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Risk Analysis and Assessment of Overtopping Concerning Sea Dikes in the Case of Storm Surge*

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ABSTRACT

Risk analysis and assessment relating coastal structures has been one of the hot topics in the area of coastal protection recently. In this paper, from three aspects of joint return period of multiple loads, dike failure rate and dike continuous risk prevention respectively, three new risk analysis methods concerning overtopping of sea dikes are developed. It is worth noting that the factors of storm surge which leads to overtopping are also considered in the three methods. In order to verify and estimate the effectiveness and reliability of the newly developed methods, quantified mutual information is adopted. By means of case testing, it can be found that different prior variables might be selected dividedly, according to the requirement of special engineering application or the dominance of loads. Based on the selection of prior variables, the correlating risk analysis method can be successfully applied to practical engineering.

Key words: *sea dikes*; *risk analysis*; *failure rate*; *mutual information*

1. Introduction

Sea dikes play an important role in a variety of protection structures relating coastal engineering. They are mainly used to resist the intrusion of waves, aiming at avoiding attacks due to spring tide, high tide and storm surge in the area of coastal or estuarine area. Once they fail to prevent the overtopping resulted from extreme sea conditions, it would bring unpredictable environmental disasters to our social and economic lives (van Gent, 2002). For example, when the 11th typhoon landed in Wenling, Zhejiang Province in 1997, high-tide period of the astronomical tide happened at the same time, leading to serious storm surge. For coastal dikes of the Ningbo Port could not withstand this kind of strong tide induced by high tide and typhoon, it consequently led to the happening of

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seawall crevasse, saltwater intrusion and flash floods for more than three days and a direct economic loss of more than 4.5 billion Chinese Yuan (Xiao *et al*., 2011). It is obvious that it is very meaningful to take the key factors of storm surge into account, when considering the risk analysis and assessment of sea dike overtopping.

The beginning of modern security analysis for coastal protection structures might go back to the well-known MAST project, which has began from the early 1990s in Western Europe. Voortman (2003) also evaluated the risk of engineering structure design related to coastal protection system in the form of risk failure rate. Mínguez *et al*. (2006) took individual dike overtopping risk into account and subsequently conducted the optimization design of dikes structure. Koç (2009) calculated the overturn probability of sea dikes in the case of single load, namely the extreme wave condition, and then carried out the risk analysis. Li (2002) conducted some kind of dike height designs with the consideration of uncertainty of tidal level, in order to analyze sea dike overtopping risks. Yang *et al.* (2012) mainly focused on the risk probability analysis about design combination method of target wind wave and tidal level. However, the phenomena of sea dike overtopping, which were induced by the considerable multi-uncertainty of complicated ocean environments happening more than once during a typhoon period, have been ignored.

When typhoon passes by, it is usually followed by strong winds, heavy rains and storm surge. If the happening of storm surge coincides with that of the astronomic tide upsurge, both the tidal level and typhoon impact might reach their maximum. On the other hand, if the boosted waves under this situation join with the down-rush surge of the passing upriver flood peaks which are caused by heavy rains, the water and tidal levels tend to soar. It can easily lead the boosted waves to overtop the sea dike, causing dike overtopping risk. In general, astronomical tide might be regarded as a steady natural phenomenon, in the case that only the tiny random fluctuations of the tide elevation are ignored. However, the tidal effects resulted from storm surge and heavy rains induced by tropical cyclone are both with great uncertainties (Liu *et al.*, 2008). Therefore, the attempt to take the joint effect of two loads, i.e. storm surge and passing upriver flood, into account is very meaningful.

In this paper, a multi-perspective analysis, including the instantaneous overtopping risk relating the conditions in which sea dikes resist several loads' impacts at the same time and the endurance risk concerning the conditions that overtopping happens continuously is performed. From the viewpoint of focusing on joint return period of multiple loads, dike failure rate and sea dike continuous risk prevention, new risk analysis methods for dike overtopping with the consideration of storm surge are developed. Furthermore, the assessment about the three risk analysis methods above is also conducted by means of quantifying the mutual information.

