China Ocean Eng., Vol. 27, No. 1, pp. 99 – 110

DOI 10.1007/s13344-013-0009-9, ISSN 0890-5487

Route Height Connection Across the Sea by Using the Vertical Deflections and Ellipsoidal Height Data*

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(Received 14 September 2011; received revised form 18 June 2012; accepted 22 August 2012)

ABSTRACT

Distance between the main land and island is so long that it is very difficult to precisely connect the height datum across the sea with the traditional method like the trigonometric leveling, or it is very expensive and takes long time to implement the height transfer with the geopotential technique. We combine the data of GPS surveying, astro-geodesy and EGM2008 to precisely connect the orthometric height across the sea with the improved astronomical leveling method in the paper. The Qiongzhou Strait is selected as the test area for the height connection over the sea. We precisely determine the geodetic latitudes, longitudes, heights and deflections of the vertical for four points on both sides across the strait. Modeled deflections of the vertical along the height connecting routes over the sea are determined with EGM2008 model based on the geodetic positions and heights of the sea segmentation points from DNSC08MSS model. Differences of the measured and modeled deflections of the vertical are calculated at four points on both sides and linearly change along the route. So the deflections of the vertical along the route over the sea can be improved by the linear interpolation model. The results are also in accord with those of trigonometirc levelings. The practical case shows that we can precisely connect the orthometric height across the Qiongzhou Strait to satisfy the requirement of order 3 leveling network of China. The method is very efficient to precisely connect the height datum across the sea along the route up to 80 km.

Key words: *height connection across sea*; *deflection of the vertical*; *astro-geodesy*; *orthometric height*; *ellipsoidal height*

1. Introduction

Heights as the basic geographical information reflect the earth surface undulations and the geopotential differences. There are two main height systems in geodesy, that is, the ellipsoidal height referenced to one reference ellipsoid and the orthometric or normal height referenced to one gravimetric-defined datum (Zhang *et al*., 2009). Orthometric or normal height is usually used in the

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* The research was financially supported by the National Natural Science Foundation of China (Grant No. 40974004), the National High-Technology Research and Development Program of China (863 Program, Grant No. 2009AA121405), the Key Laboratory of Surveying and Mapping Technology on Island and Reef of NASMG, China (Grant No. 2011A01), and the Key Laboratory of Advanced Engineering Surveying of NASMG, China (Grant No. TJES1101).

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practical engineering instead of ellipsoidal height because the latter is not of the significant geophysical meaning. The local mean sea level (MSL) determined by long-term data from one or more tide gauges is often selected as the national or regional height datum (Rapp, 1994). The height datum is commonly transferred to many leveling benchmarks to build and maintain the national or regional height reference frame with the precise leveling method. So the orthometric or normal heights are traditionally referenced to the local MSL, and there are different height datums for each country or region. Differences between different local height datums may be larger than 2 m because of the effect of the sea surface topography (Rummel and Teunissen, 1988). Because the main land and island or reef are separated by the ocean, there are usually different height datums in the land and island or reef, which makes one country or regions with islands have different height frames and cannot unify the corresponding spatial information of the land and ocean. Therefore many geodetic scientists pay more attention to the height datum unification and height transfer (Amos and Featherstone, 2009; Ardalan and Safari, 2005; Burša *et al*., 1999, 2001; Colombo, 1980; Featherstone, 2000; Grafarend and Ardalan, 2000; Heck and Rummel, 1990; Hipkin, 2002; Jekeli, 2003; Nahavandchi and Soltanpour, 2006; Pan and Sjöberg, 1998; Rapp, 1995; Rummel and Ilk, 1995; Sanso and Usai, 1995; Zhang *et al*., 2009).

Height connection across the sea can be made to unify the height datums for the land and island with methods of precise leveling, trigonometric leveling, hydrostatic leveling, oceanic dynamics, GPS/leveling, and/or geopotential difference (Xu and Bao, 2009). Precise leveling method cannot be used to connect the heights across the sea through a long distance which makes the distance difference of the fore sight and rear sight very long because the stadia between the instrument and leveling ruler is very short. Hydrostatic leveling method utilizes one connecting pipe to transfer the height across the sea (Madsen and Tscherning, 1990). This method is very expensive. The connecting pipe should be of high quality and no air pocket appears in it to hold hydrostatic balance under the condition of long distance across the sea. Meanwhile the air pressure difference and the fluid density difference also seriously affect the hydrostatic balance (Li and Jiang, 2001).

