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A Maximum-Entropy Compound Distribution Model for Extreme Wave Heights of Typhoon-Affected Sea Areas*

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ABSTRACT

A new compound distribution model for extreme wave heights of typhoon-affected sea areas is proposed on the basis of the maximum-entropy principle. The new model is formed by nesting a discrete distribution in a continuous one, having eight parameters which can be determined in terms of observed data of typhoon occurrence-frequency and extreme wave heights by numerically solving two sets of equations derived in this paper. The model is examined by using it to predict the *N*-year return-period wave height at two hydrology stations in the Yellow Sea, and the predicted results are compared with those predicted by use of some other compound distribution models. Examinations and comparisons show that the model has some advantages for predicting the *N*-year return-period wave height in typhoon-affected sea areas.

Key words: *maximum entropy principle*; *typhoon occurrence-frequency*; *N-year return period wave heights*; *maximum entropy compound distribution*

1. Introduction

In accordance with the concept of compound distribution for random variables proposed first by Feller (1958), Liu and Ma (1976) derived an explicit expression of compound distribution.

$$F(x) = \sum_{k=0}^{\infty} p_k [G(x)]^k \ , \tag{1}$$

where p_k is the probability of typhoon-occurring k times in a year, and $G(x)$ is the distribution function of the extreme value appearing in a typhoon course (one extreme value per typhoon course). To describe the probability distribution of extreme value of environmental elements in typhoon-affected sea areas, they further assigned the Poisson distribution to be the discrete one to describe the distribution of typhoon occurrence-frequency and the Gumbel, the Weibull or the Pearson-III distribution to be the continuous one to describe the distribution of the extreme values, forming the so-called

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Poisson-Gumbel, Poisson-Weibull and Poisson-Pearson-III models of compound extreme distribution (Dong *et al*., 2003; Cao *et al*., 2007; Jiang *et al*., 2008). Later on, Wang *et al*. (2010) substituted the Gumbel, the Weibull and the Pearson-III distributions in the three models by a maximum entropy distribution derived by Xu and Zhang (2004), forming the so-called Poisson-maximum entropy model of compound extreme distribution. These compound extreme distribution models have been practically used to predict the *N*-year-return-period extreme value of wave height, wind speed and storm surge level etc. in typhoon-affected sea areas and obtain some results better than those obtained by use of the traditional models such as the Pearson-III and the Gumbel distributions.

The proposition of these compound extreme distribution models is undoubtedly an improvement in determining the design values of environmental elements, for these models take into account the effect of typhoon occurrence-frequency on the distribution of environmental extremes and such an improvement is necessary for typhoon-affected sea areas (Xu and Yu, 2001).

In the present study, a distribution of discrete random variables will be derived on the maximum-entropy principle (Thomas and Thomas, 1991), and the distribution will be adopted to substitute for the Poisson distribution in the Poisson-maximum entropy model to form a new model of compound extreme distribution for describing the probability distribution of extreme wave heights of typhoon-affected sea areas. As both the discrete and continuous distributions in the new model are of maximum-entropy, the new model is less in apriority and more suitable to describe the distribution of extreme wave height, which have large uncertainty. Furthermore, as the discrete distribution derived in this paper has four parameters, this makes it more competent than the Poisson distribution for flexibly and precisely fitting the data of typhoon occurrence-frequency.

To examine its advantages, the new model has been used on trial to predict the *N*-year return-period wave height at two hydrology stations in the Yellow Sea. The results will be reported and compared with those predicted by use of the Poisson-Gumbel and the Poisson-maximum entropy models.

2. A Maximum Entropy Distribution Model Proposed for Typhoon Occurrence-Frequency

To more reasonably describe the probability distribution of annual typhoon occurrence-number of times (referred to as typhoon occurrence-frequency), a distribution for discrete random variables will be derived on the maximum-entropy (ME) principle (Dai *et al.*, 2001). The ME principle states that the prior probability assignment that describes the available information but is maximally noncommittal with regard to the unavailable information is the one with maximum entropy (Ulrych and Bishop, 1975). This principle has been successfully applied to spectral estimation for short series data in geophysics and others (Ulrych, 1972a, 1972b; Cheng and Zhu, 2010; Dong and Yao, 2010; Gao, 2007; Xu and Zhang, 2004).

