Damage Diagnosis in Steel Structures when Dimensional Structures with Dimensional Revel Structures with Different Noise in November 2016. Levels via Optimization Algorithms

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Abstract

In this paper, the main objective is to solve the inverse problem of damage identification with the help of the Imperialist Competitive Algorithm (ICA) optimization. Three different numerical cases, including a clamped-free beam, a 2D truss and a 2D plate-type structure are modelled by using Finite Element Method (FEM) and used to evaluate the proposed damage identification procedure. The proposed objective function for the optimization procedure is formed by using modal parameters. Those parameters are obtained from the damage state, where cracks are simulated with the assumption of reducing the local stiffness of the structure. Then, proposed damage detection/characterization process is performed by implementing an optimization algorithm, called Imperialist Competitive Algorithm (ICA). Results obtained from the numerical case studies show that this algorithm is trustworthy and can be used to identify the severity and location of the damage with a good accuracy. Furthermore, the effects of noise on the results of damage identification process are studied so as to investigate the tolerance of the method in the face of environmental noise. Finally, the results obtained by the ICA are compared to the ones obtained by using two commonly used algorithms, i.e. the binary genetic algorithm (BGA) and particle swarm optimization (PSO).

Keywords: damage identification, modal parameters, imperialist competitive algorithm

1. Introduction

The safety of structures or machines is always an important issue for the operators who are dealing with them. As a result, detecting cracks and flaws in structures has attracted attention over the past decades. In order to give the assurance with the safety of a structure or a machine during its operation, monitoring of its dynamic behavior is a requirement. Any change in physical properties including its mass, stiffness or damping affects the dynamic parameters of the structure (i.e. natural frequencies, mode shapes and modal damping). Vibrationbased damage identification methods have been introduced and developed based on the changes in the dynamic behavior of the structure ensue from occurring damage. The very first studies of damage detection were performed in the 1970s and early 1980s (Vandiver, 1975; Loland and Dodds, 1976; Wojnarowski et al., 1977; Duggan et al., 1980; Nataraja, 1983). Fan and Qiao (2011) presented a review in vibration-based damage detection methods.

Received September 19, 2013; accepted January 20, 2015; published online September 30, 2015 © KSSC and Springer 2015