Height datums on islands close to the main land can be connected with the trigonometric leveling method (Li and Jiang, 2001). The geometric height difference between two benchmarks on the land can be precisely determined with the trigonometric leveling method. So the height difference estimated with the method includes errors caused from the non parallelism of levels, the vertical refractive index difference, the deflection of the vertical and the precise sighting objective over the sea (Li *et al*., 2007; Guo *et al*., 2011). The weather conditions also limit its applications across seas and the determined height difference has so many errors that the method is seldom used in the height transfer across seas through a long distance more than 10 km.

Oceanic dynamic leveling method also known as the tidal observing method has been ever used to determine two MSLs' difference by using many years' tidal observations of two tide gauges to estimate the orthometric height or the normal height on the island (Mather, 1976). The method takes long-term tidal observations for computation of MSLs (Ekman, 1999). We all know that the local MSL differs with the geoid and there exists the sea surface slope (Rummel and Teunissen, 1988; Xu and Rummel, 1991). Oceanic data are too sparse. So the method should be improved to obtain more precise heights on separated islands over long distance.

The GPS/leveling method uses many benchmarks with geodetic and orthometric/normal heights to fit a local geoid/quasigeoid on the land, then extrapolates the local reference surface to the unknown point on the island, and calculates its orthometric or normal height by its known ellipsoidal height. On the one hand, the precision for the mathematical local geoid or quasigeoid is only up to the level of decimeter or centimeter (Guo *et al*., 2005). On the other hand, the extrapolating algorithm also gives additional errors. The premise for the method is to require GPS/leveling benchmarks on the land and unknown points on the island to be on the same local geoid or quasigeoid, which is very unrealistic. The premise-made error for a long distance up to 20 km can be up to the level of decimeter. Madsen and Tscherning (1990) ever successfully utilized the method to transfer the land height datum to one island separated 20 km in the Great Belt Channel. Li and Jiang (2001) ever used the method to transfer the national height datum 85 of China (CNHD85) to the Yangshan Island about 30 km away from the main land with the precision of centimeter level. Li *et al*. (2007) ever transferred CNHD85 to the Little Changshan Island about 30 km away from the main land with the precision of centimeter level.

Height datums across the sea can be connected with GPS technique on the basis of the refinement of local gravimetric geoid/quasigeoid. Rummel and Teunissen (1988) ever solved the geodetic boundary value problem to directly connect the vertical datums. Zhang *et al*. (2009) adopted the solution of the linearized fixed-gravimetric boundary value problem to compare the height datums at Shenzhen and Hong Kong. Xu and Bao (2009) proposed to employ the geopotential difference technique to transfer the height datum across the sea. Gravimetric and GPS data in the same reference frame are needed to realize the height transfer across the sea with the geopotential method, and the geodetic boundary value problem should be solved to determine the potential difference to unify the height datums (Ardalan and Grafarend, 2004; Colomo, 1980; Heck and Rummel, 1990; Rapp, 1997; Rummel and Teunissen, 1988; Sano and Usai, 1995). But local heights referenced to local height datum are needed to calculate gravity anomalies so that it is very difficult to unify gravimetric datum. Meantime more gravity data are needed and much complex algorithms are used which will take long time and more cost.

The astronomical leveling method can be used on the land instead of that over the sea because deflections of the vertical on the land can be precisely measured and those over the sea can not be directly and precisely measured. Based on the astronomical leveling principle (Guan and Ning, 1981), the geoid undulation difference of two points can be calculated if deflections of the vertical along the route connecting these two points across sea are known. Again if the geodetic height difference is known, we can estimate the orthometric height difference from the relationship of the ellipsoidal height and orthometric height. The astronomical leveling method is generally used on the land instead of over the sea because there are not precise deflections of the vertical over seas. Here a route height connection method is put forward to transfer the height datum across the sea with the improved astronomical leveling method in the paper. Geodetic coordinates and deflection of the vertical on the points near coasts on the land and island are precisely measured, and then ellipsoidal heights of all stations can be calculated. The astronomical leveling route is located on the sea and ellipsoidal heights along the route can be estimated by use of DNSC08MSS model (Andersen and Knudsen, 2009). Modeled deflections of the vertical along the route can be given by use of EGM2008 model (Pavlis *et* *al.*, 2008) and then should be improved by the measured vertical deflections at two endpoints by the astro-geodetic method with high precision smaller than 0.5″. The Qiongzhou Strait is selected as the test area for the height connection.

2. Orthometric Height Difference with the Astronomical Leveling Method

The deflection of the vertical is one separated angle between the plumb line and the normal of the reference ellipsoid, which indicates the slope degree of the geoid with respect to ellipsoid (Guan and Ning, 1981). So geoid undulation can be computed with deflections of the vertical, and astro-geodetic data are directly used to transfer the heights on the land where the astro-geodetic surveying is easily implemented. But it is very difficult to precisely take the astro-geodesy field work. The astronomical leveling method can also be used to connect the height across the sea as long as we know precise deflections of the vertical over the sea.

