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Ultimate Behavior of Steel Cable-stayed Bridges - II. Parametric Study -

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Abstract

This paper presents the characteristics of the ultimate behavior of steel cable-stayed bridges through considering various geometric parameters. Steel cable-stayed bridges show complex ultimate behavior, because of their geometric characteristics and various nonlinearities. In this study, the patterns of the ultimate behavior of steel-cable stayed bridges under the critical live load case are classified. In addition, the effects of various geometric parameters on the ultimate behavior, such as cable-arrangement type, height of the girder and mast, and area of the stay cables, are studied. For rational analytical research, the analysis method suggested in the previous paper, Ultimate behavior of steel cable-stayed bridges-I. Rational ultimate analysis method (Kim *et al*., 2016), is mainly used. Using the analysis method, the main geometric and material nonlinearities, such as the cable sag effect, beam-column effect of the girder and mast, large-displacement effect, girder-mast-cable interaction, and gradual yield effect of steel members, are reflected and considered in the analytical research. After the analytical study, the characteristics of the change of ultimate mode and load carrying capacity are investigated, with respect to the change of various geometric parameters.

Keywords: cable-stayed bridges, nonlinear analysis, initial shape analysis, refined plastic hinge method, generalized displacement control method

1. Introduction

Because of their structural efficiency combined with good aesthetics, cable-stayed bridges have become the most popular bridge system, especially for long-span bridges. The excellent structural efficiency of cable-stayed bridges results from the combination of girder, mast and stay cables, which show different structural behavior. In other words, flexural strength of the girder, compressive strength of the mast and tensile strength of stay cables are combined in cable-stayed bridges. Therefore the structure shows extreme structural performance.

But, cable-stayed bridges show various geometric and material nonlinearities, such as the beam-column effect of the girder and mast, cable sag effect, large displacement effect, gradual yield effect, and so on (Adeli and Zhang, 1994; Xi and Kuang, 1999; Ren, 1999; Freire *et al*., 2006). In addition, the effect of those nonlinearities on the general structural behavior also increases when the structure is designed for long-span bridges. Under the effects of those nonlinearities, the ultimate behavior of cable-stayed bridges can occur as various patterns, such as material yield, elastic/inelastic buckling of girder and/or mast, fatigue, or local failure. Thus, the investigation of the ultimate behavior of steel cable-stayed bridges should be studied in more detail by the rational analysis method.

Studies to investigate the structural stability of completed cable-stayed bridges have been performed by several researchers (Tang *et al*., 2001; Shu and Wang, 2001). In those studies, elastic buckling modes were introduced, and the effects of various geometric properties on structural stability were described. But there were analytical limitations, because those studies were performed based on conventional eigenvalue analysis, thus important geometric and material nonlinearities of cable-stayed bridges were not reflected. Also, the initial condition of the structure was not considered before live load analysis. It is well known that the consideration of the initial structural state is very important, because cable-stayed bridges are designed with appropriate

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initial tensile forces of cables (Chen *et al*., 2000; Cheng and Xiao, 2007; Kim and Lee, 2001; Wang *et al*., 1993; Wang and Yang, 1993).

Studies based on nonlinear analysis have been performed by several researchers to investigate the ultimate behavior of cable-stayed bridges. The ultimate behavior of concrete cable-stayed bridges was studied by Ren (1999), who considered essential nonlinearities, boundary and loading conditions. But in that study also, initial shape analysis was not performed before live load analysis. A nonlinear analysis method for obtaining the ultimate capacity of cable-stayed bridges was suggested by Song and Kim (2007). The suggested analysis method in their study is a type of two-step analysis, which consists of initial shape analysis and live load analysis. In this method, the beam-column element and conventional equivalent truss element were used to model the main members. For the incremental-iterative numerical solution technique, the Newton-Rapshon method was used, which cannot trace complex nonlinear problems. In addition, a relatively simple analytical model was dealt with for the investigation of the ultimate behavior, and the procedure of ultimate behavior wasn't described in detail.

In this study, the ultimate behavior of steel cable-stayed bridges is investigated using the analysis method proposed in the previous paper (Kim *et al*., 2016). In the previous study, the ultimate behavior of completed steel cable-stayed bridges under specific live load conditions was described in detail. In this study, governing factors of the ultimate behavior of steel cable-stayed bridges are investigated and classified. Through considering various geometric parameters, such as cable arrangement type, the flexural stiffness of the girder and mast, and the sectional area of stay cables, the effects of those parameters on the change of ultimate behavior and load carrying capacity are studied.

2. Theoretical Background

In this chapter, the theoretical background of the ultimate analysis method is briefly introduced. Firstly, the geometric nonlinear elements to model the main members of cable-stayed bridges are described, and the method to consider material nonlinearity of steel members is introduced. Secondly, a numerical strategy for incremental-iterative analysis is described. Finally, the analysis scheme of two-step ultimate analysis for steel cable-stayed bridges is proposed.

2.1. Geometric & material nonlinearity consideration

Nonlinear elements are used in this study to consider various geometric nonlinearities of the main members of cable-stayed bridges, such as the cable sag effect, beam-column effect of the girder and mast, and large displacement effect. Firstly, a nonlinear frame element, which has 2 nodes and 6 degree of freedoms, is used to model the girder and mast, which show flexural behavior and are subjected to axial forces. The element was derived based on the updated-Lagrangian formulation. The elastic, geometric and induced stiffness matrices of the element were introduced in the previous researches (Yang and Kuo, 1994; Kim, 2010; Kim *et al*., 2015; Kim *et al*., 2016).

Secondly, a nonlinear equivalent truss element is used to model the cable member. This element was developed based on the nonlinear truss element, with the equivalent modulus derived to consider the sag effect of the stay cables. In other words, the stiffness matrix of the element consists of the elastic stiffness and geometric stiffness, but the elastic modulus in the elastic stiffness is replaced by the equivalent modulus derived based on the force-elongation relationship of the elastic catenary (Ernst, 1965; Fleming, 1979; Gimsing, 1983). The equivalent modulus of the common equivalent truss element was derived, with some simplification, by Taylor's series. But, in this study, another equivalent modulus derived without any simplification of the force-elongation relationship of the elastic catenary was used (Song *et al*., 2006; Kim, 2010). The elastic and geometric stiffness matrices were introduced in the previous researches (Kim, 2010; Kim *et al*., 2015; Kim *et al*., 2016).

