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Adaptive Observer Based Backstepping Controller Design for Dynamic Ship Positioning

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Abstract
Modified adaptive observer based backstepping control system for dynamic positioning of ship is proposed. As an improvement, the adaptive observer takes the first-order wave frequency model and the bias term which represent the slowly varying environmental disturbances and the unmodeled dynamics. Thus, the wave-frequency motions are filtered out, and only the reconstructed low-frequency motions are sent as inputs of the controller. Furthermore, as the ship dynamics parameters are unknown, the adaptive estimation law is designed for both the unknown ship dynamics and the unmeasured state variables. Based on the estimated states and parameters, backstepping controller considering the integral action is designed. Global exponential stability (GES) for the total system is proved using Lyapunov direct method. Simulation results show a good performance of the observer and control system.

Key words: Dynamic positioning, backstepping control, adaptive observer, parameter estimation

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1 Introduction

Dynamic positioning (DP) system has been widely used for marine craft on station keeping, drilling and offshore offloading. Vessels equipped with the DP system can control the three horizontal motions (position and heading) by means of thrusters. The DP vessel motions are supposed as the superposition of low-frequency (LF) vessel motion and wave-frequency (WF) motion. For that there is no need for actuators to response for the WF disturbances, and the state observer (wave filtering) systems are needed to reconstruct the LF motions and estimate the unmeasured states, which are sent as the input of controller (Fossen, 2011).

The first generation DP system was designed to employ conventional PID controllers with low-pass and/or notch filters. From the middle 1970s, the optimal control theory and Kalman Filtering techniques was employed to the DP system by Balchen et al. (1976). From then on, many extensions and modifications of this work have been proposed (Julier and Uhlmann, 1997), and it has rapidly become the most extensively used method. However, the big drawbacks for the method are that it needs to linearize the kinematic and dynamic equations of motion, and is difficult and time-consuming to tune the state estimator. These limitations stimulate the researches of nonlinear controllers. Thus, Koditschek (1987) put forward backstepping control idea which does not need to linearize the system. The method can implement the control law design by constructing Lyapunov function for the subsystem of the nonlinear system, and meanwhile, ensure the system asymptotically stable at the equilibrium point.

Fossen and Grøvlen (Grøvlen and Fossen, 1996, Fossen and Grøvlen, 1998) employed vectorial observer backstepping to solve the DP problem and proved its uniform global exponential stability (UGES). However, wave filtering and bias estimation were not included in their papers. Fossen and Strand (1999) proposed an UGES passive nonlinear observer with wave filtering and bias estimation using Lyapunov methods. From then on, many extensive researches on it have been implemented (Aarset et al., 1998; Torsetnes et al., 2004; Morishita et al., 2014). However, these observers suppose that the ship dynamics is stable and all the parameters are known and constant. Under real conditions, the ship dynamics may be unstable in complicated environments, or some dynamic parameters may be unknown for lack of ship model tests. To solve this problem, Skjetne et al. (2005) proposed the adaptive backstepping method with the assumption that all the states are measurable. Calugi et al. (2003) proposed the method of backstepping control based on adaptive observer for dynamic ship position. However, the environmental disturbances and the first-or-

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der wave-frequency motions were not considered, and this will result in unnecessary responses of the thrust system for lack of wave filtering, and inaccurate state estimation for absence of the bias term.

In this paper, a modified nonlinear adaptive observer based backstepping control method is proposed. The observation model is built with consideration of the bias term and the wave-frequency motions, where the bias term represents the environmental disturbances due to wind, currents and waves, and the unmodelled system dynamic. Furthermore, under the assumption that the dynamic coefficients are unknown, the adaptive observer is designed to identify the dynamic parameters and estimate the unmeasured states, and then the integral backstepping controller is derived based on the estimated parameters and states. The global exponential stability (GES) is proved for the total observer and control system using Lyapunov direct method. The performances of the observer and control system have been determined through computer simulation at last.

2 Mathematical ship model

Models for dynamic positioning (DP) are derived under the assumption of low speed, which is up to approximately 3 m/s. Thus, the vertical motions of the heave, roll and pitch are neglected, and only the horizontal motions of the surge, sway and yaw shall be considered for the DP models. A mathematical model suitable for the adaptive observer and backstepping control design should consider the low-frequency (LF) motions, the wave-frequency (WF) motions, and the slow varying components of wind, waves and currents or the unmodelled dynamics. Reference to Fossen (2011), the mathematical model for DP vessel can be expressed as:

$$\dot{\boldsymbol{\xi}} = \boldsymbol{A}_\omega \boldsymbol{\xi} + \boldsymbol{E}_\omega \boldsymbol{w}_1; \tag{1}$$

$$\dot{\boldsymbol{\eta}} = \boldsymbol{J}(\boldsymbol{\eta})\boldsymbol{v}; \tag{2}$$

$$\dot{\boldsymbol{b}} = -\boldsymbol{T_b}^{-1}\boldsymbol{b} + \boldsymbol{w}_2; \tag{3}$$

$$\boldsymbol{M}\dot{\boldsymbol{v}} = -\boldsymbol{D}\boldsymbol{v} + \boldsymbol{\tau} + \boldsymbol{J}^{\mathrm{T}}(\boldsymbol{\eta})\boldsymbol{b} + \boldsymbol{w}_3; \tag{4}$$

$$\boldsymbol{y} = \boldsymbol{\eta} + \boldsymbol{C}_\omega \boldsymbol{\xi} + \boldsymbol{n}, \tag{5}$$

where $\boldsymbol{\xi} \in \boldsymbol{R}^6$ is the internal variable vector, $\boldsymbol{w}_i (i = 1,2,3) \in \boldsymbol{R}^3$ is the zero-mean white noise process, $\boldsymbol{\eta}_\omega = \boldsymbol{C}_\omega \boldsymbol{\xi}$ is the vessel's WF motion, $\boldsymbol{\eta} = [x, y, \psi]^{\mathrm{T}}$ is the low-frequency position vector, $\boldsymbol{v} = [u, v, r]^{\mathrm{T}}$ is the velocity vector including linear and angular velocities of the vessel, $\boldsymbol{b} \in \boldsymbol{R}^3$ represents the bias term describing the effects of low-frequency environmental disturbances and the unmodelled nonlinear dynamics, $\boldsymbol{T_b} \in \boldsymbol{R}^{3\times3}$ is the user-defined diagonal matrix of positive bias time constants, $\boldsymbol{J}(\boldsymbol{\eta})$ is the transformation rotation matrix, $\boldsymbol{M}$ is the inertia matrix including the added inertia, $\boldsymbol{D}$ is the linear damping coefficients matrix, $\boldsymbol{\tau} \in \boldsymbol{R}^3$ is the control force vector to be determined, and $\boldsymbol{y} \in \boldsymbol{R}^3$ is the measurement considered as the sum of LF and WF components, in addition to the zero-mean measurement noise $\boldsymbol{n} \in \boldsymbol{R}^3$. The matrix $\boldsymbol{M}$, $\boldsymbol{D}$, $\boldsymbol{A}_\omega$, $\boldsymbol{E}_\omega$, $\boldsymbol{C}_\omega$ and $\boldsymbol{J}(\boldsymbol{\eta})$ are defined as:

$$\boldsymbol{M} = \begin{bmatrix} m - X_{\dot{u}} & 0 & 0 \\ 0 & m - Y_{\dot{v}} & mx_G - Y_{\dot{r}} \\ 0 & mx_G - N_{\dot{v}} & I_z - N_{\dot{r}} \end{bmatrix} > 0,$$