Suppose that there are two close points A and B on geoid whose azimuth is α_{AB}, and the meridian and prime vertical components of the vertical deflection on point A are ξ_A and η_A, respectively, as shown in Fig. 1. Component θ_{AB} along the direction AB of the vertical deflection is

$$\theta_{AB} = \xi_A \cos\alpha_{AB} + \eta_A \sin\alpha_{AB} \,. \tag{1}$$

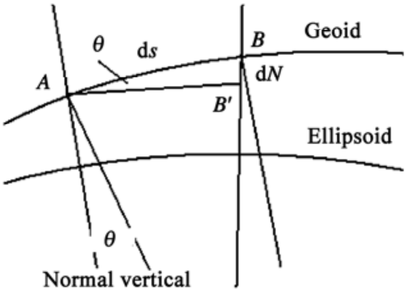

Fig. 1. Schematic chart of geoid undulation with the astro-leveling method.

In practice, θ_{AB} is the component along the direction AB of the dihedral angle between the geoid and the surface parallel to the ellipsoid passing point A. AB' is perpendicular to the ellipsoid normal passing point B. Since θ_{AB} is very small, BB' can be approximately considered as the difference $\mathrm{d}N$ of geoid undulations at points A and B, that is

$$\mathrm{d}N = -\theta_{AB}\mathrm{d}s\,, \tag{2}$$

where $\mathrm{d}s$ is the distance between points A and B. When point A is apart from point B, Eq. (2) is integrated to obtain the difference ΔN_{AB} of geoid undulations at points A and B

$$\Delta N_{AB} = N_B - N_A = -\int_A^B \theta \mathrm{d}s\,, \tag{3}$$

where N is the geoid undulation. Eq. (3) is discreted to practically calculate the difference of geoid undulations as:

$$\Delta N_{AB} = -\sum_{i=1}^{n} \theta_i \Delta s_i \ , \tag{4}$$

where n is the number of segmentation along the connecting route from A to B. From Eq. (4), we can compute the difference of geoid undulation across the sea if we know the precise deflections of the vertical along route AB. Ellipsoidal heights H_A and H_B in the same reference frame (for example ITRF2005) can be precisely estimated with the precise GPS surveying technique. We all know that the relation of ellipsoidal height H and orthometric height h is

$$H = h + N \ . \tag{5}$$

Then the orthometric height difference Δh_{AB} between points A and B is

$$\begin{aligned} \Delta h_{AB} &= h_B - h_A = (H_B - H_A) - \Delta N_{AB} \\ &= (H_B - H_A) + \int_A^B \theta \mathrm{d}s = \Delta H_{AB} + \sum_{i=1}^{n} \theta_i \Delta s_i , \end{aligned} \tag{6}$$

where ΔH_{AB} is the geodetic height difference between points A and B which can be precisely determined with the GPS technique.

Suppose that the deflection of the vertical and the distance for each segmentation are independent, based on the error propagation law, from Eq. (6) we can obtain the precision of orthometric height difference as:

$$m_{\Delta h_{AB}}^2 = m_{\Delta H_{AB}}^2 + \sum_{i=1}^{n} (\theta_i^2 m_{\Delta s_i}^2 + \Delta s_i^2 m_{\theta_i}^2) , \tag{7}$$

where $m_{\Delta h_{AB}}$ is the root-mean-square error of the orthometric height difference, $m_{\Delta H_{AB}}$ is the error of the geodetic height difference, $m_{\Delta s_i}$ is the error of the distance for i-th segmentation, and m_{θ_i} is the error of the vertical deflection component of i-th segmentation.

The deflection of the vertical is generally very small in Eqs. (6) and (7). For example, it is about $10''=4.848\times10^{-5}$ rad on the flat areas. So the effect of the distance error on the orthometric height difference is very small and can be neglected. For example, the distance error is 10 cm whose effect is only 4.8×10^{-3} mm. The precision of geodetic height difference can be smaller than 1 cm through a long session survey with the relative GPS positioning method (Guo *et al*., 2008). For a height-connecting route of 50 km, supposing $\theta = 20''$, Tables 1 and 2 list the errors of orthometric height differences for different segmentation lengths and precisions of the vertical deflection.

