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Modeling of Propagation and Transformation of Transient Nonlinear Waves on A Current*

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ABSTRACT

A novel theoretical approach is applied to predict the propagation and transformation of transient nonlinear waves on a current. The problem was solved by applying an eigenfunction expansion method and the derived semi-analytical solution was employed to study the transformation of wave profile and the evolution of wave spectrum arising from the nonlinear interactions of wave components in a wave train which may lead to the formation of very large waves. The results show that the propagation of wave trains is significantly affected by a current. A relatively small current may substantially affect wave train components and the wave train shape. This is observed for both opposing and following current. The results demonstrate that the application of the nonlinear model has a substantial effect on the shape of a wave spectrum. A train of originally linear and very narrow-banded waves changes its one-peak spectrum to a multi-peak one in a fairly short distance from an initial position. The discrepancies between the wave trains predicted by applying the linear and nonlinear models increase with the increasing wavelength and become significant in shallow water even for waves with low steepness. Laboratory experiments were conducted in a wave flume to verify theoretical results. The free-surface elevations recorded by a system of wave gauges are compared with the results provided by the nonlinear model. Additional verification was achieved by applying a Fourier analysis and comparing wave amplitude spectra obtained from theoretical results with experimental data. A reasonable agreement between theoretical results and experimental data is observed for both amplitudes and phases. The model predicts fairly well multi-peak spectra, including wave spectra with significant nonlinear wave components.

Key words: *numerical modeling*; *transient waves*; *current*; *boundary conditions*; *initial conditions*

1. Introduction

The prediction of the propagation and transformation of transient nonlinear waves on a current is a frequent necessity in coastal and ocean engineering. A reliable description of the propagation and evolution of nonlinear waves on a current and understanding of the physics of this phenomenon is of fundamental importance for the modeling of a sea state. The description of this phenomenon may also provide insight into the formation of extreme waves and wave events, not to mention that its modeling is a challenging and interesting problem from a theoretical point of view.

The first attempt to describe waves on a current focused on the modeling of the propagation of periodic waves of small amplitudes on a uniform current. The derived models demonstrated that when waves propagate into opposing currents, their group velocity reduces, leading to an increase in wave height and eventually to wave blocking (Bretherton and Garrett, 1968, Peregrine and Smith, 1979).

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These results were confirmed by a number of laboratory investigations including a series of laboratory experiments and complementary analyses of practical importance conducted by Chawla and Kirby (2002) and Suastika and Battjes (2009).

The models derived for the propagation of periodic waves of small amplitudes on a uniform current are based on linear wave theories. The propagation of weakly nonlinear waves on a current was described by applying perturbation methods or shallow-water equations (Baddour and Song, 1990; Yoon and Liu, 1989; Teng *et al*., 2001). These models provide an insight into the features of the propagation of nonlinear waves on a uniform current. The problem is that the applicability range of the models is rather limited. As it has been expected, the application of weakly nonlinear wave theories implies that the derived models can be applied only for sufficiently small Froude numbers. Moreover, as it was shown by Chen *et al*. (1998), the dispersion equation corresponding to shallow-water waves can be applied only for small wave numbers.

The classical approaches describing weakly nonlinear waves on ambient currents, including shallow water equations or the Boussinesq equations, are basically applicable to waves of small amplitude and weak currents due to their weak nonlinearity or dispersion. A wider applicability range possesses extended Boussinesq-type equations (Chen *et al*., 1998; Madsen and Schäffer, 1998; Ma *et al*., 2009). The modified Boussinesq-type equations contain extra terms to describe nonlinear and dispersive wave effects with a good accuracy. The problem is that the higher-order Boussinesq-type equations are very complex and their implementation is a tedious task. Moreover, there is no appropriate reference dispersion relation for nonlinear waves on a current to adequately modify or impose terms in the Boussinesq-type equations to obtain a model for general applications. Finally, there is a problem to verify or investigate wave propagation on a current by conducting laboratory experiments because of difficulties with input and output boundaries of waves and current or with determining initial conditions. It is a widely recognized fact that nonlinear wave-current interactions are very difficult subjects to be studied in the laboratory as it is often troublesome to generate and control a uniform current field in a wave flume, not to mention more complex situations (Chen *et al*., 1998; Chawla and Kirby, 2002; Ma *et al*., 2009). This deficiency motivates a need to develop different models, approaches and combine techniques which can be applied to describe nonlinear waves on an ambient current in order to be able to understand and predict processes arising from wave-current interactions.

In this work, a novel theoretical approach is applied to predict the propagation and transformation of transient nonlinear waves on a current. First, a semi-analytical solution is derived by applying an eigenfunction expansion method to predict the propagation of nonlinear water waves on a current. The solution is applied to study the transformation of wave profile and the evolution of wave components in a wave train propagating on a current. The main attention is paid to the transformation and evolution of wave profile and energy spectrum arising from the nonlinear interactions of wave components in a wave train which may lead to the formation of very large waves. Then, theoretical results are compared with experimental data and conclusions are specified.

2. Theoretical Formulation

2.1 Statement of Problem

We consider the propagation and transformation of transient nonlinear water waves on a current of speed U. A right-hand Cartesian coordinate system is selected such that the xy plane is horizontal and coincides with the undisturbed free surface, and z points vertically upwards.