$$\boldsymbol{D} = \begin{bmatrix} -X_u & 0 & 0 \\ 0 & -Y_v & -Y_r \\ 0 & -N_v & -N_r \end{bmatrix} > 0$$

$$\boldsymbol{A}_\omega = \begin{bmatrix} 0_{3\times3} & \boldsymbol{I}_{3\times3} \\ -\boldsymbol{\Omega}_{3\times3} & -\boldsymbol{\Lambda}_{3\times3} \end{bmatrix}, \boldsymbol{E}_\omega = \begin{bmatrix} 0_{3\times1} \\ \boldsymbol{I}_{3\times1} \end{bmatrix},$$

$$\boldsymbol{C}_\omega = [\, 0_{3\times1} \quad \boldsymbol{I}_{3\times1} \,], \ \boldsymbol{J}(\boldsymbol{\eta}) = \boldsymbol{J}(\psi) = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

where m is the vessel mass, I_z is the moment of the inertia, $\{X_{\dot{u}}, Y_{\dot{v}}, Y_{\dot{r}}, N_{\dot{v}}, N_{\dot{r}}\}$ are the added mass hydrodynamic coefficients, $\{X_u, Y_v, Y_r, N_v, N_r\}$ are the hydrodynamic damping coefficients, $\boldsymbol{\Omega} = \mathrm{diag}\{\omega_{01}^2, \omega_{02}^2, \omega_{06}^2\}$, $\boldsymbol{\Lambda} = \mathrm{diag}\{2\zeta_1\omega_{01}, 2\zeta_2\omega_{02}, 2\zeta_6\omega_{06}\}$, ω_{0i} is the dominating wave frequency, ζ_i is the relative damping ratio, i=1, 2, 6 represents the surge, sway and yaw directions, respectively.

Generally, the hydrodynamic coefficients can be gained through ship model tests. However, the coefficients may be unknown for lack of experiments. At this time, the added mass coefficients can be calculated by theoretical formula derived through the strip theory (Fossen, 1994), whereas there are no relatively accurate formula for the damping coefficients calculation. Thus, in this paper, we suppose that the hydrodynamic damping parameters $\{X_u, Y_v, Y_r, N_v, N_r\}$ are unknown, and the parameters will be identified by the design of adaptive nonlinear observer.

Here, we define $\boldsymbol{\theta} = [X_u, Y_v, Y_r, N_v, N_r]^{\mathrm{T}}$ representing the unknown parameters, and then Eq. (4) can be rewritten as:

$$\boldsymbol{M}\dot{\boldsymbol{v}} = \boldsymbol{\varphi}(\boldsymbol{v})\boldsymbol{\theta} + \boldsymbol{\tau} + \boldsymbol{J}^{\mathrm{T}}(\boldsymbol{\eta})\boldsymbol{b} + \boldsymbol{w}_3, \tag{6}$$

where

$$\boldsymbol{\varphi}(\boldsymbol{v}) = -\begin{bmatrix} u & 0 & 0 & 0 & 0 \\ 0 & v & r & 0 & 0 \\ 0 & 0 & 0 & v & r \end{bmatrix} \tag{7}$$

3 Nonlinear adaptive observer design

When designing backstepping controller, an observer is needed in the feedback loop to provide the control system with LF signals which have filtered the WF signals, and with the estimated signals such as velocity, and acceleration, which are not measured. Here, the adaptive observer also provides the estimated unknown parameters to the control system.

3.1 Adaptive observer equations

According to Fossen and Strand (1999), the following assumptions are also made to guarantee the Lyapunov stability.

Assumption (i): $w_i (i = 1,2,3) = 0$. The terms are omitted in the analysis of an observer and with estimation error instead. Furthermore, the measurement noise is not included ($\boldsymbol{n}$=0) since it is negligible compared with $\boldsymbol{\eta}_\omega$.

Assumption (ii): $\boldsymbol{J}(\boldsymbol{\eta})=\boldsymbol{J}(\boldsymbol{y})$ or $\boldsymbol{J}(\psi)=\boldsymbol{J}(\psi+\psi_\omega)$. This is equal since the magnitude of wave-induce yaw disturbance ψ_ω is normally smaller than 5° in extreme weather situations and smaller than 1° in normal operation.

For general nonlinear systems, Tyukin et al. (2013) proposed the method for the adaptive observer and parameter estimation. Applied the method into DP ship observer and parameter identification, according to Eqs. (1)–(6), the observer equations with parameters adaptive estimation can be written as follows:

$$\dot{\hat{\boldsymbol{\xi}}}=\boldsymbol{A}_\omega\hat{\boldsymbol{\xi}}+\boldsymbol{K}_1\tilde{\boldsymbol{y}}; \quad (8)$$

$$\dot{\hat{\boldsymbol{\eta}}}=\boldsymbol{J}(\boldsymbol{y})\hat{\boldsymbol{v}}+\boldsymbol{K}_2\tilde{\boldsymbol{y}}; \quad (9)$$

$$\dot{\hat{\boldsymbol{b}}}=-\boldsymbol{T_b}^{-1}\hat{\boldsymbol{b}}+\boldsymbol{K}_3\tilde{\boldsymbol{y}}; \quad (10)$$

$$\begin{aligned}\boldsymbol{M}\dot{\hat{\boldsymbol{v}}}&=-\hat{\boldsymbol{D}}\hat{\boldsymbol{v}}+\boldsymbol{J}^{\mathrm{T}}(\boldsymbol{y})\hat{\boldsymbol{b}}+\boldsymbol{\tau}+\boldsymbol{J}^{\mathrm{T}}(\boldsymbol{y})\boldsymbol{K}_4\tilde{\boldsymbol{y}}\\&=\boldsymbol{\varphi}(\hat{\boldsymbol{v}})\hat{\boldsymbol{\theta}}+\boldsymbol{J}^{\mathrm{T}}(\boldsymbol{y})\hat{\boldsymbol{b}}+\boldsymbol{\tau}+\boldsymbol{J}^{\mathrm{T}}(\boldsymbol{y})\boldsymbol{K}_4\tilde{\boldsymbol{y}};\end{aligned} \quad (11)$$

$$\dot{\hat{\boldsymbol{\theta}}}=\boldsymbol{\Gamma}\boldsymbol{\varphi}^{\mathrm{T}}(\hat{\boldsymbol{v}})\tilde{\boldsymbol{v}}; \quad (12)$$

$$\hat{\boldsymbol{y}}=\hat{\boldsymbol{\eta}}+\boldsymbol{C}_\omega\hat{\boldsymbol{\xi}}, \quad (13)$$

where $\tilde{\boldsymbol{y}}=\boldsymbol{y}-\hat{\boldsymbol{y}}$ and $\tilde{\boldsymbol{v}}=\boldsymbol{v}-\hat{\boldsymbol{v}}$ are the estimation errors for position and velocity, respectively; $\boldsymbol{K}_1\in\boldsymbol{R}^{6\times3}$, $\boldsymbol{K}_{2,3,4}\in\boldsymbol{R}^{3\times3}$, and $\boldsymbol{\Gamma}\in\boldsymbol{R}^{5\times5}\left(\boldsymbol{\Gamma}=\boldsymbol{\Gamma}^{\mathrm{T}}>0\right)$ are the observer gain matrixes to be interpreted. The tuning of observer gains $\boldsymbol{K}_1$, $\boldsymbol{K}_2$, $\boldsymbol{K}_3$ and $\boldsymbol{K}_4$ may refer to Fossen and Strand (1999).