Table 1 Errors of orthometric height differences for $m_{\Delta H_{AB}} = 10$ mm and $m_{\Delta s_i} = 1000$ mm , unit in mm

Δs	$m_\theta = 0.1''$	$m_\theta = 0.3''$	$m_\theta = 0.5''$	$m_\theta = 1.0''$	$m_\theta = 2.0''$
50 km	26.2	73.4	121.6	242.6	484.9
25 km	19.8	52.4	86.3	171.7	343.0
10 km	14.8	34.0	55.1	108.9	217.0
5 km	12.6	25.1	39.6	77.3	153.6
2 km	11.1	17.7	26.2	49.5	97.5
1 km	10.6	14.4	19.9	35.7	69.3

From Table 1, we can find that the error of geodetic height difference is the main factor affecting the height connection across the sea when $m_{\Delta h_{AB}} = 10\ \text{mm}$ and the precision of the vertical deflection on the sea is smaller than $0.5''$ for the segments of 2 km and 1 km, respectively. From Table 2, we can also find that the error of geodetic height difference is the main factor affecting the height connection across the sea when $m_{\Delta h_{AB}} = 50\ \text{mm}$ and the precision of vertical deflection on the sea is smaller than $1''$ for the segments of 5 km, 2 km, and 1 km, respectively.

Table 2 Errors of orthometric height differences for $m_{\Delta H_{AB}} = 50\ \text{mm}$ and $m_{\Delta s_i} = 1000\ \text{mm}$, unit in mm

Δs	$m_\theta = 0.1''$	$m_\theta = 0.3''$	$m_\theta = 0.5''$	$m_\theta = 1.0''$	$m_\theta = 2.0''$
50 km	55.6	88.3	131.1	247.5	487.4
25 km	52.9	71.7	99.2	178.6	346.4
10 km	51.2	59.6	73.7	119.4	222.5
5 km	50.6	55.0	63.0	91.5	161.3
2 km	50.2	52.1	55.6	69.6	109.1
1 km	50.1	51.1	52.9	60.6	84.9

Based on the state leveling specifications of China (GB/T 12897-2006 and GB 12898-91), the tolerances for the leveling of orders 1, 2, 3, and 4 along the leveling route of 50 km are 12.7 mm, 28.3 mm, 84.9 mm, and 141.4 mm, respectively. From Tables 1 and 2, we can find that it is very difficult to obtain the precisions of order 1 leveling using the GPS geodetic data and the vertical deflections. But the precision for order 2 leveling may be acquired, and it is easy to obtain the precisions of order 3 or 4 leveling for the segments of 5 km, 2 km, and 1 km along the connecting route.

3. Deflections of the Vertical Along the Connecting Route over the Sea

In Eq. (6), the vertical deflection is the most important element affecting the precision of the calculated height. It is very difficult or even impossible to implement the traditional astro-geodetic surveying over the sea to obtain precise deflections of the vertical. This is the key reason that the astronomical leveling is not used over the sea. There are two points *B* and *C* near the coast across the sea, as shown in Fig. 2. We can precisely measure the meridian and prime vertical components of the vertical deflections with the traditional astro-geodetic method, that is, ξ_B, η_B, ξ_C, and η_C, respectively. There is a sea route from *B* to *C* along which it is impossible to directly obtain the deflections of the vertical over the sea. The sea is shallow near the land and island, where data precision of the satellite altimeter is very low and the precisions of vertical deflections determined with satellite altimetric data are very poor (Guo *et al*., 2010). Therefore, we cannot use the satellite altimetry-derived vertical deflections in the height connection project. The vertical deflections along the sea route can be calculated from the high-degree earth gravity field model like EGM2008 (Pavlis *et al*., 2008) based on the geopotential theory (Moritz, 1980; Guan and Ning, 1981). Of course, their precisions should be improved for the astronomical leveling utilization. Let the components of vertical deflections for points *B* and *C* determined with EGM2008 be ξ'_B, η'_B, ξ'_C, and η'_C, respectively.

Then there exist differences between the measured and calculated vertical deflections of *B* and *C*, that is

$$\Delta\xi_B = \xi_B - \xi_B', \quad \Delta\eta_B = \eta_B - \eta_B', \quad \Delta\xi_C = \xi_C - \xi_C', \quad \Delta\eta_C = \eta_C - \eta_C'. \tag{8}$$

Supposing that the difference between measured and calculated vertical deflections is linear for the small sea area, we can obtain the linear model as:

$$\Delta\xi = a_{10} + a_{11}s, \quad \Delta\eta = a_{20} + a_{21}s, \tag{9}$$

where $a_{10} = \Delta\xi_B$, $a_{11} = \dfrac{\Delta\xi_C - \Delta\xi_B}{s_{BC}}$, $a_{20} = \Delta\eta_B$, $a_{21} = \dfrac{\Delta\eta_C - \Delta\eta_B}{s_{BC}}$, and s is the distance from the interesting point to point *B*.