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Rytter (1993) divided damage detection procedures into four steps. The first step is identifying the defective structures, the second step is determining the geometrical location of the damage, the third step is determining the severity of the damage, and the fourth step is predicting the remaining service life of the structure. The inverse problem of the damage detection consists of the second and third steps. Although Genetic Algorithm (GA) was introduced in the mid-1960s (Mitchell, 1995), structural health monitoring process entered a new stage when researchers started using this optimization technique to formulate damage detection problems. Mares and Surace (1996) proposed a new formulation for objective function of GA by using a residual force method. They found the location and size of the crack in an elastic structure and evaluated the reliability of their results using different case studies. Ruotolo and Surace (1997) proposed a procedure based on the modal parameters to identify damages and its severity in a multiple cracked beam using genetic algorithm. They evaluated their method by using both numerical simulations and experimental tests for a cantilevered steel beam. Friswell et al. (1998) used a twolevel procedure in order to find damage location and its extent. They first applied GA to an objective function which was formed based on some modal parameters and then employed a standard eigensensitivity method to determine damage extent. A GA based technique was proposed by Chou and Ghaboussi (2001), where the static displacements at a few degrees of freedom are measured and then GA is used to find out the unmeasured DOFs. This technique is useful for detecting damages when the number of measured degrees of freedom is limited due to the experimental restrictions. Hao and Xia (2002) used three various parameters, namely, frequency changes, mode shape changes and a combination of the two to form an objective function. They compared the changes in their measurements before and after occurring damage and then used GA for minimization process. They evaluated their approach by using both numerical results and laboratory tests and showed that their technique is practical, even if the analytical model is not accurate. He and Hwang (2006) proposed a hybrid algorithm which is beneficial in terms of the synergistic advantages of GA and simulated annealing. They used finite element models of different damaged beam structures with different boundary conditions and damage scenarios to obtain displacements due to the static response and changes in natural frequencies with the aim of evaluating their method. Vakil-Baghmishe et al. (2008) put forward a technique to monitor the changes in frequencies of a structure using GA and also proposed a new cost function. They reported the average errors for their numerical and experimental studies. Perera et al. (2007) formulated damage detection problem based on modal flexibility and used a multiobjective GA to localize and quantify the damage. They also compared the results of some multiobjective GA based on Pareto optimality (Perera et al., 2009). Niemann et al. (2010) used topology optimization feature of the MSC Nastran software to develop a damage detection technique. They not only numerically evaluated their approach but also used a composite laminate specimen to experimentally validate the technique. Jaishi and Ren (2006) used GA to minimize the objective function consisting of the modal flexibility residuals, and then identified location and damage size for a simply supported beam. They propose a damage detection technique procedure based on finite element (FE) model updating and modal flexibility residual. Using other optimization techniques is also reported. including ant colony optimization (Majumdar et al., 2012), particle swarm optimization (Begambre and Laier, 2009; Perera et al., 2010), hybrid particle swarm optimization (Sandesh and Shankar, 2010; Vakil-Baghmisheh et al., 2012) and simulated annealing (Ruotolo et al., 1997; He and Hwang, 2006), which is inspired by the way that metals cool and anneal or liquids freeze and crystallize. Recently, a new optimization technique, Imperialist Competitive Algorithm (ICA), has been proposed to solve optimization problems. This algorithm was first proposed by Atashpaz-Gargari and Lucas (2007). In addition, Atashpaz-Gargari et al. (2008) applied this algorithm for PID controller design.

In the present study, damage localization and its severity estimation problems are solved for three different numerical case studies, where a meta-heuristic optimization algorithm with global search capability is used. Contaminated with different noise levels, vibration data (frequencies and mode shapes) are used to form the objective function. Essential experimental data, which are generally obtained from the real modal testing, are provided by contaminating the simulation data using different noise levels. Not only five different damage scenarios are considered to evaluate the proposed method, but also results of the ICA based algorithm are compared to the results obtained from the two other conventional optimization algorithms, which are frequently used in damage detection, i.e. GA and particle swarm optimization (PSO).

2. An Overview of Imprialist Competitive Algorithm

Most of the optimization methods, which are inspired by natural phenomena (i.e. GA, PSO,…), does not take the humanitarian aspects of life (i.e. political, cultural and social revolutions) into consideration. ICA as a cultural based algorithm adds the ability of the cultural revolution (i.e. providing the possibility of exchanging the information among the population) to current optimization techniques and as a result increases the convergence rate. The first step to implement this algorithm is to choose an initial number of populations, which are called countries. In this algorithm, each country is a $1 \times N$ array which is equal to a chromosome in GA. The cost for a country is found by evaluating the f, which is a function of $(p_1, p_2, p_3, ..., p_{N \text{var}})$ variables:

$$
\cos t_i = f(\text{country}_i) = f(p_1, p_2, p_3, ..., p_{N_{\text{var}}})
$$
(1)

From social-political viewpoint, the unknown variables of the cost function are historical and cultural identities of a country. For determining the initial empires, some of the best countries are selected as imperialists and the remaining countries form colonies (Fig. 1(a)). Next step is to adopt the assimilation policy with the aim of evaluating the cultural and social structure of the colonies. In fact, a central government tries to assimilate the colonies. One example of these policies, which an imperialist uses to incorporate the colonies as part of itself, is shown in Fig. 1(b).

$$
x \sim U(0, \beta \times d) \tag{2}
$$

$$
\theta \sim U(-\gamma, \gamma) \tag{3}
$$

In Fig. $1(b)$, d is the distance between imperialist and colony and x is a uniformly distributed random variable. In equation (2), β is larger than one and close to two. In equation (3), θ is a random angle, which is added to avoid the probable deviation during the assimilation process and ν is an arbitrary parameter which its increase results in an increase in the searching process around the