3.2 Observer error dynamics and stability properties

Firstly, define the low frequency and wave frequency motion estimation errors $\tilde{\boldsymbol{\eta}}=\boldsymbol{\eta}-\hat{\boldsymbol{\eta}}$ and $\tilde{\boldsymbol{\xi}}=\boldsymbol{\xi}-\hat{\boldsymbol{\xi}}$, the system dynamic parameter estimation errors $\tilde{\boldsymbol{D}}=\boldsymbol{D}-\hat{\boldsymbol{D}}$ and $\tilde{\boldsymbol{\theta}}=\boldsymbol{\theta}-\hat{\boldsymbol{\theta}}$. For that the unknown parameters are supposed to be constant or slowly changed, and then its derivative $\dot{\boldsymbol{\theta}}=0$. With the **Assumptions** (i) and (ii), subtracting Eqs. (8)–(11) and (13) respectively from Eqs. (1)–(3), (6) and (5) yields the following observer errors dynamics:

$$\dot{\tilde{\boldsymbol{\xi}}}=\boldsymbol{A}_\omega\tilde{\boldsymbol{\xi}}-\boldsymbol{K}_1\tilde{\boldsymbol{y}}; \quad (14)$$

$$\dot{\tilde{\boldsymbol{\eta}}}=\boldsymbol{J}(\boldsymbol{y})\tilde{\boldsymbol{v}}-\boldsymbol{K}_2\tilde{\boldsymbol{y}}; \quad (15)$$

$$\dot{\tilde{\boldsymbol{b}}}=-\boldsymbol{T_b}^{-1}\tilde{\boldsymbol{b}}-\boldsymbol{K}_3\tilde{\boldsymbol{y}}; \quad (16)$$

$$\begin{aligned}\boldsymbol{M}\dot{\tilde{\boldsymbol{v}}}&=-\tilde{\boldsymbol{D}}\hat{\boldsymbol{v}}-\boldsymbol{D}\tilde{\boldsymbol{v}}+\boldsymbol{J}^{\mathrm{T}}(\boldsymbol{y})\tilde{\boldsymbol{b}}-\boldsymbol{J}^{\mathrm{T}}(\boldsymbol{y})\boldsymbol{K}_4\tilde{\boldsymbol{y}}\\&=-\boldsymbol{D}\tilde{\boldsymbol{v}}+\boldsymbol{\varphi}^{\mathrm{T}}(\hat{\boldsymbol{v}})\tilde{\boldsymbol{\theta}}+\boldsymbol{J}^{\mathrm{T}}(\boldsymbol{y})\tilde{\boldsymbol{b}}-\boldsymbol{J}^{\mathrm{T}}(\boldsymbol{y})\boldsymbol{K}_4\tilde{\boldsymbol{y}};\end{aligned} \quad (17)$$

$$\dot{\tilde{\boldsymbol{\theta}}}=-\boldsymbol{\Gamma}\boldsymbol{\varphi}^{\mathrm{T}}(\hat{\boldsymbol{v}})\tilde{\boldsymbol{v}}; \quad (18)$$

$$\tilde{\boldsymbol{y}}=\tilde{\boldsymbol{\eta}}+\boldsymbol{C}_\omega\tilde{\boldsymbol{\xi}}. \quad (19)$$

Furthermore, by defining the new vector $\tilde{\boldsymbol{x}}=[\tilde{\boldsymbol{\xi}}^{\mathrm{T}},\tilde{\boldsymbol{\eta}}^{\mathrm{T}},\tilde{\boldsymbol{b}}^{\mathrm{T}}]^{\mathrm{T}}$, the error dynamics Eqs. (14)–(17) can be rewritten as:

$$\dot{\tilde{\boldsymbol{x}}}_0=\boldsymbol{A}_0\tilde{\boldsymbol{x}}_0+\boldsymbol{B}_0\boldsymbol{J}(\boldsymbol{y})\tilde{\boldsymbol{v}}; \quad (20)$$

$$\boldsymbol{M}\dot{\tilde{\boldsymbol{v}}}=-\boldsymbol{D}\tilde{\boldsymbol{v}}-\boldsymbol{\varphi}^{\mathrm{T}}(\hat{\boldsymbol{v}})\tilde{\boldsymbol{\theta}}-\boldsymbol{J}^{\mathrm{T}}(\boldsymbol{y})\boldsymbol{C}_0\tilde{\boldsymbol{x}}_0, \quad (21)$$

where

$$\boldsymbol{A}_0=\begin{bmatrix}\boldsymbol{A}_\omega-\boldsymbol{K}_1\boldsymbol{C}_\omega & -\boldsymbol{K}_1 & 0\\ -\boldsymbol{K}_2\boldsymbol{C}_\omega & -\boldsymbol{K}_2 & 0\\ -\boldsymbol{K}_3\boldsymbol{C}_\omega & -\boldsymbol{K}_3 & -\boldsymbol{T_b}^{-1}\end{bmatrix},$$

$$\boldsymbol{B}_0=\begin{bmatrix}0\\ \boldsymbol{I}\\ 0\end{bmatrix}, \boldsymbol{C}_0=[\boldsymbol{K}_3\boldsymbol{C}_\omega \quad \boldsymbol{K}_3 \quad -\boldsymbol{I}_{3\times3}].$$

Consider the following Lyapunov function candidate for the observer:

$$V_{\mathrm{obs}}=\tilde{\boldsymbol{v}}^{\mathrm{T}}\boldsymbol{M}\tilde{\boldsymbol{v}}+\tilde{\boldsymbol{x}}_0^{\mathrm{T}}\boldsymbol{P}\tilde{\boldsymbol{x}}_0+\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\boldsymbol{\Gamma}^{-1}\tilde{\boldsymbol{\theta}}>0, \forall\tilde{\boldsymbol{v}},\tilde{\boldsymbol{x}}_0,\tilde{\boldsymbol{\theta}}\neq0 \quad (22)$$

where $\boldsymbol{P}=\boldsymbol{P}^{\mathrm{T}}>0$ is a constant matrix. Time differentiation of V_{obs} along the solution of Eqs. (18), (20) and (21) yields:

$$\begin{aligned}\dot{V}_{\mathrm{obs}}&=-\tilde{\boldsymbol{v}}^{\mathrm{T}}(\boldsymbol{D}+\boldsymbol{D}^{\mathrm{T}})\tilde{\boldsymbol{v}}+2\tilde{\boldsymbol{v}}^{\mathrm{T}}\boldsymbol{\varphi}\tilde{\boldsymbol{\theta}}-2\tilde{\boldsymbol{v}}^{\mathrm{T}}\boldsymbol{J}^{\mathrm{T}}(\boldsymbol{y})\boldsymbol{C}_0\tilde{\boldsymbol{x}}_0\\&\quad+\tilde{\boldsymbol{x}}_0^{\mathrm{T}}(\boldsymbol{P}\boldsymbol{A}_0+\boldsymbol{A}_0^{\mathrm{T}}\boldsymbol{P})\tilde{\boldsymbol{x}}_0+2\tilde{\boldsymbol{x}}_0^{\mathrm{T}}\boldsymbol{P}\boldsymbol{B}_0\boldsymbol{J}(\boldsymbol{y})\tilde{\boldsymbol{v}}-2\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\boldsymbol{\varphi}^{\mathrm{T}}\tilde{\boldsymbol{v}}.\end{aligned} \quad (23)$$

Defining that

$$\boldsymbol{D}+\boldsymbol{D}^{\mathrm{T}}=\boldsymbol{Q}_1>0 \quad (24)$$

and the matrix $\boldsymbol{P}$ is required to satisfy the following Eqs. (25) and (26)

$$\boldsymbol{P}\boldsymbol{A}_0+\boldsymbol{A}_0^{\mathrm{T}}\boldsymbol{P}=-\boldsymbol{Q}_2<0; \quad (25)$$

$$\boldsymbol{C}_0=\boldsymbol{B}_0^{\mathrm{T}}\boldsymbol{P}, \quad (26)$$

then, we will have

$$\dot{V}_{\mathrm{obs}}=-\tilde{\boldsymbol{v}}^{\mathrm{T}}\boldsymbol{Q}_1\tilde{\boldsymbol{v}}-\tilde{\boldsymbol{x}}_0^{\mathrm{T}}\boldsymbol{Q}_2\tilde{\boldsymbol{x}}_0<0, \forall\tilde{\boldsymbol{v}},\tilde{\boldsymbol{x}}_0\neq0. \quad (27)$$

Thus, under the Assumptions (i) and (ii), and when the matrix satisfies Eqs. (24)–(26), the global exponential stability (GES) of the observer is satisfied according to Lyapunov direct method.