It is assumed that:

(1) The fluid is inviscid and incompressible.

(2) The fluid motion is irrotational.

(3) The sea bottom is impervious.

According to the assumptions, the velocity vector, $\boldsymbol{V}(x, z, t)$, has a potential $\Phi(x, z, t)$, such that $\boldsymbol{V}=\nabla\Phi$. The fluid motion is governed by the Laplace equation

$$\nabla^2\Phi = 0 \tag{1a}$$

and the Bernoulli equation

$$\Phi_t + \frac{1}{\rho}P + gz + \frac{1}{2}\left|\nabla\Phi\right|^2 = 0\,, \tag{1b}$$

where ρ is the fluid density, P is the pressure, and g is the gravitational acceleration.

At the free surface, the velocity potential, $\Phi(x, z, t)$, has to satisfy the kinematic boundary condition

$$\eta_t + \Phi_x\eta_x - \Phi_z = 0, \quad z = \eta(x,t) \tag{2a}$$

and the dynamic boundary condition

$$\Phi_t + g\eta + \frac{1}{2}\left|\nabla\Phi\right|^2 = 0, \quad z = \eta(x,t)\,. \tag{2b}$$

At the sea bottom, the following boundary condition must be satisfied:

$$\Phi_z = 0, \ \ z = -h\,. \tag{2c}$$

Moreover, the velocity potential must satisfy boundary conditions at infinity and initial conditions (Wehausen, 1960; Kinsmann, 1965).

The propagation and transformation of transient nonlinear water waves on a current is a complex phenomenon and the solution of Eqs. (1) and (2) is troublesome because the free-surface boundary conditions contain nonlinear terms. Moreover, the boundary conditions are applied on the free surface which is unknown and is a part of a final solution. In order to achieve a solution, various techniques and simplifications are applied. One of the most popular solution techniques is to expand the kinematic free-surface boundary condition and the dynamic free-surface boundary condition in a Taylor series about a mean position

$$\sum_{n=0}^{\infty}\frac{\eta^n}{n!}\frac{\partial^n}{\partial z^n}(\eta_t + \Phi_x\eta_x - \Phi_z) = 0, \ \ z = 0\,; \tag{3a}$$

$$\sum_{n=0}^{\infty}\frac{\eta^n}{n!}\frac{\partial^n}{\partial z^n}\left(\Phi_t + g\eta + \frac{1}{2}\left|\nabla\Phi\right|^2\right) = 0, \ \ z = 0\,, \tag{3b}$$

which helps to obtain a solution because the boundary conditions are applied on z=0 (Sulisz and Hudspeth, 1993).

By expanding the free-surface boundary conditions in a Taylor series, one can obtain the following boundary value problems:

$$\nabla^2 \Phi = 0\,; \tag{4a}$$

$$\sum_{n=0} \frac{\eta^n}{n!}\frac{\partial^n}{\partial z^n}(\eta_t + \Phi_x \eta_x - \Phi_z) = 0,\ z = 0\,; \tag{4b}$$

$$\sum_{n=0} \frac{\eta^n}{n!}\frac{\partial^n}{\partial z^n}\left(\Phi_t + g\eta + \frac{1}{2}|\nabla \Phi|^2\right) = 0,\ z = 0\,; \tag{4c}$$

$$\Phi_z = 0,\ z = -h\,, \tag{4d}$$

where the summation in Eqs. (4b)–(4c) is limited to third-order terms in wave amplitude. Moreover, the velocity potential must satisfy boundary conditions at infinity and initial conditions.

2.2 Solution Technique

The solution is sought by applying eigenfunction expansion method. This method is a widely recognized technique applied to solve boundary-value problems in mathematics and theoretical physics and has been shown to be an efficient method in the modeling of the propagation and transformation of nonlinear waves (Fenton, 1999; Sulisz and Paprota, 2004, 2008). According to the method, the free-surface elevation, η, and the velocity potential, Φ, are sought in the following forms:

$$\eta = \eta_0 + \sum_{n=1}^{N}(a_n \cos \lambda_n x + b_n \sin \lambda_n x)\,; \tag{5a}$$

$$\Phi = \Phi_0 + \sum_{n=1}^{N}\frac{\cosh[\lambda_n (z+h)]}{\cosh(\lambda_n h)}(A_n \cos \lambda_n x + B_n \sin \lambda_n x)\,, \tag{5b}$$

where η_0 and Φ_0 are known functions related to imposed initial conditions, and

$$\lambda_n = \frac{2\pi(n-1)}{b}\,, \tag{5c}$$

in which b is the length of a sector over which the solution is assumed to be periodic.