Figure 1. (a) Procedures followed to form the initial empires, (b) moving of a colony towards imperialist.

imperialist and its decrease causes the colony to come close to the vector, connecting the imperialist and colony. In order to prevent the algorithm from trapping in a local minimum, it is equipped with a mechanism of revolution. Generating a random number varying from 0 to 1 for each colony, the algorithm compares this value with the probability of the revolution to decide whether the revolution should occur or not. For a colony, revolution will occur provided that its generated random number is lower than the probability of the revolution rate. For each iteration, this process is carried out to improve the cost of the competent colonies. The next step to model the algorithm is to replace the imperialist with a colony, which is approaching the imperialist, when this colony achieves a better situation compared to its imperialist (i.e. this colony finds a point in which it has a lower cost in comparison to the imperialist). In this situation, imperialist and colony will change their positions with each other and new imperialist starts to apply the assimilation policy to the colonies. The power of an empire is equivalent to the power of the imperialist, plus a percent of the power of its colonies. Then, empires start to compete and each empire that cannot increase its power during the imperialist competitive will be eliminated. During the imperialist competitive, power of the potent imperialists (which have been more successful in assimilating the colonies of the other imperialists) increases and feeble empires are eliminated. In order to increase their power, imperialists have to develop their colonies. Over time, colonies come close to the imperialists in terms of power and a kind of convergence process is observed. The algorithm continues to progress until the convergence condition is satisfied. Having a unique empire in the world along with the colonies which are very close to the empire in terms of position is the ultimate goal of the imperialist competitive algorithm. The pseudo code for the ICA is shown in Fig. 2.

3. Theoritical Formulation

3.1. Governing equation

The equation of motion of a multi-degree of freedom (MDOF) system without damping is as follows:

$$
[M]{\ddot{X}(t)} + [K]{X(t)} = 0
$$
 (4)

In the above equation, $[M]_{n \times n}$ and $[K]_{n \times n}$ are the mass and stiffness matrices, respectively. $\{X(t)\}\$ is the displacement vector and n is the number of degrees of freedom in the system. Frequencies and mode shapes of MDOF system without damping can be obtained by solving the following eigenvalue problem

$$
([K] - \lambda_i [M]) \{ \varphi_i \} = 0 \tag{5}
$$

here, λ_i and φ_i are eigenvalues and eigenvectors, respectively. They are the modal parameters, which are

1) Select some random points on the function and initialize the empires.
2) Move the colonies toward their relevant imperialist (Assimilating).
3) If there is a colony in an empire which has lower cost than that of imperialist, exchange the positions of that colony and the imperialist.
4) Compute the total cost of all empires (Related to the power of both imperialist and its colonies).
5) Pick the weakest colony (colonies) from the weakest empire and give it (them) to the empire that has the most likelihood to possess it (Imperialistic competition).
6) Eliminate the powerless empires.
7) If there is just one empire, stop, if not go to 2.

Figure 2. The pseudo-code for ICA.

functions of the physical properties of the system. In general, normalized eigenvectors are used in the dynamic analysis of the structure.

3.2. Numerical modelling of the damage

Crack occurrence in any structure affects the physical properties of the system such as mass and stiffness. The impact of these changes would be minor on the mass and it can be neglected, however it reduces the bending stiffness of the structure. This assumption is almost true in many practical implementations.

$$
\beta_e = \frac{(E I)_e^d}{(E I)_e} \tag{6}
$$

In equation (6), β_e is the stiffness reduction ratio, and it is defined as the ratio of the bending stiffness of the cracked element $(ED_e^d$ over the primary value of the bending stiffness $(EI)_e$ for each element. Needless to say that the value of this parameter is between 0 and 1.

$$
[K^d] = \sum_{e=1}^n (1 - \beta_e)[K_e]
$$
 (7)

In equation (7), $[K^d]$ is the global stiffness matrix for cracked structure. Substituting $[K^d]$ with $[K]$ in equation (5) results in eigenvalues and eigenvectors of the cracked structure. In this equation, n is the number of the elements of the FE model. Also, it can be notified that β =0 signifies no damage in the element and β =1 represents that the element is completely destroyed.