4 Backstepping controller design

4.1 Backstepping design

Now, the notation $\boldsymbol{\eta}_{\mathrm{d}}=[x_{\mathrm{d}},y_{\mathrm{d}},\psi_{\mathrm{d}}]^{\mathrm{T}}$ is used to represent the demanded position and heading, where $\boldsymbol{\eta}_{\mathrm{d}}$ is a smooth reference trajectory. To introduce integral action into the control law, we define the error integral term

$$\dot{\boldsymbol{e}}_{\mathrm{I}}=\hat{\boldsymbol{\eta}}-\boldsymbol{\eta}_{\mathrm{d}}. \quad (28)$$

Step 1: the backstepping error variable $\boldsymbol{z}_1$ is defined as $\boldsymbol{z}_1=\boldsymbol{e}_{\mathrm{I}}$, thus

$$\dot{\boldsymbol{z}}_1=\dot{\boldsymbol{e}}_{\mathrm{I}}=\hat{\boldsymbol{\eta}}-\boldsymbol{\eta}_{\mathrm{d}}. \quad (29)$$

Firstly, the term $\hat{\boldsymbol{\eta}}$ is chosen as the virtual control of $\boldsymbol{z}_1$. By calculating the time differentiation of the Lyapunov function $V_1=\boldsymbol{z}_1^{\mathrm{T}}\boldsymbol{z}_1/2$, and then the stabilizing function $\boldsymbol{\alpha}_1$ is defined as:

$$\boldsymbol{\alpha}_1=-\boldsymbol{K}_{\mathrm{I}}\boldsymbol{z}_1+\boldsymbol{\eta}_{\mathrm{d}}, \quad (30)$$

where $\boldsymbol{K}_{\mathrm{I}}\in\boldsymbol{R}^{3\times3}\left(\boldsymbol{K}_{\mathrm{I}}=\boldsymbol{K}_{\mathrm{I}}^{\mathrm{T}}>0\right)$ is an integral gain matrix.

Step 2: define the second backstepping error variable $\boldsymbol{z}_2$ as:

$$\boldsymbol{z}_2=\hat{\boldsymbol{\eta}}-\boldsymbol{\alpha}_1. \quad (31)$$

Recalculating $\dot{\boldsymbol{z}}_1$ by substituting Eqs. (30) and (31) into Eq. (29) yields

$$\dot{\boldsymbol{z}}_1=-\boldsymbol{K}_{\mathrm{I}}\boldsymbol{z}_1+\boldsymbol{z}_2. \quad (32)$$

Taking time differentiation of Eq. (31) and inserting Eqs. (9) and (29) into it results in:

$$\dot{z}_2 = \boldsymbol{J}(\boldsymbol{y})\hat{\boldsymbol{\nu}} + \boldsymbol{K}_2\tilde{\boldsymbol{y}} + \boldsymbol{K}_{\rm I}(\hat{\boldsymbol{\eta}} - \boldsymbol{\eta}_{\rm d}) - \dot{\boldsymbol{\eta}}_{\rm d}. \tag{33}$$

Now, choose the term $\boldsymbol{J}(\boldsymbol{y})\hat{\boldsymbol{\nu}}$ as the virtual control of z_2. Calculate the time differentiation of the Lyapunov function $V_2 = V_1 + z_2{}^{\rm T}z_2/2$, and then the stabilizing function $\boldsymbol{\alpha}_2$ is defined as:

$$\boldsymbol{\alpha}_2 = -z_1 - \boldsymbol{C}_1 z_2 - \boldsymbol{D}_1 z_2 + \dot{\boldsymbol{\eta}}_{\rm d} - \boldsymbol{K}_{\rm I}(\hat{\boldsymbol{\eta}} - \boldsymbol{\eta}_{\rm d}), \tag{34}$$

where $\boldsymbol{C}_1, \boldsymbol{D}_1 \in \boldsymbol{R}^{3\times3}$ are the strictly positive diagonal gain matrices. Elements of $\boldsymbol{D}_1$ are defined as:

$$\boldsymbol{D}_1 = {\rm diag}\left\{d_1 \boldsymbol{k}_1^{\rm T}\boldsymbol{k}_1, d_2 \boldsymbol{k}_2^{\rm T}\boldsymbol{k}_2, d_3 \boldsymbol{k}_3^{\rm T}\boldsymbol{k}_3\right\}, \tag{35}$$

where $[\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3] = \boldsymbol{K}_2^{\rm T}$, d_i (i=1, 2, 3) are positive design constants.

Step 3: the third backstepping error variable z_3 is defined as:

$$z_3 = \boldsymbol{J}(\boldsymbol{y})\hat{\boldsymbol{\nu}} - \boldsymbol{\alpha}_2. \tag{36}$$

Recalculating $\dot{z}_2$ by substituting Eqs. (34) and (36) into Eq. (33) yields

$$\dot{z}_2 = -z_1 - \boldsymbol{C}_1 z_2 - \boldsymbol{D}_1 z_2 + z_3 + \boldsymbol{K}_2\tilde{\boldsymbol{y}}. \tag{37}$$

Calculating the time differentiation of Eq. (36) and submitting Eqs. (11), (29) and (37) into it result in

$$\dot{z}_3 = \boldsymbol{J}(y)\hat{\boldsymbol{M}}^{-1}\boldsymbol{\tau} + \boldsymbol{\Phi} + \boldsymbol{\Omega}_1\tilde{\boldsymbol{y}} + \boldsymbol{\Omega}_2\tilde{\boldsymbol{\nu}} + \boldsymbol{\Omega}_3\tilde{\boldsymbol{\xi}}, \tag{38}$$

where

$$\begin{aligned}\boldsymbol{\Phi} = {} & \boldsymbol{J}(\boldsymbol{y})\boldsymbol{M}^{-1}\left[-\hat{\boldsymbol{D}}\hat{\boldsymbol{\nu}} + \boldsymbol{J}^{\rm T}(\boldsymbol{y})\hat{\boldsymbol{b}}\right] + \boldsymbol{J}(\boldsymbol{y})\boldsymbol{S}(\hat{\boldsymbol{\rho}})\hat{\boldsymbol{\nu}} \\ & - (\boldsymbol{C}_1 + \boldsymbol{D}_1)^2 z_2 + (\boldsymbol{C}_1 + \boldsymbol{D}_1)(z_3 - z_1) + (\hat{\boldsymbol{\eta}} - \boldsymbol{\eta}_{\rm d}) \\ & + \boldsymbol{K}_{\rm I}[\boldsymbol{J}(\boldsymbol{y})\hat{\boldsymbol{\nu}} + \boldsymbol{K}_2\tilde{\boldsymbol{y}}] - \boldsymbol{K}_{\rm I}\dot{\boldsymbol{\eta}}_{\rm d} - \ddot{\boldsymbol{\eta}}_{\rm d};\end{aligned} \tag{39}$$