The coefficients of the eigenfunction expansions in Eq. (5) are determined by applying a time-stepping procedure. The procedure is based on the Adams–Bashford–Moulton predictor-corrector method that enables prediction of the value of a function f from its time derivatives f' (Press *et al.*, 1988). Accordingly, the boundary conditions, Eqs. (4b)–(4c), are combined with the Adams-Bashford predictor

$$f(t+\Delta t) = f(t) + \frac{\Delta t}{24}[55 f_t(t) - 59 f_t(t-\Delta t) + 37 f_t(t-2\Delta t) - 9 f_t(t-3\Delta t)] \tag{6a}$$

and the Adams–Moulton corrector

$$f(t+\Delta t) = f(t) + \frac{\Delta t}{24}[9 f_t(t+\Delta t) + 19 f_t(t) - 5 f_t(t-\Delta t) + f_t(t-2\Delta t)] \tag{6b}$$

to predict the free-surface elevation, η, and the velocity potential, Φ, at a new time level. Then, in analogy to the approach applied by Sulisz and Paprota (2004), the coefficients a_n, b_n, A_n, and B_n are determined by applying a fast Fourier transform.

The solution procedure requires an initial space distribution of the free-surface elevation and the velocity. The problem is that data available in coastal and offshore engineering are based on wave records provided by waverider buoys or stationary wave gauges. In order to obtain a spatial distribution of the free-surface elevation and the velocity potential, a Fourier transform of a recorded time series of the free-surface elevation and a linear wave theory is applied. Accordingly, the following formulas are applied to calculate initial conditions (t<0):

$$\Phi_0 = \sum_n \frac{g}{\omega_n - Uk_n} \frac{\cosh[k_n(z+h)]}{\cosh(k_n h)} \{a_{0n} \sin[k_n(x-x_0)-\omega_n t] + b_{0n} \cos[k_n(x-x_0)-\omega_n t]\}\,; \quad (7a)$$

$$\eta_0 = \sum_n \{a_{0n} \cos[k_n(x-x_0)-\omega_n t] - b_{0n} \sin[k_n(x-x_0)-\omega_n t]\}\,, \quad (7b)$$

where a_{0n} and b_{0n} are the amplitudes arising from the Fourier transform of the free-surface elevation recorded at x_0, ω_n is the wave frequency (ω_n=2π/T_n), and k_n is the corresponding wave number (k_n=2π/L_n).

The described solution technique is very efficient. The application of eigenfunction expansions and a Fourier transform makes it possible to obtain reliable results even for large spatial or time domains. This provides an opportunity to analyze the propagation of nonlinear waves on a current for realistic times and to assess the interaction of wave components in a wave train in large domains, where the application of numerical techniques may provide inaccurate results due to round-off errors.

3. Results

The model derived to predict the propagation of transient nonlinear waves on a current was applied to investigate the effect of wave parameters on the evolution of wave components in a wave train propagating on a uniform current. The evolution of transient nonlinear waves on a current is predicted for a wide range of wave parameters and currents. The analysis of the results is conducted to evaluate the effect of wave frequency and wave steepness on the propagation and transformation of transient nonlinear waves on a current. The analysis concentrates on wave profiles, wave amplitude spectrums, and the changes of wave profile and wave amplitude spectrum due to the interaction of wave components in the wave train propagating on a current. Moreover, some attention is paid to conditions for which freak waves may be formed in a wave train. The nonlinear interaction of wave components in a wave train propagating on a current is believed to be one of the potential sources of the formation of freak waves. This indicates eyewitness reports on formations or propagations of large waves on an ambient current.

The selected basic features of waves on a current and some fundamental differences in wave profiles arising from the opposing and following currents are demonstrated in Fig. 1. The plots show wave trains corresponding to waves of kh=0.5, kh=1, and kh=2 at $Fr=U/(gh)^{0.5}=0$. The results are intuitive and show that water waves on an opposing current are shorter than waves propagating without current. The following current causes further expansion of a wave train in space, which shows the results plotted in Figs. 1b and 1c. The dependence on wave frequencies is complex and the results show that the effects of wave frequencies on the waves are more pronounced for opposing currents. Additional calculations conducted for shorter waves indicate that in drastic cases i.e. for relatively short waves, the propagation of a wave train may be completely blocked, which confirms laboratory experiments and field observations (Chawla and Kirby, 2002; Suastika and Battjes, 2009).

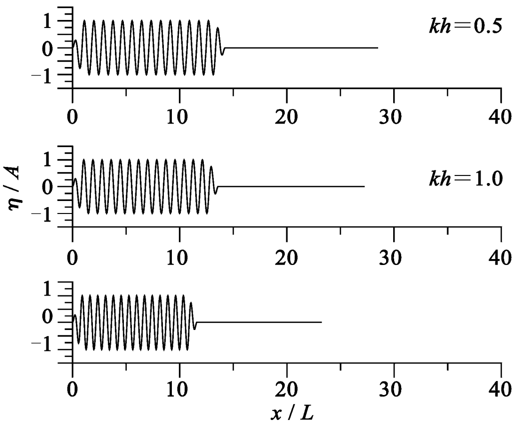

Fig. 1a. Wave trains at *Fr*= –0.1.

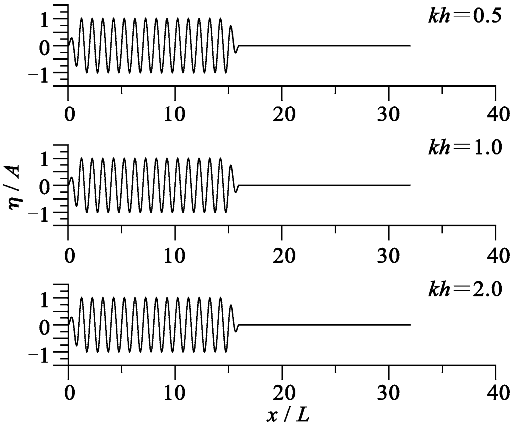

Fig. 1b. Wave trains at *Fr*=0.