4. Objective Function

4.1. Frequency changes and modal assurance criterion (MAC)

Friswell and mottershed (1995) offered a method based on a sensitivity function. In this method, they used the minimization of the difference between measured modal parameters (natural frequencies and mode shapes) and the ones predicted by the model. Detecting the location and the severity of a crack based on the vibration data, this method can be implemented for any structure. The objective function helps us to detect the location of the crack by creating a relation between the two groups of vibration data. This can be defined as follows in a form of minimization of J function that is kind of a summation (a hybrid of frequency changes and MAC (HFCMAC)) of modal parameters.

$$
J(\{\beta\}) = \sum_{i=1}^{n} \left(\frac{\lambda_i^A(\{\beta\}) - \lambda_i^D}{\lambda_i^D} \right)^2 + \sum_{i=1}^{n} (1 - MAC(\varphi_i^A(\{\beta\})), \varphi_i^D)
$$
(8)

In equation (8) , superscript A shows modal parameters which are predicted with the help of finite element model of the structure and D expresses the modal parameters obtained from damaged structure. Also, n is the number of measured modes. MAC in the formula is defined as follows:

$$
MAC = \frac{\left| {\{\phi_i^A\}}^T {\{\phi_i^D\}} \right|^2}{(\{\phi_i^A\}}^T {\{\phi_i^A\}})(\{\phi_i^D\}}^T {\{\phi_i^D\}})
$$
(9)

MAC represents the level of correlation between two modal vectors, where it is equal to one for two perfectly correlated mode shapes and equal to zero for completely uncorrelated ones. When damage occurs, this index shows that a change in a component of the mode shape. which is related to the degree of freedom close to the damage location, is larger than the other components.

4.2. The effect of noise

Data in any measurement are usually contaminated by environmental noise. Also, other factors such as human error, the accuracy of measurement devices, systematic errors, and etc. can affect measured data. These effects are taken into consideration for eigenvalues and eigenvectors with the help of the following formulas:

$$
\varphi_{ij}^k = \varphi_{ij} (1 - \eta \zeta_{ij}^k) \tag{10}
$$

$$
\lambda_{ij}^k = \lambda_{ij} (1 - \eta \zeta_{ij}^k), \ i = j \tag{11}
$$

where, φ_{ii}^k is the j^{th} component of i^{th} noise contaminated mode for k^{th} measurement. λ_{ij}^{k} is the i^{th} eigenvalue while η is the noise level, and ζ_{ii}^* is a random number in the range of [0,1]. φ_{ii}^k λ_{ii}^k ζij k

5. Numerical Examples

The first example, is a clamped-free steel beam with the length of 0.6 m, cross-sectional area of 0.03×0.02 m², Young's modulus of 207 Gpa and density of 7,860 kg/m³, which is simulated by the finite element model of the Euler-Bernoulli beam with 10 elements (Fig. 3). The second example is a 2D-truss structure, modelled with 13 elements, where the cross-sectional area of each member is 0.03×0.025 m², the elastic modulus is 200 Gpa, and the density is $7,800 \text{ kg/m}^3$ (Fig. 4). The third example is a plate-type structure with the dimension of 0.5×0.5 m² and thickness of 0.05 m, which is fully clamped. The physical properties of this structure are as follows: elastic modulus

Figure 3. FE model of the clamped-free beam.

Figure 4. FE model of the 2D-truss structure.

