. This relationship is described through a statistical model that can be exploited, in an initial phase, to understand the influence of each predictor (input of the model) on the dependent variable (output of the model) and then, to predict future values of the response when only the predictors are known. In the context of SHM, a first set of data (usually referred as estimation phase) is used to construct the model and afterwards, the developed model is used to predict the outputs (natural frequencies, crack opening, etc.) taking into account the measured independent variables (temperature, relative humidity, loading conditions, etc.)

The predicted outputs (usually referred as validation phase) are subsequently compared with the values directly estimated from acquired time series or recorded static data, calculating the residuals between actual data and model predictions. In this way, it is possible to remove the environmental factors and provide a preliminary judgement on the health conditions of the structure.

Among the regression models available in literature, in this work multivariable ARX models [12] are used: they comprehend an auto-regressive output and an exogenous input part and are ideal for representing monitored parameters when they depend (linearly) on the rate of change or trend in temperature as well as the present temperature. These models have the advantage to be able to take into account the thermal inertia of the structure, including some dynamics: the current output and input are related to outputs and inputs at previous time instants [13, 14].

The multivariable ARX model with n inputs u and one output y is presented by

$$A_q y_k = B_q u_{k-nk}^{\text{env}} + e_k, \quad (1)$$

where A_q is a scalar with the delay operator q^{-1} , B_q is a matrix $1 \times n$ and e_k is the unknown residual, a white noise term indicating that the input–output relation is not perfect. The ARX model is characterized by three numbers: n_a , the auto-regressive order, n_b the exogenous order and n_k the pure time delay between input and output. The coefficients of operator polynomials A_q and B_q can be estimated with the simple linear least square method. ARX models are commonly indicated by the orders n_a , n_b and n_k , e.g. ARX142 with $[n_a, n_b, n_k] = [1, 4, 2]$.

Before applying the model, a correlation analysis [15] helps in the identification of the input time series (environmental factors) presenting higher correlation coefficients with the time series containing the evolution of monitored parameters (static data or natural frequencies). Through this analysis it is possible to study the correlation between environmental effects x_k and measured outputs y_k , calculating for each temperature–frequency pair the so-called correlation coefficient r_{xy} which represents the normalized measure of the strength of linear relationship between variables, given by

$$\hat{r}_{xy} = \frac{\text{cov}(x_k, y_k)}{\sigma_x \sigma_y}, \quad (2)$$

where $\text{cov}(x_k, y_k)$ is the estimated covariance:

$$\text{cov}(x_k, y_k) = \frac{1}{N-1} \sum_{k=1}^N (x_k - \bar{x})(y_k - \bar{y}), \quad (3)$$

and σ_x and σ_y are the estimated standard deviation:

$$\sigma_x = \sqrt{\frac{1}{N-1} \sum_{k=1}^N (x_k - \bar{x})^2} \text{ and} \quad (4)$$

$$\sigma_y = \sqrt{\frac{1}{N-1} \sum_{k=1}^N (y_k - \bar{y})^2}.$$

The absolute value of the correlation coefficient varies from zero to one, indicating, respectively, a weak or strong correlation between the two variables.

After this preliminary selection of the predictor variables based on correlation coefficients, it is possible to create an input–output ARX model. Some quality criteria to define the model order, as proposed by Peeters and De Roeck [13], can be used to assess and compare the quality of models: the value of the loss function λ_0 and the Akaike's final prediction error (FPE) [15] defined as

$$\lambda_0 = \frac{1}{N} \sum_{k=1}^N e_k^2 \quad \text{FPE} = \lambda_0 \frac{1 + d/N}{1 - d/N}, \quad (5)$$

where e_k is the residual error calculated between measured data and the ARX model of Eq. (1), and d is the number of estimated parameters. Another quality criterion used is the coefficient of determination R^2 defined as the ratio between the variances of the fitted values of the model (\hat{y}_k) and the measured values of the dependent variable (y_k^m):

$$R^2 = \frac{\sum (\hat{y}_k - \bar{y})^2}{\sum (y_k^m - \bar{y})^2}. \quad (6)$$

Changing the parameters n_a, n_b, n_k the 'best' ARX model, able to explain and fit better the measured data, is the one with the lower λ_0 , the lower FPE and the higher R^2 .

To evaluate whether the ARX model found adequately describes the phenomenon in question, another quality criterion proposed by [13] consists in investigating the auto-correlation function of its prediction error e_k . If the prediction error is zero-mean white noise a good ARX model is obtained. The auto-correlation function of the prediction error is estimated as

$$\lambda_i = \frac{1}{N} \sum_{k=1}^N e_{k+i} e_k. \quad (7)$$

Once good ARX models are constructed for each monitored parameter based on data of the estimation period, they can be used to simulate the response (outputs) based on the new measured environmental parameters (fresh data as inputs) during the so-called validation period.

The final phase of the proposed procedure consists in the application of a residual analysis, able to detect the outliers and provide the identification of possible damages [16]. This is to compare simulated monitored data and their identified counterparts, so that one can calculate the

standard deviation of these data. The standard deviations can be used to establish confidence intervals around the predicted values. For instance, if \hat{y} is the predicted output and $\hat{\sigma}_y$ the estimated standard deviation on a new observation, the $(100 - \alpha)\%$ confidence interval on \hat{y} is given by

$$[\hat{y} - t_{\alpha/2, v} \hat{\sigma}_y, \hat{y} + t_{\alpha/2, v} \hat{\sigma}_y], \quad (8)$$

where the value $t_{\alpha/2, v}$ is found from a statistical table of the t Student distribution and for a large number of data (as in this case) and $\alpha = 0.05\%$ (leading to 95% confidence intervals), we have $t_{\alpha/2, v} = 1.96$.

The confidence intervals defined in Eq. (8) can be used as an objective criterion to detect damage under the varying environmental conditions.

3 L'Aquila earthquake and the SHM network

The earthquake that occurred on April 6, 2009, in the Abruzzo Region of Italy seriously hit the Cultural Heritage patrimony with major destructive effects on l'Aquila, a city of 70,000 inhabitants with the size and the historical and strategic importance of the capital of the Region. The severity and the extent of damages caused by the earthquake to historical buildings and monument were never reached before in the recent Italian earthquakes history. The emergency activities to protect the CH structures have been developed following two parallel levels: (1) damage survey and (2) design and implementation of temporary safety measures.

Some major monuments were soon equipped with SHM systems in order to control the progression or stationariness of the damage pattern already assessed. In a second step, further systems were applied on other CH buildings, also during the stabilization works execution, in order to appraise the effectiveness of the interventions carried out, or to denounce their inadequacy. In this framework, a small network of SHM systems (Fig. 1) was set up by the Department of Civil, Architectural and Environmental Engineering and the Nagoya City University, whose data—also containing several after-shocks records—will constitute a sound database for investigation in the field of CH structures response and strengthening procedures.

In the next paragraph two monuments (i.e. the Spanish Fortress and the Civic Tower) belonging to the SHM network are described in detail, focusing on damage survey activities, visual inspections and dynamic identification tests performed during the emergency phase.



Fig. 1 L'Aquila structural health monitoring (SHM) network installed after the earthquake

4 Case studies description

4.1 I: The Spanish Fortress

The Spanish Fortress of L'Aquila (Fig. 2a) is one of the most impressive Renaissance castles in Central and Southern Italy [17].

In the fifteenth century, L'Aquila became the second most powerful city in the Kingdom of Naples, under the Spanish domination. In 1528, Viceroy Filiberto d'Orange ordered to build a fortress in the highest North spot of the city, according to the project of a famous Spanish architect, Don Pirro Aloisio Escrivà [18]. The construction started in 1534; Escrivà designed a giant fortress, composed by four bastions connected through heavy walls, 60 m long, with a thickness of 30 m at the bottom and 5 m at the top (Fig. 2b). All around the fortress there was a ditch (never filled with water) 23 metres wide and 14 m deep, aimed at defending the foundations from the enemy artillery.

4.1.1 Earthquake-induced damages and provisional interventions

The Spanish Fortress was seriously damaged by the earthquake of the 6th of April 2009. The most relevant damages and collapses involved especially the upper floors of the castle.

According to the damage survey template for palaces used in the technical inspections [19], overturning and flexural mechanisms on the external walls, shear damage in the external and internal walls, damages to vaults and arches, local collapses of floors and vaults, correspond to the most worrying observations. Damages were remarkable both for intensity and distribution, and were considered so serious to be likely menacing the overall stability of some large parts of the building. The seismic event caused the most severe damages on South-East and South-West wings of the fortress. On the external front it can be noted the overturning of the upper masonry façade due to the activation of a hammering mechanism induced by the presence of a stiff reinforced concrete floor (Fig. 3). In the internal front of the same side the pillars of the porch arcade show crushing failure mechanisms. Other damages are located in vaults at the ground floor; shear cracks were surveyed on internal transverse walls as well as local collapses on the SW wing due to the poor masonry quality (Fig. 3b).