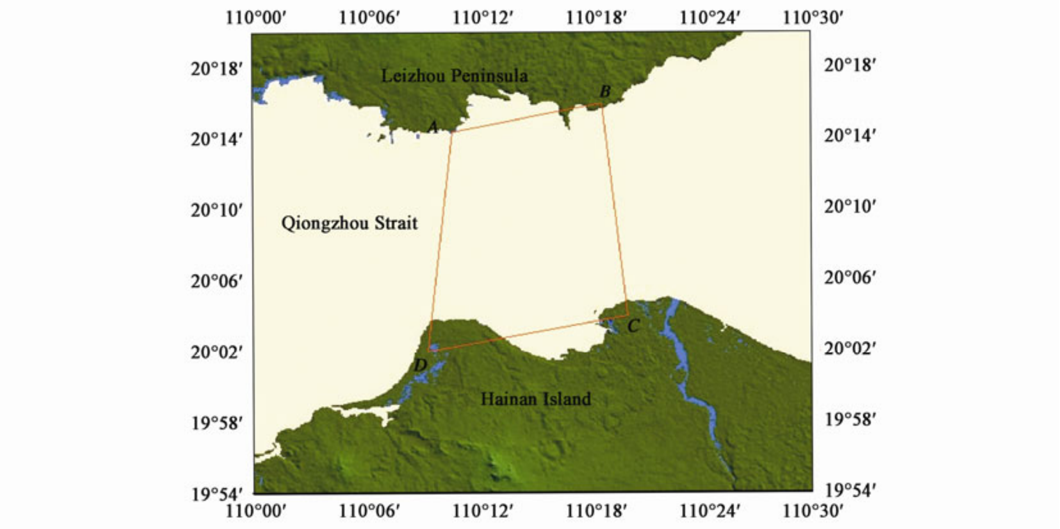

Fig. 2. Schematic chart of orthometric height connection across the Qiongzhou Strait.

Components of the vertical deflection of point *i* along the sea route determined from EGM2008 (Pavlis *et al.*, 2008) are ξ_i' and η_i', and the distance of point *i* to point *B* is s_{Bi}. The corrected components calculated by Eq. (9) are $\Delta\xi_i = a_{10} + a_{11}s_{Bi}$ and $\Delta\eta_i = a_{20} + a_{21}s_{Bi}$. Then the improved vertical deflections of point *i* are

$$\xi_i = \xi_i' + \Delta\xi_i, \quad \eta_i = \eta_i' + \Delta\eta_i. \tag{10}$$

In order to verify the precisions of the vertical deflections determined by EGM2008 on both sides across the Qiongzhou Strait, we collected the measured vertical deflections on nine astro-geodetic points of order 1 as shown in Fig. 3 and the modeled vertical deflections are calculated from EGM2008 up to 2190°. These checked points are located from 19°39'N to 20°25'N and 109°59'E to 110°44'E. Precision of the vertical deflection on these checked points is up to $0.5''$. We found that the precisions of meridian and prime vertical components determined from EGM2008 are $1.6''$ and $1.7''$, respectively. So the interpolated method can improve the modeled deflection of the vertical.

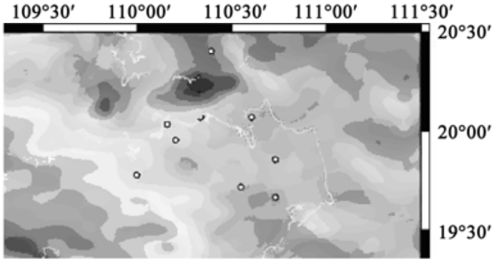

Fig. 3. Distribution of checked astro-geodetic points in which circles stand for their positions.

4. Optimal Determination of the Segment Number

Supposing that the geodetic height difference is independent of the effect of the vertical deflection and neglecting the effect of the distance error in Eq. (6), we can obtain the optimal precision of the height difference when the geodetic height error is equal to the error effect of the vertical deflection:

$$m_{\Delta H_{AB}}^2 \approx \sum_{i=1}^{n} \Delta s_i^2 m_{\theta_i}^2 \,. \tag{11}$$

In general, all segments are equal. So we can obtain

$$n = l^2 \frac{m_\theta^2}{m_{\Delta H_{AB}}^2} \,, \tag{12}$$

where l is the route distance. For example, giving $l = 50$ km, $m_\theta = 1''$ and $m_{\Delta H_{AB}} = 5$ cm, we can obtain $n \approx 24$. So we finally select the distance of each segment to be 2 km.

Precision of the geodetic height difference determined with the relative GPS technique can be smaller than 1 cm. But it is very difficult to obtain the precise vertical deflections with the precision smaller than $1''$ on the sea. Thus Eq. (12) can be rewritten as:

$$n = l^2 \frac{m_\theta^2}{k m_{\Delta H_{AB}}^2} \,, \tag{13}$$

where k is the adjustable factor to make the segment number in accord with the precisions of geodetic height differences and vertical deflections on the sea surface. For example, giving $l = 50$ km, $m_\theta = 1''$ and $m_{\Delta H_{AB}} = 1$ cm, and considering the effect of more segments and distance errors, we can adopt k=20 and then obtain $n \approx 30$. Therefore we can select the distance of each segment to be 1.5~2 km.