To consider the material nonlinearities of steel members modeled by line elements, there are several methods, such as the plastic zone method, plastic hinge method and refined plastic hinge method. In fact, the plastic zone method is the most accurate method among those three methods. But, it requires substantial calculation times. So, in this study, the refined plastic hinge method is adopted and used, because of its efficiency and accuracy. The refined plastic hinge method uses the tangential modulus E_t to consider the effect of the gradual yield by axial force, and the scalar parameter η to consider the effect of the gradual yield and plastic hinge occurrence by applied axial force and bending moment (Liew *et al*., 1993; Song and Kim, 2007; Kim, 2010; Kim *et al*., 2016). Using the tangential modulus and scalar parameter, the elastic stiffness matrices of the nonlinear frame element and nonlinear equivalent truss element are modified. Modified elastic stiffness matrices of the elements were introduced in the previous paper.

2.2. Numerical solution strategy for nonlinear finite element analysis

An appropriate numerical technique should be used to trace the nonlinear response by nonlinear analysis. There are several numerical methods, such as the Newton-Rapshon method, arc-length method, and generalized displacement control method. Because cable-stayed bridges have various geometric and material nonlinearities, complex nonlinear responses may occur when the structure is subjected to external forces. The numerical methods can be classified into several categories, such as the force-control method, displacement-control method, and work control method. In general, a force-control method, such as the Newton-

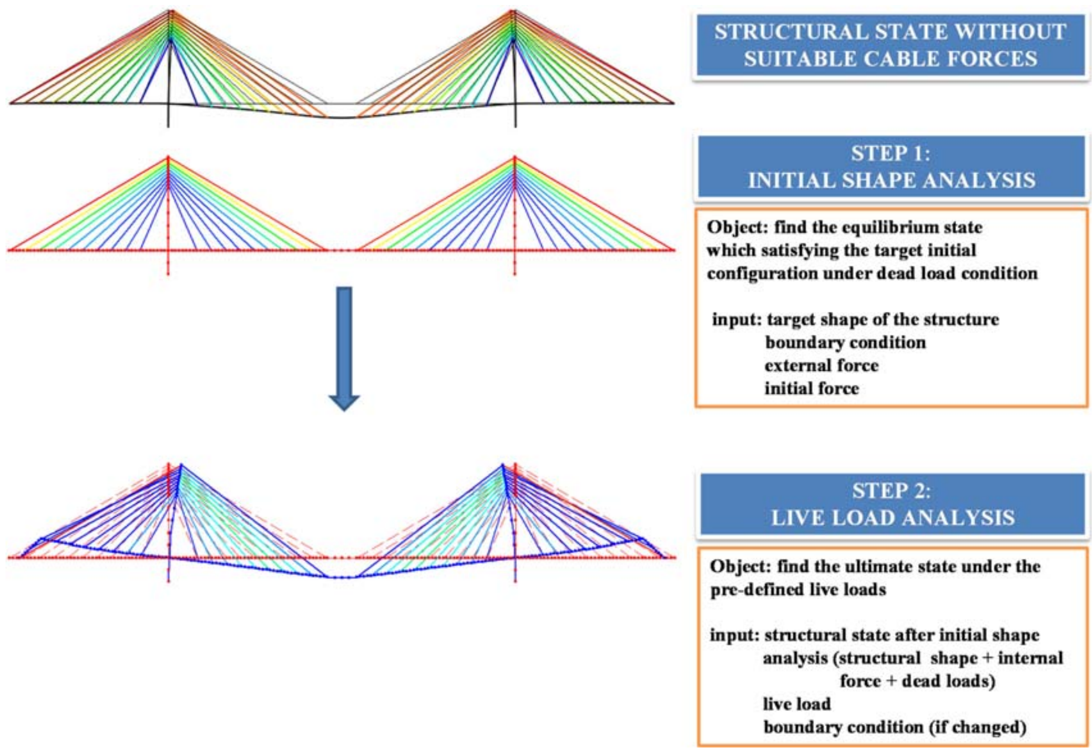


Figure 1. The analysis strategy of ultimate analysis for steel cable-stayed bridges that was used (Kim *et al*., 2016).

Rapshon method, is not appropriate for tracing the complex nonlinear response. For this response, a displacement or work control method is widely used. Among these methods, the arc-length method (Crisfield, 1983) has been typically used. But, there is a problem in the constrain equation of the method. In the constrain equation of the method, the units of each term are not the same, and the inequality of those units may induce numerical instability when the structural response reaches the ultimate state (Yang and Kuo, 1994). So, a generalized displacement control method (Yang and Kuo, 1994) is used in this study. Using this method, the incremental-iterative load factors are calculated and applied during nonlinear analysis, so that the complex nonlinear response of cable-stayed bridges can be traced with numerical stability.

2.3. Analysis strategy of the ultimate analysis for steel cable-stayed bridges

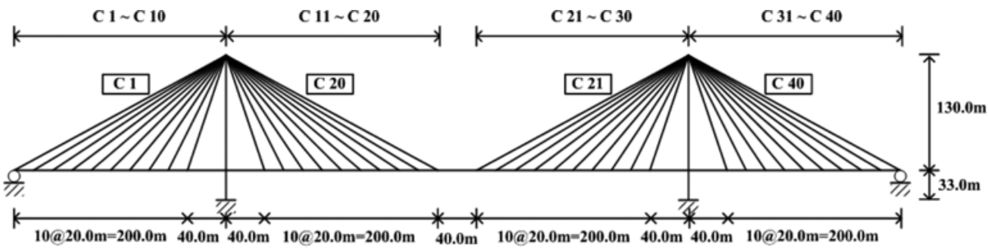
The ultimate behavior of steel cable-stayed bridges is analyzed by a two-step analysis scheme, as shown in Fig. 1. Firstly, initial shape analysis is performed to determine the optimal tensile forces of stay cables, which ensure the structure suffers minimal deformation under the dead load condition, and to reflect the structural state before live load analysis. After that, live load analysis is performed to trace the nonlinear response, and to find the ultimate behavior under specific live load condition. These two analyses, initial shape analysis and live load analysis, are performed based on the theory of nonlinear finite element analysis, thus geometric and material nonlinearities of steel cable-stayed bridges are considered during the structural analyses.