$$\boldsymbol{\Omega}_1 = \boldsymbol{J}(\boldsymbol{y})\hat{\boldsymbol{M}}^{-1}\boldsymbol{J}^{\rm T}(\boldsymbol{y})K_4 + (\boldsymbol{C}_1 + \boldsymbol{D}_1)\boldsymbol{K}_2; \tag{40}$$

$$\boldsymbol{\Omega}_2 = -\boldsymbol{JS}(\hat{\boldsymbol{\nu}})\boldsymbol{L};\ \boldsymbol{\Omega}_3 = -\boldsymbol{JS}(\hat{\boldsymbol{\nu}})\boldsymbol{N};\ \boldsymbol{\rho} = [0,0,r+\dot{\psi}_w]^{\rm T}; \tag{41}$$

$$\boldsymbol{S}(\boldsymbol{\theta}) = \boldsymbol{S}\begin{pmatrix}\theta_1\\ \theta_2\\ \theta_3\end{pmatrix} = \begin{bmatrix}0 & -\theta_3 & \theta_2\\ \theta_3 & 0 & -\theta_1\\ -\theta_2 & \theta_1 & 0\end{bmatrix};$$

$$\boldsymbol{L} = \begin{bmatrix}0&0&0\\0&0&0\\0&0&1\end{bmatrix}; \boldsymbol{N} = \begin{bmatrix}0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&1\end{bmatrix}.$$

In Eq. (38), the term $\boldsymbol{J}(\boldsymbol{y})\hat{\boldsymbol{M}}^{-1}\boldsymbol{\tau}$ is chosen as the virtual control of z_3. Calculate the time differentiation of the Lyapunov function $V_3 = V_2 + z_3{}^{\rm T}z_3/2$, and then the feedback backstepping control law is defined as:

$$\boldsymbol{J}(\boldsymbol{y})\hat{\boldsymbol{M}}^{-1}\boldsymbol{\tau} = -z_2 - \boldsymbol{C}_2 z_3 - \boldsymbol{D}_2 z_3 - \boldsymbol{\Phi}, \tag{42}$$

where $\boldsymbol{C}_2, \boldsymbol{D}_2 \in \boldsymbol{R}^{3\times3}$ are the strictly positive diagonal gain matrices. The elements of $\boldsymbol{D}_2$ are defined as:

$$\boldsymbol{D}_2 = {\rm diag}\left\{d_4 \sum_{i=1,4,7}\omega_i^{\rm T}\omega_i, d_5 \sum_{i=2,5,8}\omega_i^{\rm T}\omega_i, d_6 \sum_{i=3,6,9}\omega_i^{\rm T}\omega_i\right\}, \tag{43}$$

where d_i (i=4, 5, 6) are the positive design constants and

$$[\omega_1, \omega_2, \omega_3] = \boldsymbol{\Omega}_1^{\rm T}; [\omega_4, \omega_5, \omega_6] = \boldsymbol{\Omega}_2^{\rm T}; [\omega_7, \omega_8, \omega_9] = \boldsymbol{\Omega}_3^{\rm T}. \tag{44}$$

Recalculating $\dot{z}_3$ by substituting Eq. (42) into Eq. (38) yields

$$\dot{z}_3 = -z_2 - \boldsymbol{C}_2 z_3 - \boldsymbol{D}_2 z_3 + \boldsymbol{\Omega}_1\tilde{\boldsymbol{y}} + \boldsymbol{\Omega}_2\tilde{\boldsymbol{\nu}} + \boldsymbol{\Omega}_3\tilde{\boldsymbol{\xi}}. \tag{45}$$

4.2 Stability analysis for the total system

By taking into account Eqs. (18), (20), (21), (32), (37) and (45), error dynamics for both the observer and backstepping control system can be expressed as:

$$\dot{z} = -\boldsymbol{C}_z z - \boldsymbol{D}_z z + \boldsymbol{E}z + \boldsymbol{W}_1\tilde{\boldsymbol{y}} + \boldsymbol{W}_2\tilde{\boldsymbol{\nu}} + \boldsymbol{W}_3\tilde{\boldsymbol{\xi}}; \tag{46}$$

$$\dot{\tilde{\boldsymbol{x}}}_0 = \boldsymbol{A}_0\tilde{\boldsymbol{x}}_0 + \boldsymbol{B}_0 J(\boldsymbol{y})\tilde{\boldsymbol{\nu}}; \tag{47}$$

$$M\dot{\tilde{\boldsymbol{\nu}}} = -\boldsymbol{D}\tilde{\boldsymbol{\nu}} - \boldsymbol{\varphi}^{\rm T}\tilde{\boldsymbol{\theta}} - \boldsymbol{J}^{\rm T}(y)K_3\boldsymbol{C}_0\tilde{\boldsymbol{x}}_0; \tag{48}$$

$$\dot{\tilde{\boldsymbol{\theta}}} = -\boldsymbol{\Gamma}\boldsymbol{\varphi}^{\rm T}(\dot{\hat{\boldsymbol{\nu}}}, \hat{\boldsymbol{\nu}}, \hat{\boldsymbol{\eta}})\tilde{\boldsymbol{\nu}}, \tag{49}$$

where

$$z = \left[z_1^{\rm T}, z_2^{\rm T}, z_2^{\rm T}\right]^{\rm T},\ \boldsymbol{C}_z = {\rm diag}\{\boldsymbol{K}_{\rm I}, \boldsymbol{C}_1, \boldsymbol{C}_2\},$$

$$\boldsymbol{D}_z = {\rm diag}\{0_{3\times3}, \boldsymbol{D}_1, \boldsymbol{D}_2\}; \tag{50}$$

$$\boldsymbol{E} = \begin{bmatrix}0 & \boldsymbol{I} & 0\\ -\boldsymbol{I} & 0 & \boldsymbol{I}\\ 0 & -\boldsymbol{I} & 0\end{bmatrix}, \boldsymbol{W}_1 = \begin{bmatrix}0\\ \boldsymbol{K}_2\\ \boldsymbol{\Omega}_1\end{bmatrix},$$

$$\boldsymbol{W}_2 = \begin{bmatrix}0\\0\\ \boldsymbol{\Omega}_2\end{bmatrix}, \boldsymbol{W}_3 = \begin{bmatrix}0\\0\\ \boldsymbol{\Omega}_3\end{bmatrix}. \tag{51}$$

Considering the following Lyapunov function for the total system:

$$V = V_{\rm con} + V_{\rm obs} = z^{\rm T}z/2 + V_{\rm obs}, \forall z \neq 0. \tag{52}$$

By taking into account of Eqs. (27) and (46), its time derivative is:

$$\begin{aligned}\dot{V} & = z^{\rm T}\dot{z} + \dot{V}_{\rm obs} \\ & = z^{\rm T}(-\boldsymbol{C}_z z - \boldsymbol{D}_z z + \boldsymbol{E}z + \boldsymbol{W}_1\tilde{\boldsymbol{y}} + \boldsymbol{W}_2\tilde{\boldsymbol{\nu}} + \boldsymbol{W}_3\tilde{\boldsymbol{\xi}}) \\ & \quad - \tilde{\boldsymbol{\nu}}^{\rm T}\boldsymbol{Q}_1\tilde{\boldsymbol{\nu}} - \tilde{x}_0^{\rm T}\boldsymbol{Q}_2\tilde{\boldsymbol{x}}_0.\end{aligned} \tag{53}$$