2. New Risk Analysis Methods of Sea Dike Overtopping

2.1 Risk Analysis Based on Joint Return Period of Multiple Loads

Generally speaking, the overtopping risk of sea dike mentioned here refers to that which is induced by the fact that the temporal mean water level is higher than the local breakwater height. With $Z_{\rm d}$ denoting the local breakwater height, Z_0 denoting the base water level, $Z'_{\rm d}=Z_{\rm d}-Z_0$ denoting the altitude difference, and H_l denoting the rising value of the mean water level during the typhoon period, the prerequisite for overtopping risk of sea dike is that $H_l \geqslant Z'_{\mathrm{d}}$, where l denotes the external factors which cause the rise of the mean water level in front of the dikes during a typhoon period. As the dike failure value means the exact probability value of the occurrence that the water level in front of the dike is higher than Z'_{d} during the N-year return-period, the overtopping risk of sea dike in N-year return-period can be defined as:

$$P_N = P(H_l > Z'_{\mathrm{d}}) = \int_{Z'_{\mathrm{d}}}^{+\infty} f(z)\mathrm{d}z\,. \tag{1}$$

Since two key load factors, i.e. storm surge X and water level Y of passing upriver flood are considered, the crucial step to calculate the overtopping risk of sea dike is to clarify the joint probability properties of the above two-dimensional random variables.

Based on the assumption that the two-dimensional random variables (X,Y) consist of storm surge X and water level Y of passing upriver flood, their joint probability density function $f(x,y)$ can be expressed as follows:

$$f(x,y) = f(x\,|\,y)f(y) = f(y\,|\,x)f(x)\,, \tag{2}$$

where $f(x\,|\,y)$ denotes conditional probability density function of (x, y), and the expression $H_l = H_x + H_y$ denotes the random variable of the rising water level in front of the sea dikes, and then the overtopping risk of sea dike based on N-year joint return period of storm surge and passing upriver flood can be defined as:

$$\begin{aligned} F(x,y) = P((x+y) \geq Z'_{\mathrm{d}}) &= \iint\limits_{\substack{x+y\geq Z'_{\mathrm{d}} \\ y\geq y_0}} f(x,y)\mathrm{d}x\mathrm{d}y \\ &= \iint\limits_{\substack{x+y\geq Z'_{\mathrm{d}} \\ y\geq y_0}} f(x\,|\,y)f(y)\mathrm{d}x\mathrm{d}y. \end{aligned} \tag{3}$$

2.2 Risk Analysis Based on Dike Failure Rate

The so-called dike failure rate refers to the probability that a sea dike still keeps working when water level exactly reaches the dike height, while it begins to cease to be effective when water level overtops the dike height. By far, only the single load has been usually considered, when we conduct some problems relating dike failure rate. In this section, both storm surge and passing upriver flood will be taken into account and be regarded as two key loads which mainly induce the sea dike failure rate. Here, the sea dike risk can be defined as follows.

Based on the assumption that the probability distribution function of the two-dimensional random variable (X,Y) is $F(x,y)$, and its corresponding probability density function is $f(x,y)$, the dike overtopping risk can be defined as Eq. (4) for the given $\mu < x^*, \upsilon < y^*$, where (x^*, y^*) is the upper bound of function $F(x,y)$'s define domain. $\overline{F}(x,y)$ is expressed as $\overline{F}(x,y) = 1 - F_x(x) - F_y(y) + F(x,y)$, where $F_x(x)$ and $F_y(y)$ are the marginal distribution functions of $F(x,y)$.

$$q = \frac{f(\mu,\ \upsilon)}{\bar{F}(\mu,\ \upsilon)}. \quad (4)$$

Here the obtained calculation result is just the failure value of the dike overtopping risk.

2.3 Risk Analysis of Dike Overtopping Prevention Relating Continuous Loads

In practical dike risk prevention works, severely high water level which is induced by storm surge maybe happens more than once, probably leading to catastrophic damages on sea dikes subsequently. For this reason, the risk analysis of dike's continuous prevention ought to take into account the probability that the sea dikes can continue to keep working, when the water level continues to rise, following once dash caused by high water level.