3. Ultimate Behavior of Steel Cable-stayed Bridges

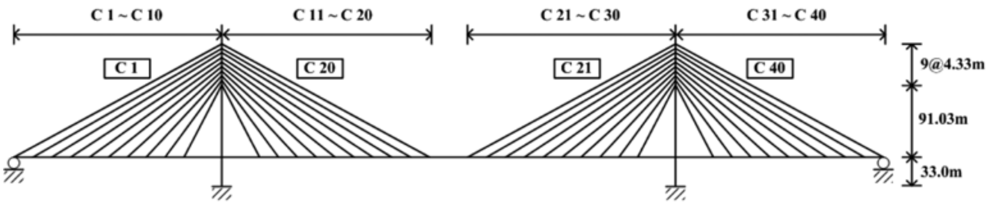
3.1. Analysis model

In this chapter, the characteristics of the ultimate behavior of long-span steel cable-stayed bridges under specific live load case are studied by considering various geometric parameters. As mentioned previously, various geometric characteristics, such as cable-arrangement type, flexural stiffness of the girder and mast, and area of stay cables are considered as the main parameters for this parametric study, and the effects of those parameters on the ultimate behavior are investigated and described.

Figure 2 shows the analysis models considered in this study. As shown in the figure, basically two different cable-arrangement types are considered. Also, both ends of the girder was vertically supported by rollers while the bottom of the masts were restrained as a fixed boundary condition. Figure 3 shows the live load case considered in this study. The load case, designed by reference to the Korean Specification for Highway Bridges, is a vertically distributed load acting on the girder of only the center span. The live loads assumed by traffic load are 12.7 kN/m/lane, and a six-lane road condition is assumed in each bridge. The live loads are applied as the distributed load at the girder.



(a) Radiating type model



(b) Fan type model

Figure 2. Analysis model.

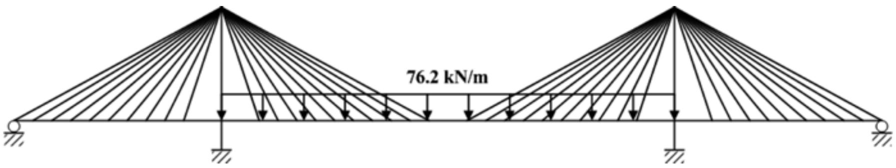


Figure 3. The considered live load case.

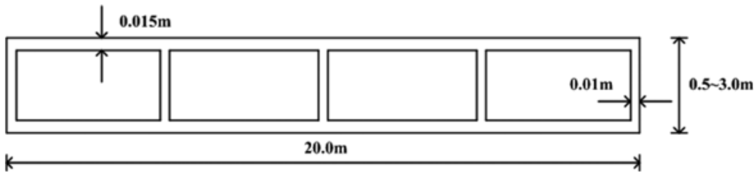


Figure 4. Section of the girder.

In fact, dead loads calculated by unit weight and sectional area shown in Table 1 are also applied as the distributed loads at the girder, masts, and stay cables. According to the previous paper (Kim *et al*., 2016), this live load case was more critical for completed cable-stayed bridges than the live load case, which is acting vertically distributed on the whole span. In this study, the investigation of the ultimate behavior under the live load acting on only the center span is focused on.

As shown in Fig. 4, the girder is designed with a four-cell box section. It is assumed that there are sufficient stiffeners and ribs for preventing local buckling (plate buckling due to locally applied compressive or shear stress components) in the section. The section of the mast is designed as a one-cell box, and the same assumption about the local failure is also adopted.

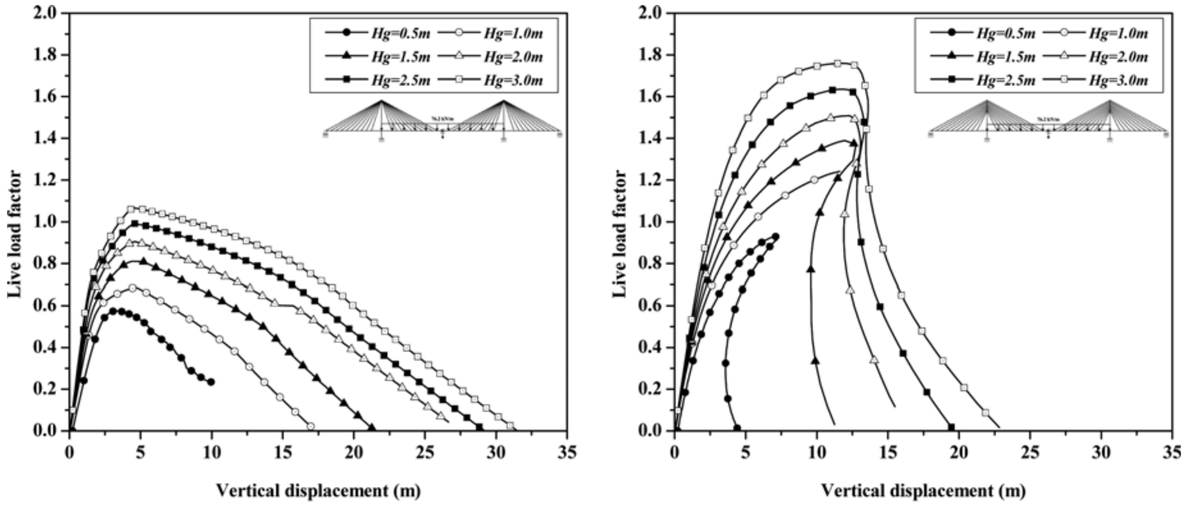
Table 1 represents the material and geometric properties of the girder, mast, and cables. In the Table, the range of the height of the girder and mast are considered, and the area of the stay cable is introduced. By considering those variable parameters, the governing ultimate modes of steel cable-stayed bridges are classified, and the effects of those parameters on the ultimate behavior are investigated.