Figure 5. FE model of the 2D Plate-type structure.

of 210 Gpa, mass density of $7,850$ kg/m³ and Poisson's ratio of 0.3. This plate is modelled by using the Mindlin plate theory and it has 25 nodes and 16 elements (Fig. 5).

6. Results and Discussion

The noise-contaminated data can affect the optimizationbased identification process. Therefore, eigenvalues are contaminated with 2% noise by using equation (11), and eigenvectors are contaminated with 5% and 10% noises by using equation (10). Five damage scenarios with various severities and locations are considered for three different structures (Table 1). Figure 6 shows the general flowchart of the crack identification procedure with the help of the optimization algorithms. Here, we primarily

Table 1. Simulated damage scenarios for examples 1-3

Case No.	Scenario
	25% in third element of beam
2	40% in fifth element plus 30% in ninth element of heam
3	30% in seventh element of truss
4	50% in fourth element plus 35% in eleventh element of truss
5	30% in seventh element of 2D plate-type structure

use ICA to identify damages and also employ two commonly used optimization techniques, including PSO and Binary Genetic Algorithm (BGA) to compare the results. For damage detection using optimization techniques and vibration measurements, there exist several parameters that can be modified to achieve better and more accurate outcomes, namely the initial population, the mutation rate of the GA/revolution rate of ICA, the number of iterations, and etc.. The above mentioned parameters should be determined according to the geometry of the structure, number of the elements of the model and type of the objective functions. Table 2 shows those values which are used to run the three optimization algorithms. Figures 7(a) and 7(b) show the searching process of the imperialist competitive algorithm to find an optimum solution for the objective function and Figs. 8 and 9 show the effect of noise in the damage detection process for the beam and 2-D truss examples. In Tables 3 and 4, the stiffness reduction ratio obtained by ICA is compared to those found by two other algorithms (BGA and PSO) which are commonly used in damage identification studies.

Figure 10(a) shows the process of searching for an optimum solution, for plate-type structure. The results of the procedure are shown in Fig. 10(b), both for contaminated-

Figure 6. Flowchart of the damage diagnosis process by using optimization algorithms.

Algorithm	Parameters	Values for Ex.1	Values for Ex.2	Values for Ex.3	
	Initial Countries	50	50	50	
	Initial Imperialists	10	13	10	
ICA	Number of Decades	50	100	250	
	Revolution rate	0.3	0.3	0.25	
	Population size	100	120	120	
BGA	Number of bits in each parameter	8	10	10	
	Number of iterations	350	450	500	
	Mutation rate	0.1	0.07	0.07	
PSO	Population size	50	50	50	
	Number of iterations	80	100	250	

Table 2. Run parameters used for ICA, BGA, and PSO

Figure 7. ICA feature selection searching for optimal solutions.

Figure 8. Damage percentage obtained by ICA for different noise levels.

free and for noise-contaminated data. Furthermore, data contaminated with 10% noise are used to compare the outcomes of the procedure for three different optimization techniques. It can be seen that ICA has a slightly better tolerance when it comes to the noise. It should be noted that first four frequencies and mode shapes of the structures were used for these simulations and the results were obtained by utilizing the introduced hybrid objective function (HFCMAC), formulated by equation (8).

One of the key elements of damage identification using vibration measurements is to define an appropriate and practical index, which has a high sensitivity to damages.

Figure 9. Damage percentage obtained by ICA for different noise level.

In practice, it is easy to find the natural frequencies of a structure, compared to the other modal parameters and that is the reason why they are mostly used for damage identification procedures. However, this parameter is mostly used to monitor the availability of the crack and it is not easy to use it for higher level processes such as damage localization and severity detection. Based on the discussions provided in section 4.1, HFCMAC function

Element No.	Case 1				Case 2			
	Simulated	BGA	PSO	ICA	Simulated	BGA	PSO	ICA
		0.9765	0.9465					
		0.9472		0.9415		0.9608		
	0.75	0.7804	0.7796	0.7600		0.9725	0.9502	0.9600
						0.9960		0.9700
					0.6	0.6275	0.6050	0.6050
6						0.9882	0.9991	
		0.9851				0.9372		
8		0.9921		0.9646		0.9960	0.9800	0.9900
9		0.9568	0.9534	0.9410	0.7	0.7000	0.7000	0.7060
10							0.9900	

Table 3. Results obtained for β with 10% noise.