Based on the previous assumption that the probability distribution function of the two-dimensional random variables (X,Y) is $F(x,y)$, and their corresponding probability density function is $f(x,y)$, the following Eq. (5) can be obtained for the given $\mu < x^*, \upsilon < y^*$, where (x^*, y^*) is the upper bound of function $F(x,y)$'s define domain.

$$F_{\mu,\upsilon}(x,y) = P(X-\mu \le x, Y-\upsilon \le y \mid X>\mu, Y>\upsilon)$$
$$= \frac{F(x+\mu, y+\upsilon) - F_x(x+\mu,\upsilon) - F_y(\mu, y+\upsilon) + F(\mu,\upsilon)}{\bar{F}(\mu,\upsilon)}. \quad (5)$$

Thus the risk of dike continuous risk prevention can be defined as:

$$f_{\mu,\upsilon}(x,y) = \frac{f(x+\mu, y+\upsilon)}{1-F_x(\mu)-F_y(\upsilon)+F(\mu,\upsilon)} = \frac{f(x+\mu, y+\upsilon)}{\bar{F}(\mu,\upsilon)}, x \ge 0, y \ge 0 \quad (6)$$

In practical engineering application, (μ,υ) denotes the joint return levels calculated under the condition of various prior variables. Moreover, it is often used to calculate risk failure values when being inserted into Eq. (6).

3. Assessment of Risk Analysis Methods Relating Dike Overtopping

Risk assessment means to select a unified risk standard to evaluate the obtained results, after finishing the quantitative calculation of risk failure values concerning the three kinds of risk analysis methods developed in Section 2. During a typhoon period, the joint effect of storm surge and passing upriver flood leads to sea dike overtopping risk. In other words, the risk failure values calculated by the above three methods are relevant to the overtopping probability $1/N$ (where, N is the joint return period) of the joint effect of storm surge and passing upriver flood. The correlation between them can be described by mutual information (Li and Li, 2005). In this paper, the assessment to those three methods will be conducted by means of mutual information.

The method to estimate the mutual information between overtopping probability $1/N$ due to the joint effect of storm surge and passing upriver flood, and the joint risk failure values of the above two loads during typhoon period are described as follows. First of all, it is worth noting here that the joint risk failure values due to the two loads are calculated by using three kinds of risk analysis methods.

Based upon the hypothesis that X and Y are both discrete random variables and $M = \{x_1, x_2, \cdots, x_n\}$ and $N = \{y_1, y_2, \cdots, y_n\}$ are samples of X and Y respectively, and $\boldsymbol{Z} = \{X, Y\}$ denotes the two-dimensional random vector, The sample of $\boldsymbol{Z}$ $(= \{X, Y\})$ can be expressed as $L = \{(x_i, y_i) | x_i \in M, y_i \in N\}$. On the other hand, the distance $d(z, z')$ between two points $z = (x, y)$ and $z' = (x', y')$ in $\boldsymbol{Z}$ define domain may be expressed as $d(z, z') = \max\{|x - x'|, |y - y'|\}$. For any point z_i in $\boldsymbol{Z}$ define domain, it is possible to easily find its most nearest *k* points, where *k* denotes a parameter belonging to the define domain $k \in \{2, 3, 4\}$ (Kraskov *et al*., 2004). It is worth noting that the value of *k* in this paper is set to be 2. If marking the distance between $z_i = (x_i, y_i)$ and its the most nearest *k*-th point z_k as $\varepsilon_1/2$, that is $d(z_1, z_k) = \varepsilon_1/2$, it is easy to find some points with total number $n_x(i)$ in M define domain whose distance to point x_i is smaller than $\varepsilon_1/2$ and that some points with total number $n_y(i)$ in N whose distance to point y_i is also smaller than $\varepsilon_1/2$, where $i = 1, 2, \cdots, n$. Therefore, the mutual information can be estimated as:

$$I(X, Y) = \psi(k) + \psi(n) - \frac{1}{n}\sum_{i=1}^{n}\left\{\psi\left[n_x(i) + 1\right] + \psi\left[n_y(i) + 1\right]\right\}, \tag{7}$$

where $\psi(x)$ is a digamma function which satisfies the following formula (Li *et al*., 2008):

$$\psi(x+1) = \begin{cases} \psi(x) + \dfrac{1}{x}, & x > 0 \\ -C, & x = 0 \end{cases} \qquad C = 0.5772156\,. \tag{8}$$

It is worth noting that the so-called mission reliability denotes the ability of risk analysis method to complete the risk analysis in limitative period (Gao and Zhang, 2002). Here, mutual information could exactly manifest the reliability of these three risk analysis methods to conduct the risk analysis. The larger the mutual information is, the more reliable the risk analysis method becomes, if the same *N*-year joint return period has been chosen in three risk analysis methods.