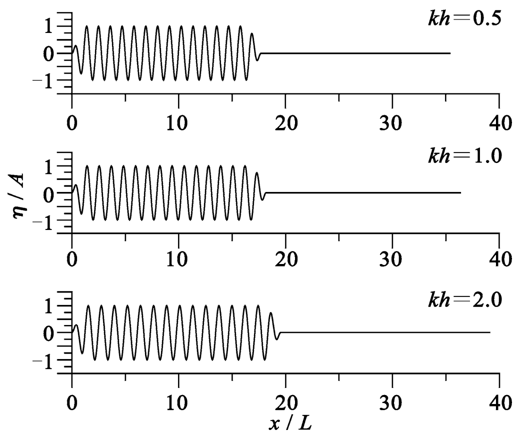

Fig. 1c. Wave trains at *Fr*=0.1.

The effects of the propagation of the wave trains presented in Fig. 1 are demonstrated in Fig. 2 and Fig. 3. The results are presented for waves of moderate steepness and correspond to A/h=0.06 for kh=0.5, A/h=0.05 for kh=1, and A/h=0.04 for kh=2. Figs. 2 and 3 also show the results predicted by applying the solution derived within the frame of linear wave theory. The outcome of the Fourier analysis is included because information obtained by applying a Fourier analysis is helpful in the analysis of the interaction of waves in a wave train and the wave evolution process.

The results shown in Figs. 2 and 3 indicate that the propagation of wave trains is significantly affected by a current. A relatively small current may significantly affect wave train components and the wave train shape. This is observed for both opposing and following currents. The plots in Figs. 2 and 3 indicate that the application of the nonlinear model has a substantial effect on the shape of a predicted wave spectrum. A train of originally linear and very narrow-banded waves changes its one-peak spectrum to a multi-peak one in a fairly short period of time. The nonlinear effects are more pronounced in shallow water. The analysis shows that for waves of intermediate lengths theoretical results provided by the linear model are in reasonable agreement with the results obtained by applying the nonlinear approach often even for fairly steep waves. This indicates that the model derived for linear waves can be applied far beyond its expected range of applicability.

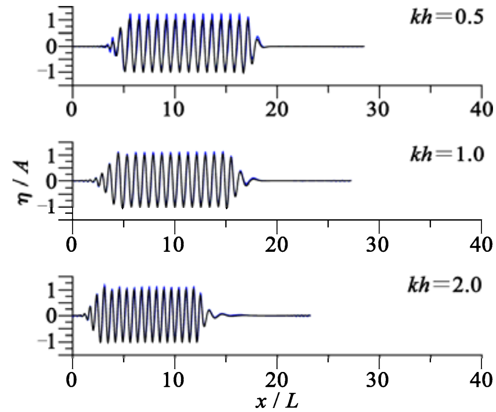

Fig. 2a. Effect of wave length on nonlinear wave train propagation on a current, Fr= –0.1,— linear solution, — nonlinear solution.

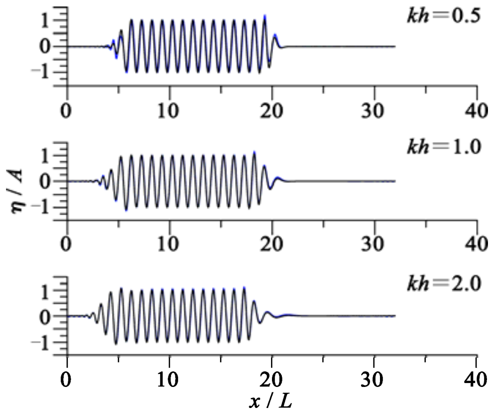

Fig. 2b. Effect of wave length on nonlinear wave train propagation on a current, Fr=0,— linear solution, — nonlinear solution.

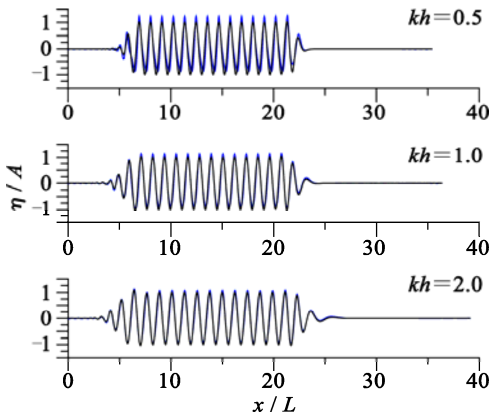

Fig. 2c. Effect of wave length on nonlinear wave train propagation on a current, Fr = 0.1, — linear solution, — nonlinear solution.