Table 1. Material and geometric properties of the main members

	Girder	Mast	Cable
Elastic modulus E (kN/m^2)	2.1×10^8	2.1×10^8	2.1×10^8
Sectional area A (m^2)	0.62~0.75	0.79~0.86	0.02~0.14
2nd moment of inertia I (m^4)	0.04~1.45	0.39~8.21	-
Unit weight γ (kN/m^3)	218.27	76.90	76.90
Yield stress f_y (MPa)	380.0	Elastic	1,800.0

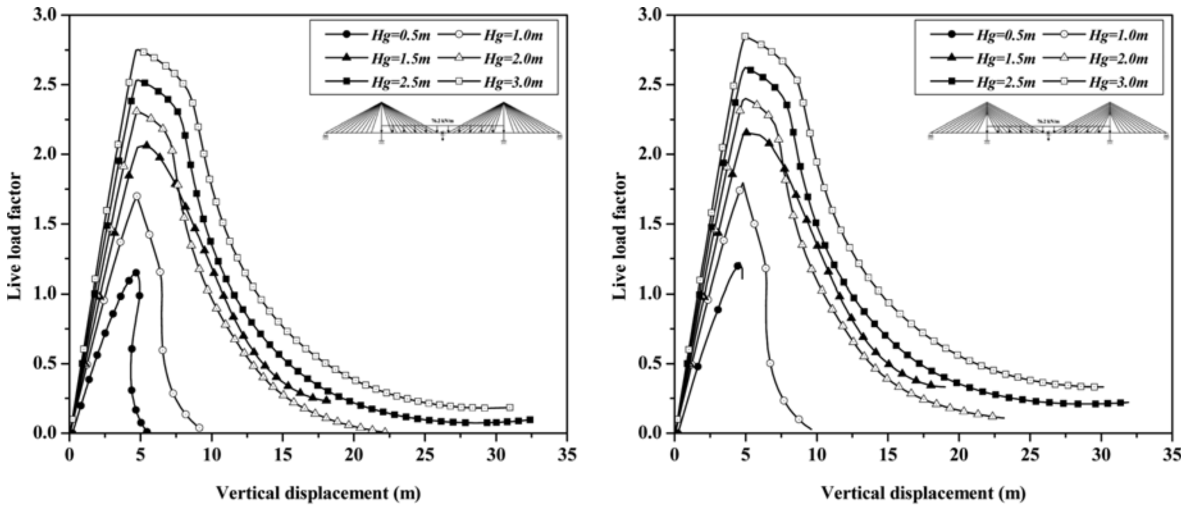
3.2. Effect of the flexural stiffness of the girder (height of the girder)

First of all, the effect of the flexural stiffness of the girder on the ultimate behavior is described in this section. Figures 5 and 6 show the load-displacement curves at the



(a) Radiating type model (b) Fan type model

Figure 5. Load-displacement curve (I_m=0.390 m^4, A_c=0.02 m^2)



(a) Radiating type model (b) Fan type model

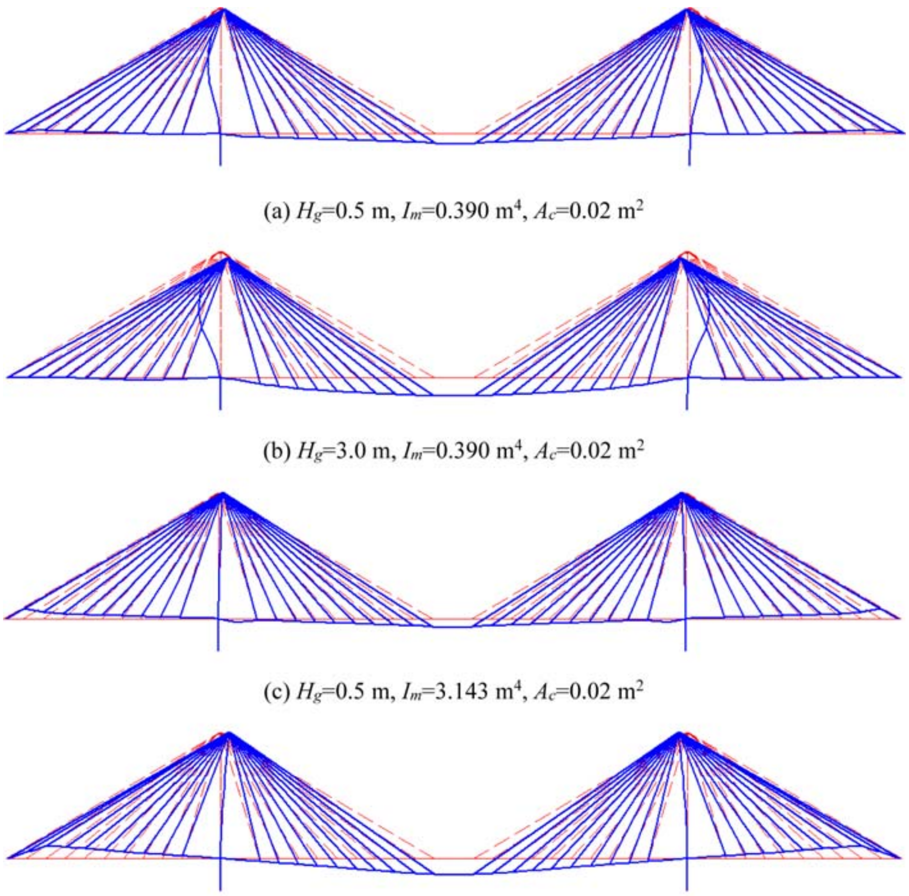
Figure 6. Load-displacement curve (I_m=3.140 m^4, A_c=0.02 m^2)

center of the center span of each analysis models.

As shown in these figures, there are significant ultimate points that show peak live load factors. So, it can be said that the ultimate point is the ultimate capacity under the considered live load case. The load-displacement curves clearly show that the ultimate live load factor increases as the height of the section of the girder increases. In addition, the initial slope of the curve, which means the initial stiffness for the live load case, also increases as the height increases, because of the increase of the flexural stiffness of the girder. The increase of the ultimate live load factor and the initial structural stiffness are caused by the increases of the flexural strength and stiffness of the girder. As the height of the section increases, the section area, 2nd moment of inertia, sectional modulus and plastic modulus also increase. Thus, the peak point and initial slope of the load-displacement curve also increase.

Figures 7 and 8 show the deformation shapes, which mean the ultimate mode shapes. When the vertically distributed load is applied to the center span, the center span deflects vertically, and both masts suffer horizontal movement to the center. The horizontal movement of both masts causes the uplift of both side spans, because of the girder-mast-cable connectivity. Also, the upward deformation of the side span is amplified by the beam-column effect, which is caused by the flexural deformation with applied compressive force by the cables.