4. Examinations and Applications

In order to systematically examine these three kinds of risk analysis methods proposed in this paper, mutual information has been adopted to assess their effectiveness to study the risk analysis of sea dike overtopping in Shanghai. Here, the observed data of storm surge, passing upriver flood and the corresponding typhoon occurrence frequency which are used as the statistical studying samples were selected from Datong Hydrological Station (31°24′, 121°30′) during 1970–1989.

4.1 Risk Analysis of Sea Dike Overtopping

The procedure for sea dike overtopping risk analysis may be described as follows:

(1) With the consideration of typhoon occurrence frequency, quantify the relationship between extreme storm surge and passing upriver flood during typhoon period by means of conditional

probability. In order to exactly describe various possible risk statuses which may emerge during typhoon period, and the uncertain effect of some given random variables on other ones, the maximum-entropy compound distribution model is chosen as the joint probability distribution for storm surge and passing upriver flood (Liu *et al*., 2006; Wang *et al*., 2008, 2011, 2012).

By regarding storm surge and passing upriver flood as prior variable respectively, their joint return levels under different return periods are given in Tables 1 and 2.

Table 1 Joint return levels derived from the assumption regarding extreme storm surge as prior variables (m)

Joint return period	Storm surge	Passing upriver flood
10	1.085	0.34
50	1.358	0.37
100	1.465	0.38
200	1.570	0.40

Table 2 Joint return levels derived from the assumption regarding passing upriver flood as prior variables (m)

Joint return period	Storm surge	Passing upriver flood
10	1.310	0.315
50	1.465	0.405
100	1.508	0.420
200	1.525	0.445

(2) By inserting the joint return levels derived from different prior variables which are listed in Tables 1 and 2 into Eqs. (3), (4) and (6) respectively, different risk failure values from the above three methods can be obtained, as listed in Tables 3 and 4.

Table 3 Risk failure values of joint return levels with storm surge as prior variables (m)

Joint return period	Method 1	Method 2	Method 3
10	9.82e–02	2.755e–01	1.465e–01
50	3.51e–02	2.008e–01	1.110e–01
100	1.78e–02	1.894e–01	9.357e–02
200	1.06e–02	1.337e–01	6.499e–02

Table 4 Risk failure values of joint return levels with passing upriver flood as prior variables (m)

Joint return period	Method 1	Method 2	Method 3
10	2.650e–01	1.879e–01	2.198e–01
50	9.849e–02	4.740e–02	8.280e–02
100	7.210e–02	5.023e–02	8.120e–02
200	6.237e–02	4.341e–02	6.842e–02

The dike overtopping risk q in Eq. (4) can be regarded as the risk attributed to the fact that the sea dike subjects to the design value of joint return level about storm surge and passing upriver flood reaching u and v, respectively.

The dike overtopping risk shown in Eq. (6) can be regarded as the probability that the sea dike can continue to keep working, after the two-dimensional variables (X, Y) composed of storm surge and passing upriver flood successfully resisted the risk (u, v).

The risk failure values of joint return levels calculated from the three risk analysis methods with different variables as prior variables are manifested in Fig. 1, which directly illustrates the results in Tables 3 and 4.

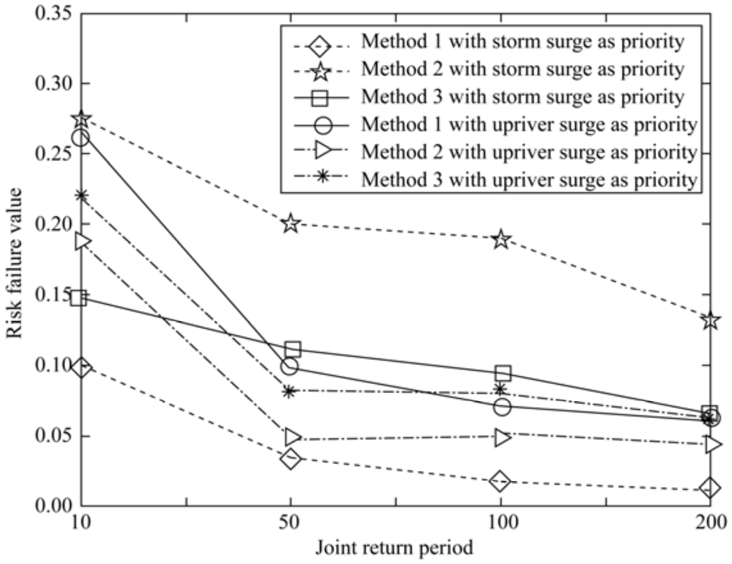

Fig. 1. Risk failure values of three kinds of methods based on different joint return periods.