The two façades of the North-West part showed a greater resistance to the overturning mechanism; there are no large detachments of the floors from the perimeter walls. In fact, in this area of the fortress a system of tie rods connecting the perimeter walls had been inserted before the earthquake. This intervention was effective and avoided collapses and irreversible damages to the structure. The structural stability was provided by relying on the remaining strength of the resisting elements, e.g. by connecting the internal and external façades of the damaged wings by means of stainless steel cables (Fig. 4), in order to avoid the observed

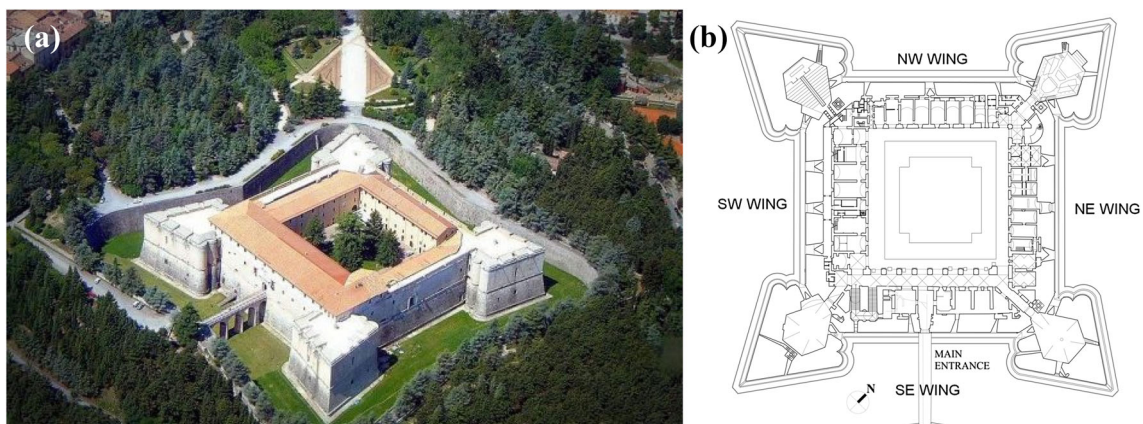


Fig. 2 a Aerial view of the Spanish fortress of L'Aquila, before the earthquake; b plan of the ground floor

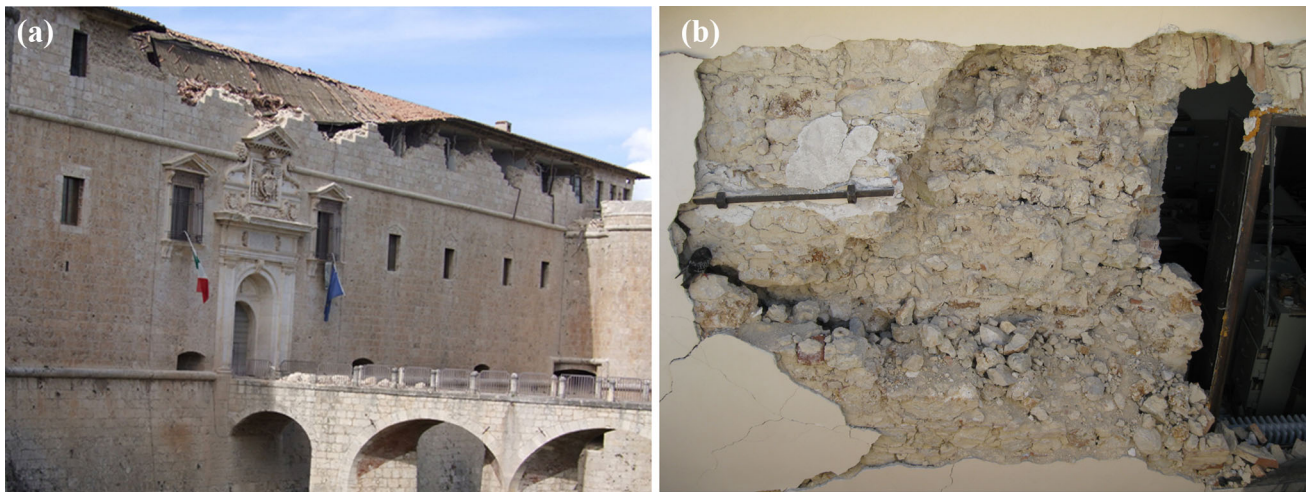


Fig. 3 **a** Out-of-plane overturning of the upper part of the main façade (SE wing); **b** local collapses on the SW wing due to poor masonry quality



Fig. 4 Provisional interventions carried out on SE and SW wings: connection of the internal and external façades by means of steel cables and frames

overturning mechanisms evolution, especially taking into account that non-negligible aftershocks occurred for several months. In the South-East wing, it was necessary to rebuild the roof, by using hollow section steel trusses and a light covering structure made of wood. The substitution of the original wooden structure of the roof with a heavy and stiff reinforced concrete structure, without any strengthening interventions on the underlying masonry walls, caused the collapse of the upper part of the façade. In the South-West wing steel frames were positioned in contrast to the external and internal façades before tensioning the cables.

4.1.2 Modal identification tests

Ambient vibration tests (AVT) were performed on the most damaged part of the fortress, i.e. the SE wing with the aim

of evaluating the dynamic response after the overturning mechanisms activated by the earthquake. From a structural point of view, it was in fact important to verify if the two longitudinal walls (also thanks to the provisional interventions, i.e. tie-rod systems) had still a unitary dynamic behaviour.

A parallel objective was the identification of principal mode shape of the wing to optimize the number and locate acceleration transducers used in the dynamic SHM system.

Five configuration setups were performed with 27 points of acquisition (Fig. 5), using high sensitivity piezoelectric seismic accelerometers. For each setup, no more than eight sensors were used, including two fixed reference sensors, arranged in two directions (X and Y) parallel to the ground and perpendicular to each other (channels 1 and 2 at the second floor). Acceleration transducers were placed along



Fig. 5 Layout of the uniaxial accelerometers for the four setups (S1-red, S2-yellow, S3-green and S4-blue) used for the execution of AVT on the first (a) and second (b) floors of the monument (colour figure online)

the out-of-plane direction on both sides of the wing. The records concentrated mainly on the perimeter walls as the aim was to investigate any dissimilar dynamic behaviour of the two façades due to the heavy state of damage of the structure. For each setup three records (65'536 points each) were acquired with a sampling rate of 100 SPS (samples per second).

Data series acquired have been processed using FDD—frequency domain decomposition [20], EFDD—enhanced frequency domain decomposition [21] methods. Peaks in the frequency domain related to structural modes were selected and the corresponding mode shapes extracted (Fig. 6). The acquisitions of the various setups were first analysed separately and then put together in one global identification. The extraction of modal parameters clearly indicates four mode shapes orthogonal to the façade in the frequency range 2.9–5.6 Hz, higher local mode at 8.8 Hz as well as many other peaks—at higher frequencies—difficult to assess. In Table 1 modal parameters extracted with different operational modal analysis (OMA) techniques are listed and compared. In this paper, it was decided to limit the analysis in the range 0–8 Hz, excluding the local mode from the processing phase of dynamic monitoring data.

The identification of the global vibration modes of the structure indicates that the building, in spite of the high level of damage and the disconnection of the perimeter masonry walls, has still a unitary dynamic response, probably thanks to the provisional emergency interventions that provide a certain level of confinement to the out-of-plane overturning mechanism activated by the earthquake.

4.1.3 The monitoring system

Once the investigation campaign was concluded, a dynamic monitoring system has been installed on the SE wing of the fortress. The system complements a static monitoring installed immediately after the earthquake by the ISCR (Istituto Superiore per la Conservazione ed il Restauro—National Conservation and Restoration Institute) of Rome, devoted to the control of the crack pattern evolution and of the environmental parameters. The dynamic system is composed by an acquisition unit connected to eight high sensitivity piezoelectric uniaxial accelerometers. The central unit, located at the second floor of the fortress, is provided with a Wi-Fi router for remote data transmission.

A couple of reference sensors is fixed at the base of the structure (at the foot of one of the massive pillars on the inner courtyard, with CH1 orthogonal to the façade and CH2 parallel to it) for the record of the ground acceleration both in operational conditions and during seismic events. The positioning of the acceleration sensors on the elevation of the South-East was decided according to the results of the dynamic identification. Sensors were fixed orthogonally to the façade, following vertical and horizontal lines, on the internal and external façades, with an increased number of sensor at the second level (Fig. 7).

Dynamic data are collected both at fixed time intervals ('long' acquisitions, corresponding to 131'072 points, or to 21'51" of record at a sampling frequency of 100 Hz, each 24 h) to allow successive dynamic identification of the structure with different environmental conditions, and on a

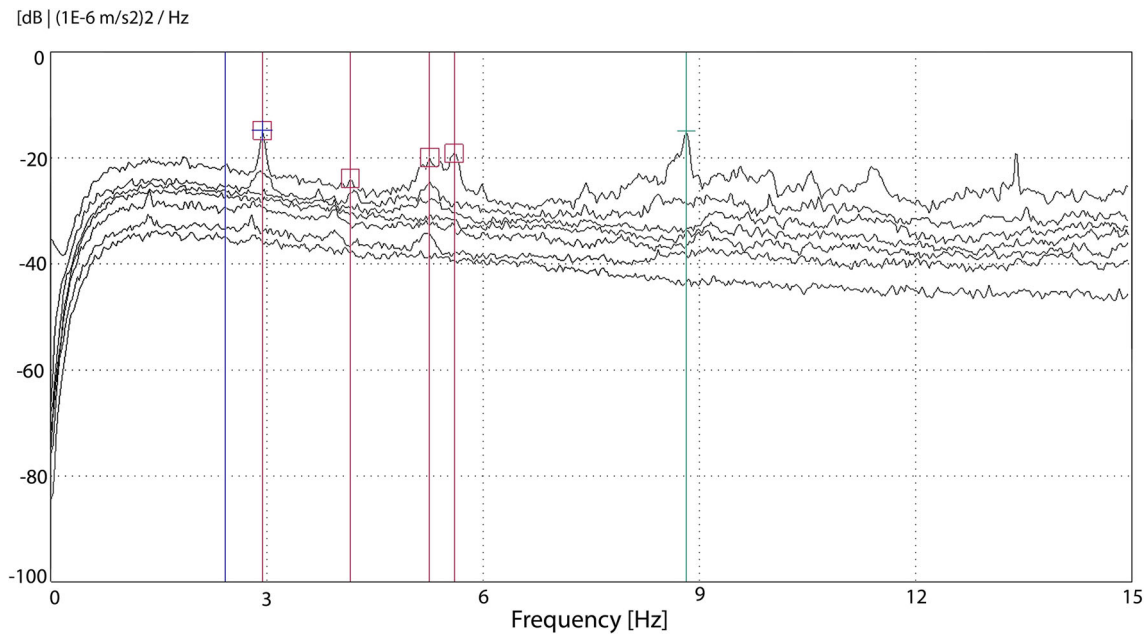


Fig. 6 Frequency domain decomposition—peak picking. Average of the normalized singular values of the spectral density matrix: peaks indicate the structural frequencies