5. Practical Case of the Height Connection across the Qiongzhou Strait

The Qiongzhou Strait connects the Hainan Island to the south and the Leizhou Pensinsula to the north. The minimum distance over the Qiongzhou Strait is about 20 km and the mean depth is about 44 m. CNHD85 is used in the Leizhou Peninsula and the datum should be precisely transferred to the Hainan Island. So four points near the coast at both sides across the Qiongzhou Strait are carefully selected, in which points *A* and *B* are located at the Leizhou Peninsula and points *C* and *D* are situated at the Hainan Island, as shown in Fig. 2. The distances of *AD* and *BC* are about 23 km and 22 km, respectively. We made the static relative GPS survey on these four points to obtain their geodetic latitudes, longitudes and heights whose precisions are smaller than 1 cm. And the astro-geodetic observation was also made on these four points to obtain their astronomical latitudes and longitudes to compute the deflections of the vertical whose precisions are smaller than $0.4''$.

Modeled deflections of the vertical along the sea connecting the routes are calculated from EGM2008 up to 2190° based on the geopotential theory. In the calculation ellipsoidal heights of all points over the sea are needed and determined with DNSC08MSS model (Andersen and Knudsen, 2009). Eqs. (8), (9) and (10) are then used to improve the qualities of vertical deflections along the routes over the sea. Finally, Eq. (6) is used to solve the orthometric height differences across the sea. Tables 3, 4 and 5 list the results of the connected heights for different segmentations of 5 km, 2 km and 1 km along the routes over the sea, respectively.

Table 3 Results of connected height for segmentation of 5 km

Route	S (m)	ΔH (m)	ΔN (m)	Δh (m)
A-B	15998.68	2.912	0.6894	3.6014
B-C	21885.50	36.710	0.3248	37.0348
C-D	18880.53	2.129	−0.8153	1.3137
D-A	23006.84	−41.751	−0.2026	−41.9536
A-C	25251.94	39.622	0.9903	40.6123
B-D	31216.85	38.849	−0.5346	38.3144

Table 4 Results of connected height for segmentation of 2 km

Route	S (m)	ΔH (m)	ΔN (m)	Δh (m)
A-B	15998.68	2.912	0.6902	3.6022
B-C	21885.50	36.710	0.3228	37.0328
C-D	18880.53	2.129	−0.8145	1.3145
D-A	23006.84	−41.751	−0.2001	−41.9511
A-C	25251.94	39.622	0.9891	40.6111
B-D	31216.85	38.849	−0.5374	38.3116

Table 5 Results of connected height for segmentation of 1 km

Route	S (m)	ΔH (m)	ΔN (m)	Δh (m)
A-B	15998.68	2.912	0.6904	3.6024
B-C	21885.50	36.710	0.3225	37.0325
C-D	18880.53	2.129	−0.8144	1.3146
D-A	23006.84	−41.751	−0.1996	−41.9506
A-C	25251.94	39.622	0.9889	40.6109
B-D	31216.85	38.849	−0.5422	38.3068

Table 6 lists the closure errors for all close loops in the height connecting project. Based on the state leveling specifications of China (GB/T 12897-2006 and GB 12898-91), the closure errors for all close loops are smaller than the tolerances of order 3 leveling. We find that the connected orthometric height across the Qiongzhou Strait for segmentations of 5 km, 2 km and 1 km can all satisfy the requirement of order 3 leveling, among which precisions for segmentation of 2 km are the best, in general.

Table 6 Closure errors for all loops

Loop	Distance (m)	Closure error (mm)			Allowed error (mm)	
		5 km	2 km	1 km	Order 2 leveling	Order 3 leveling
ABCDA	79771.55	−3.8	−1.5	−1.2	35.7	107.2
ABCA	63136.12	+23.8	+23.9	+24.0	31.8	95.4
ABDA	70222.37	−37.8	−37.3	−41.5	33.5	100.6
ACDA	67139.30	−27.6	−25.5	−25.2	32.8	98.3
BCDB	71982.88	+34.0	+35.7	+40.3	33.9	101.8

Another index to evaluate the precision of the connected height is the full root-mean-square error (FRMSE) of the height difference per kilometer as:

$$w = \pm\sqrt{\frac{1}{M}\left(\frac{WW}{L}\right)}, \tag{14}$$

where W is the closure error in mm, L is the loop distance in km, and M is the number of loops. Substituting the data listed in Table 6 into Eq. (14), we can calculate the FRMSEs per kilometer for segmentations of 5 km, 2 km, and 1 km are 3.37 mm, 3.35 mm, and 3.62 mm, respectively. Allowed FRMSEs per kilometer for order 2 and 3 leveling are 2 mm and 6 mm respectively according to the state leveling specifications of China. So the connected orthometric height across the Qiongzhou Strait for segmentations of 5 km, 2 km and 1 km can all satisfy the requirement of order 3 leveling, among which the precisions for the segmentation of 2 km are the best.