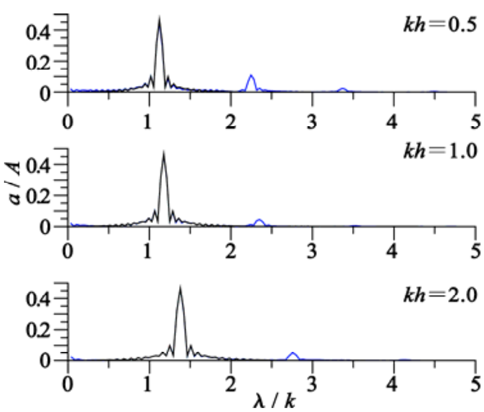

Fig. 3a. Effect of wave length on the amplitudes of Fourier series, Fr= –0.1, — linear solution, — nonlinear solution.

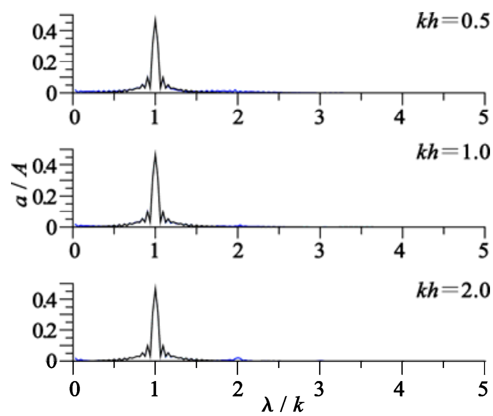

Fig. 3b. Effect of wave length on the amplitudes of Fourier series, Fr=0, — linear solution, — nonlinear solution.

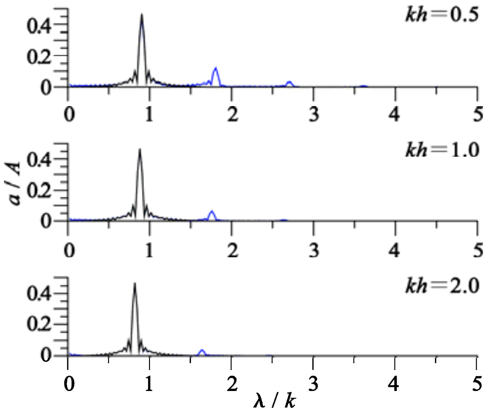

Fig. 3c. Effect of wave length on the amplitudes of Fourier series, Fr = 0.1, — linear solution, — nonlinear solution.

The features of the nonlinear model can be further demonstrated by applying the model to wave trains of different wave steepnesses. The outcome of the model for wave trains of low, moderate, and high waves is presented in Figs. 4 and 5. The results correspond to initial wave trains of A/h=0.01 for low waves and double moderate wave heights for high waves. Figs. 4 and 5 also show the results predicted by applying the solution derived within the frame of linear wave theory.

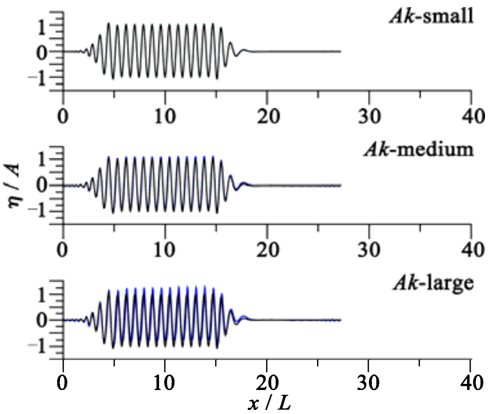

Fig. 4a. Effect of wave steepness on nonlinear wave train propagation on a current, Fr= –0.1, — linear solution, — nonlinear solution.

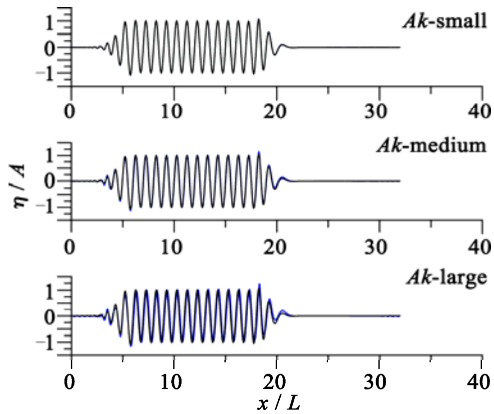

Fig. 4b. Effect of wave steepness on nonlinear wave train propagation on a current, Fr=0, — linear solution, — nonlinear solution.

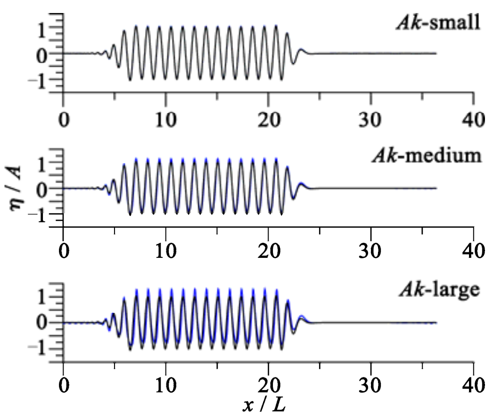

Fig. 4c. Effect of wave steepness on nonlinear wave train propagation on a current, Fr=0.1,— linear solution, — nonlinear solution.