For that the matrix $\boldsymbol{E}$ is skew-symmetric, the term $z^{\rm T}\boldsymbol{E}z$ is equal to zero. Now, add the following zero terms into Eq. (53)

$$\frac{1}{4}\left(\tilde{\boldsymbol{y}}^{\rm T}\boldsymbol{G}_1\tilde{\boldsymbol{y}} - \tilde{\boldsymbol{y}}^{\rm T}\boldsymbol{G}_1\tilde{\boldsymbol{y}}\right) = 0;$$

$$\frac{1}{4}\left(\tilde{\boldsymbol{\nu}}^{\rm T}\boldsymbol{G}_2\tilde{\boldsymbol{\nu}} - \tilde{\boldsymbol{\nu}}^{\rm T}\boldsymbol{G}_2\tilde{\boldsymbol{\nu}}\right) = 0;$$

$$\frac{1}{4}\left(\boldsymbol{\xi}^{\rm T}\boldsymbol{G}_3\boldsymbol{\xi} - \boldsymbol{\xi}^{\rm T}\boldsymbol{G}_3\boldsymbol{\xi}\right) = 0. \tag{54}$$

where $\boldsymbol{G}_i = g_i\boldsymbol{I} (i = 1,2,3)$.

$$g_1 = \frac{1}{4}\sum_{i=1}^{3}\left(\frac{1}{d_i} + \frac{1}{d_{i+3}}\right), g_2 = g_3 = \frac{1}{4}\sum_{i=1}^{3}\frac{1}{d_{i+3}}. \tag{55}$$

And by defining that $\tilde{\boldsymbol{y}} = \boldsymbol{C}_y\tilde{\boldsymbol{x}}_0$, $\tilde{\boldsymbol{\xi}} = \boldsymbol{C}_\xi\tilde{\boldsymbol{x}}_0$ where $\boldsymbol{C}_y$ and $\boldsymbol{C}_\xi$ are diagonal positive matrixes, it will yield

$$\begin{aligned}\dot{V} = & -z^{\rm T}\boldsymbol{C}_z z + z^{\rm T}\left(-\boldsymbol{D}_z z + \boldsymbol{W}_1\tilde{\boldsymbol{y}} + \boldsymbol{W}_2\tilde{\boldsymbol{\nu}} + \boldsymbol{W}_3\tilde{\boldsymbol{\xi}}\right) \\ & - \frac{1}{4}\left(\tilde{\boldsymbol{y}}^{\rm T}\boldsymbol{G}_1\tilde{\boldsymbol{y}} + \tilde{\boldsymbol{\nu}}^{\rm T}\boldsymbol{G}_2\tilde{\boldsymbol{\nu}} + \boldsymbol{\xi}^{\rm T}\boldsymbol{G}_3\boldsymbol{\xi}\right) \\ & - \tilde{\boldsymbol{x}}_0^{\rm T}\left(\boldsymbol{Q}_2 - \frac{1}{4}\boldsymbol{C}_y{}^{\rm T}\boldsymbol{G}_1\boldsymbol{C}_y - \frac{1}{4}\boldsymbol{C}_\xi{}^{\rm T}\boldsymbol{G}_3\boldsymbol{C}_\xi\right)\tilde{\boldsymbol{x}}_0 \\ & - \tilde{\boldsymbol{\nu}}^{\rm T}\left(\boldsymbol{Q}_1 - \frac{1}{4}\boldsymbol{G}_2\right)\tilde{\boldsymbol{\nu}}.\end{aligned} \tag{56}$$

With the definition of $\boldsymbol{D}_z$, $\boldsymbol{W}_1$, $\boldsymbol{W}_2$ and $\boldsymbol{W}_3$ in Eqs. (50) and (51), the second and third terms of Eq. (56) can be rewritten as:

$$\lambda = z_2{}^{\mathrm{T}}(-\boldsymbol{D}_1 z_2+\boldsymbol{K}_2\tilde{\boldsymbol{y}}) + z_3{}^{\mathrm{T}}(-\boldsymbol{D}_2 z_3+\boldsymbol{\Omega}_1\tilde{\boldsymbol{y}}+\boldsymbol{\Omega}_2\tilde{\boldsymbol{v}}+\boldsymbol{\Omega}_3\tilde{\boldsymbol{\xi}})$$
$$-\frac{1}{4}\left(\tilde{\boldsymbol{y}}^{\mathrm{T}}\boldsymbol{G}_1\tilde{\boldsymbol{y}}+\tilde{\boldsymbol{v}}^{\mathrm{T}}\boldsymbol{G}_2\tilde{\boldsymbol{v}}+\boldsymbol{\xi}^{\mathrm{T}}\boldsymbol{G}_3\boldsymbol{\xi}\right). \tag{57}$$

With Eqs. (35), (43), (44) and (55), it can be proved that the term λ is the negative semi-definite function, that is

$$\lambda = -\sum_{i=1}^{3}\left\{d_i\left[k_i z_2(i)-\frac{1}{2d_i}\tilde{y}\right]^{\mathrm{T}}\left[k_i z_2(i)-\frac{1}{2d_i}\tilde{y}\right]\right.$$
$$+d_{i+3}\left[w_i z_3(i)-\frac{1}{2d_{i+3}}\tilde{y}\right]^{\mathrm{T}}\left[w_i z_3(i)-\frac{1}{2d_{i+3}}\tilde{y}\right]$$
$$+d_{i+3}\left[w_{i+3} z_3(i)-\frac{1}{2d_{i+3}}\tilde{v}\right]^{\mathrm{T}}\left[w_{i+3} z_3(i)-\frac{1}{2d_{i+3}}\tilde{v}\right]$$
$$\left.+d_{i+3}\left[w_{i+6} z_3(i)-\frac{1}{2d_{i+3}}\tilde{\xi}\right]^{\mathrm{T}}\left[w_{i+6} z_3(i)-\frac{1}{2d_{i+3}}\tilde{v}\right]\right\}\leqslant 0 \tag{58}$$

Now, if the matrixes $\boldsymbol{Q}_1$ and $\boldsymbol{Q}_2$ satisfy

$$\boldsymbol{Q}_2-\frac{1}{4}\boldsymbol{C}_y{}^{\mathrm{T}}\boldsymbol{G}_1\boldsymbol{C}_y-\frac{1}{4}\boldsymbol{C}_\xi{}^{\mathrm{T}}\boldsymbol{G}_3\boldsymbol{C}_\xi>0,\ \ \boldsymbol{Q}_1-\frac{1}{4}\boldsymbol{G}_2>0, \tag{59}$$

it can be proved that

$$\dot{V}\leqslant -z^{\mathrm{T}}\boldsymbol{C}_z z-\tilde{\boldsymbol{x}}_0^{\mathrm{T}}\left(\boldsymbol{Q}_2-\frac{1}{4}\boldsymbol{C}_y{}^{\mathrm{T}}\boldsymbol{G}_1\boldsymbol{C}_y-\frac{1}{4}\boldsymbol{C}_\xi{}^{\mathrm{T}}\boldsymbol{G}_3\boldsymbol{C}_\xi\right)\tilde{\boldsymbol{x}}_0$$
$$-\tilde{\boldsymbol{v}}^{\mathrm{T}}\left(\boldsymbol{Q}_1-\frac{1}{4}\boldsymbol{G}_2\right)\tilde{\boldsymbol{v}}<0,\ \ \forall z,\tilde{\boldsymbol{x}}_0,\tilde{\boldsymbol{v}}\neq 0. \tag{60}$$

Thus, with the Lyapunov direct method, the adaptive observer based backstepping system is proved to be Globally Exponentially Stable (GES) under the Assumptions (i) and (ii), when matrixes $\boldsymbol{Q}_1$ and $\boldsymbol{Q}_2$ satisfy Eq. (59).