With p_i denoting the probability of typhoon-occurring i times in a year in an assigned sea area, the entropy of the random variable i is defined as:

$$E = -\sum_{i=1}^{\infty} p_i \ln p_i \,. \tag{2}$$

According to the ME principle, we want to find such a p_i as to maximizing E expressed in Eq. (2). Obviously, this is a variation problem whose solution exists subject to certain constraints. The constraints for the present problem are proposed as:

$$\sum_{i=1}^{\infty} p_i = 1\,; \tag{3}$$

$$\sum_{i=1}^{\infty} i^c \cdot p_i < \infty\,; \tag{4}$$

$$\sum_{i=1}^{\infty} \ln i \cdot p_i < \infty\,, \tag{5}$$

where c is a positive constant. It is worth noting that these constraints are requisite and rational rather than *a priori*. The constraint shown in Eq. (3) is inevitable to any random variable; the constraint shown in Eq. (4) is generally true because i is always finite; and the constraint shown in Eq. (5) characterizes a general fact that $p_i \to 0$ when $i \to \infty$ or $i \to 0$.

Subject to the above three constraints, when $i = 0$, $p_i = 0$; and when $i \neq 0$, the probability p_i maximizing E must satisfy the Eular equation

$$\frac{\partial L}{\partial p_i} = 0 \quad i = 1, 2, \cdots$$

where

$$\begin{aligned} L &= -\sum_{i=1}^{\infty} p_i \ln p_i + a\sum_{i=1}^{\infty} p_i - b\sum_{i=1}^{\infty} i^c \cdot p_i + d\sum_{i=1}^{\infty} \ln i \cdot p_i \\ &= \sum_{i=1}^{\infty} (-p_i \ln p_i + ap_i - bi^c \cdot p_i + d \ln i \cdot p_i), \end{aligned} \tag{6}$$

in which a, b, c, and d are constant parameters to be determined by the constraint conditions and observed data (boundary conditions).

From Eq. (6) we have

$$-\ln p_i - 1 + a - bi^c + d \ln i = 0 \tag{7}$$

It follows that

$$p_i = \exp\{a - 1 - bi^c + d \ln i\},\ i = 1, 2, \cdots \tag{8}$$

This is the ME distribution of the discrete random variable i derived in this paper. As this distribution is derived on the ME principle, it is prior in theory to the artificially assigned Poisson distribution for describing the typhoon occurrence-frequency that has large uncertainty.

To determine these parameters in Eq. (8), the distribution moments of i, $\overline{i}^m (m = 0, 1, 2, \cdots)$ are expressed in terms of these parameters, and then these parameters are solved in terms of $\overline{i}^m$, where $\overline{i}^m$ is the expectation of i^m, i.e.,

$$\overline{i}^m = \sum_{i=1}^{\infty} i^m p_i \qquad m = 0, 1, 2, \cdots \tag{9}$$

With A_m denoting $\overline{i}^{\,m}$, it follows from Eq. (8) that

$$A_m = \sum_{i=1}^{\infty} i^{m+d} \mathrm{e}^{-bi^c} \mathrm{e}^{a-1} \tag{10}$$

The above expression can be rewritten as (see Appendix I for proof):

$$A_m = \mathrm{e}^{a-1} b^{\frac{1+m+d}{c}} \frac{1}{c} \int_0^{\infty} y^{\frac{m+d+1}{c}-1} \mathrm{e}^{-y} \mathrm{d}y \tag{11}$$