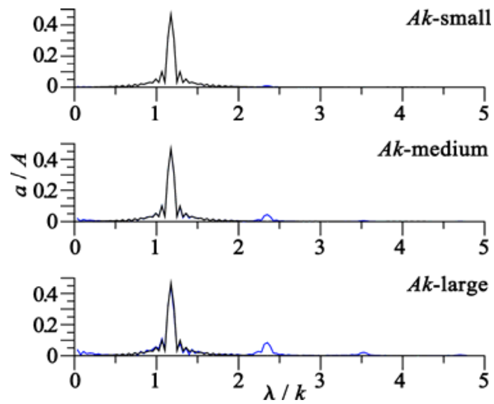

Fig. 5a. Effect of wave steepness on the amplitudes of Fourier series, *Fr*=–0.1,— linear solution, — nonlinear solution.

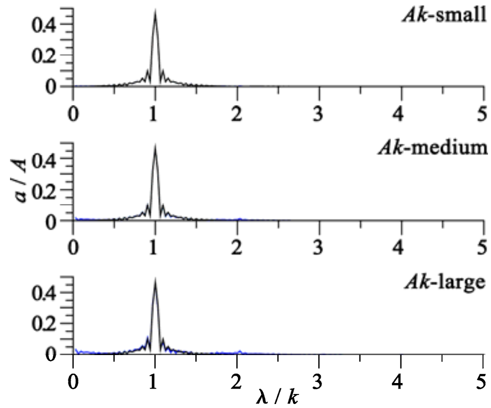

Fig. 5b. Effect of wave steepness on the amplitudes of Fourier series, *Fr*=0, — linear solution, — nonlinear solution.

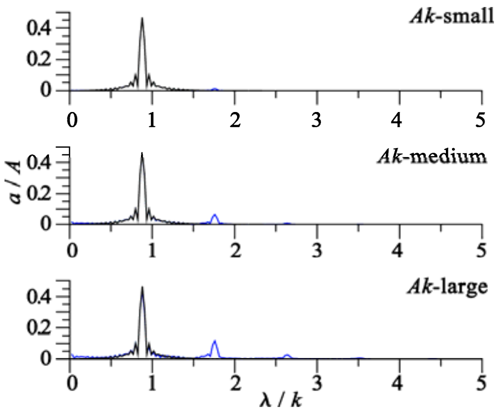

Fig. 5c. Effect of wave steepness on the amplitudes of Fourier series, *Fr*=0.1,— linear solution, — nonlinear solution.

The results in Figs. 4 and 5 show that, for waves of very low steepness, the nonlinear effects and the changes of wave profile and wave spectrum arising from nonlinear interactions of wave components in a wave train may be neglected. Accordingly, linear or weakly nonlinear wave theories can be applied to describe the propagation of a wave train on a current. The higher the waves, the stronger the nonlinear effects arising from nonlinear interactions of wave components in a wave train. As a consequence, a train of originally linear and very narrow-banded one-peak wave spectrum drastically changes its form in a fairly short distance from its initial position. The results in Figs. 4 and 5 indicate a need to apply nonlinear approaches in the modeling of waves on current for steep waves and demonstrate the significance of nonlinear terms in the free-surface boundary conditions.

Finally, the effect of current speed on wave dispersion is analyzed. The results for *Ak*=0.05 are presented in Fig. 6. The plots show the results corresponding to the linear solution reported by Dean and Dalrymple (1984) and the outcome of the present nonlinear model. It can be seen that linear solution underpredicts the phase speed of the regular waves. The results indicate that the longer the waves, the larger the differences become. Additional analysis indicate that wave dispersion also depends on wave steepness. The discrepancies between linear and nonlinear solutions increase with the increasing wave steepness.

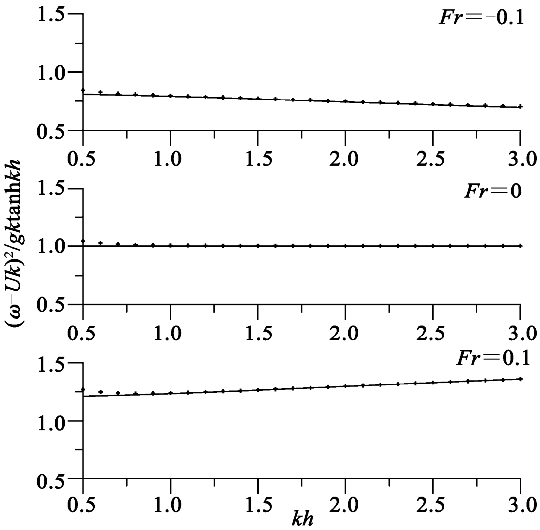

Fig. 6. Effect of current speed on dispersion, — linear solution, + nonlinear solution.

4. Experimental Verification

4.1 Laboratory Experiments

In this study, a series of laboratory experiments were conducted to verify theoretical results obtained from the nonlinear wave model. Because it is not easy to generate a uniform steady current field with waves, and experimental data with adequate information to formulate an initial boundary value problem for starting calculations are not available, at this stage only the verification of the derived model corresponding to its applications related to the prediction of wave propagation and transformation was carried out. As a matter of fact, difficulties to verify wave-current interaction models for uniform currents are well known and are widely recognized in coastal and ocean communities, as it is hard to generate and control a uniform current field with waves in a wave flume (Chen *et al.*, 1998; Chawla and Kirby, 2002; Ma *et al.*, 2009). On the other hand, for nonlinear wave models for simple current fields, more advanced approaches are necessary to be developed valid for a wider class of currents which can be eventually verified.