5 Simulation analysis and results

To evaluate the performance of the proposed adaptive observer based backstepping control method, a model of Cybership II (CSII) in the Marine Systems Simulator (MSS) toolbox (Fossen and Perez, 2007) was used for simulations. CSII ship is a 1:70 scale model of a supply ship. The ship was disturbed by waves with a peak frequency of ω_0=0.8 rad/s (the significant wave height H_s=2.5 m, moderate wave), and its first-order wave frequency disturbances are added on the ship as a random noise. The initial position of the simulated ship was set at $\boldsymbol{\eta}_0$=[0 m, 0 m, 0°]$^{\mathrm{T}}$, and the ship is required to move to the position of $\boldsymbol{\eta}_{\mathrm{d}}$= [1 m, 0.5 m, 30°]$^{\mathrm{T}}$. The dynamics of CSII can be described by:

$$\boldsymbol{M}=\begin{bmatrix}25.8 & 0 & 0\\ 0 & 33.8 & 1.0115\\ 0 & 1.0115 & 2.76\end{bmatrix},\ \boldsymbol{D}=\begin{bmatrix}2 & 0 & 0\\ 0 & 7 & 0.1\\ 0 & 0.1 & 0.5\end{bmatrix}$$

Here, we suppose that the parameters in the matrix $\boldsymbol{D}$ are unknown, according to the adaptive observer based backstepping controller designed above, the gains used in the system are chosen as: $\boldsymbol{K}_1$=[–2.34$\boldsymbol{I}$; 1.8$\boldsymbol{I}$], $\boldsymbol{K}_2$=1.3$\boldsymbol{I}$, $\boldsymbol{K}_4$=diag {0.1, 0.1, 0.01}, $\boldsymbol{\Gamma}$=diag {150, 1000, 100, 100, 100}, $\boldsymbol{K}_1$=0.2$\boldsymbol{I}$, $\boldsymbol{C}_1$=0.1$\boldsymbol{I}$, d_i (i=1, 2, 3)=0.1, $\boldsymbol{C}_2$=10$\boldsymbol{I}$, d_i (i=4, 5, 6)=10. The simulation results are shown in Figs. 1–4. From Fig. 1, we can see a good control performance of the proposed controller, and the ship tracks well to the settled position and heading. Fig. 2 shows that most of the first-order

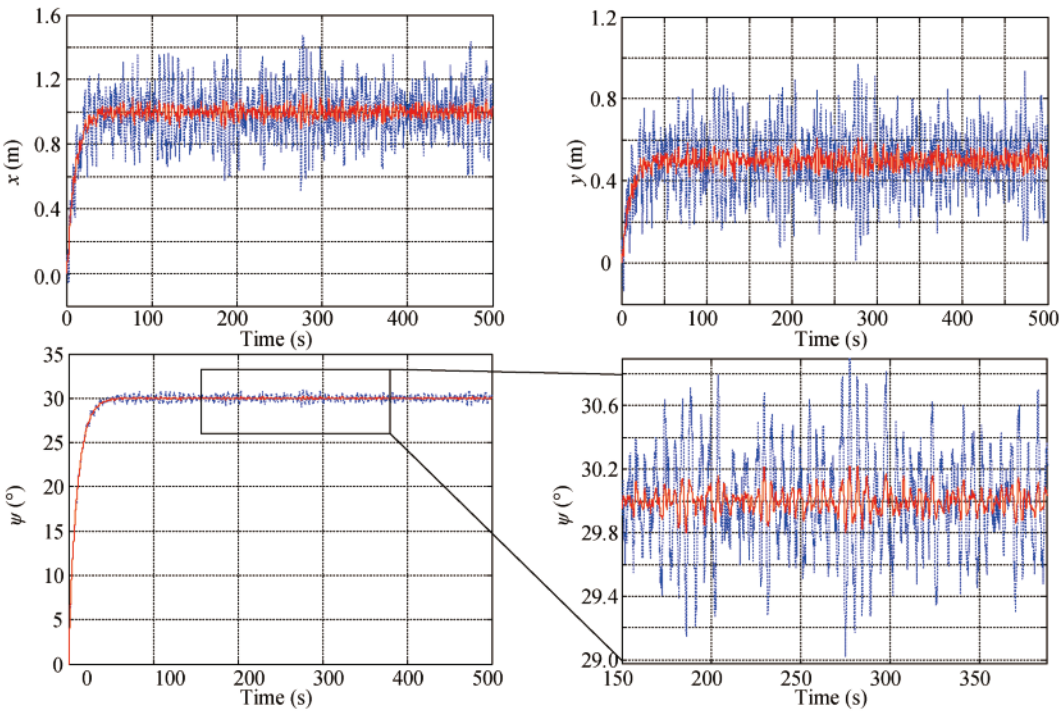

Fig. 1. Control output: estimated low-frequency (solid) and measured (dashed) position(x, y)/heading(ψ).

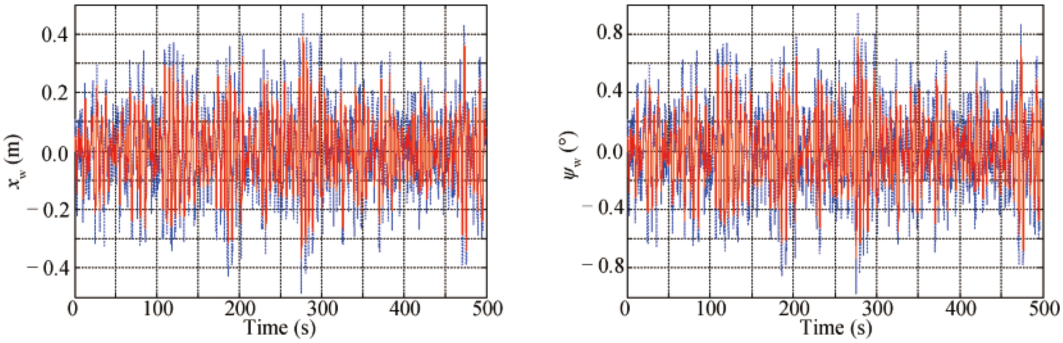

Fig. 2. Actual (dashed) and filtered (solid) wave-frequency positions (x_w, y_w) and heading (ψ_w)

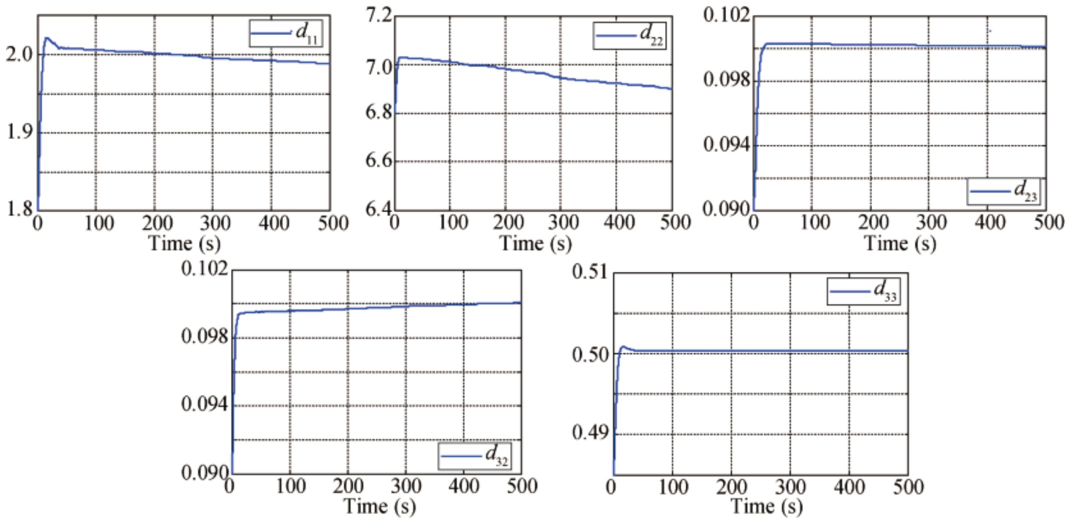

Fig. 3. Adaptive parameter estimates of elements of ***D***.