Completing the integration gives

$$A_m = \mathrm{e}^{a-1} b^{-\frac{1+m+d}{c}} \frac{1}{c} \Gamma\left(\frac{m+d+1}{c}\right), \tag{12}$$

where $\Gamma(\cdot)$ is the well-known Γ-function. Taking $m = 0, 1, 2, 3, 4$ in Eq. (12) results in, respectively,

$$A_0 = \mathrm{e}^{a-1} b^{-\frac{d+1}{c}} \frac{1}{c} \Gamma\left(\frac{d+1}{c}\right); \tag{13}$$

$$A_1 = \mathrm{e}^{a-1} b^{-\frac{d+2}{c}} \frac{1}{c} \Gamma\left(\frac{d+2}{c}\right); \tag{14}$$

$$A_2 = \mathrm{e}^{a-1} b^{-\frac{d+3}{c}} \frac{1}{c} \Gamma\left(\frac{d+3}{c}\right); \tag{15}$$

$$A_3 = \mathrm{e}^{a-1} b^{-\frac{d+4}{c}} \frac{1}{c} \Gamma\left(\frac{d+4}{c}\right); \tag{16}$$

$$A_4 = \mathrm{e}^{a-1} b^{-\frac{d+5}{c}} \frac{1}{c} \Gamma\left(\frac{d+5}{c}\right), \tag{17}$$

and thus we have

$$\frac{A_0 A_3}{A_1 A_2} = \frac{\Gamma\left(\frac{d+4}{c}\right)\Gamma\left(\frac{d+1}{c}\right)}{\Gamma\left(\frac{d+2}{c}\right)\Gamma\left(\frac{d+3}{c}\right)}; \tag{18}$$

$$\frac{A_1 A_4}{A_2 A_3} = \frac{\Gamma\left(\frac{d+2}{c}\right)\Gamma\left(\frac{d+5}{c}\right)}{\Gamma\left(\frac{d+3}{c}\right)\Gamma\left(\frac{d+4}{c}\right)}; \tag{19}$$

$$b = \left[A_1 \Gamma\left(\frac{d+1}{c}\right) \Big/ \Gamma\left(\frac{d+2}{c}\right) \right]^{-c}; \tag{20}$$

$$a = \ln\left[b^{\frac{d+1}{c}} c \Big/ \Gamma\left(\frac{d+1}{c}\right) \right] + 1\,. \tag{21}$$

It can be seen from the above four equations that so far as A_m are known, c and d can be numerically solved from Eq. (18) and Eq. (19) by iteration, and a and b are then obtained from Eq. (20) and Eq. (21), respectively.

As usually done in practical statistics, A_m is estimated from observed data by use of the formula

$$A_m = \frac{1}{N}\sum_{n=1}^{N} I_n^{\ m} \ , \tag{22}$$

where I_n denotes the annual typhoon-occurrence times in a assigned sea area and N is the total number of years.

3. Maximum Entropy Distribution Model for Extreme Wave Heights

The ME distribution model derived by Zhang and Xu (2004) will be adopted as the continuous distribution to form the ME compound distribution model proposed in the next section to describe the probability of extreme wave heights in Typhoon affected sea areas. For convenience in description, this model is briefly introduced in the following.

On the ME principle, Zhang and Xu (2004) derived the model of probability density function for extreme wave height (as a positive random variable)

$$f(x) = \alpha x^{\gamma} \mathrm{e}^{-\beta x^{\zeta}} \ , \tag{23}$$

where α, β, γ and ζ are constant parameters to be determinated. This model has been rather widely adopted to predict the N-year return-period wave height and water level (Dong *et al.*, 2009; Zhang *et al.*, 2006; Zhou *et al.*, 2005) and gave results better than those predicted by use of the Pearson-III and the Gumbel models.

With the distribution moments of a positive continuous random variable defined as:

$$A_m = \int_0^{\infty} x^m f(x)\mathrm{d}x,$$

the equations from which the parameters in Eq. (23) is solved in terms of A_m are derived as follows:

$$\frac{A_3}{A_1^3} = \frac{\Gamma^2\left(\dfrac{\gamma+1}{\zeta}\right)\Gamma\left(\dfrac{\gamma+4}{\zeta}\right)}{\Gamma^3\left(\dfrac{\gamma+2}{\zeta}\right)}; \tag{24}$$