Laboratory experiments were conducted in the wave flume at the Institute of Hydroengineering, Polish Academy of Sciences, Gdańsk. The wave flume at the Institute of Hydroengineering is 64 m long, 0.6 m wide and 1.4 m deep. It is equipped with a programmable piston wave generator. A wave absorber is supplied at the end of the wave flume. The absorber is built from porous material and can efficiently dissipate even large waves. Wave reflection from the absorber is usually smaller than 5% for typical experiments (Sulisz, 2003).

The experiments in the wave flume were conducted at a water depth h=0.6 m. The wavemaker generated trains of waves of different frequencies to verify the derived model. Additional series of experiments were conducted to analyze the effect of wave steepness on the propagation and transformation of nonlinear waves. The generation of waves of higher wave steepness was achieved by increasing the amplitude of the wavemaker motion. The experimental verification was restricted to the free-surface elevation $\eta(x,t)$.

A group of resistance-type wave gauges were installed in the wave flume to measure the free-surface elevation. The distances of the gauges from the wavemaker mean position were 4 m, 8 m, 12 m, 16 m, 20 m, 24 m, and 28 m, respectively (Fig. 7). The free-surface elevation was measured for each wave train for about 100 s and was sampled at the rate of 100 Hz. Data recording started simultaneously with the wavemaker oscillation.

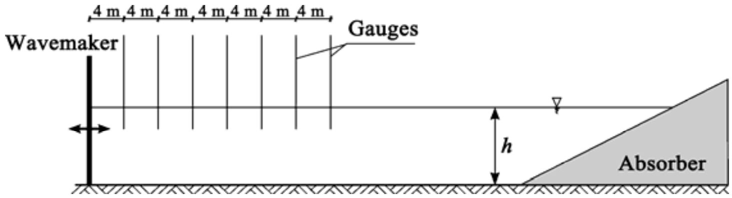

Fig. 7. Wave flume with a system of wave gauges.

The measured time series of free-surface elevations, the evolution of wave components in the generated wave trains, and the measured wave energy spectrum were used to conduct verification of the theoretical approach. The recorded free-surface elevation was analysed by applying a Fourier method and a Kalman filter.

4.2 Comparisons with Experimental Data

The comparisons between theoretical results and experimental data are shown in Fig. 8. The plots show time series of the free-surface elevation predicted by the derived model and corresponding experimental data. Theoretical results and comparisons with experimental data are presented for seven locations along the wave flume where the free-surface elevation was recorded by installed wave gauges. A standard laminar damping was incorporated in the theoretical model to calculate theoretical results for comparisons with experimental data. This is because water waves in a wave flume are exposed to laminar damping and it is recommended to include a damping in a theoretical modeling, especially, if comparisons between theoretical results and experimental data are conducted for locations far away from the wavemaker.

The comparisons between theoretical results and measurements presented in Fig. 8 show that the results obtained by the application of the derived model are in good agreement with experimental data. The plots show that a reasonable agreement between the theoretical results and experimental data is observed for both amplitudes and phases. It is important to note that a good agreement between theoretical results and experimental data is observed even after the formation of a large wave. Usually, wave models poorly predict complex changes of a wave train due to the formation of a large wave, especially, a wave train formed after a large wave event. The plots indicate that the model can predict interaction of wave components in a wave train with sufficient accuracy as well as fairly accurately highly nonlinear wave events.

The ability of the model to adequately predict nonlinear wave transformation may be further demonstrated by applying a Fourier analysis and comparing wave amplitude spectra obtained from theoretical results and experimental data. This type of comparisons of theoretical results and experimental data provides information about the ability of the model to properly describe nonlinear interactions of wave components in a wave train which may lead to the formation of very dangerous large waves. The outcome of the Fourier analysis is presented in Fig. 9. The plots in Fig. 9 show the amplitudes of wave components for the time series presented in Fig. 8.

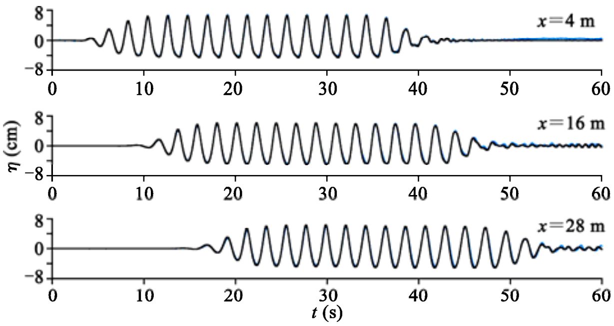

Fig. 8a. Free-surface elevation, L/h=4, — theoretical results, — experimental data.

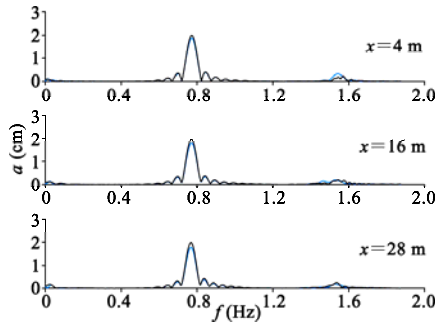

Fig. 9a. Outcome of Fourier analysis, L/h=4, — theoretical results, — experimental data.