Table 4. Results obtained for β_e with 10% noise

Element No.	Case 3				Case 4			
	Simulated	BGA	PSO	ICA	Simulated	BGA	PSO	ICA
		0.9921						
			0.9800			0.9084	0.9350	
		0.9885	0.9800			0.9921		
					0.5	0.5080	0.5072	0.5264
						0.9804		0.9612
						0.9804		0.9743
	0.7	0.7095	0.7000	0.7062		0.9254	0.9602	
8		0.9960						
9		0.9843	0.9809			0.9800	0.9600	
10		0.9451						
11				0.9400	0.65	0.6592	0.6574	0.6506
12		0.9921		0.9522		0.9668		0.9220
13		0.9980						

Figure 10. (a) ICA feature selection searching for optimal solutions in example 3, (b) Damage percentage obtained by ICA for different noise levels, for 2D plate-type structure.

(equation (8)) is introduced to overcome this issue. This function is a combination of natural frequency changes and MAC criterion to facilitate the localization process as well as the severity estimation. Figures 8 and 9 show the single damage and double-damage cases for the beam and 2-D truss. As it can be seen, there are intact elements which are identified as damaged due to the considered noise in the vibration data. Furthermore, noise caused some error in damage localization results.

There are three factors, which are important in damage detection using optimization techniques: (1) The objective function used for the case, (2) The choice of the suitable algorithm based on the physics of the problem, and (3) The accuracy of the data used for the procedure. First two factors can be controlled based on the previous experience for different cases, however, measuring the accurate data might be challenging in most cases. In practice, the extracted modal parameters are affected by different factors such as environmental conditions, numerical errors in curve fitting, and limitations in the measurements of higher vibration modes. The error in natural frequencies and mode shapes of the structure can be up to 5 and 15% respectively. In Tables 3 and 4 different values of stiffness reduction (β_e) have been estimated by using three different optimization algorithms. The average value obtained for the first case is 77%, for the second case is 61 and 70%, respectively, for the third case is 71%, and for the fourth case is 51 and 66% respectively. All three algorithms are showing damages in some intact elements which is caused by the noise in the objective function. These errors are more noticeable for GA. The values listed in Table 3 and 4 are the best outcomes which are found after 10 times running these algorithms.

Figure 10(b) provides the results of ICA for the 2D truss problem. In order to investigate the accuracy of estimations using ICA, Fig. 11 provides a good comparison. It shows that ICA is capable of providing better outcomes

Figure 11. The damage percentage obtained by using three different algorithms with 10% noise for 2D platetype structure.

compared to the other methods when it comes to contaminated data. However, PSO could also provide a good estimation of the damage severity. Finally, it can be seen that the number of elements which are falsely identified as damaged elements by GA is more than that of PSO and ICA. This shows that the GA is more sensitive to the noise and contaminated data for damage identification procedure in a 2D plate-type structure.

7. Conclusion

In this study, the effect of the structural geometry on optimization-based damage detection, localization, and severity identification was studied by using ICA optimization. Three numerical case studies were used to apply the proposed procedure and a hybrid objective function was introduced and used for all cases. The objective function was a hybrid function of natural frequencies and mode shapes of the structure. Several damage scenarios with different locations and severities were induced. Then the effect of environmental noises was taken into consideration

for modal parameters and the inverse problem of damage detection was solved by the single-objective ICA. The results were compared with two other commonly used evolutionary algorithms, i.e. PSO and BGA.

Acknowledgments

The authors gratefully acknowledge the comments and advice of Dr. Mahdi Bamdad from Shahrood University of Technology.

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