$$\frac{A_4}{A_1^4} = \frac{\Gamma^3\left(\dfrac{\gamma+1}{\zeta}\right)\Gamma\left(\dfrac{\gamma+5}{\zeta}\right)}{\Gamma^4\left(\dfrac{\gamma+2}{\zeta}\right)}; \tag{25}$$

$$\alpha = \zeta\beta^{\frac{\gamma+1}{\zeta}} \Big/ \Gamma\left(\frac{\gamma+1}{\zeta}\right); \tag{26}$$

$$\beta = \left[\Gamma\left(\frac{\gamma+2}{\zeta}\right) \Big/ \left(A_1\Gamma\left(\frac{\gamma+1}{\zeta}\right)\right)\right]^{\zeta} . \tag{27}$$

Thus, so long as A_m (m=0, 1, 2, 3, 4) are known, γ and ζ can be numerically solved from Eqs. (24) and (25) by iteration and then β is obtained from Eq. (27), and further α is obtained from Eq. (26). The moment A_m are estimated from observed data with the formula

$$A_m = \frac{1}{N}\sum_{i=1}^{N} x_i^{\,m} \quad m = 0,\ 1,\ 2,\ \cdots,\ n\ , \tag{28}$$

where x_i denotes the observed value of extreme wave height and N denotes total number of observed values.

4. A Proposed Maximum-entropy Compound Distribution Model for Extreme Wave Heights in Typhoon-Affected Sea Areas

The new compound distribution model proposed in this paper for describing the distribution of extreme wave heights in typhoon-affected sea areas is formed on the basis of Eq. (1) by taking p_k shown in Eq. (8) as the discrete distribution p_k and $f(x)$ shown in Eq. (23) as the continuous distribution $G(x)$, respectively, that is,

$$F(x) = \sum_{\mathrm{k}=0}^{\infty} \exp(a - 1 - bk^c + d\ln k)\left(\int_0^x \alpha x^{\gamma} \mathrm{e}^{-\beta x^{\zeta}} \mathrm{d}x\right)^k\ , \tag{29}$$

where k is the annual typhoon occurrence-frequency, that is, the number of typhoon-occurrence times in a year, and $f(x)$ is the probability density function of the extreme wave height appearing in a typhoon course.

As seen from Eq. (29), there are eight parameters in the new compound distribution model. In practical application, these parameters are determined by use of observed data.

For an assigned sea area, parameters α, γ, β and ζ are determined by the values of extreme wave height observed there, x_i. At first, the moments of extreme wave height, $A_m\,(m = 0, 1, 2, 3, 4)$ are calculated according to Eq. (28) and substituted into Eqs. (24) and (25). Secondly, the two equations are solved by iteration to obtain γ and ζ, and then α and β are obtained from Eqs. (26) and (27), respectively. Parameters $a, b,\ c$ and d are determined by use of the data of annual typhoon-occurrence times in the sea area. At first the moments $A_0 \sim A_4$ are calculated according to Eq. (22) and substituted into Eqs. (18) and (19). Secondly, the two equations are solved to obtain c and d, and b and a are obtained from Eqs. (20) and (21), respectively.

The compound distribution model shown in Eq. (29) has the following advantages:

(1) The formation of this model is based on the explicit expression of compound distribution which is strictly derived by Liu and Ma (1976) according to the concept proposed first by Feller (1958).

(2) Both the two distributions forming this model are of maximum entropy and thus more suitable to describing the distribution of extreme wave heights in typhoon-affected sea areas, which has large uncertainty.

(3) There are two groups of parameters in this model, each for a distribution, and these parameters can be determined by observed data. This makes the model competent for more flexibly and precisely fitting observed data.

Because of the above mentioned advantages, it is expected that this model is more competent for describing the distribution of extreme wave heights in typhoon-affected sea areas.

5. Examinations and Applications

To examine the ME compound distribution model proposed for describing the distribution of extreme wave heights in typhoon-affected sea areas, the model is used to predict the N-year return-period extreme wave heights at two hydrology stations in the Yellow Sea (referred to as Station I and Station II, respectively). The observed data of annual typhoon occurrence-frequency and extreme height at Station I from 1963 to 1989 and at Station II from 1961 to 1980 are used as statistical samples for estimating the parameters in Eq. (29).