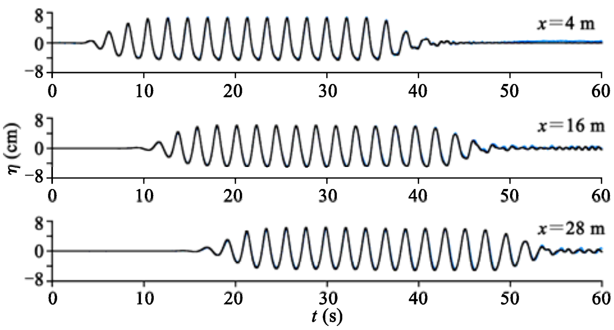

Fig. 8b. Free-surface elevation, L/h=8, — theoretical results, — experimental data.

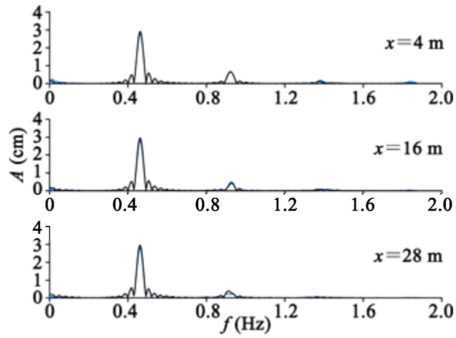

Fig. 9b. Outcome of Fourier analysis, L/h=8, — theoretical results, — experimental data.

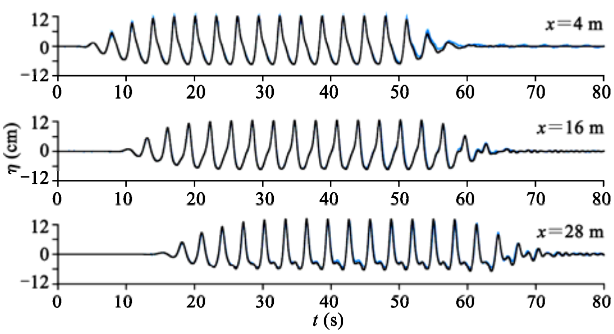

Fig. 8c. Free-surface elevation, L/h=12, — theoretical results, — experimental data.

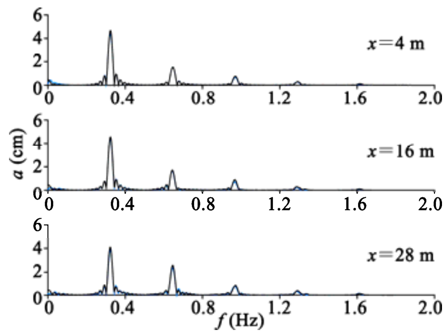

Fig. 9c. Outcome of Fourier analysis, L/h=12, — theoretical results, — experimental data.

The comparisons between theoretical results and measurements presented in Fig. 9 show that the results obtained by the application of the derived model are in good agreement with experimental data. A fairly good agreement between predicted results and the experimental data is observed for a wide range of wave frequencies. The model predicts fairly well multi-peak spectra, including wave spectra with significant nonlinear wave components. It is worth to note that it is not an easy task to predict nonlinear wave transformation effects with such a good accuracy as it is achieved by applying the derived model. On the other hand, the ability to accurately predict wave evolutions in a wave train is of significant practical importance because nonlinear effects may lead to resonant interactions of wave components in a wave train, formation of extreme waves, substantial changes of wave spectrum etc., and the understanding of these processes is of vital interest to scientists and engineers.

5. Conclusions

A novel theoretical approach is applied to predict the propagation and transformation of transient nonlinear waves on a current. The problem was solved by applying an eigenfunction expansion method. The derived semi-analytical solution was employed to study the transformation of wave trains and the evolution of wave components in a wave train propagating on a current. Substantial attention is paid to the changes of wave profile and energy spectrum arising from the nonlinear interactions of wave components in a wave train.

The results show that the propagation of wave trains is significantly affected by a current. Relatively small currents may significantly affect wave train components and the wave train shape. This is observed for both opposing and following currents. The results demonstrate that the application of the nonlinear model has a substantial effect on the shape of a predicted wave spectrum. A train of originally linear and very narrow-banded waves changes its one-peak spectrum to a multi-peak one in a fairly short period of time. The discrepancies between the results predicted by applying the linear and nonlinear models increase with the increasing wavelength and become significant for long waves. The analysis shows that for waves of intermediate lengths theoretical results provided by the linear model are in reasonable agreement with the results obtained by applying the nonlinear approach often even for fairly steep waves. This indicates that the linear model can be applied far beyond its expected range of applicability.

Laboratory experiments were conducted in a wave flume to verify theoretical results. The free-surface elevations recorded by a system of wave gauges were compared with the results provided by the nonlinear model. Additional verification was achieved by applying a Fourier analysis and comparing wave amplitude spectra obtained from theoretical results and experimental data. The comparisons show that theoretical results are in good agreement with experimental data. A reasonable agreement between theoretical results and experimental data is observed for both amplitudes and phases. The model predicts fairly well multi-peak spectra, including wave spectra with significant nonlinear wave components.

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