Table 1 Modal parameters estimation of the Spanish Fortress

Mode	FDD	EFDD		MAC
	f (Hz)	f (Hz)	ζ (%)	
1	2.930	2.939	1.21	0.99
2	4.150	4.151	0.90	0.71
3	5.249	5.302	1.55	0.98
4	5.591	5.467	2.53	0.96
5	8.813	8.801	1.07	0.97

trigger basis (shorter records, 3'35" at a sampling frequency of 100 Hz), when the signal, on one of the acceleration channels, exceeds the predefined threshold (meaningful event, e.g. earthquake), either in frequency or time domain. One dataset of long time series per day is processed by algorithms for automated OMA and used to trace the variation of the modal parameters over time.

Temperature data are collected every 30 min by seven thermocouples, belonging to the static system managed by ISCR institute of Rome. Sensors are installed at the ground and first floor (both inside and outside) in correspondence to the displacement sensors that control crack patterns activated by the earthquake.

4.2 Case study II: the Civic Tower

The Civic Tower is an emblematic historic structure, part of the l'Aquila City Hall. The complex is composed by two bodies, characterized by a different historical-constructive evolution: the Civic Tower and the Margherita Palace (Fig. 8a).

The first one was constructed before the foundation of the city in 1254, and originally conceived as an isolated structural element. The Civic Tower, at the time of its construction, was 70 m tall, but in 1703 it was lowered after the earthquake and its height reduced again in 1838 because of the presence of diffused cracks and the collapse of the upper part of the structure. Nowadays the tower is 42 m high. The construction of the Margherita Palace began in 1294 and the building underwent several restorations and reconstructions over centuries. The most important intervention dates back to 1572 when Margherita of Austria, governor of the city, chose it for her residence. Then, three centuries later (1836–46), another restoration changed drastically the spatial configuration of the building that appeared more modest and in some way more coherent with his new function as location of the Judicial District [22].

Palace and tower experienced several strong earthquakes over their long history: the most catastrophic and destructive ones happened in 1349, 1461 and in 1703. After each seismic event the buildings were strongly damaged and subsequent restoration works have been required to consolidate their structural members.

4.2.1 Earthquake-induced damages and provisional interventions

The structure was severely damaged by the strong earthquake occurred on the 6th of April 2009. The tower (6.5 m \times 7 m base and 43 m high) is a structural element particularly vulnerable due to its tall and slender shape and

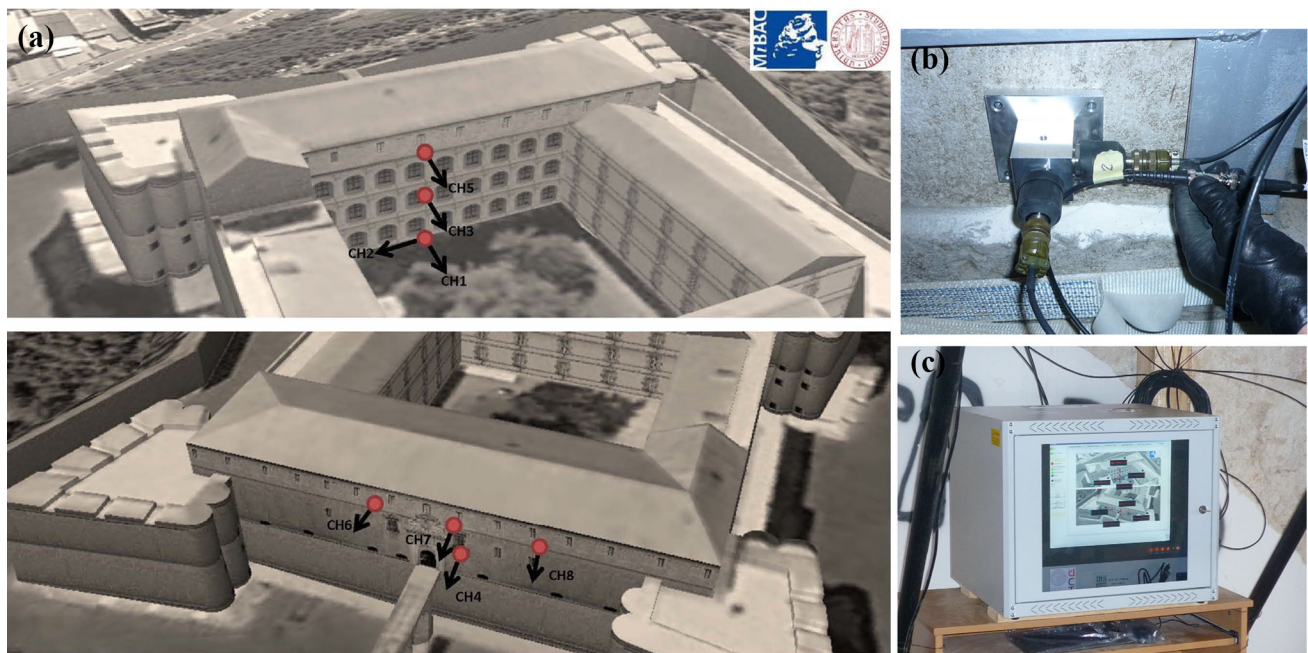


Fig. 7 a Layout of the dynamic monitoring system installed on the SE wing of the fortress at the ground floor, first and second levels; b accelerometers installed at the base; c acquisition unit on the second floor

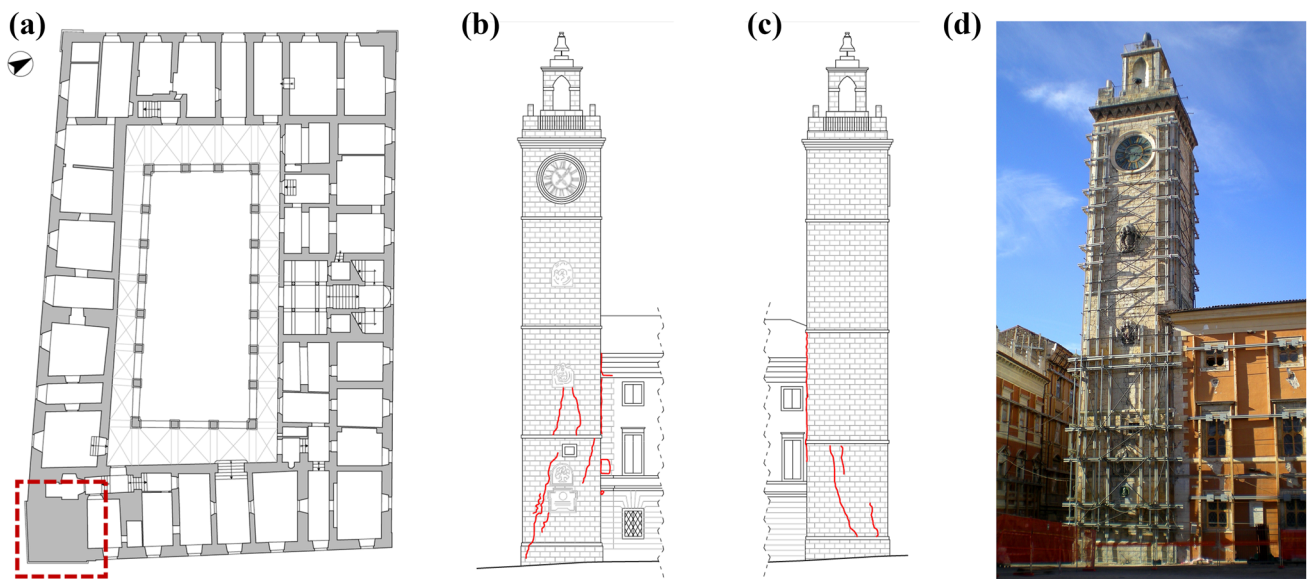


Fig. 8 a Plan of Palazzo Margherita with the Civic Tower at the SE corner. Crack pattern survey on the East (b) and South (c) elevations. d Provisional interventions after the earthquake

the scarce mechanical characteristics of the constitutive masonry walls. The earthquake induced a diffused and serious crack pattern along the entire shaft of the tower with damage concentration at its basement (Fig. 8b, c).

The tower was heavily damaged due to its tall and slender shape that made it more vulnerable to base settlements and movements induced by seismic forces. The tower slenderness and cantilever beam-type boundary conditions have made it unsuitable for redistributing

stresses and dissipating energy. The earthquake caused a concentration of stresses and damages at the basement, on the South and East sides, worsened by the low mechanical properties of masonry. Vertical cracks were surveyed at the tower–palace interface, denoting a relative displacement and disconnection between the two structures.