The proceeding of examinations is as follows:

(1) The parameters a, b, c, and d in Eq. (8) are determined by numerically solving Eqs. (18)~(21) in terms of A_m that are estimated from the observed annual occurrence-frequency at the two stations by use of the formula shown in Eq. (22), and the parameter values so determined are listed in Table 1. For comparison, the parameter λ (the averaged annual occurrence-frequency) in the Poisson distribution is also estimated from the same data with Eq. (22) (in which m is taken to be 1) and listed in Tables 2 and 3, respectively. With the above determined parameters the resulting Poisson and ME distributions are shown in Fig. 1 for Station I and Station II. As can be seen from the two figures, the resulting ME distributions are rather close to the Poisson ones while the former fits the data somewhat better than the latter so far as the sum of square deviation is concerned.

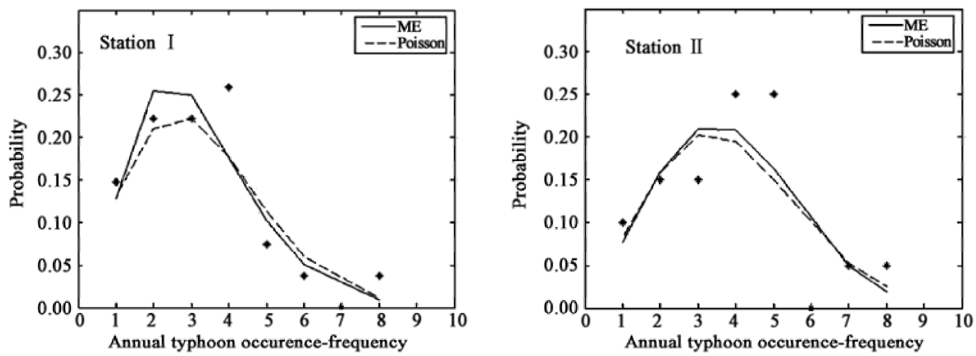

Fig. 1. Probabilities of annual typhoon occurrence-frequency.

Table 1 Values of the parameters in the ME distribution shown in Eq. (8) for typhoon occurrence-frequency. These values are estimated from the data observed at Station I from 1963 to 1989 and at Station II from 1961 to 1980 through by solving Eqs. (18)~(21)

Station	a	b	c	d
Station I	0.2088	0.4868	1.3050	2.0240
Station II	0.0792	0.0195	2.5437	1.1739

Table 2 Typhoon annual occurrence-frequencies at Station I from 1963 to 1989 and their averages λ

Annual occurrence-frequency	Total number of years	λ	
1 2 3 4 5 6 7 8	27	Estimated value	Confidence interval ($\alpha = 0.01$)
Occurrence number of year 4 6 6 7 2 1 0 1	Total number of times 86	3.1852	[2.3701, 4.1814]

Table 3 Typhoon annual occurrence-frequency at Station II from 1961 to 1980 and its average λ

Annual occurrence-frequency	Total number of years	λ	
1 2 3 4 5 6 7 8	20	Estimated value	Confidence interval ($\alpha = 0.01$)
Occurrence number of year 2 3 3 5 5 0 1 1	Total number of times 77	3.8500	[3.03841, 4.8118]

(2) The parameters α, γ, β, and ζ in Eq. (23) are determined by numerically solving Eqs. (24) ~(27) in terms of A_m that are estimated from the observed annual extreme wave heights at the two stations by use of the formula shown in Eq. (28), and the parameter values so determined are listed in Table 4.

(3) With a, b, c, and d as listed in Table 1 and α, β, γ, and ζ as listed in Table 4 the ME compound distribution shown in Eq. (29) is used to predict the values of N-year return-period wave height at Station I and Station II. The predicted values are listed in Table 5 together with those predicted by use of the Poisson-ME, Poisson-Gumbel and Poisson-Pearson-III distributions. Also listed in the two tables are the values of the sum of squared deviations where the deviation refers to the difference between the value predicted by one distribution and the value averaged over those predicted by all the four distributions listed in Table 5.