First of all, when the flexural stiffness of the mast is insufficient, the mast may buckle. As shown in Figs. 8(a), 8(b), 9(a) and 9(b), the structures collapse as mast buckling occurs, with significant flexural deformation of the girder. Because of the structural characteristics, the girder and



(a) H_g=0.5 m, I_m=0.390 m^4, A_c=0.02 m^2

(b) H_g=3.0 m, I_m=0.390 m^4, A_c=0.02 m^2

(c) H_g=0.5 m, I_m=3.143 m^4, A_c=0.02 m^2

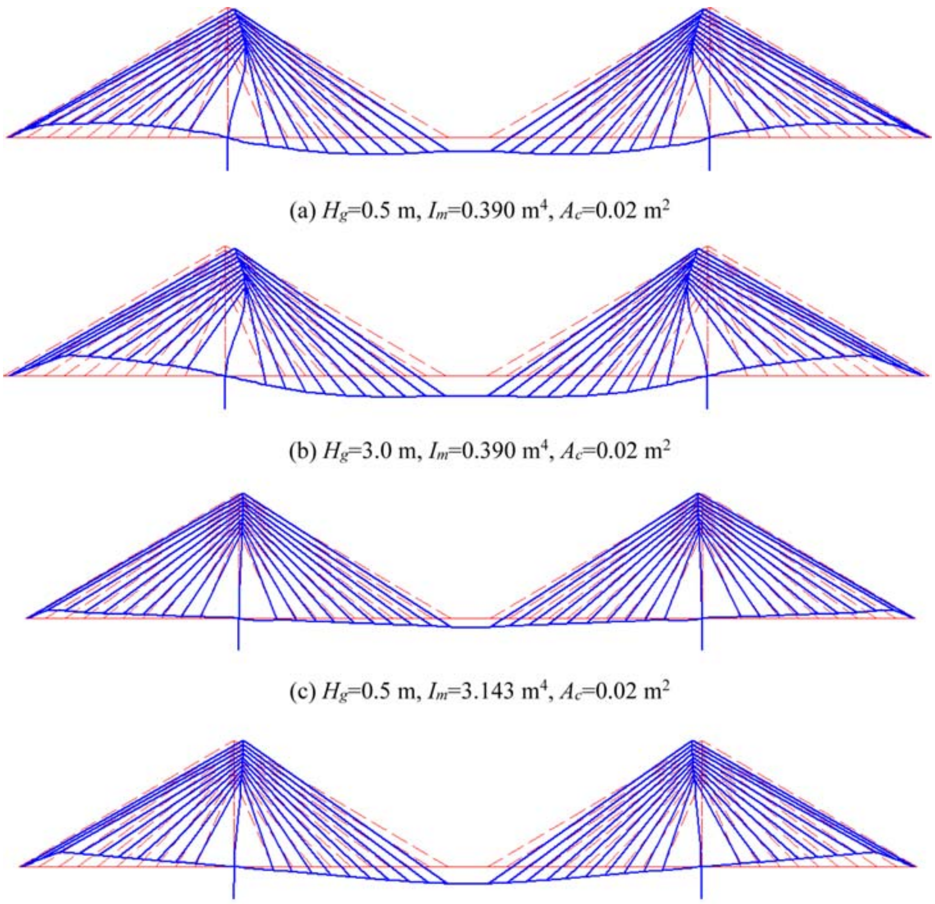
(d) H_g=3.0 m, I_m=3.143 m^4, A_c=0.02 m^2

Figure 7. Deformed shapes of the radiating type models (Scale factor: 2.0).

mast of cable-stayed bridges are always subjected to compressive forces by stay cables. As the external force is vertically applied to the girder, the girder is deflected, and tensile forces of the cables increase, to resist the deflection. The increase of tensile forces of the cables leads to the increase of compressive forces acting on the mast, as well as on the girder. When the mast has sufficient flexural stiffness, it is difficult for elastic buckling of the mast to occur. But, if sufficient flexural stiffness of the mast is not designed, the mast may buckle, due to increasing compressive forces. The buckling of the mast is affected and induced by the beam-column effect. Firstly, the mast suffers horizontal movement when the vertically distributed load is applied to the center span. The horizontal movement and flexural deformation of the mast are amplified, due to the beam-column of the mast. The effect is induced by the interaction of the flexural deformation with the compressive force. Consequently, the buckling of the mast may occur due to the beam-column effect. Therefore, rational flexural stiffness of the mast should be designed by considering the beam-column effect of the mast.

In contrast, when the mast has sufficient flexural stiffness, there is no significant flexural deformation of the mast. So, in this case, material yield of the specific sections of the girder mainly affects the ultimate capacity. As shown in Figs. 7(c), 7(d), 8(c) and 8(d), there are visible plastic hinges near the ends at the side spans. According to the previous paper, the occurrence of plastic hinges at the side span eventually leads to load decrease under the live load case. In this analytical case also, when plastic hinges occur at those points, the live load factor starts to decrease, and the structure reaches the ultimate state. Table 2 below describes the main causes of the ultimate behavior of each analysis model.

Figures 9 shows the change of the ultimate live load factor with respect to the change of flexural stiffness of the girder. To observe the ultimate behavior in detail, the results of geometric nonlinear analysis (G.N.A) were indicated together in the graphs. As shown in the figures, the increase rate of the ultimate load factor in the ultimate modes, which show the mast buckling, is quite lower than the rate in the ultimate modes, which don't show the mast buckling. In other words, the increase of the flexural stiffness of the girder cannot cause a significant increase in the ultimate load carrying capacity when mast buckling occurs, and mainly affects the ultimate behavior. Also, in