Provisional interventions were designed and implemented during the emergency phase in order to prevent further failure and collapses. The main strengthening

Table 2 Modal parameters estimation of the tower

Mode	FDD	EFDD	ζ (%)	p-LSCF		MAC	Comment
	f (Hz)	f (Hz)		f (Hz)	ζ (%)		
1	1.563	1.563	0.984	1.564	0.424	0.99	First bend. EO
2	1.582	1.583	0.907	1.582	0.432	0.98	First bend. NS
3	3.301	3.292	1.352	3.288	0.605	0.89	Second bend. NS
4	3.711	3.699	1.143	3.699	0.702	0.87	Second bend. EO
5	4.512	4.521	0.877	4.515	0.697	0.92	First torsion
6	5.527	5.457	0.581	5.534	0.771	0.78	Third bend. NS
7	6.445	6.434	1.229	6.419	0.462	0.76	Third bend. EO

solutions designed and implemented to the structure are: (1) positioning of a confining system of the tower composed by steel beams, ties and wooden frames at different heights along the shaft in order to stabilize the structural element (Fig. 8d); (2) improvement of the connection of the tower with the palace; (3) propping system of the wings of the palace by means of steel beams, plates and ties in order to avoid out-of-plane overturning mechanisms and collapses.

4.2.2 Modal identification tests

The precarious structural conditions of the building suggested installing, in addition to the temporary emergency interventions, a permanent structural health monitoring system. The system gives interesting information on the progression or stability of the assessed damage pattern, with reference to the already carried out stabilization actions and the foreseen strengthening interventions.

Before the installation of the SHM system, ambient vibration tests (AVT) were performed in July 2010. The dynamic behaviour was evaluated in the damaged conditions of the tower, after the earthquake. AVT were executed on the tower and the two wings of the palace (Eastern and Southern facades) directly connected with the tower in order to evaluate the dynamic behaviour of the whole structural complex and appraise if the tower-palace system had still an unitary dynamic response even after the high level of damage and disconnection induced by the earthquake. Other important objectives of dynamic tests are the definition of the optimal layout of the SHM system sensors and the characterization of the dynamic properties for the calibration of numerical models.

It was decided to use 32 single-axial piezoelectric acceleration transducers. Once fixed the transducers to the structure in the selected positions, tests consisted in acquiring data in three different registrations over a pre-determined period, at a specific sampling rate. A typical acquisition consisted in a record length of 144'000 samples, resulting in an acquisition time of approximately 30 min at a sample rate of 80 Hz. For the identification of the modal

parameters (natural frequencies, damping ratios and mode shapes), output only identification techniques were used.

The signal-processing phase was performed using dedicated software for OMA: SVS ARTeMIS Extractor 4.0, 2007 [23] and MACEC 3.2, 2011 [24]. It was decided to use three frequency-domain system identification techniques: FDD [20], EFDD [21] and pLSCF—poly-reference Least Squares Complex Frequency-domain [25]. Peaks in the frequency domain related to structural frequencies were selected and the corresponding mode shapes defined (Table 2, Fig. 10).

Modal analysis results show that the first two orthogonal bending modes have very closely spaced frequencies (in the range of 0.02 Hz) around 1.5 Hz. Implementing various identification techniques and in particular the p-LSCF, it was possible to separate clearly these closely spaced modes (Fig. 9). This phenomenon is pretty common for symmetric structures like the Civic Tower. It is interesting to notice that also the second-order bending modes are rather closely spaced (in a higher range of 0.4 Hz between 3.3 and 3.7 Hz).

Table 2 and Fig. 10 summarize the seven estimated modal parameters through three frequency-domain output-only techniques, in terms of resonant frequencies, damping coefficients and mode shapes. Looking at natural frequencies, the values range from 1.5 to 6.4 Hz and no significant differences could be found between the three methods.

Observing the MAC values calculated between mode shapes extracted from EFDD and p-LSCF methods, it is possible to state that the first two bending modes and the torsional mode are highly correlated (values higher than 0.87). For the higher modes (3rd bending modes) the MAC index decreases to a minimum of 0.76, meaning that the estimation of higher modes is more difficult with ambient vibration measurements.

4.2.3 The monitoring system

The monitoring system installed in the Civic Tower is composed by (1) static sensors to control the damage and

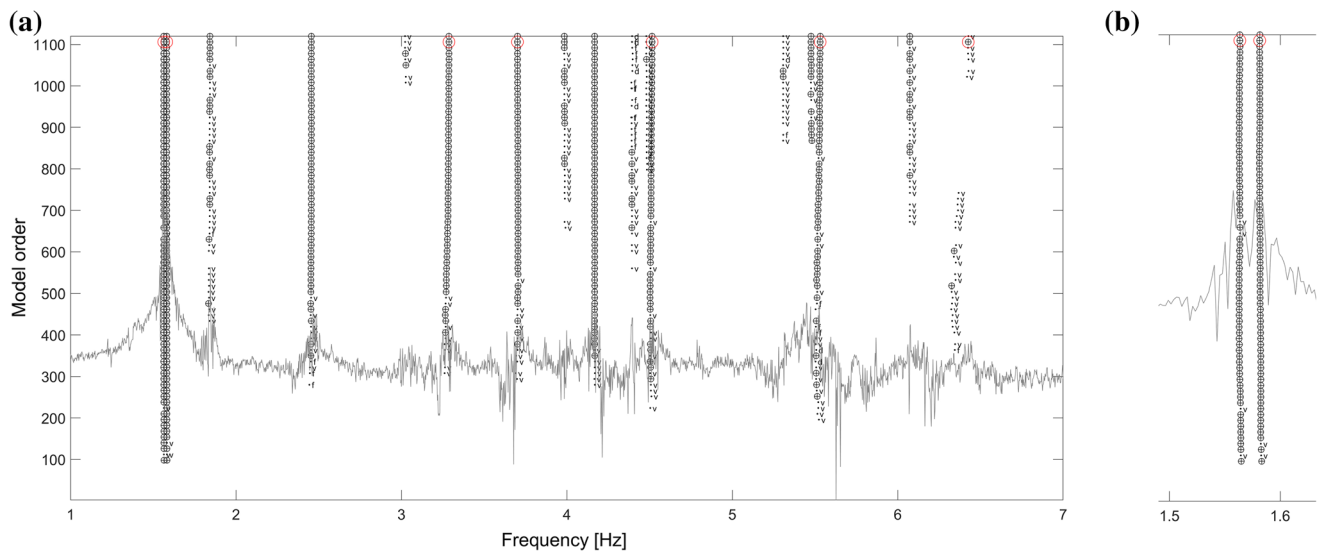


Fig. 9 p-LSCF method: **a** stabilization diagram (left). **b** Closely spaced bending modes around 1.5 Hz

crack pattern of the structure and (2) accelerometers to measure ambient vibrations and capture possible aftershocks and seismic events.

The static system includes devices to measure a set of displacements and strains at critical points of the building. Three displacement transducers were installed in the lower part of the tower to control the crack pattern and two on the crack between tower and palace to control the relative displacements of the two structures. Other devices complete the static system: one inclinometer (Fig. 11c) to control the displacement of the tower top along two in-plane orthogonal directions, six strain gauges (ST1 TO ST6) on the existing metal ties of the tower to control the strain variation and six thermocouples (T1–T6) to control both air and walls temperatures in different points of the structure. Data from the static system are registered every 30 min.

The dynamic monitoring system (Fig. 11a) is composed by eight high sensitivity piezoelectric accelerometers (Fig. 11b) connected to an acquisition unit. Three reference sensors are fixed at the base of the structure to record the ground acceleration during possible aftershocks. The positioning of the other acceleration sensors was decided on the results of the dynamic identification. Unlike the dynamic monitoring system of the Spanish Fortress, this system is not trigger-based but continuous and high-density dynamic information is continuously recorded. Time series are continuously collected at a sampling frequency of 80 Hz and records are split into files of 1 h length. 1 dataset per day is processed by algorithms for automated OMA and used to trace the variation of the modal parameters over time. Given its significance, particular

attention is given to the monitoring of the variation of temperature in the building by means of six thermocouples. All of them are installed inside the tower: three record wall temperatures and three air temperature. Their position is indicated in Fig. 11. The system is equipped with a router for remote data transmission.

5 Structural assessment and damage detection

This paragraph describes the procedures implemented to analyse and interpret monitoring data of the two SHM systems. The aim was to develop data-driven approaches to filter out the environmental effect and establish objective criteria for damage detection. In this research ARX models with single input and single output (SISO) are applied. This choice was made considering that the monitoring systems of both the Civic Tower and the Spanish Fortress have been active for 2.5 and 3 years, respectively, without any significant system malfunctioning and loss of data. Such monitoring periods are long enough to collect data related to a large range of variation.

In particular for both case studies features extracted from the first year of monitoring are implemented to establish the best statistical model of the considered parameters, i.e. the model that better described the influence of the independent inputs (environmental factors) on the recorded outputs. Afterwards, in the second step, the constructed model is exploited to predict the outputs in the following years, comparing the predicted response with the actual measured parameters [26].