Table 4 Values of parameters in the ME distribution shown in Eq. (23) for annual extreme wave heights. These values are estimated from the data observed at Station I from 1963 to 1989 and at Station II from 1961 to 1980 by use of Eqs. (24)~(27)

Station	α	β	ζ	γ
Station I	253413209.5	43.2035	0.44027	41.56657
Station II	0.00000001521	0.04316	2.3890	10.9349

Table 5 N-year return-period wave heights predicted by four compound distribution models

Return-period (year)	ME (proposed in this paper)		Poisson-ME		Poisson-Gumbel		Poisson-Pearson III	
	Station I	Station II	Station I	Station II	Station I	Station II	Station I	Station II
10	7.30	2.97	7.25	2.97	7.31	3.03	7.08	2.99
20	8.49	2.99	8.23	3.07	8.52	3.12	8.16	3.11
50	9.06	3.20	8.82	3.18	9.09	3.19	8.40	3.26
100	10.3	3.31	9.46	3.25	10.4	3.24	8.92	3.36
Sum of squares	1.0118	0.6346	1.0731	0.6644	5.2393	0.7954	1.3381	0.6367

As can be seen from Table 5, the sum of squared deviation of the value predicted by ME compound distribution is the least for both Station I and Station II. This indicates, to a certain extent, that the ME compound distribution model proposed in this paper is more suitable than the other models to predict the *N*-year return-period wave height of typhoon-affected sea areas.

6. Discussions and Conclusions

A maximum-entropy compound distribution model for extreme wave heights in typhoon-affected sea areas has been proposed in this paper. The model is formed by nesting a maximum-entropy discrete distribution in a maximum-entropy continuous one. This model has the following advantages.

(1) The model takes into account the effect of typhoon occurrence-frequency on the distribution of extreme wave heights in typhoon-affected sea areas.

(2) Both the discrete and continuous distribution in the model are of maximum entropy, which makes the model less *a priori* and hence more suitable to predict the *N*-year return-period wave height in typhoon-affected sea areas, with large uncertainty.

(3) The model includes eight parameters, and this makes the model competent to flexibly and precisely fitting observed data. Moreover, the equations relating these parameters with the distribution moments have been derived, from which these parameters can be determined in terms of observed data.

Examinations of the model have been conducted by using it to predict the *N*-year return-period wave height at two hydrology stations in the Yellow Sea and the results have been compared with those predicted by the Poisson-ME, the Poisson-Gumbel and the Poisson-Pearson-III compound distribution models. The examinations and comparisons showed some merits of the model.

Appendix Ⅰ: proof of Eq. (11)

$$A_m = \mathrm{e}^{a-1} N^{m+d} \sum_{i=1}^{N} \frac{i^{m+d}}{N^{m+d}} \mathrm{e}^{-b\frac{i^c}{N^c}N^c} = \mathrm{e}^{a-1} N^{m+d+1} \sum_{i=1}^{N} \left(\frac{i}{N}\right)^{m+d} \mathrm{e}^{-bN^c\left(\frac{i}{N}\right)^c} \frac{1}{N}$$

$$= \mathrm{e}^{a-1} N^{m+d+1} \int_0^1 x^{m+d} \mathrm{e}^{-bN^c x^c} \mathrm{d}x,$$

where $N \to \infty$. Let $y = b(Nx)^c$, then $x = \frac{1}{N}\left(\frac{y}{b}\right)^{\frac{1}{c}} = \frac{y^{\frac{1}{c}}}{Nb^{\frac{1}{c}}}$, $\mathrm{d}x = \frac{1}{cNb^{\frac{1}{c}}} y^{\frac{1}{c}-1} \mathrm{d}y$, when $x \to 0$, $y \to 0$ and $x \to 1$, $y \to \infty$. We then have

$$A_m = \mathrm{e}^{a-1} b^{-\frac{1+m+d}{c}} \frac{1}{c} \int_0^{\infty} y^{\frac{m+d+1}{c}-1} \mathrm{e}^{-y} \mathrm{d}y .$$

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