We also made the trigonometric leveling across the Qiongzhou Strait to check the above route connecting heights. The results indicate that all satisfy the requirement of order 3 leveling.

6. Conclusions

It is firstly a successful attempt to use the astronomical leveling method to connect the height across the sea instead of on the land. We combine the data of GPS survey, astro-geodesy and EGM2008 to precisely connect the orthometric height across the Qiongzhou Strait which satisfies the requirement of order 3 leveling according to the state leveling specifications of China (GB/T 12897-2006 and GB 12898-91) in this paper. From Eq. (7) and Tables 1 and 2, we can find that the main error sources of the method are from the measuring error and representative error of the deflection of the vertical and the interpolated model error in Eq. (9). So how to improve the quality of the vertical deflection over the sea is very important to obtain more precise results with the method. We also find that we can obtain the best connecting results for the segmentation of 2 km over the sea. It is very easy to implement the method to obtain much more precise results than those with the traditional method like the trigonometric leveling and the geopotential method. The loop of *ABCDA* with the distance of about 80 km only has the closure error of −3.8 mm, −1.5 mm, and −1.2 mm for segmentations of 5 km, 2 km and 1 km, respectively. Compared with the trigonometric leveling results, the route connecting height can also satisfy the requirement of order 3 leveling.

References

Amos, M. J. and Featherstone, W. E., 2009. Unification of New Zealand's local vertical datums: Iterative gravimetric quasigeoid computations, *J. Geod.*, **83**(1): 57~68. doi: 10.1007/s00190-008-0232-y

Andersen, O. B. and Knudsen, P., 2009. DNSC08 mean sea surface and mean dynamic topography models, *J. Geophy. Res.*, **114**(C11): C11001. doi:10.1029/2008JC005179

Ardalan, A. A. and Grafarend, E. W., 2004. High-resolution regional geoid computation without applying Stokes's formula: a case study of the Iranian geoid, *J. Geod.*, **78**(1-2): 138~156. doi: 10.1007/s00190-004-0385-2

Ardalan, A. A. and Safari, A., 2005. Global height datum unification: A new approach in gravity potential space, *J. Geod.*, **79**(9): 512~523. doi: 10.1007/s00190-005-0001-0

Burša, M., Kouba, J., Kumar, M., Müller, A., Raděj, K., True, S. A., Vatrt, V. and Vorríšková, M., 1999. Geoidal geopotential and world height system, *Studia Geophysica et Geodaetica*, **43**(4): 327~337. doi:10.1023/A:1023273416512

Burša, M., Kouba, J., Müller, A., Raděj, K., True, S. A., Vatrt, V. and Vorríšková, M., 2001. Determination of geopotential differences between local vertical datums and realization of a world height, *Studia Geophysica et Geodaetica*, **45**(2): 127~132. doi:10.1023/A:1021860126850

Colombo, O., 1980. *A World Vertical Network*, Report No. 296, Department of Geodetic Science, Ohio State University Columbus.

Ekman, M., 1999. Using mean sea surface topography for determination of height system differences across the Baltic sea, *Mar. Geod.*, **22**(1): 31~35. doi: 10.1080/014904199273588

Featherstone, W. E., 2000. Towards unification of the Australian height datum between the Australian mainland and Tasmanja using GPS and the AUSGeoid98 geoid model, *Geomatics Research Australasia*, (73): 33~54.

Grafarend, E. W. and Ardalan, A. A., 1999. World geodetic datum 2000, *J. Geod.*, **73**(11): 611~623. doi:10.1007/s001900050272

Guan, Z. L. and Ning, J. S., 1981. *Earth Shape and Its Exterior Gravity Field*, Beijing, Surveying and Mapping Press, 358~364. (in Chinese)

Guo, J. Y., Chang, X. T. and Yue, Q., 2005. Study on curved surface fitting model using GPS and leveling in local area, *Transaction of Nonferrous Metals Society of China*, **15**, 140~144.