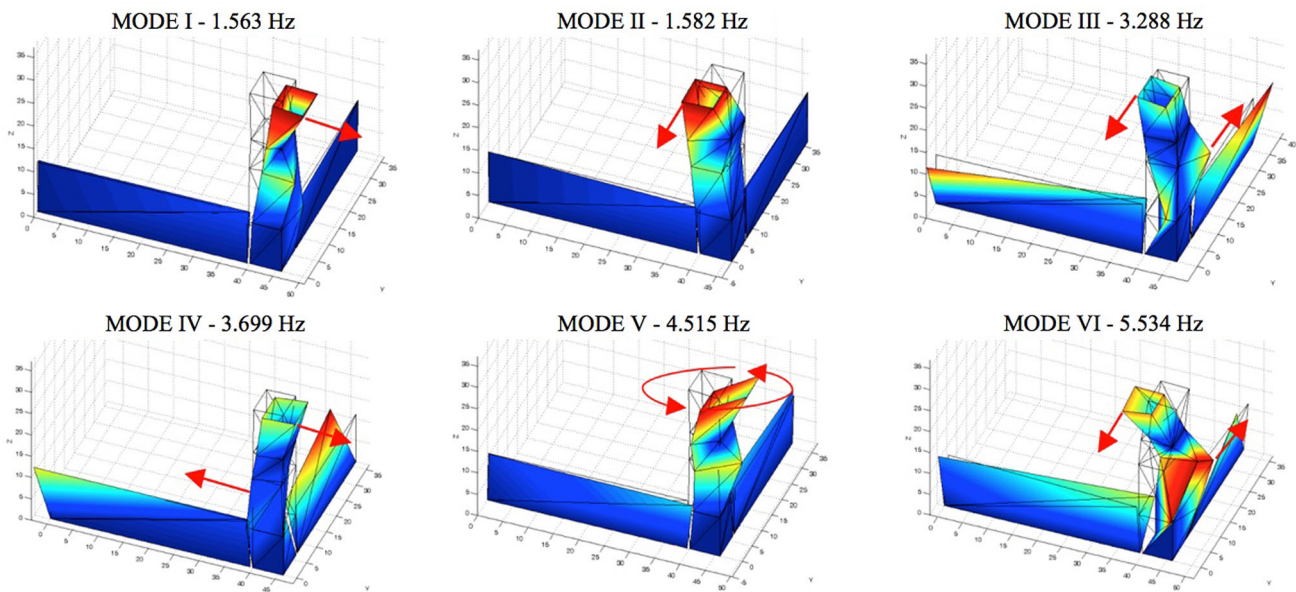


Fig. 10 Mode shapes identified for the first seven natural frequencies of the tower

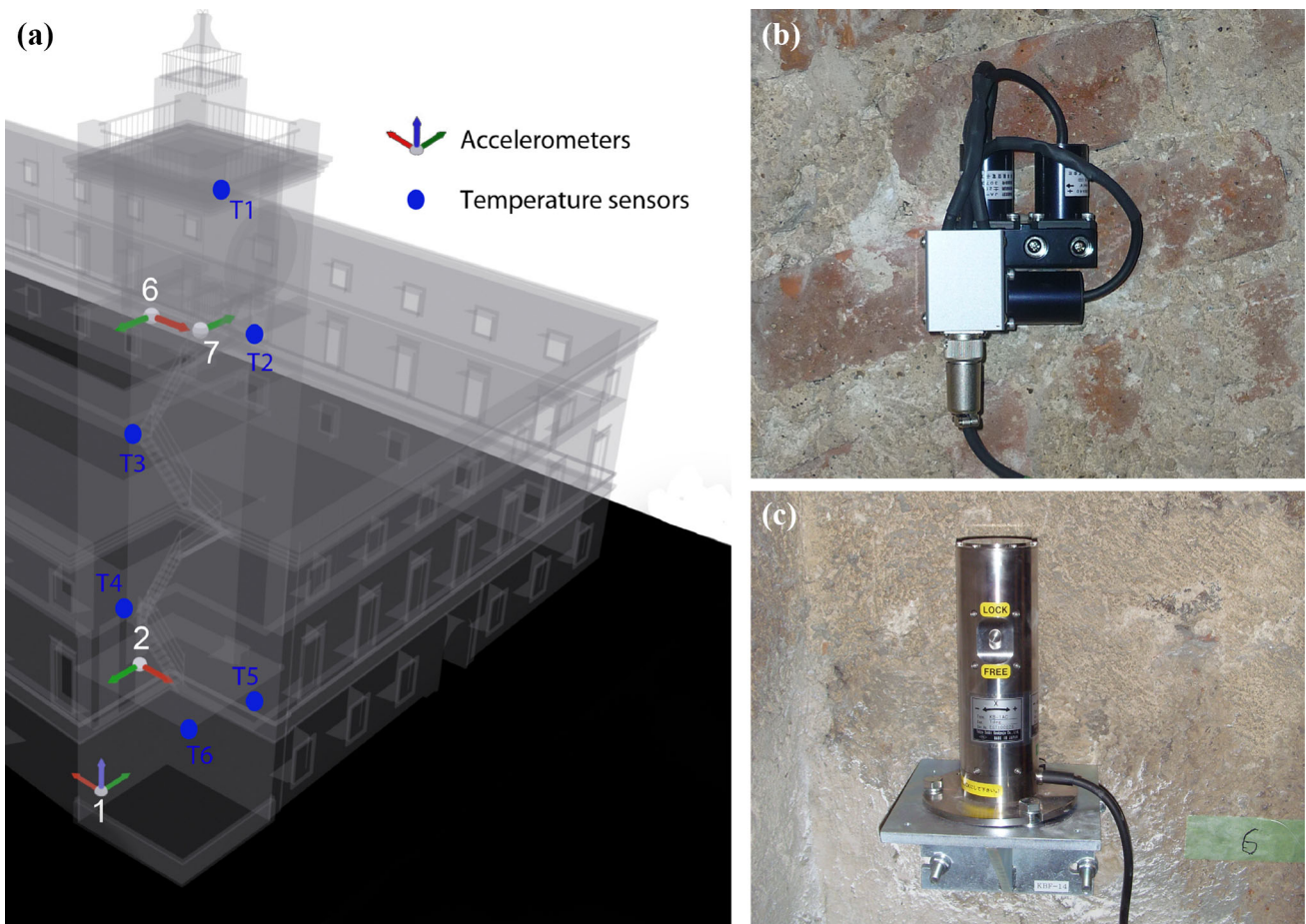


Fig. 11 a Layout of the dynamic system with eight high sensitivity piezoelectric accelerometers (b) and six temperature sensors; c inclinometer at tower top

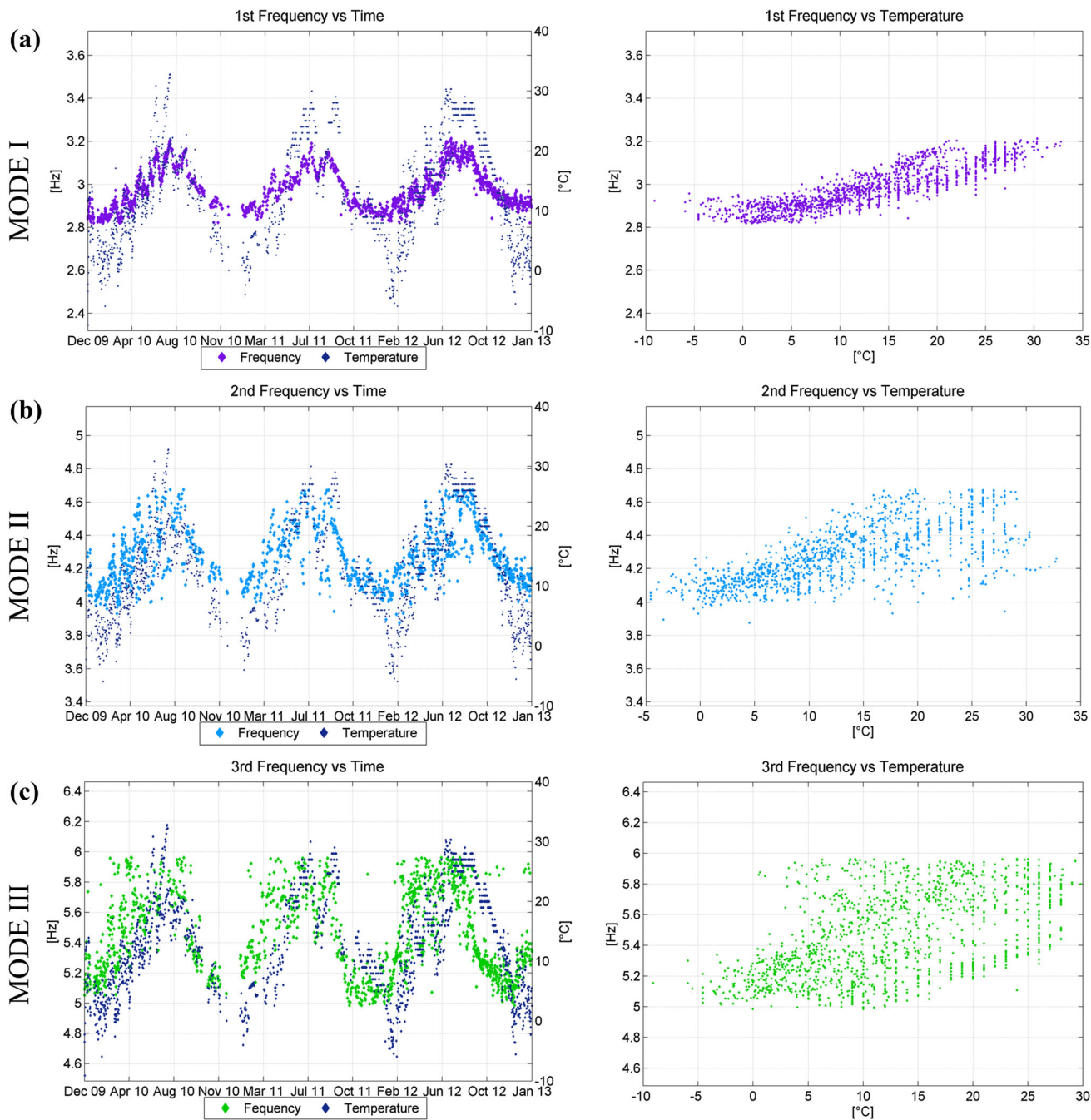


Fig. 12 Variation of the first (a), second (b) and third (c) natural frequencies of the Spanish Fortress during the monitoring period in function of time (left) and temperature (right)

Table 3 Statistical results of the first five natural frequencies of the Spanish Fortress

Mode #	f_{max} (Hz)	f_{min} (Hz)	f_{mean} (Hz)	f_{change} (%)	f_{std} (Hz)	f_{cv} (%)
1	3.213	2.819	2.982	13.98	0.097	3.26
2	4.675	3.876	4.274	20.60	0.172	4.02
3	5.963	4.985	5.457	19.63	0.278	5.10
4	6.201	5.542	5.872	11.91	0.153	2.61
5	9.668	7.069	8.963	36.77	0.505	5.80

Time series, collected by both systems are transmitted to the central server of the University of Padova and automatically processed on arrival. Here a processing software for automated OMA based on the parametric frequency domain pLSCF method [25] was recently developed [27]. The automated routine includes the execution of the following tasks:

- Pre-processing of data to eliminate the offset and decimate the sampling frequency in the range of interest (usually 0–20 Hz).
- System identification using the p-LSCF method and creation of stabilization diagrams.
- Analysis of the obtained stabilization diagrams through a hierarchical clustering algorithm and automatic extraction of modal parameters.
- Creation of a database with the results of the processing and display of plots with the most relevant results using a graphical user interface.