(a) H_g=0.5 m, I_m=0.390 m^4, A_c=0.02 m^2

(b) H_g=3.0 m, I_m=0.390 m^4, A_c=0.02 m^2

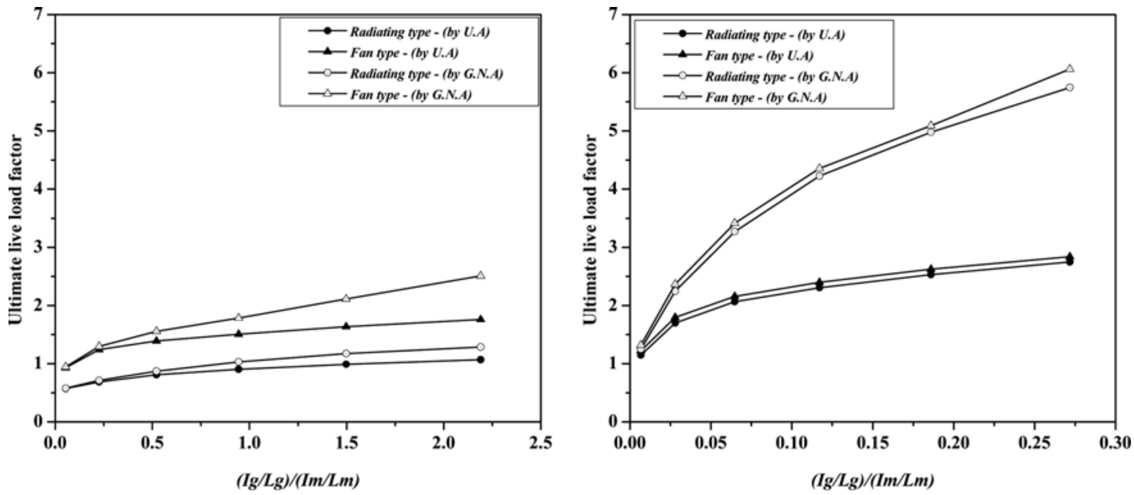
(c) H_g=0.5 m, I_m=3.143 m^4, A_c=0.02 m^2

(d) H_g=3.0 m, I_m=3.143 m^4, A_c=0.02 m^2

Figure 8. Deformed shapes of the fan type models (Scale factor: 2.0).

Table 2. Main causes of the ultimate behavior

Models		Main causes of the ultimate behavior
Radiating type model	H_g=0.5 m, I_m=0.390 m^4	Buckling of the mast Material yield at the section of the girder near the junction between the girder and mast
	H_g=3.0 m, I_m=0.390 m^4	Buckling of the mast Material yield at the section of the girder near the junction between the girder and mast
	H_g=0.5 m, I_m=3.143 m^4	Material yield at the sections supported by C2 and C39 in both side spans and near the junction between the girder and mast
	H_g=3.0 m, I_m=3.143 m^4	Material yield at the section supported by C3 and C38 in both side spans
Fan type model	H_g=0.5 m, I_m=0.390 m^4	Buckling of the mast Material yield at the section of the girder near the junction between the girder and mast
	H_g=3.0 m, I_m=0.390 m^4	Buckling of the mast Material yield at the section supported by C4 and C37 in both side spans
	H_g=0.5 m, I_m=3.143 m^4	Material yield at the sections supported by C2 and C39 in both side spans and near the junction between the girder and mast
	H_g=3.0 m, I_m=3.143 m^4	Material yield at the section supported by C2 and C39 in both side spans



(a) I_m=0.390 m^4, A_c=0.02 m^2 (b) I_m=3.143 m^4, A_c=0.02 m^2

Figure 9. Flexural stiffness ratio between the girder and mast vs. Ultimate Live load factor (Denominator: the flexural stiffness parameter of the mast).

this ultimate mode, both ultimate load factors obtained by ultimate analysis (U.A) and geometric nonlinear analysis (G.N.A) are similar. In particular, when the flexural stiffness ratio of the girder section of the radiating type model is less than 0.52 (height of the girder H_g=1.5 m), and the flexural stiffness ratio of the fan type model is less than 0.23 (height of the girder H_g=1.0 m), ultimate load factors obtained by U.A and G.N.A are almost the same. This indicates that mast buckling may govern the ultimate behavior of the structure when the mast doesn't have sufficient flexural stiffness. Moreover, in this ultimate mode, the load carrying capacity of the fan type model is higher than that of the radiating type model. In general, it is well known that, as the horizontal angle of the stay cables is higher, structural efficiency is also higher. But, it may not be adopted in this ultimate mode, which shows the mast buckling. In this ultimate mode, every part of the mast of the radiating type model is subjected to same compressive force. In other words, every part of the mast is subjected to the maximum compressive force. But, for the fan type model, the range from the junction between the girder and mast to the point where the lowest cable is attached is subjected to the maximum compressive force. Therefore, the effective buckling length of the mast of the radiating type model is longer than the effective length of the mast of the fan type model. Because of the longer effective buckling length of the mast, the radiating type model shows a smaller ultimate load capacity than the fan type model, in this ultimate mode.

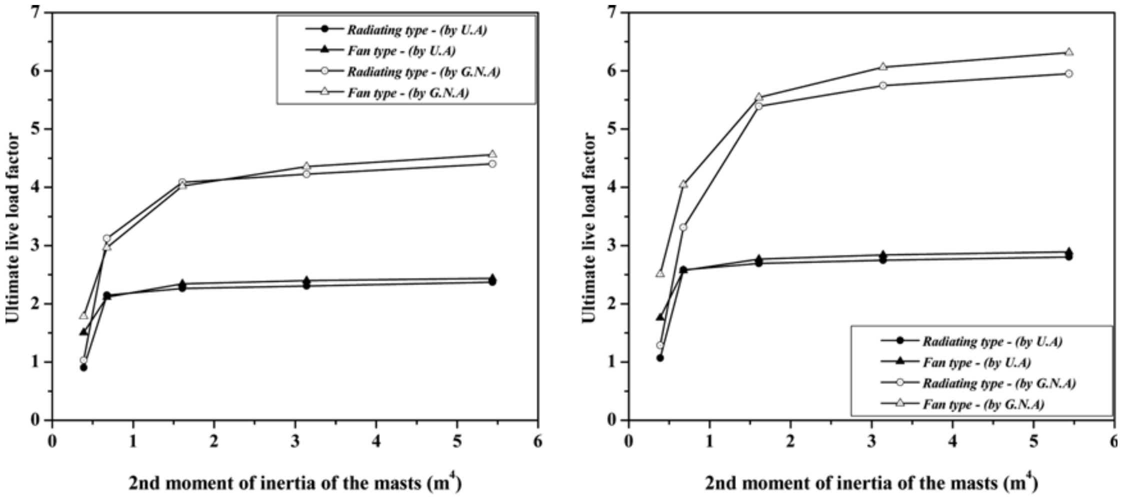
Figures 9(b) shows the ultimate live load factors when the mast has sufficient flexural stiffness. In this ultimate mode, material yield of specific sections of the mast mainly govern the ultimate behavior. By the way, in this mode, when the flexural stiffness of the section of the girder is quite low, the analytical results obtained by the U.A and G.N.A are almost the same. This indicates that if the girder is quite slender, elastic buckling of the girder, induced by the beam-column effect, may also govern the ultimate behavior. As the stiffness of the section increases, the difference between the load carrying capacity obtained by U.A and G.N.A also increases. So, the governing ultimate mode becomes clear as the section of the girder becomes large. When the vertically distributed force is applied to the center span, both side spans suffer negative flexural deformation, amplified by the beam-column effect. As the deformation grows, some section of the side span eventually yields, and that determines the ultimate state of the structure.