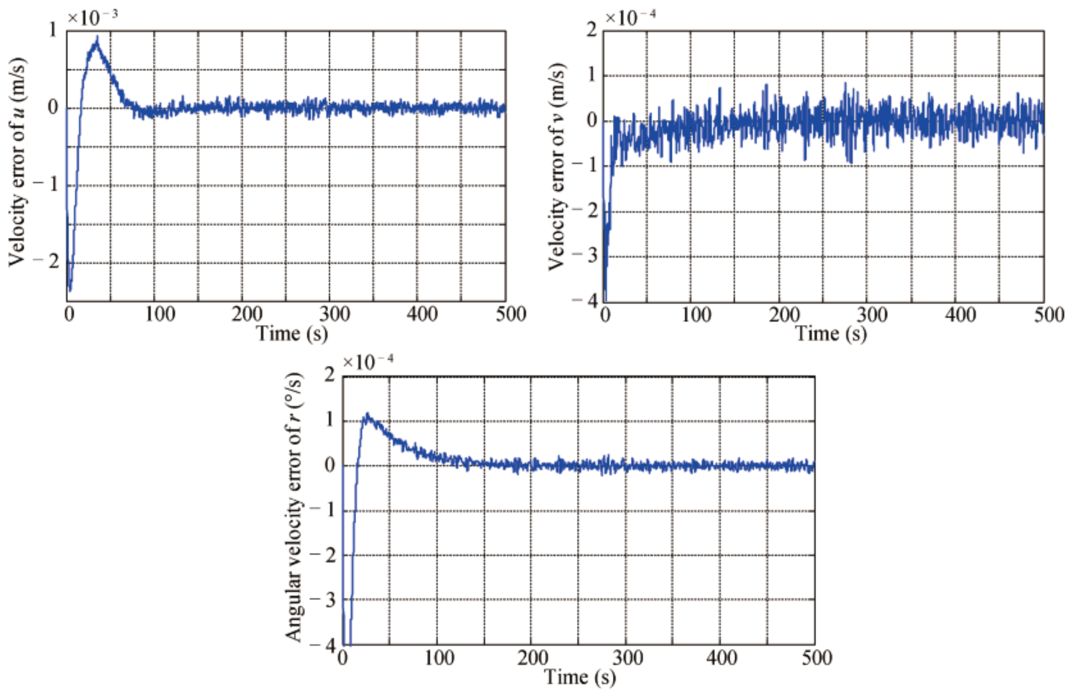

Fig. 4. Error between the plant output velocity and the estimated velocity.

WF are filtered out from the measured positions and heading, which result in a smooth estimate measurement. Fig. 3 shows the adaptive parameter estimates of $\boldsymbol{D}$ can converge to their real value. Fig. 4 shows a good estimate for velocities. Furthermore, simulations for sea states from slight wave (ω_0=1.25 rad/s, H_s=1.0 m) to high wave (ω_0=0.67 rad/s, H_s=3.5 m) were implemented, and the results similar to Fig. 3 show a good estimation of $\boldsymbol{D}$. This demonstrates that the adaptive algorithm has a good adaptability for different sea states.

6 Conclusion

In this paper, an adaptive observer and the parameter estimation based backstepping control method have been proposed. The approach considers unstable ship dynamics and parameter uncertainties. The improvement is that the proposed observer takes into account the first-order wave disturbances filtering as well as the bias term coursed by the slowly varying environmental disturbances and the unmodelled dynamics. Thus, this solution can identify the unknown dynamic parameters online, and provide more complete state estimation for the backstepping controller and will maintain smoother control outputs to introduce wave filtering. The Global Exponential Stability for both the adaptive observer and the backstepping control law has been proven using Lyapunov direct method. The simulation results show the good wave filtering, state observer and parameter estimation performance, and also good position control ability.

References

Aarset, M.F., Strand, J.P. and Fossen, T.I., 1998. Nonlinear vectorial observer backstepping with integral action and wave filtering for ships, *Proceedings of the IFAC Conference on Control Applications in Marine Systems*, Fukuoka, Japan.

Balchen, J.G., Jenssen, N.A. and Sælid, S., 1976. Dynamic positioning using Kalman filtering and optimal control theory, *Proceedings of the IFAC/IFIP Symposium on Automation in Offshore Oil Field Operation*, Holland, Amsterdam.

Calugi, F., Robertsson, A. and Johansson, R., 2003. An adaptive observer for dynamical ship position control using vectorial observer backstepping, *Proceedings of the 42nd IEEE Conference on Decision and Control*, IEEE, Maui, HI, pp. 3262–3267.

Fossen, T.I., 1994. *Guidance and Control of Ocean Vehicles*, John Wiley & Sons Inc., New York, USA.

Fossen, T.I., 2011. *Handbook of Marine Craft Hydrodynamics and Motion Control*, John Wiley & Sons, Chichester, UK.

Fossen, T.I. and Grøvlen, Å., 1998. Nonlinear output feedback control of dynamically positioned ships using vectorial observer backstepping, *IEEE Transactions on Control Systems Technology*, 6(1), 121–128.

Fossen, T.I. and Perez, T., 2007. *Marine Systems Simulator* (*MSS*), http://www.marinecontrol.org

Fossen, T.I. and Strand, J.P., 1999. Passive nonlinear observer design for ships using Lyapunov methods: full-scale experiments with a supply vessel, *Automatica*, 35(1), 3–16.

Grøvlen, Å. and Fossen, T.I., 1996. Nonlinear control of dynamic positioned ships using only position feedback: An observer backstepping approach, *Proceedings of the 35th IEEE Conference on Decision and Control*, IEEE, Kobe, Japan, pp. 3388–3393.

Julier, S.J. and Uhlmann, J.K., 1997. A new extension of the Kalman filter to nonlinear systems, *Proceedings of SPIE3068*, *Signal Processing*, *Sensor Fusion*, *and Target Recognition VI*, SPIE, Orlando, FL, USA.

Koditschek, D.E., 1987. Adaptive techniques for mechanical systems, *Proceedings of the 5th Yale Workshop on Applications of Adaptive Systems Theory*, Yale University, Yale, pp. 259–265.

Morishita, H.M. and Souza, C.E.S., 2014. Modified observer backstepping controller for a dynamic positioning system, *Control Engineering Practice*, 33, 105–114.

Skjetne, R., Fossen, T.I. and Kokotović, P.V., 2005. Adaptive maneuvering. with experiments, for a model ship in marine control laboratory, *Automatica*, 41(2), 289–298.

Torsetnes, G., Jouffroy, J. and Fossen, T.I., 2004. Nonlinear dynamic positioning of ships with gain-scheduled wave filtering, *Proceedings of the 43rd IEEE Conference on Decision and Control*, IEEE, Nassau, pp. 5340–5347.

Tyukin, I.Y., Steur, E., Nijmeijer, H. and van Leeuwen, C., 2013. Adaptive observers and parameter estimation for a class of systems nonlinear in the parameters, *Automatica*, 49(8), 2409–2423.