5.1 I: Spanish Fortress

For the Spanish Fortress a database with the variation of the modal parameters extracted from vibration signatures over more than 3 years is available (Fig. 12). The statistical results of natural frequency variations are presented in Table 3. It is possible to observe that natural frequencies are strictly correlated with temperature changes: in particular frequencies decrease during cold periods and increase during summer. Looking at these results one can notice that changes are significant (0.4 Hz for mode #1, 0.8 Hz for mode #2, 1 Hz for mode #3) and temperature is the major factor that influence the annual fluctuations.

Since continuous temperature measurements are available, ARX models are tested. Data collected during the first year of monitoring (from 20/12/2009 to 20/12/2010) are used to build the regression models and then data collected during the remaining 2 years of monitoring (from 21/12/2010 to 22/01/2013) were used to validate the quality of the forecasts provided by the models, trying to detect through residual analysis any changes in the response, possibly linked to damage.

In this procedure, it is assumed that during the first period (estimation phase) the damage pattern induced by the earthquake to the building is stable, also thanks to the provisional strengthening interventions performed by fireman during the emergency phase. The idea is to validate the long-term effectiveness of the performed interventions.

As explained in §2, as a first step, correlation analysis was performed to calculate the correlation coefficient between natural frequencies and temperature data (Table 4). It is possible to state that the correlation is very well fulfilled until the fourth frequency (with maximum

correlation coefficients equal to 0.90 for mode #1, 0.81 for mode #2, 0.64 for mode #3 and 0.66 for mode #4).

After the preliminary selection of the prediction variables, ARX models were created selecting temperatures T6 or T7 as input variables and the time series of the natural frequencies (f_1, f_2, f_3, f_4) as outputs. Means are removed from the input (x_k) and output (y_k) data (data normalization):

$$x_{k,\text{norm}} = \frac{x_k^m - \bar{x}_k}{\sigma_{x_k}} \quad y_{k,\text{norm}} = \frac{y_k^m - \bar{y}_k}{\sigma_{y_k}},$$

where x_k^m and y_k^m are the measured inputs and outputs, \bar{x}_k and \bar{y}_k are the mean values and σ_{x_k} and σ_{y_k} are the standard deviations.

The strategy for the selection of the best ARX model that can be fitted to the automatically identified natural frequency is the following: A MATLAB subroutine was implemented in order to calculate ARX models for increasing order of n_a, n_b and n_k . The maximum order was fixed at 10: ARX $[n_a, n_b, n_k] = [1 : 10, 1 : 10, 1 : 10]$. Among the 10^3 possible ARX models obtained, the best one was selected according to quality criteria defined in §2 (Table 5). In particular the ARX model with the lower λ_0 , the lower FPE and the higher R^2 was finally selected as the ‘best’ model able to explain and fit better the measured data (i.e. natural frequencies).

From the comparison of the simulated natural frequencies and their identified counterparts, the changes in identified natural frequencies caused by structural damage can be distinguished from those caused by varying

Table 4 Correlation coefficients between all the frequency–temperature pairs

	f_1	f_2	f_3	f_4	f_5
T1	0.88	0.79	0.61	0.63	0.40
T2	0.89	0.79	0.61	0.63	0.40
T3	0.88	0.78	0.60	0.63	0.40
T4	0.88	0.78	0.60	0.62	0.40
T5	0.88	0.78	0.61	0.63	0.40
T6	0.90	0.81	0.64	0.66	0.41
T7	0.90	0.80	0.64	0.66	0.41

Table 5 Selection of ARX models for the first four natural frequencies of the fortress

Mode #	n_a	n_b	n_k	λ_0	FPE	R^2
1	9	7	0	0.0004	0.0004	0.85
2	4	10	0	0.0075	0.0081	0.68
3	10	5	0	0.0130	0.0141	0.70
4	10	8	0	0.0036	0.0039	0.73

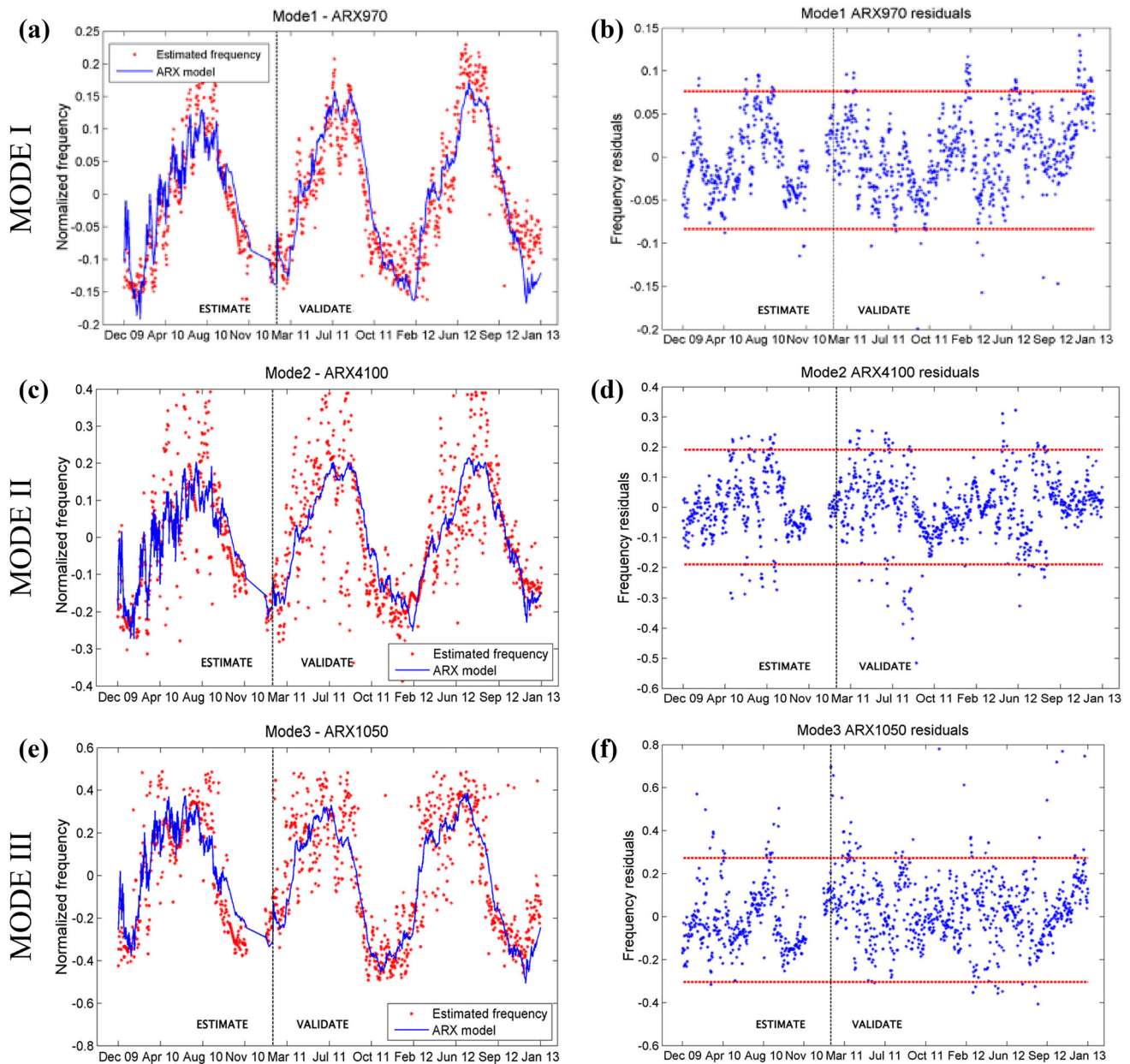


Fig. 13 Comparison between identified (red dots) and simulated (continuous blue line) frequencies for the first three modes of the structure (a–c). Analysis of residuals (blue dots) between measured

environmental conditions. The standard deviations can be used to establish confidence intervals (see Eq. 8) around the predicted values.

Figure 13 (left side) shows the comparison between the estimated frequencies, automatically identified by the subroutine, and the same modal parameters simulated by the ARX model. The vertical line splits data into two parts: an estimation period, where both input and output data (measurements) are used to estimate the statistical model and the validation period, where fresh input data related to environmental parameters are used to predict the response in terms of natural frequencies.

and predicted values of the same modal parameters with 95% confidence intervals (d–f) (colour figure online)

In Fig. 13 (right side) also the residuals, defined as the measured values minus the predicted values, with the 95% confidence intervals, are given. If the measured natural frequencies stand constantly outside the confidence intervals, it means that other parameters (in addition to temperatures) are influencing their behaviour or it is likely that the structure has been damaged.