3.3. Effect of the flexural stiffness of the mast

In this section, the effect of the flexural stiffness of the mast is studied and described. Figures 10 and 11 show the change of the ultimate live load factor, with respect to the change of the 2nd moment of inertia of the mast.

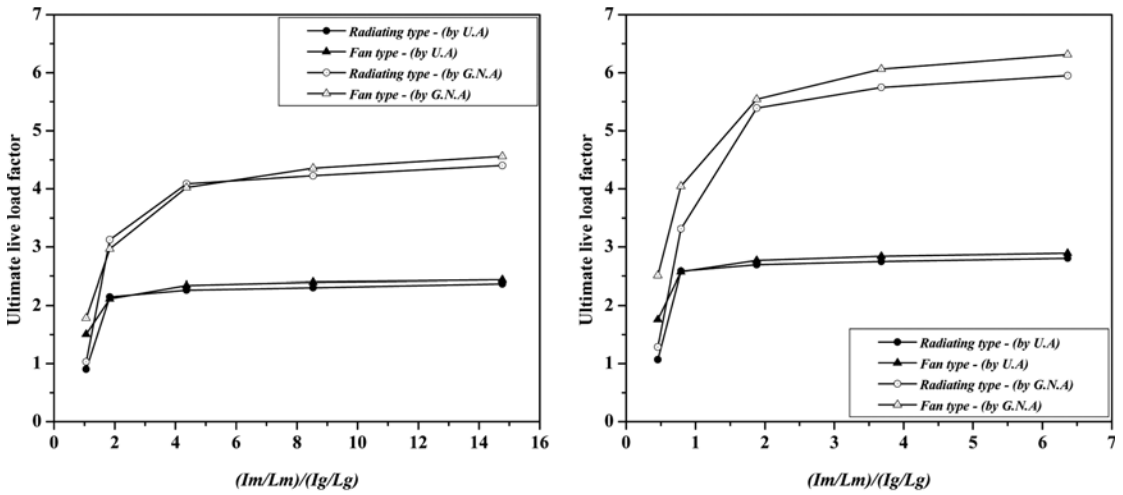
To study the effect of only the flexural stiffness of the mast, the height of the section of the girder is assumed as 2.0 and 3.0 m. As shown in those figures, the load carrying capacity converges to specific values, as the 2nd moment inertia of the mast continuously increases. In the ultimate mode, which shows the mast buckling, the ultimate live load rapidly increases, as the flexural stiffness of the mast increases. Because the mast earns sufficient stiffness for resisting mast buckling, the load carrying capacity quickly increases, as the 2nd moment of inertia of the mast increases.

But, as the stiffness increases more and approaches a certain value, the increase rate of the load carrying capacity rapidly decreases, and finally the ultimate live load factor converges. This is caused by the change of the ultimate mode. As the flexural stiffness of the mast increases, the governing ultimate mode changes from mast buckling to



(a) H_g=2.0 m, A_c=0.02 m^2 (b) H_g=3.0 m, A_c=0.02 m^2

Figure 10. 2[nd] moment of inertia of the mast vs. the Ultimate Live load factor.



(a) H_g=2.0 m, A_c=0.02 m^2 (b) H_g=3.0 m, A_c=0.02 m^2

Figure 11. Flexural stiffness ratio between the girder and mast vs. Ultimate Live load factor (Denominator: the flexural stiffness parameter of the girder).

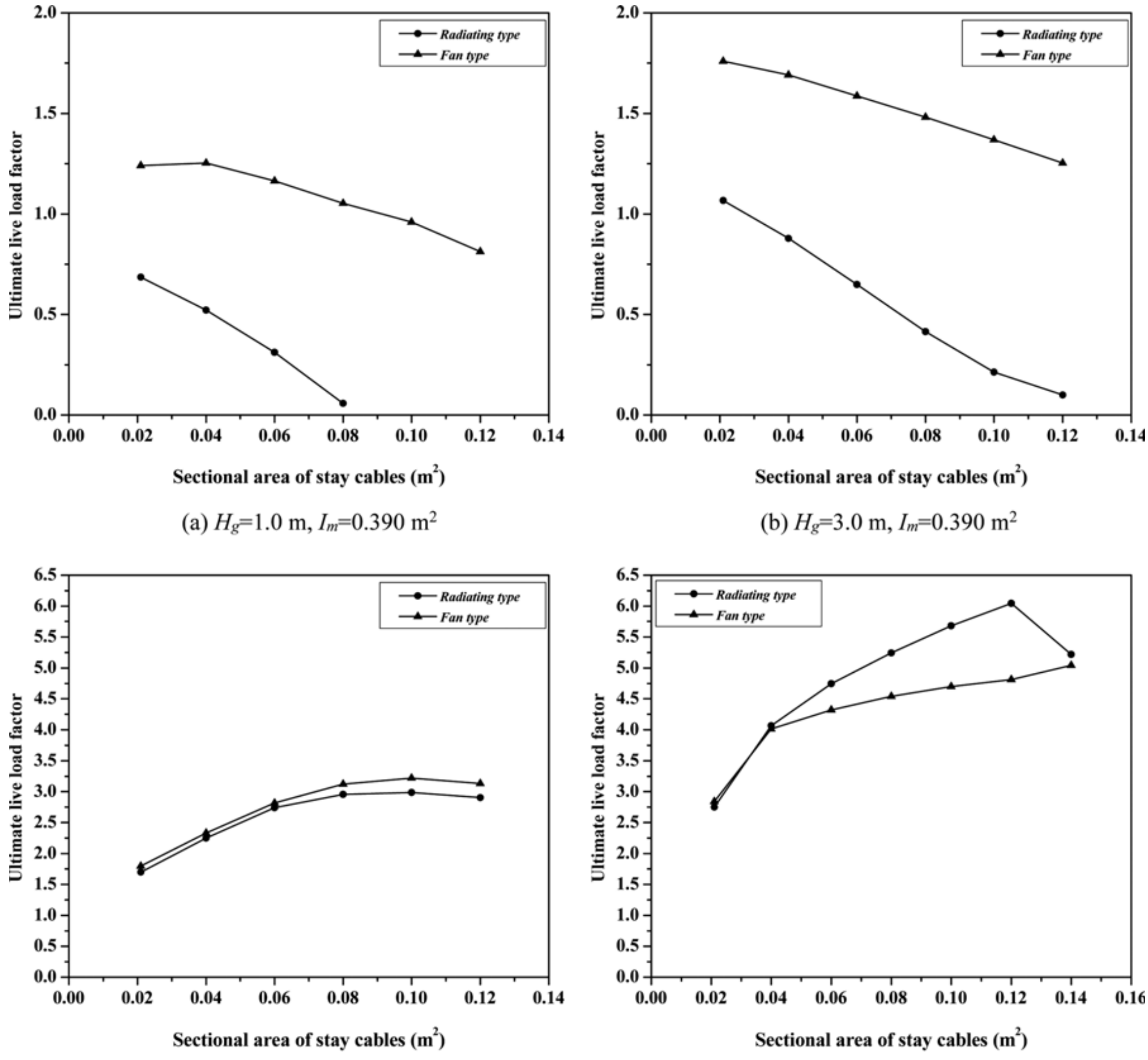
material yield of the girder. In this ultimate mode, the most influential factor on the ultimate behavior is material yield at the side span, amplified by the beam-column effect. Therefore, additional increase of the flexural stiffness of the mast doesn't affect the increase of the load carrying capacity under this live load condition.