Guo, J. Y., Gao, Y. G., Hwang, C. W. and Sun, J. L., 2010. A multi-subwaveform parametric retracker of the radar satellite altimetric waveform and recovery of gravity anomalies over coastal oceans, *Science China Earth Sciences*, **53**(4): 610~616. doi:10.1007/s11430-009-0171-3

Guo, J. Y., Hwang, C. W., Tseng, Z. P. and Han, Y. B., 2008. Determination of high-rate clock correction for GPS satellites from observation of IGS stations, *Geomatics and Information Science of Wuhan University*, **33**(7): 706~710. (in Chinese)

Guo, J. Y., Yu, L., Liu, X., Kong, Q. L. and Li, G. W., 2011. Automatic trigonometric leveling system based on GPS and ART, *Applied Mechanics and Materials*, **90-93**, 2897~2902. doi:10.4028/www.scientific.net/AMM.90-93.2897

Heck, B. and Rummel, R., 1990. Strategies for solving the vertical datum problem using terrestiral and satellite geodetic data, in: *Sea Surface Topography and the Geoid*, Sunkel, H., Baker, T. eds., Springer-Verlag, New York, 104, 116~128.

Hipkin, R. G., 2002. Vertical datum defined by $W_0=U_0$: Theory and practice of a modern height system, in: *Proceedings of Third Meeting of the International Gravity and Geoid Commision*, Thessaloniki, Greece, 26~30 Aug. 2002.

Jekeli, C., 2003. On monitoring a vertical datum with satellite altimetry and water-level gauge data on large lakes, *J. Geod.*, **77**(7-8): 447~453. doi: 10.1007/s00190-003-0345-2

Li, F. B., Liu, G. K., Wang, X. L., Tang, S. Y. and Du, M. C., 2007. Long distance cross sea elevation datum transmission method and accuracy, *Modern Surveying and Mapping*, **30**(2): 7~16. (in Chinese)

Li, J. C. and Jiang, W. P., 2001. Height datum transference within long distance across sea, *Geomatics and Information Science of Wuhan University*, **26**(6): 514~517. (in Chinese)

Madsen, F. and Tscherning, C. C., 1990. The use of height differences determined by the GPS in the construction process of the fixed link across the Great Belt, *Proceedings of FIG XIX International Congress*, Helsinki, Finland.

Mather, R. S., 1976. *Some Possibilities for Recovering Oceanographic Information from the SEASAT Missions*, Unisurv G24.

Moritz, H., 1980. *Advanced Physical Geodesy*, New York, Abacus Press.

Nahavandchi, H. and Soltanpour, A., 2006. Improved determination of heights using a conversion surface by combine gravimetric quasi-geoid/geoid and GPS-levelling height differences, *Studia Geophysica et Geodaetica*, **50**(2): 165~180. doi:10.1007/s11200-006-0010-3

Pan, M. and Sjöberg, E., 1998. Unification of vertical datums by GPS and gravimetric geoid models with application to Fennoscandia, *J. Geod.*, **72**(2): 64~70. doi: 10.1007/s001900050149

Pavlis, N. K., Holmes, S. A., Kenyon, S. C., Factor, J. K., 2008. *An Earth Gravitational Model to Degree 2160: EGM2008*, presented at the 2008 General Assembly of the European Geosciences Union, Vienna, Austria, April 13~18.

Rapp, R. H., 1994. Separation between reference surfaces of seleceted vertical datums, *Bulletin Géodésique*, **69**(1): 26~31.

Rapp, R. H., 1995. A world vertical datum proposal, *Allgemeine Vermessungs-Nachrich-ten*, **102**(8-9): 297~304.

Rapp, R. H., 1997. Use of potential coefficient models for geoid undulation determinations using a spherical harmonic representation of the height anomaly/geoid undulation difference, *J. Geod.*, **71**(5): 282~289.

Rummel, R. and Ilk, K. H., 1995. Height datum connection-the ocean part, *Allgemeine Vermessungs-Nachrich-ten*, **102**, 321~330.

Rummel, R. and Teunissen, P., 1988. Height datum definition, height datum connection and the role of the geodetic boundary value problem, *Bulletin Géodésique*, **62**(4): 477~498.

Sanso, F. and Usai, S., 1995. Height datum and local geodetic datums in the theory of geodetic boundary value problems, *Allgemeine Vermessungs-Nachrich-ten*, **102**, 343~355.

Xu, H. Z. and Bao, L. F., 2009. *Precise Transfer of Height Datum Across Ocean with Gravimetric Potential Difference Method*, Report of Institute of Geodesy and Geophysics, CAS, Wuhan. (in Chinese)

Xu, P. and Rummel, R., 1991. *A Quality Investigation of Global Vertical Datum Connection*, Nederlandse Commissie voor Geodesie, Delft, The Netherlands.

Zhang, L., Li, F., Chen, W. and Zhang, C., 2009. Height datum unification between Shenzhen and Hong Kong using the solution of the linearized fixed-gravimetric boundary value problem. *J. Geod.*, **83**(5): 411~417. doi:10.1007/s00190-008-0234-9