Based on these figures, it is observed that the identified natural frequencies are included, in general, within the confidence intervals, meaning that no further damages occurred on the structure during the validation period

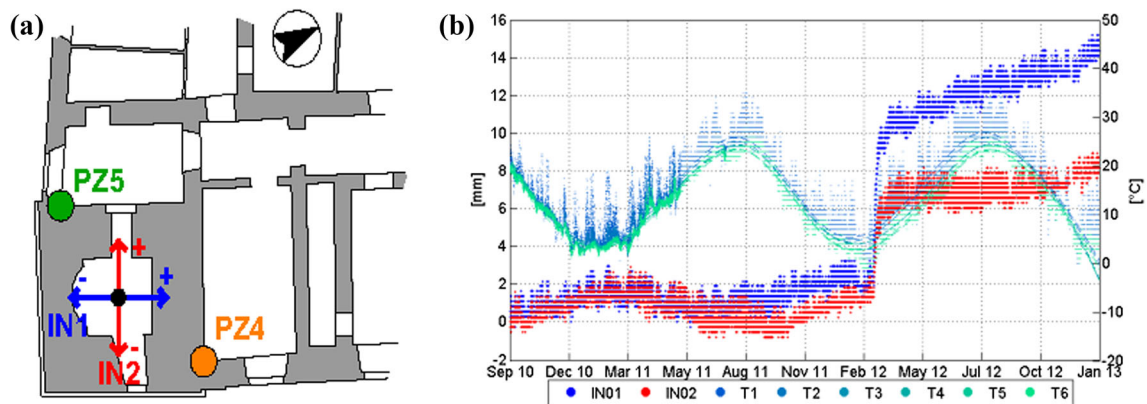


Fig. 14 Static monitoring of the Civic Tower: **a** layout of inclinometers (IN1 and IN2) and displacement transducers (PZ4 and PZ5); **b** top tower displacements recorded by inclinometers along N-S (IN1-blue) and E-W (IN2-red) directions [26] (colour figure online)

(Fig. 13). This conclusion is consistent with visual inspections performed on the Spanish Fortress that confirm that damage and crack patterns induced by the earthquake are rather stable also thanks to the provisional strengthening measures implemented immediately after the seismic event.

However, it is also observed that the identified natural frequencies during some short periods move out of the bounds of the confidence intervals. This could be due to the fact that other parameters influence natural frequencies, e.g. loading conditions. Therefore, these ARX models are not always capable of completely removing the environmental effects and they could be improved and validated by means of the introduction of other input parameters that might influence the structural response.

5.2 II: Civic Tower

The same procedure for the analysis of the influence of environmental parameters on the dynamic characteristics is applied on the Civic Tower.

Before implementing statistical models to simulate and predict the response of the monitored structure, main results of the static monitoring are discussed, as they help understanding the outcomes of the identified modal parameters during more than 2.5 years of structural controls.

During the first 1.5 years of monitoring the crack and damage pattern of the tower kept rather stable, also thanks to the provisional interventions implemented immediately after the earthquake. The strengthening measures concentrated in particular on the shaft of the tower, where a system of metal and wooden frames was installed to create a strong confinement along the entire height of the structural element. Additional ties were inserted to improve the tower-palace connection.

In February 2012 the equilibrium conditions of the tower underwent a significant change due to a slight

rotation/displacement of the tower toward the palace, as recorded both by the inclinometer (Fig. 14b) and by the displacement transducers placed at the tower-palace interface, in correspondence to the big lesion that separate the two structures.

The dynamic system has been working from the 22/07/2010 to 09/01/2013. Data recorded by the system are transmitted to the central server of the University of Padova and automatically processed on arrival by the algorithm for automated SHM. Data processing allowed to extract the first seven natural frequencies.

Figure 15 shows the evolution over 2.5 years of monitoring of the first, third and fourth natural frequencies of the tower and the dependency on environmental factors through frequency vs. temperature plots.

It is possible to note that starting from February 2012 all the natural frequencies of the tower tend to slightly increase, exactly in the same period when the anomalous static displacements of the structure were recorded. The frequency change can be appreciated also looking at frequency vs. temperature plots, where for almost all modes it is possible to distinguish two parallel clouds of points, demonstrating the frequency shift not correlated with ambient factors.

Also in the present application, as continuous temperature measurements are available, ARX models are tested. Data collected during the first year of monitoring (from 22/07/2010 to 22/07/2011) were used to build the regression models and then data collected during the remaining 1.5 years of monitoring (from 23/07/2010 to 09/01/2013) were used to validate the quality of the forecasts provided by the models, trying to detect through residual analysis any changes in the response, possibly linked to damage.

The statistical results of the evolution of natural frequencies over the entire period of monitoring are presented in Table 6. The results of the correlation analysis (Table 7) show that the variation of the first two frequency (first

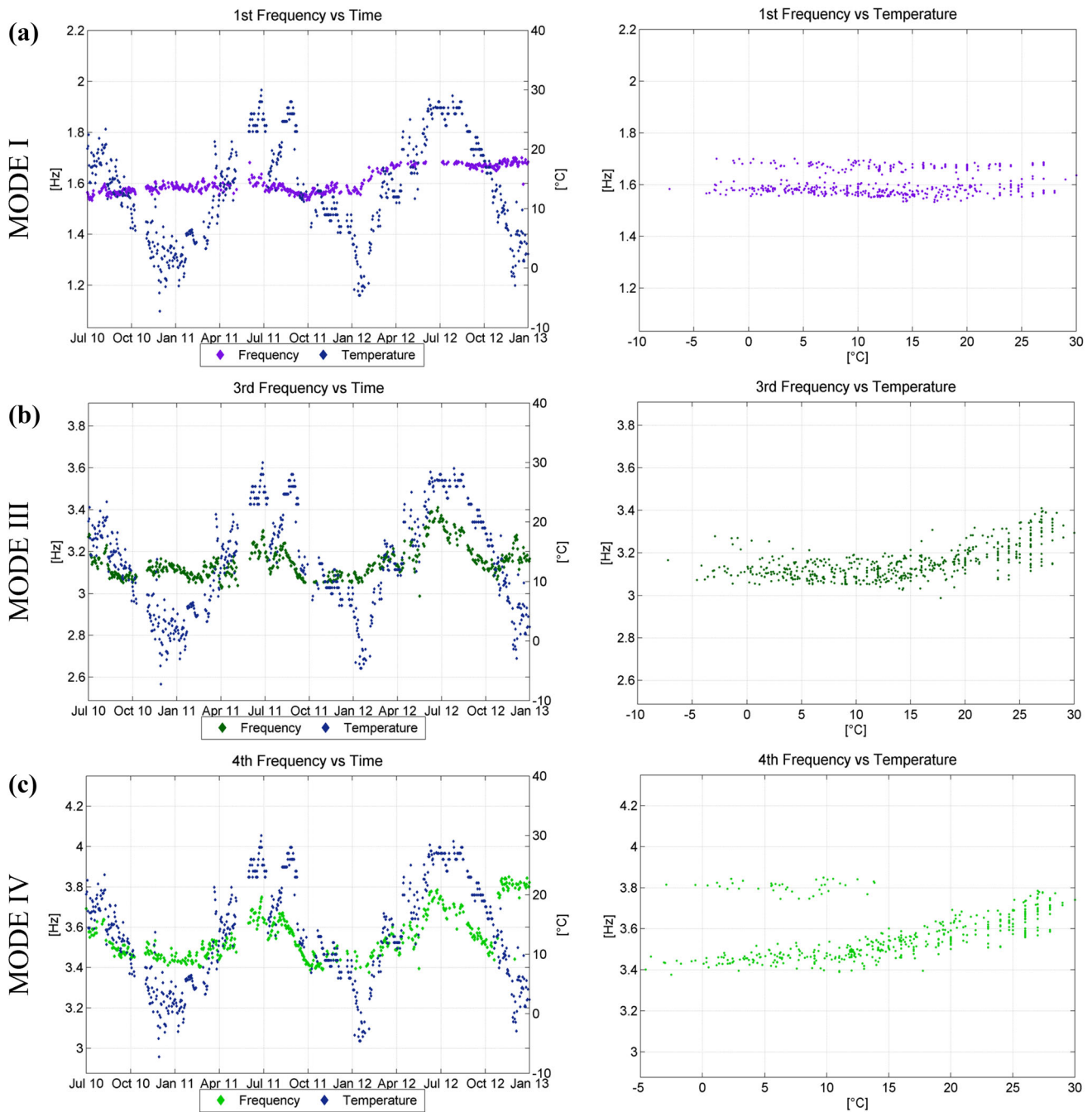


Fig. 15 Variation of the first (a), third (b) and fourth (c) natural frequencies of the Civic tower during the monitoring period in function of time (left) and temperature (right)

order bending modes) is poorly correlated with temperature outputs, meaning that temperature do not fully describe the evolution of the modal parameters. Natural frequencies are much better correlated to temperatures for modes #3 and #4 (second order bending modes) and for the torsional mode (#5). The last two eigenfrequencies (third-order bending) present the lowest correlation coefficients and thus are excluded from the analysis. The main reason can be

attributed again to the poor quality of the estimates for the higher frequencies.

Following this analysis, it was decided to develop statistical ARX models only for the first five natural frequencies of the tower. The statistical procedures applied here give also the possibility to select $T1$ and $T5$ as the best predictors.