Also, as shown in those figures, the difference of the ultimate load factor obtained by U.A and G.N.A is much smaller in the ultimate mode, which shows the mast buckling, while the difference in the ultimate mode governed by the material yield at the side spans is relatively quite large. This also proves the change of the ultimate mode as the flexural stiffness of the mast increases.

Consequently, the graph indicates that if the mast has sufficient flexural stiffness to resist the mast buckling, additional increase of the flexural stiffness of the mast doesn't ensure the continuous increase of the ultimate capacity of the structure. Also, the flexural stiffness of the mast, which causes the ultimate load carrying capacity to converge to a specific value, can be the minimum required flexural stiffness of the mast, if the considered live load case is the most critical live load case.

3.4. Effect of the area of stay cables

In this chapter, the effect of the area of stay cables is introduced. For cable-stayed bridges, stay cables are designed as intermediate supports for the girder. So, it can be supposed that the larger the sectional area of stay cables, the larger the stiffness of the supporting system for the girder. Actually, this might be correct for the elastic stiffness of the axial member, because the elastic stiffness of the axial members is proportional to the



(a) H_g=1.0 m, I_m=0.390 m^2 (b) H_g=3.0 m, I_m=0.390 m^2

(c) H_g=1.0 m, I_m=3.143 m^2 (d) H_g=3.0 m, I_m=3.143 m^2

Figure 12. Area of stay cables vs. Ultimate Live load factor.

sectional area of the member. But other effects made by the increase of the sectional area of stay cables should be considered. Firstly, the increase of the sectional area of the cable leads to the increase of the weight of the cable. As the weight of stay cables increases, compressive force applied to the mast also increases. As studied previously, compressive forces acting on the mast also affect the ultimate behavior, especially the mast buckling mode. In addition, the sag effect of the stay cables also increases. So, there are positive and negative effects from the increase of sectional area of the stay cables.

Figure 12 shows the change of the ultimate live load factor with respect to the change of sectional area of the stay cables.

As shown in Fig. 12, the effect of the sectional area of stay cables is clearly revealed. Firstly, in the ultimate behavior, which shows the mast buckling mode, the ultimate load carrying capacity decreases as the area of stay cables increases. In this ultimate behavior, the applied compressive force to the mast is quite important, because mast buckling is the governing factor of this ultimate behavior. Therefore, as the sectional area of stay cables increases, negative effects, such as increase of the weight and sag effect, lead to the decrease of the ultimate load carrying capacity, although the elastic stiffness of axial members increases.

In the ultimate behavior when mast buckling is prevented, it can be said that there is an optimal sectional area of stay cables, which ensures the maximum ultimate carrying capacity. The graphs that show this tendency can be divided into two parts by the peak point of the ultimate live load factor. In the first part, the positive effect of the section increase affects ultimate behavior more than the negative effect (increase of the elastic stiffness of axial member > increase of the weight and sag effect of stay cables). In the second part, it is the opposite. Of course, the curve of the fan type model (which is shown in Fig.

12(d)) doesn't show the peak point in the range considered in this study. But, a similar tendency can be supposed, considering the shape of the curve.

Stay cables are quite important members for cable-stayed bridges, because they are the intermediate support. Because of stay cables, the structure can eventually be designed as a long-span bridge. As shown in this section, the optimal sectional area of stay cables should be determined, because there are positive and negative effects together as the sectional area is enlarged.

4. Conclusion

In this study, the ultimate behavior of completed steel cable-stayed bridges under the specific live load case was studied. For rational ultimate analysis, a two-step ultimate analysis proposed in the previous paper was used. According to intensive analytical study, governing factors which affect the ultimate behavior can be sorted as follows.

(A) Mast buckling mode

(B) Material yield of the section of the side span

Of course, there may be additional factors, such as material yield of the section of the center span, buckling of the girder, material yield of stay cables, and so on. According to the former paper, these additional factors affect the ultimate behavior under other live load cases, which show quite larger ultimate live load factors than the value under the live load case considered in this study.

It is common sense that for efficient use of materials, the buckling of any main member should be prevented. In addition, the mast is quite an important support system of cable-stayed bridges, because it is the main support of the structure. The buckling of the mast means the direct collapse of the structure. So, the mast should be designed with sufficient flexural stiffness, as well as sufficient sectional area. According to this study, there is a minimum required flexural stiffness of the mast, because larger stiffness than the minimum stiffness may not lead to additional increase of the ultimate load carrying capacity.

An additional interesting tendency was studied of the stay cables, which form the intermediate support system. According to this study, there is an optimal sectional area of stay cables that ensures the ultimate load carrying capacity becomes the maximum. The difference of the positive and negative effects varies according to the sectional area of the stay cables. So, the optimal sectional area of the stay cables should be determined in bridge design, through performing intensive ultimate analyses.

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