As shown in Fig. 1, it is obvious that the risk failure values calculated depending on the three kinds of risk analysis methods gradually decrease with the increase of joint return period *N*. It can been found that the risk failure value calculated by Method 1 regarding storm surge as the prior variable is lower than that with passing upriver flood as the prior variable. On the contrary, it also can been found that the risk failure value calculated by Method 2 regarding storm surge as the prior variable is higher than that with passing upriver flood as the prior variable. Otherwise, the contrast result of the risk failure value becomes sensitive in the case of applying Method 3. If the joint return period is set to be 10-year, the risk failure value which is calculated with regard to storm surge as the prior variable is lower than that with passing upriver flood as the prior variable. If the joint return period is set to be 50-year or 100-year, the former is higher than the latter; and, if the joint return period is set to be 200-year, the former is almost equivalent to the latter.

4.2 Assessment of the Dike Overtopping Risk Analysis Methods

The mutual information between the probability $1/N$ of dike overtopping risk induced by the joint effect of storm surge and passing upriver flood during typhoon period and the joint risk failure values of the two loads calculated by the above three kinds of risk analysis methods is estimated by Eq. (7). The calculated results are listed in Table 5.

Table 5 Mutual information calculations of the three kinds of risk analysis methods (bit)

	Method 1	Method 2	Method 3
Storm surge priority	1.4583	1.2083	1.2083
Passing upriver flood priority	0.7500	1.0833	1.3333

Since the risk failure values obtained from the three kinds of risk analysis methods are calculated based on the same joint return period of storm surge and passing upriver flood, the amount of the original information, namely basic information, is the same. However, as it is demonstrated in Table 5, when storm surge is regarded as the prior variable, the mutual information amount calculated by Method 1 is larger than that of Methods 2 and 3. It means that the larger the information amount relating sea dike overtopping risk with N-year joint return period of the two loads is, the more reliable the risk failure value calculated by Method 1 becomes, when we conduct risk analysis with regard to storm surge as prior variable. However, the risk analysis ability of Methods 2 and 3 is almost equivalent to each other. When conducting risk analysis with passing upriver flood as the prior variable, it can be found that the mutual information amount increases from Methods 1, 2 and 3 gradually, so does the information amount relating sea dike overtopping risk with N-year joint return period of the two loads through risk failure values calculated by the three kinds of methods. As a rule, the reliability of the three risk analysis methods increases from the first to the third, and Method 3 is the most reliable one; when the effects of storm surge and passing upriver flood are nearly equivalent, Method 2 becomes more reliable than the other two.

5. Conclusions

(1) The overtopping risk of sea dike induced by the join effect of multiple loads (such as storm surge and passing upriver flood) which happen once or more times is systematically analyzed in this paper. Three analysis methods for sea dike overtopping risk are developed in the form of regarding the joint return period of storm surge and upper reaches flood, dike failure rate, and dike continuous risk prevention as considered factors, respectively.

(2) Mutual information between the sea dike overtopping probability $1/N$ attributed to the joint effect of storm surge and passing upriver flood during typhoon period and the joint risk failure values of the two loads is established. Furthermore, it is also used to assess the three risk analysis methods. The effectiveness and reliability of mutual information to manifest the risk analysis method are well verified. When the same N-year joint return period is given in the three risk analysis methods, one can draw a conclusion that the larger the mutual information amount is, the more reliable the risk analysis method becomes.

(3) According to the effectiveness and reliability examinations for the risk analysis methods and assessment of the correlating mutual information, one can conclude that the selection of risk analysis methods for multiple-load overtopping has tight relationship with the impact of these multiple loads on it; when regarding different loads as the prior variable, the selection of risk analysis method should be different; and for the practical application, the selection of the prior variables and the correlating risk analysis methods for risk analysis of dike overtopping might be determined according to the requirement of engineering project or the dominance of some loads. The three newly developed risk analysis methods and the assessment method of mutual information are meaningful to analyze the sea dike overtopping risk. In other words, their theoretical and practical significances are both appreciable. Moreover, they also provide an effective technique for the construction of sea dikes and the risk

analysis assessment of dike overtopping in the future.

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