Applying the same procedure described for the previous case study SISO ARX models have been constructed.

Table 6 Statistical results of the first seven natural frequencies of the Civic Tower

Mode #	f_{\max} (Hz)	f_{\min} (Hz)	f_{mean} (Hz)	f_{change} (%)	f_{std} (Hz)	f_{cv} (%)
1	1.701	1.533	1.604	10.92	0.047	2.93
2	1.752	1.531	1.642	14.44	0.060	3.64
3	3.410	2.988	3.150	14.09	0.076	2.42
4	3.849	3.377	3.558	14.00	0.118	3.32
5	5.291	4.391	4.692	20.48	0.173	3.69
6	5.989	5.328	5.566	12.41	0.152	2.73
7	7.251	5.786	6.305	25.32	0.232	3.68

Table 7 Correlation coefficients between all the frequency–temperature pairs

	f_1	f_2	f_3	f_4	f_5	f_6	f_7
T1	0.23	0.20	0.50	0.84	0.41	0.13	0.14
T2	0.15	0.27	0.49	0.81	0.48	0.10	0.14
T3	0.05	0.35	0.44	0.76	0.54	0.04	0.14
T4	0.04	0.35	0.44	0.75	0.54	0.04	0.13
T5	0.03	0.36	0.43	0.75	0.55	0.04	0.13
T6	0.09	0.33	0.46	0.78	0.53	0.05	0.15

The results are presented in Table 8 where the best fitting ARX models for the selected five natural frequencies of the tower are presented, compared with the results of the corresponding static regression models. The analysis of results indicates acceptable quality of the developed statistical models, especially for modes #3, #4 and #5. As can be expected the quality of the ARX models of the first two frequencies is lower due to the poor correlation between inputs and outputs.

Figure 16 (left side) shows the comparison between the estimated frequencies, automatically identified by the subroutine, and the same modal parameters simulated by ARX models, whereas in Fig. 16 (right side) also the residuals, defined as the measured minus the predicted value, with the 95% confidence intervals, are given.

The analysis of these plots demonstrates that until February 2012 the earthquake-induced damages were rather stable, as the frequency residuals are always included within the confidence intervals. Starting from February 2012, as also highlighted by the analysis of static parameters, the equilibrium conditions of the tower changed significantly due to a substantial displacement of the structure toward the palace in both horizontal directions. Changes in the structural and boundary conditions of the tower led to an evident variation also of the dynamic response. The slight rotation/displacement of the tower toward the palace caused in fact an increase of the degree of interconnection between the two structures and

Table 8 Selection of ARX models for the first five natural frequencies of the Civic Tower

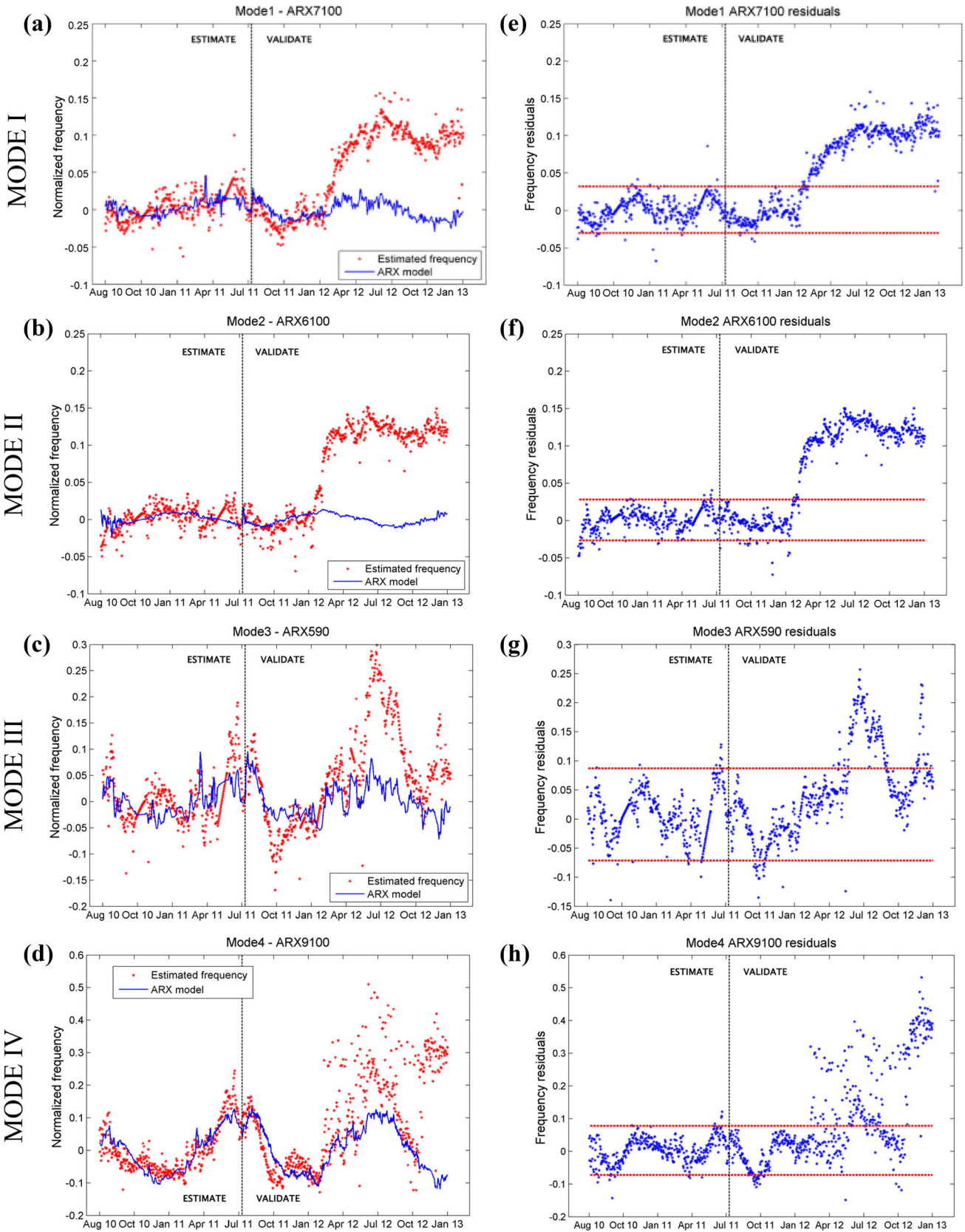
Mode #	n_a	n_b	n_k	λ_0	FPE	R^2
1	7	10	0	0.0001	0.0001	0.52
2	6	10	0	0.0001	0.0001	0.38
3	5	9	0	0.0004	0.0004	0.54
4	9	10	0	0.0004	0.0005	0.85
5	0	10	0	0.0051	0.0054	0.54

modifications of the restraint system offered by the palace. The new mechanical equilibrium conditions, characterized by stiffer constraints, are responsible of the increment of natural frequencies, clearly visible from the residuals of all analysed vibration modes. This phenomenon is particularly evident for the first two closely spaced bending modes of the tower, characterized by the highest quality of the modal estimates.

Thanks to the adopted statistical-based procedures for damage identification it was possible to detect successfully significant changes in the structural conditions of the building, cross correlating static monitoring outputs with the outcomes of modal parameters estimation.

6 Conclusions

The paper reports the application of SHM techniques to cultural heritage buildings severely damaged by an earthquake. After the execution of visual inspections, damage surveys and OMA, a network of static and dynamic sensors, connected to an acquisition unit, was installed on two case studies in I'Aquila: the Spanish Fortress and the Civic Tower. The aim of monitoring is to evaluate the effectiveness of the provisional interventions performed immediately after the seismic event and detect any changes in the structural response caused by a worsening of the damage and crack pattern.



◀ **Fig. 16** Comparison between identified (red dots) and simulated (continuous blue line) frequencies for the first four modes of the structure (a–d). Analysis of residuals (blue dots) between measured and predicted values of the same modal parameters with 95% confidence intervals (e–h) (colour figure online)

It has been demonstrated that static and dynamic parameters are strongly influenced by environmental effects that may shadow an ongoing damaging process on the structure. A data-driven approach is tested to verify the possibility to apply robust statistical methods and damage detection algorithms able to filter out environmental factors. To this aim regression analysis and black-box models (i.e. single input single output ARX models) were applied. Natural frequencies variations of the two analysed buildings were used for the application of the proposed methodology, since they provide information on the global structural behaviour.

Promising results have been obtained. In one case (i.e. Spanish Fortress) it was demonstrated that the damage pattern induced by the earthquake is rather stable also thanks to the provisional strengthening measures applied immediately after the seismic event. In the other case (i.e. Civic Tower) a significant variation of the equilibrium and boundary conditions of the structure were revealed during the monitoring period, combining and cross correlating static and dynamic information.

The application of damage detection algorithms based on vibration signatures on masonry structures provides interesting results. However, it is necessary to highlight some drawbacks and limitations. SISO ARX models are not always capable of completely removing the environmental effects. This means that temperature is not the unique parameter that influences natural frequencies variation (relative humidity and loading conditions could play for example a significant role). A possible improvement could be the implementation of more complex statistical models with the introduction of other input parameters. Masonry buildings are usually massive and sometimes the frequency content and the amplitude of ambient vibrations (exploited by dynamic monitoring) are not always sufficiently appropriate to excite uniformly all the structural modes in the frequency band of interest. For this reason the extraction of modal parameters is not trivial and it is affected by large uncertainties. Moreover, damage on masonry structures is usually spread and diffused, not concentrated on specific points or nodes (as in the case of reinforced concrete or steel structures) and the application of damage detection algorithms based on natural frequency variation can be unsuccessful.

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